Why sensitivity analysis (or: How not to do a sensitivity analysis)

Andrea Saltelli

i4Driving summer school, Anacapri,
June 30th – July 2nd 2025

Where to find this talk: www.andreasaltelli.eu

Andrea Saltelli

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Latest papers

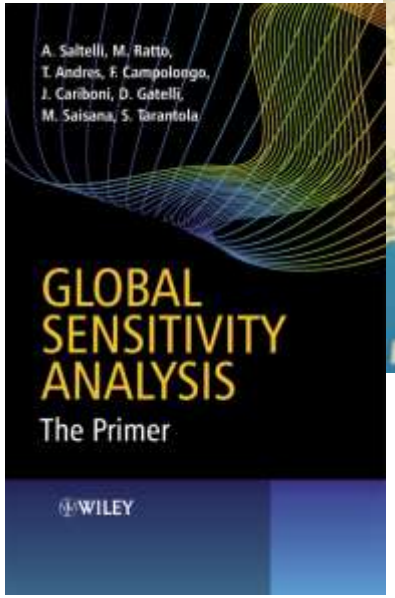
Models and the common good

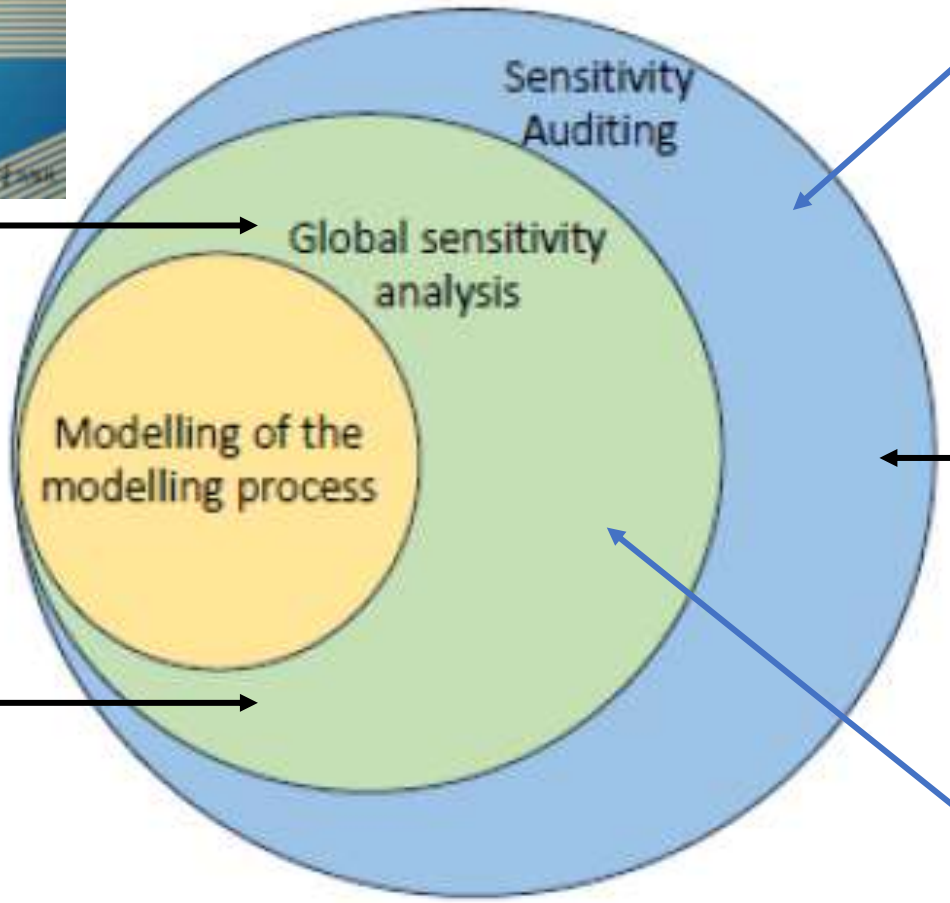
A short analysis of norms and counter-norms in modelling, discussing how models may misuse their epistemic authority for reasons of occasion, opportunity and interest.

Outline:
Narratives and counter-narratives,
State of exception,
Seeking Cinderella,
Reformation

Ranking the Rankers

Ranking the Rankers? We investigate using global sensitivity analysis the properties of the science-wide author databases' standardised citation indicators produced since 2015 by a team of researchers led by John P. A. Ioannidis.





Normative quality

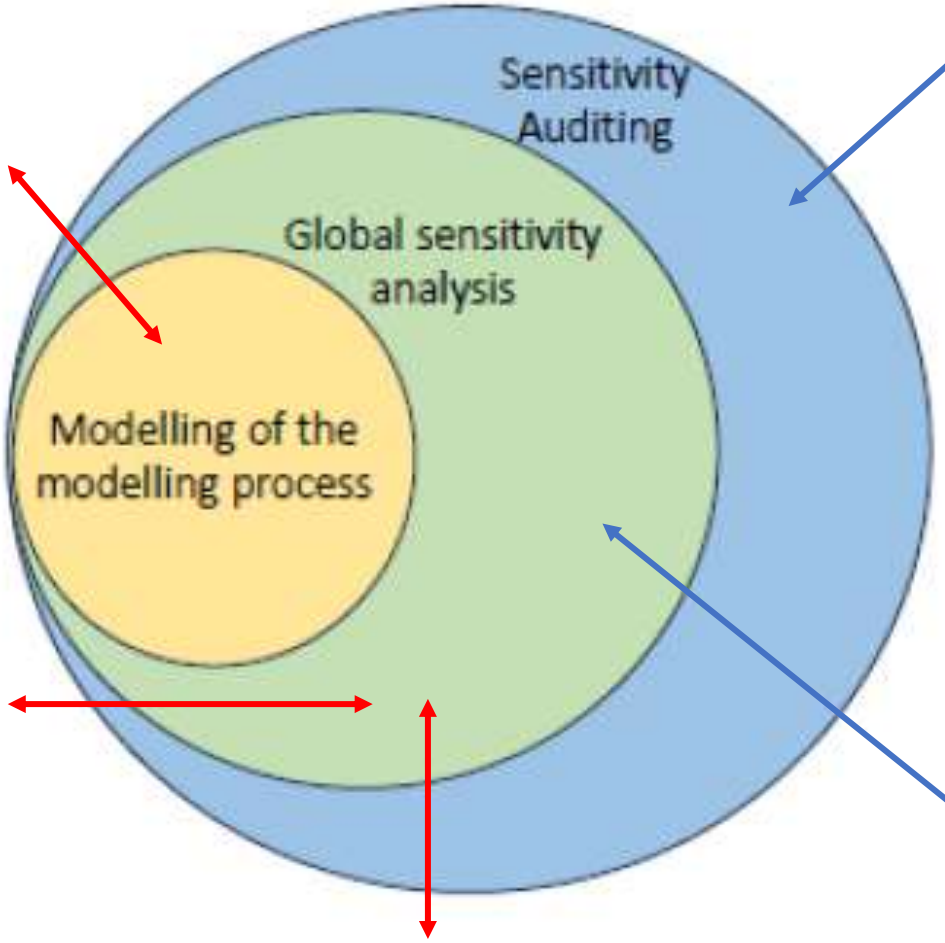
Technical Quality

Sensitivity auditing : “Sensitivity auditing is a wider consideration of the effect of all types of uncertainty, including structural assumptions embedded in the model, and subjective decisions taken in the framing of the problem” (European Commission, 2021).

European Commission. November 2021. “Better Regulation: Guidelines and Toolbox.” https://ec.europa.eu/info/law/law-making-process/planning-and-proposing-law/better-regulation-why-and-how/better-regulation-guidelines-and-toolbox_en

Normative quality

Multiverse analysis



Perturbed Physics Ensemble

Technical Quality

Exploratory modelling analysis (EMA)

Futures
Volume 144, December 2022, 103041

Unpacking the modelling process via sensitivity auditing

Samuele Lo Piano [a], Razi Sheikholeslami [b], Arnald Puy [c d e], Andrea Saltelli [f]

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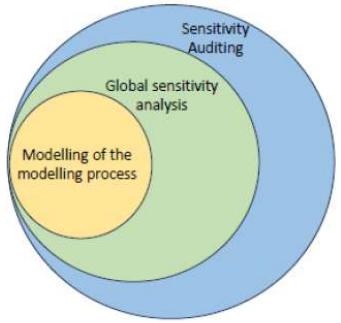

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Robustification of structural equation modelling via global sensitivity analysis

Original Paper | Published: 11 March 2025

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Statistical Methods & Applications

2025

Alessio Lachi, Josep Llach, Jordi Perramon, Michela Baccini & Andrea Saltelli

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Environmental Science & Policy
Volume 142, April 2023, Pages 99-111

Impact assessment culture in the European Union. Time for something new?

Andrea Saltelli [a b], Marta Kuc-Czarnecka [c], Samuele Lo Piano [d], Máté János Lőrincz [d], Magdalena Olczyk [c], Arnald Puy [e], Erik Reinert [f g], Stefán Thor Smith [d], Jeroen P. van der Sluijs [b h]

2023

Risk Analysis
AN INTERNATIONAL JOURNAL
An Official Publication of the Society for Risk Analysis

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Unpacking the modeling process for energy policy making

Samuele Lo Piano, Máté János Lőrincz, Arnald Puy, Steve Pye, Andrea Saltelli, Stefán Thor Smith, Jeroen van der Sluijs

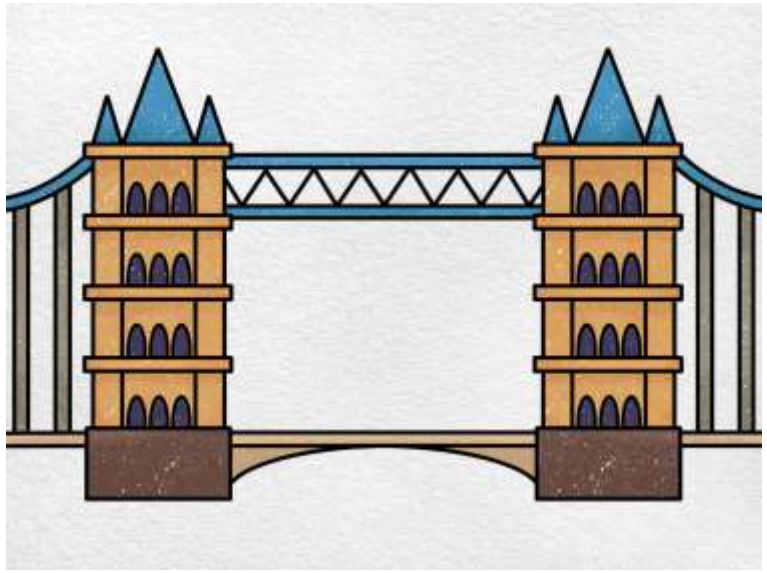
First published: 14 November 2023 | https://doi.org/10.1111/risa.14248

Figures Refer

2023

Why a sociology of models?

- State of exception
- Overinterpreted and underexplored
- High epistemic authority
- Purported neutrality and ‘speaking truth to power’
- Opacity
- Asymmetry users/developers
- Silos / bridges needed



Source: https://helloartsy.com/

2025

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Environmental Modelling & Software

Volume 188, April 2025, 106430

EnviroFutures Invited Opinion

Models and the common good ☆

Andrea Saltelli ✉

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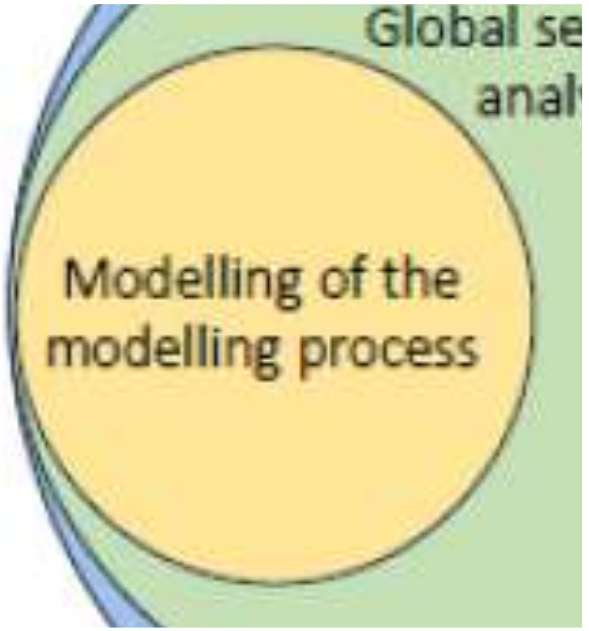
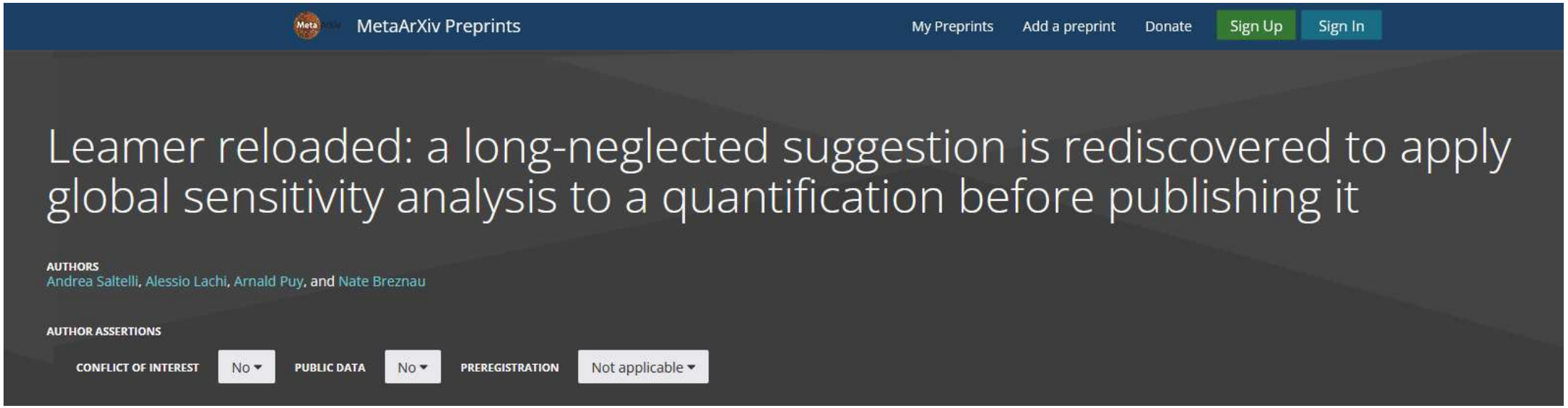
2024

The multiverse

The universe of uncertainty that didn't hide: reaching out to multiverse analysis

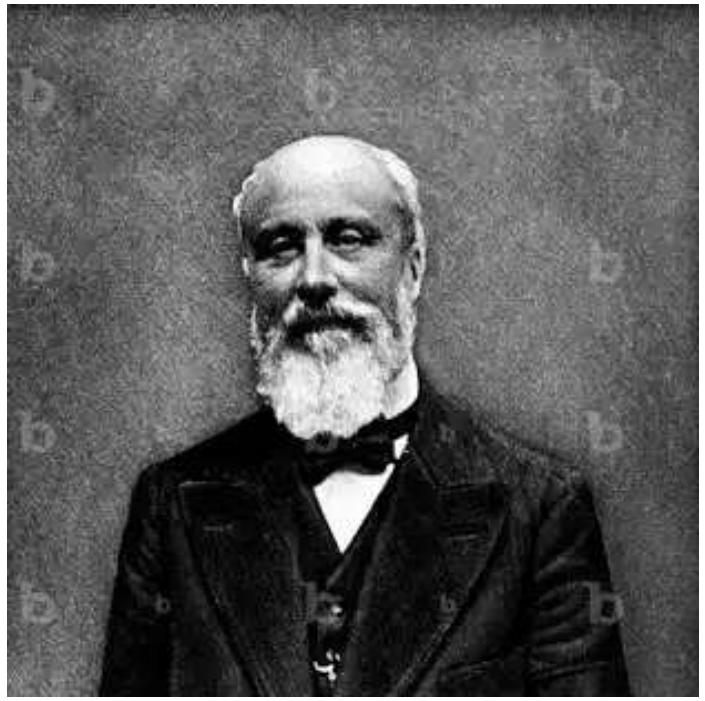




Models are fragile

"… inferences from mathematical models of phenomena to real physical applications must also be demonstrated to be approximately correct when the assumptions of the model are only approximately true" [1]

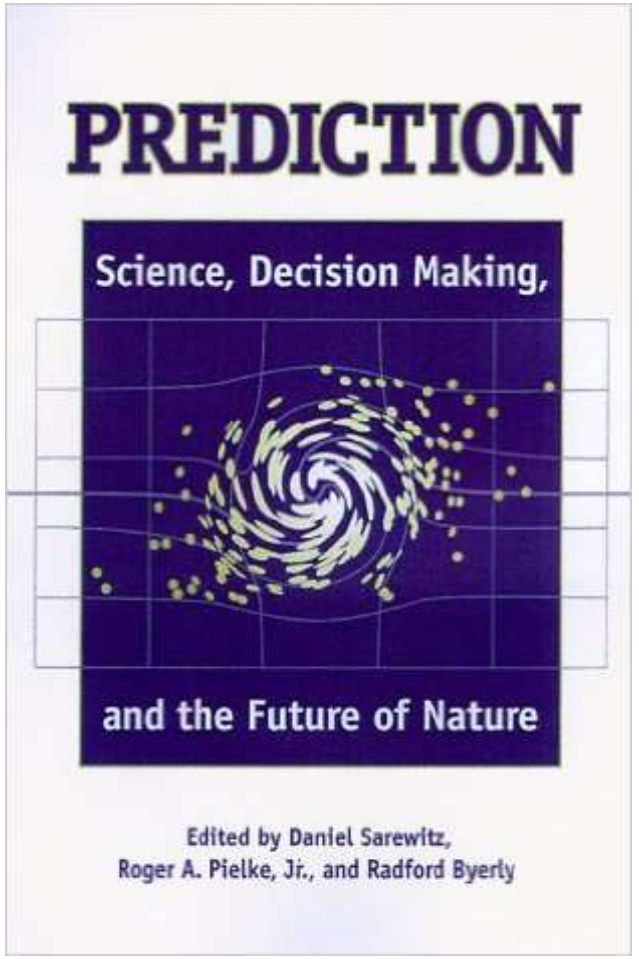
Pierre Duhem (1861–1916) and his 'Principle of stability'



[1] S. C. Fletcher, 'The Principle of Stability', Philosopher's Imprint, vol. 20, no. 3, 2020, Accessed: Sep. 08, 2024. [Online].

“models are most useful when they are used to challenge existing formulations, rather than to validate or verify them”

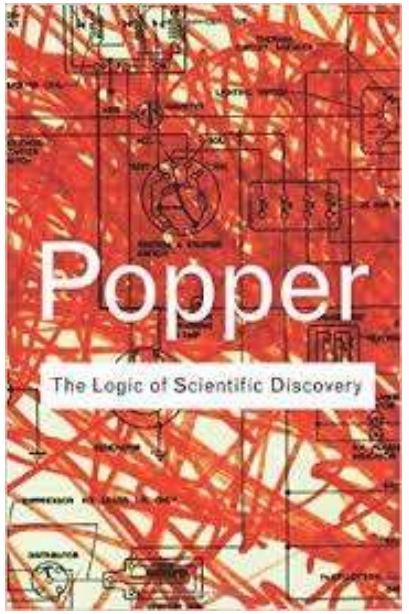
Naomi Oreskes



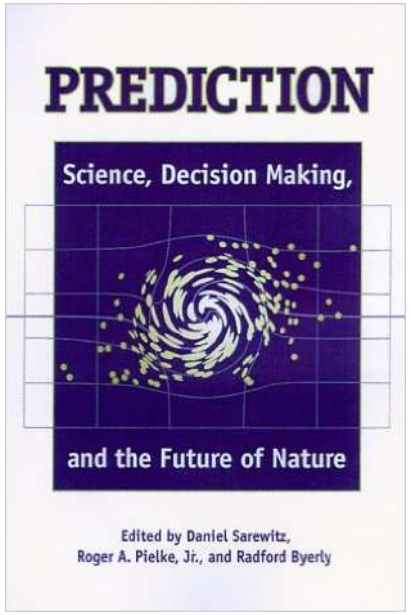
N. Oreskes, K. Shrader-Frechette, and K. Belitz, “Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences,” Science, 263, no. 5147, 1994.

Oreskes, N., 2000, Why predict? Historical perspectives on prediction in Earth Science, in Prediction, Science, Decision Making and the future of Nature, Sarewitz et al., Eds., Island Press, Washington DC

"[…] to be of value in theory testing, the predictions involved must be capable of refuting the theory that generated them"



"When a model generates a prediction, of what precisely is the prediction a test? The laws? The input data? The conceptualization?" (N. Oreskes)

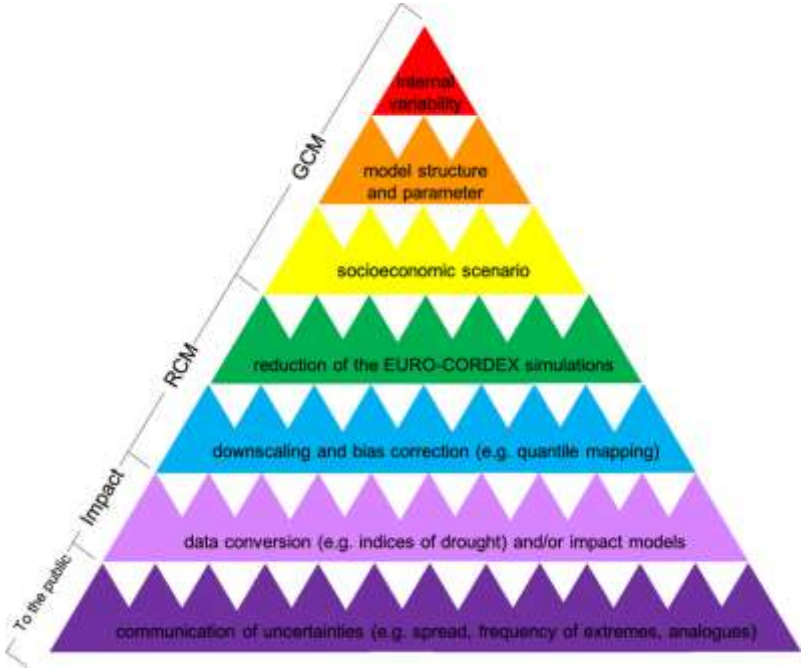


Models are fragile

Duhem's principle of stability [1], and the occurrence of either Butterfly [2] or Hawkmoth effects [3].

The accumulation of parametric error in a model, the so-called uncertainty cascade [4], that is the subject of global sensitivity analysis studies.

[1] S. C. Fletcher, 'The Principle of Stability', Philosopher's Imprint, vol. 20, no. 3, 2020, Accessed: Sep. 08, 2024. [Online].
[2] H. G. Schuster, Deterministic Chaos: An Introduction, 2nd Rev edition. Weinheim: Vch Pub, 1998.
[3] E. Winsberg, 'Appendix: Structural Stability and the "Hawkmoth Effect"', in Philosophy and Climate Science, Cambridge: Cambridge University Press, 2018, pp. 232–246. doi: 10.1017/9781108164290.016.
[4] M. Christie, A. Cliffe, P. Dawid, and S. S. Senn, Simplicity, Complexity and Modelling. Wiley, 2011.



"I have proposed a form of organised sensitivity analysis that I call 'global sensitivity analysis' in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified.
Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful."

Edward E. Leamer, 1990, Let's Take the Con Out of Econometrics, *American Economics Review*, 73 (March 1983), 31-43.

"One reason these methods are rarely used is their honesty seems destructive;"

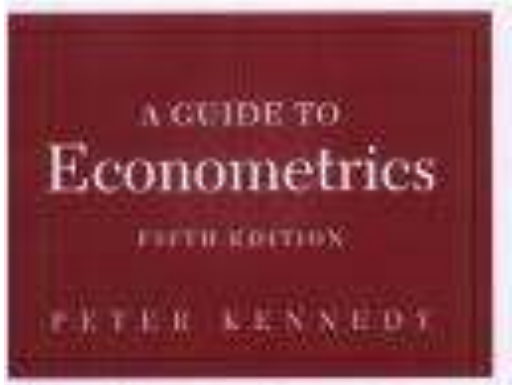
Tantalus on the Road to Asymptopia, Edward E. Leamer, 2010 *Journal of Economic Perspectives*, 24, (2), 31-46.

27 years

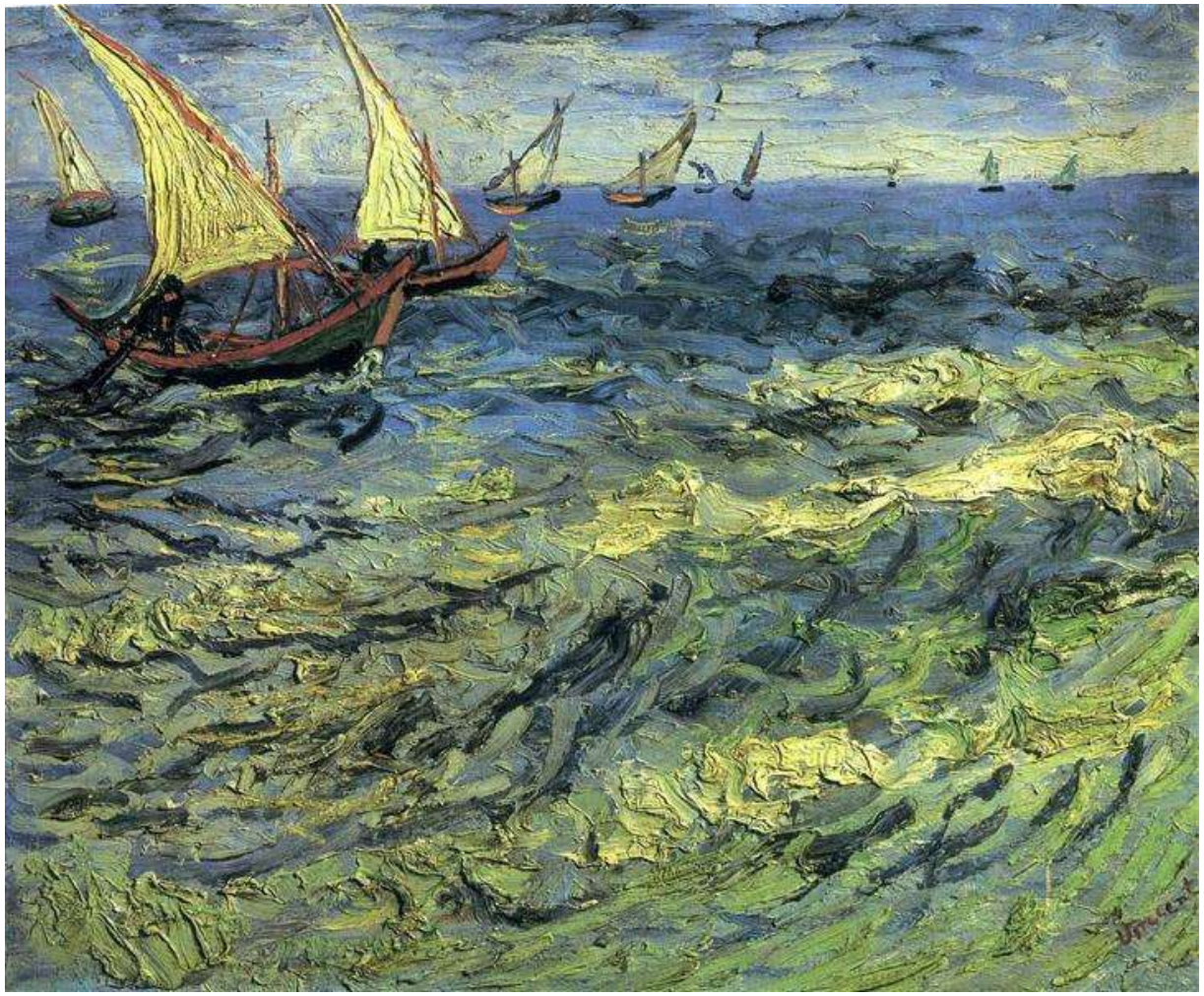
Edward E. Leamer, 1990, Let's Take the Con Out of Econometrics, *American Economics Review*, 73 (March 1983), 31-43.

Peter Kennedy, A Guide to Econometrics.
Anticipating criticism by applying sensitivity analysis. This is one of the ten commandments of applied econometrics according to Peter Kennedy:

<<Thou shall confess in the presence of sensitivity.
Corollary: Thou shall anticipate criticism >>



Fishing expeditions and forking paths …



Jorge Luis Borges
(1899-1986)

Taking different narratives within the same novel like Ts'ui Pên

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no "fishing expedition" or "p-hacking" and the research hypothesis was posited ahead of time*

Andrew Gelman† and Eric Loken‡

14 Nov 2013

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no "fishing expedition" or "p-hacking" and the research hypothesis was posited ahead of time*

Andrew Gelman† and Eric Loken‡

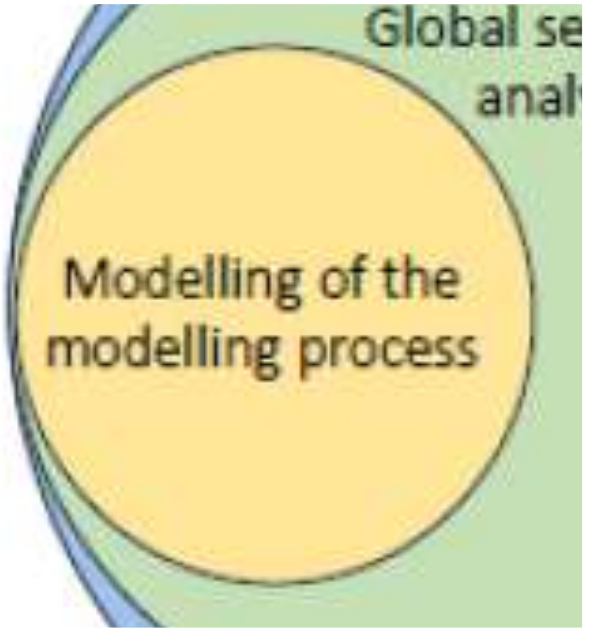
14 Nov 2013

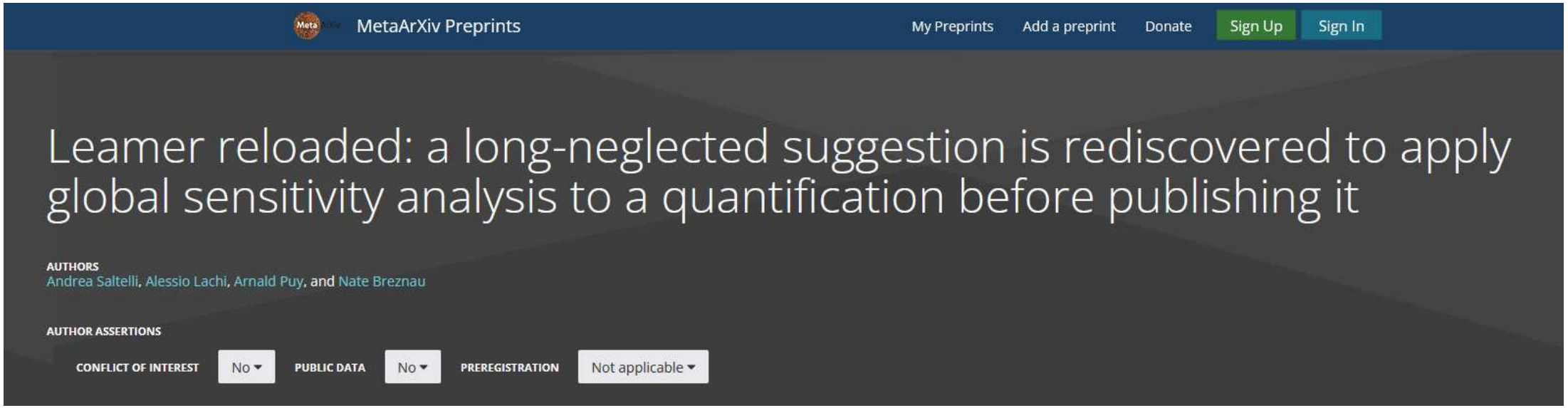
Can global SA help?

PNAS RESEARCH ARTICLE | SOCIAL SCIENCES OPEN

Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty

The universe of uncertainty that didn't hide: reaching out to multiverse analysis





Many multi-analyst studies ongoing, e.g.:

"Are soccer players with dark skin tone more likely to receive red cards from referees?", Silberzahn et al. 2018

"Is there an impact of compulsory schooling on teenage pregnancy rates?", Huntington-Klein et al. (2021)

Source: BBC https://www.bbc.com/sport/football/68435064

"There are at least twenty multi-analyst experiments brewing"

… some large! 343 authors of Menkveld et al. (2024)"

HUNTINGTON-KLEIN, N., ARENAS, A., BEAM, E., BERTONI, M., BLOEM, J. R., BURLI, P., CHEN, N., GRIECO, P., EKPE, G., PUGATCH, T. et al. (2021). The influence of hidden researcher decisions in applied microeconomics. Economic Inquiry 59 944–960.
MENKVELD, A. J., (2024). Nonstandard Errors. The Journal of Finance 79 2339–2390. https://doi.org/10.1111/jofi.13337.
SILBERZAHN, R., UHLMANN, E. L., MARTIN, D. P., ANSELMI, P., AUST, F., AWTREY, E., BAHNÍK, Š., BAI, F., BANNARD, C., BONNIER, E. et al. (2018). Many analysts, one data set: Making transparent how variations in analytic choices affect results. Advances in Methods and Practices in Psychological Science 1, 337–356.

We take this →

PNAS RESEARCH ARTICLE | SOCIAL SCIENCES OPEN

Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty

Source: Britannica https://www.britannica.com/topic/human-migration

The question to be answered was the effect of migration on public attitude toward social policies

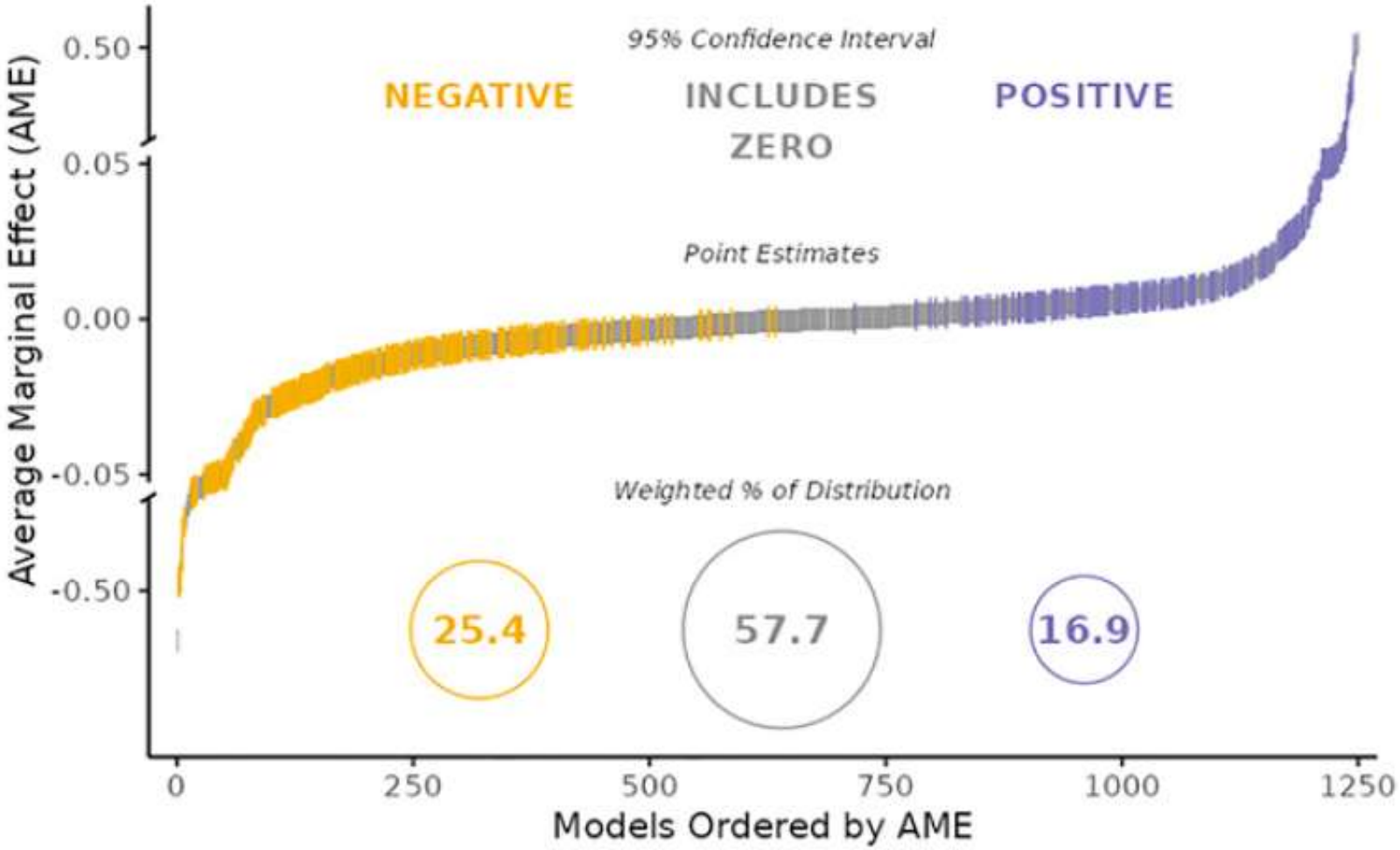
Literature says: negative correlation but Brady and Finnegan (2014) test 3 hypotheses:

- ethnic and linguistic heterogeneity make class-based solidarity more difficult
- competition for jobs increases support for social policy (compensation hypothesis)
- the chauvinism hypothesis, no to policies that benefit immigrants

BRADY, D. and FINNIGAN, R. (2014). Does immigration undermine public support for social policy? American sociological review 79 17–42.

BREZNAU, N., RINKE, E. M., WUTTKE, et al. (2022). Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty. Proceedings of the National Academy of Sciences 119 e2203150119.

ENGZELL, P. (2023). A universe of uncertainty hiding in plain sight. Proceedings of the National Academy of Sciences 120 e2218530120.



"Will different researchers [73 teams] converge on similar findings when analyzing the same data?

…teams' results varied greatly, ranging from large negative to large positive effects"

"A universe of uncertainty hiding in plain sight" (Engzell, 2023)

In the original multi-analyst study the normative (political) orientation of the analysts was found **not** to explain the variance

The modelling variables were found to explain at most a few percent of the variance of the results … **where does the variance come from?**

https://www.istockphoto.com/

k_i	Variable	Description
1	age	Inclusion or non-inclusion of the covariates age and square age of participants
2	$all - ethnic$	Inclusion or non-inclusion of a covariate that takes into account all ethnic categories
3	$bootstrap$	Inclusion or non-inclusion of the bootstrap procedure
4	$cont - europe$	Inclusion or non-inclusion of continental Europe countries in the analysis
5	$continent$	Inclusion or non-inclusion of different continents in the analysis
6	$education$	Inclusion or non-inclusion of the covariates level of education
7	$female$	Inclusion or non-inclusion of the covariates gender as self-reported by respondents in the survey
8	$GINI$	Inclusion or non-inclusion of the covariates related to the Gini indexes
9	$income$	Inclusion or non-inclusion of the covariate related to the income
10	$pop - wb$	Inclusion or non-inclusion of the covariate related to the information about country-specific numerosity of population
11	$rich - dems$	Inclusion or non-inclusion of rich democracies in the analysis
12	SE	Inclusion or non-inclusion of clustered standard error
13	$socx - oecd$	Inclusion or non-inclusion of the covariate related to social expenditure
14	$X1$	Inclusion or non-inclusion of the covariates related to the migration indexes
15	Y	Choice of policy dependent variable

Table 1. Descriptions of variables included in the analysis.

We simulate different paths using triggers then we do a global sensitivity analysis

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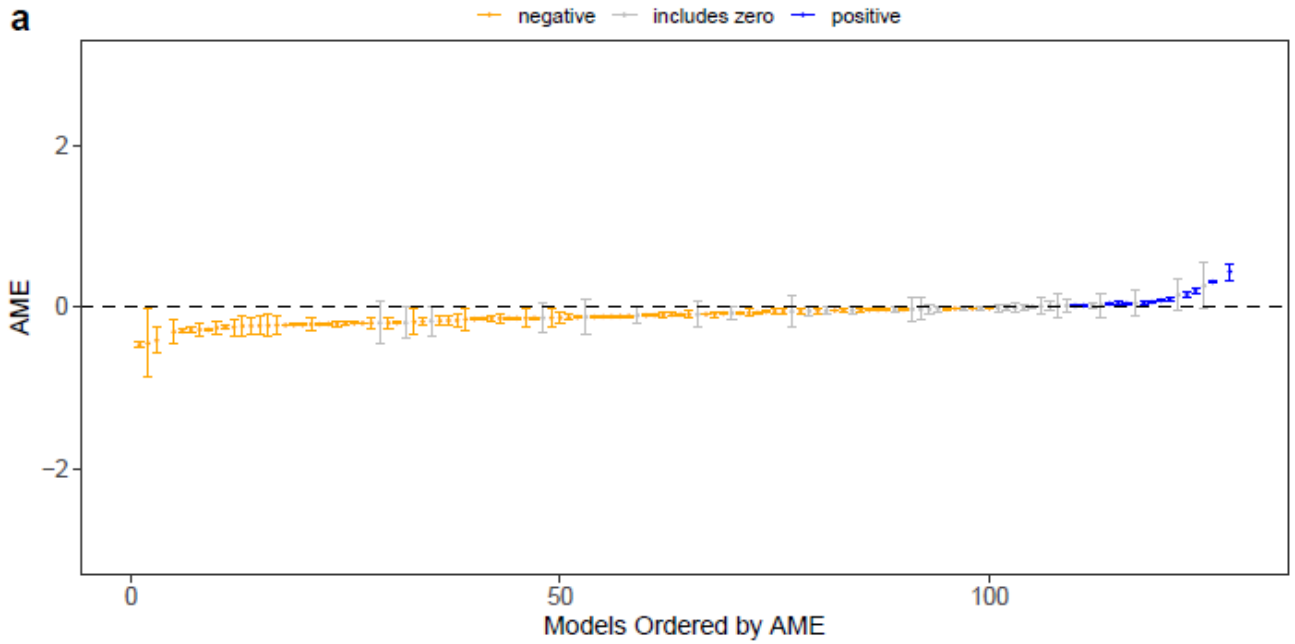
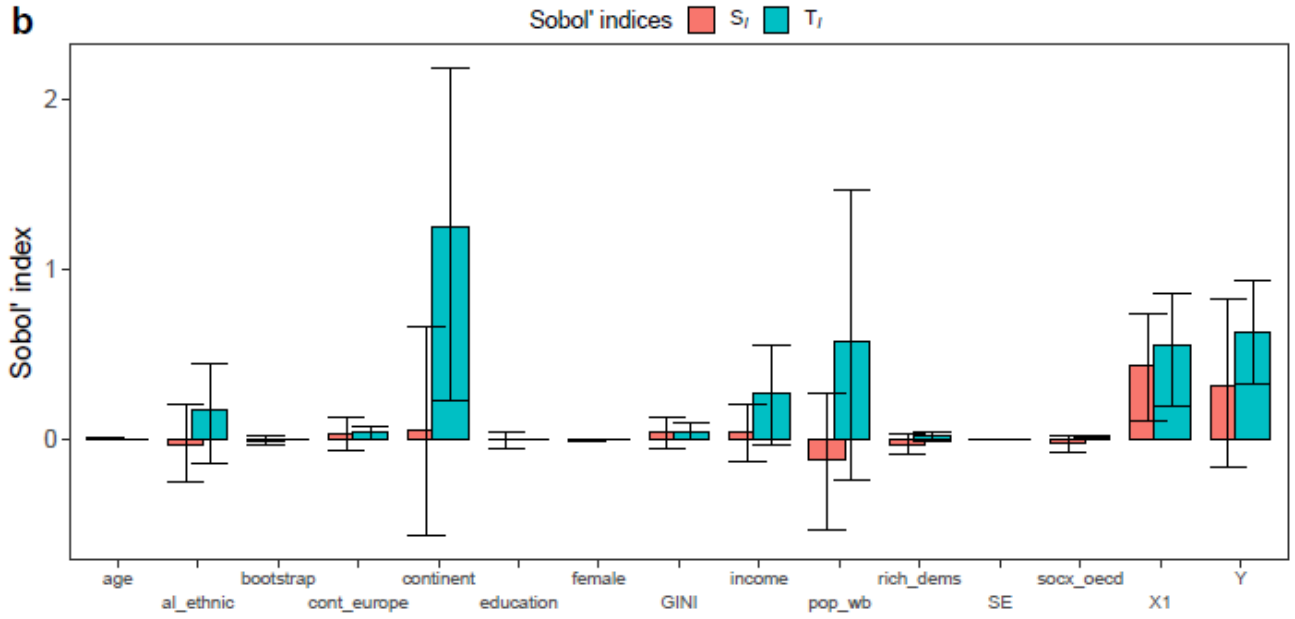


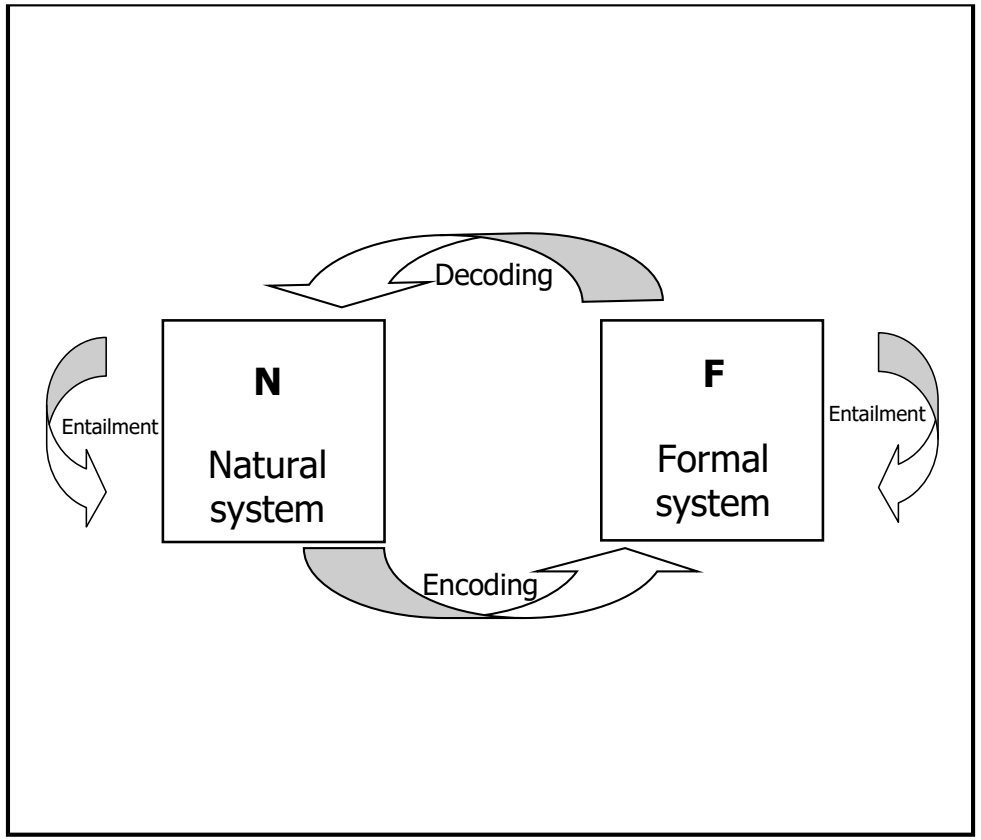


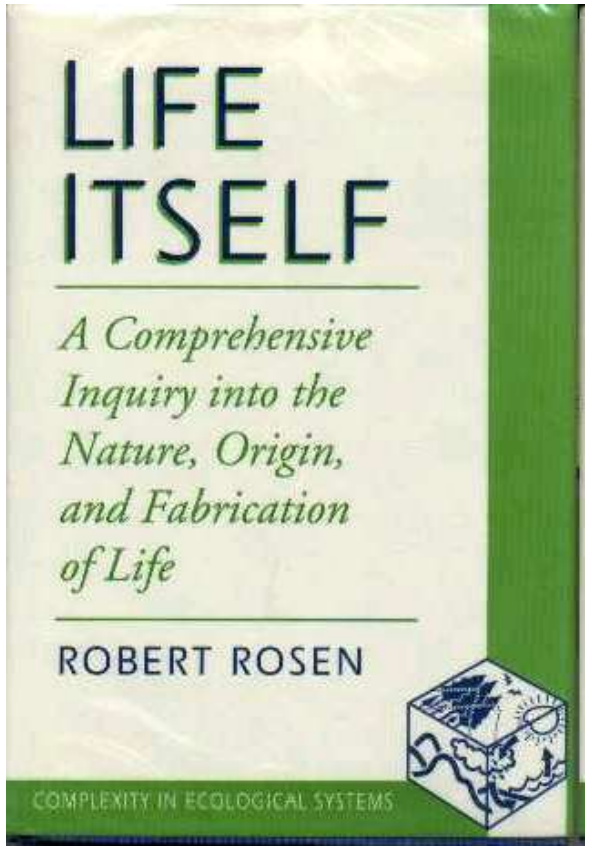
Uncertainty analysis reproduces the results of the multi-analyst experiment

Our sensitivity analysis explains where the missing variance went: **interactions**

Modelling is a craft more than a science

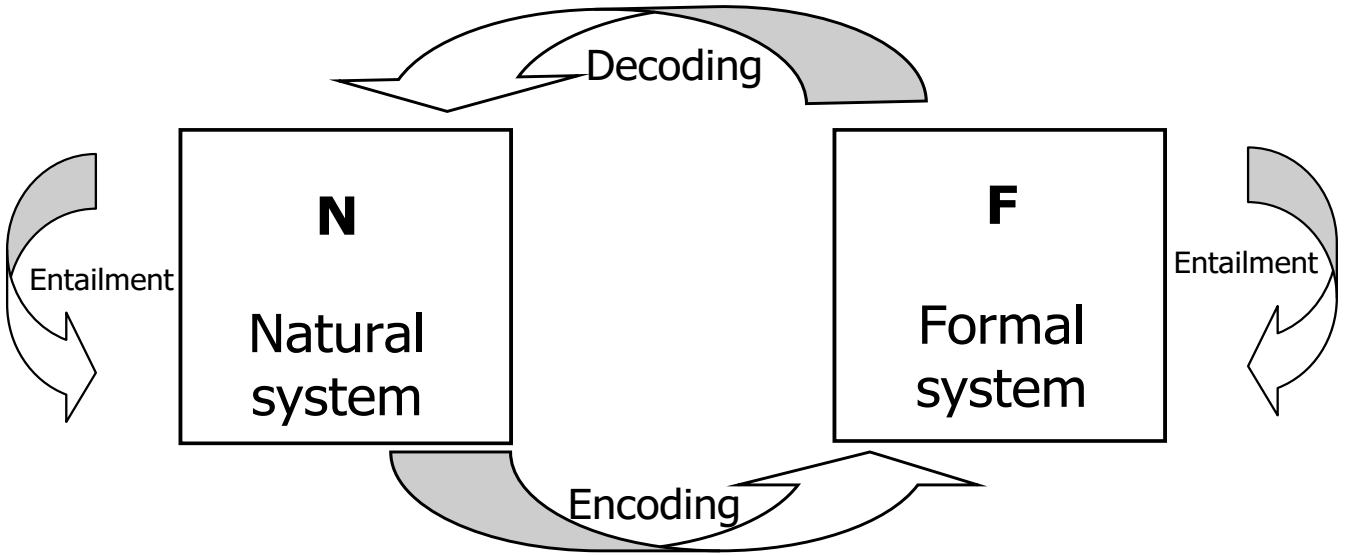
Modelling as a craft rather than as a science for Robert Rosen





R. Rosen, Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life. Columbia University Press, 1991.

Louie, A.H. 2010. “Robert Rosen’s Anticipatory Systems.” Edited by Riel Miller. Foresight 12 (3): 18–29. https://doi.org/10.1108/14636681011049848.



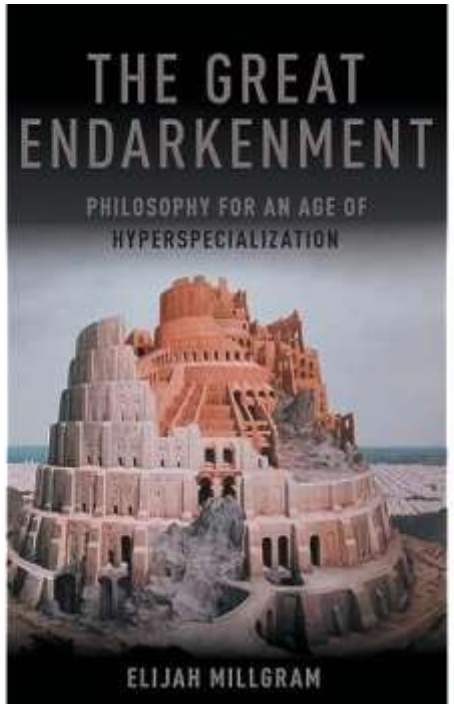
What is a model ?

Robert Rosen
(1934-1998)

Models have little memory

“[…] The process of constructing and validating [value-at risk] models is time consuming and detail oriented; normally even the people who produced the model will not remember many of the assumptions incorporated into it, short of redoing their work, which means that the client cannot simply ask then what went into it.”

E. Millgram The Great Endarkenment, p. 29

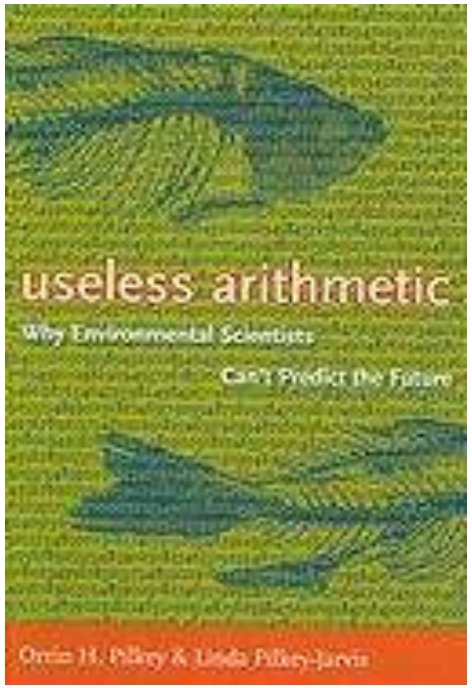


Don’t confuse the map with the territory

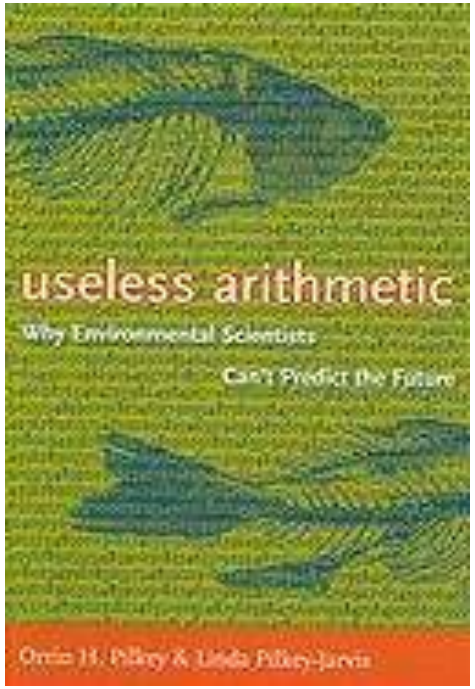
If you do, sensitivity analysis will not save you

<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature>>

Orrin H. Pilkey



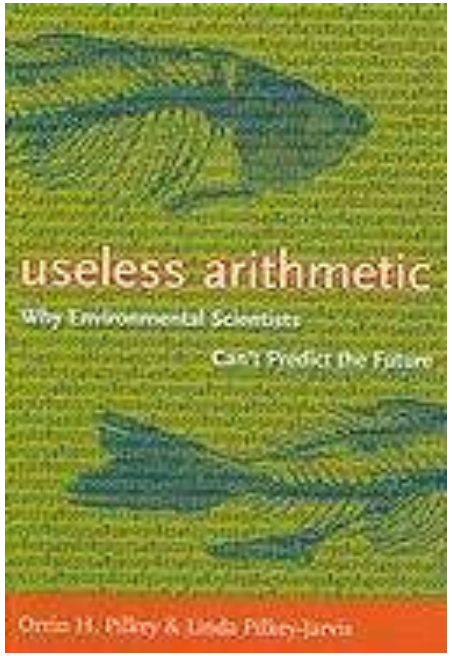
Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

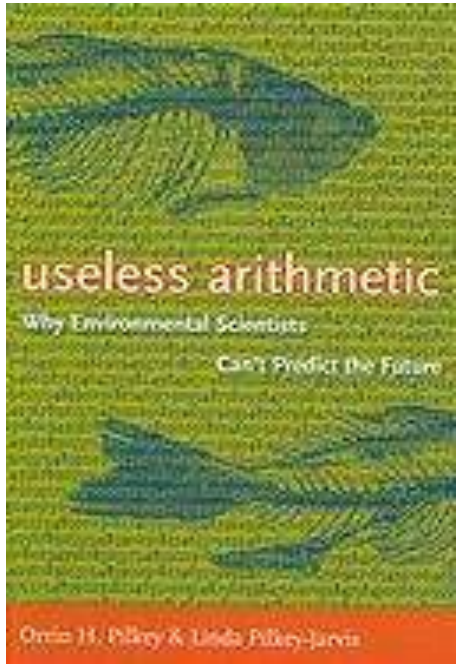


<<…If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit>>

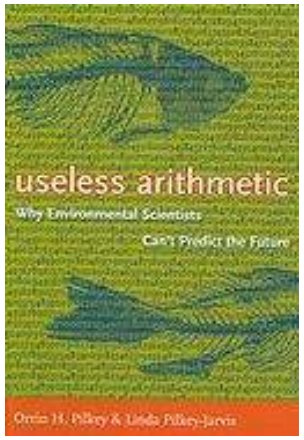
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

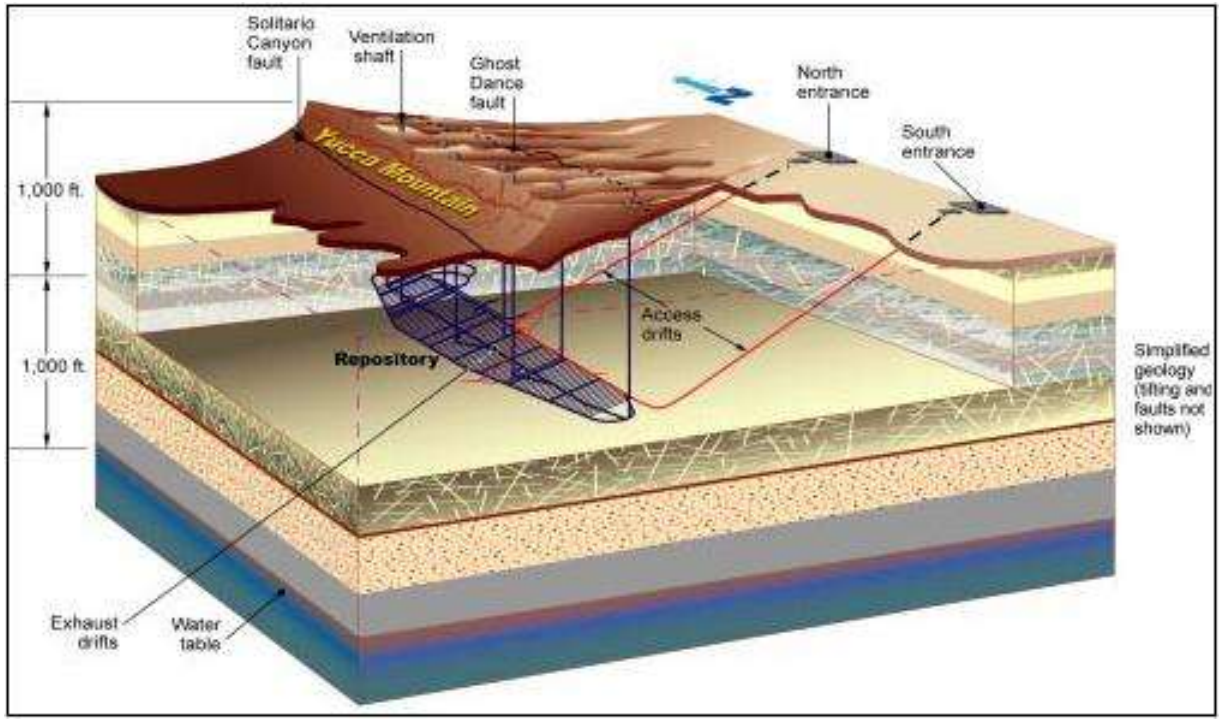
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal

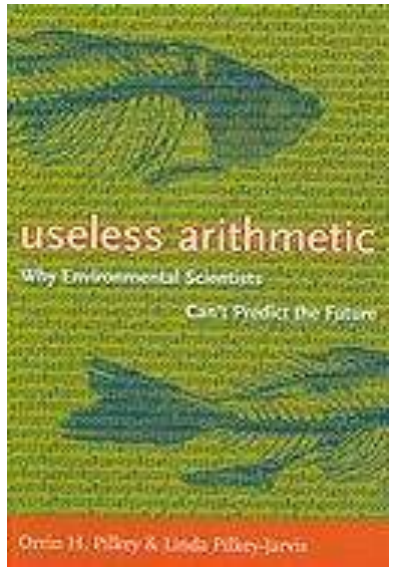




The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter
(the ^{36}Cl story)

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→… SA useless if it is instead ~ 3,000 millimetres per year.



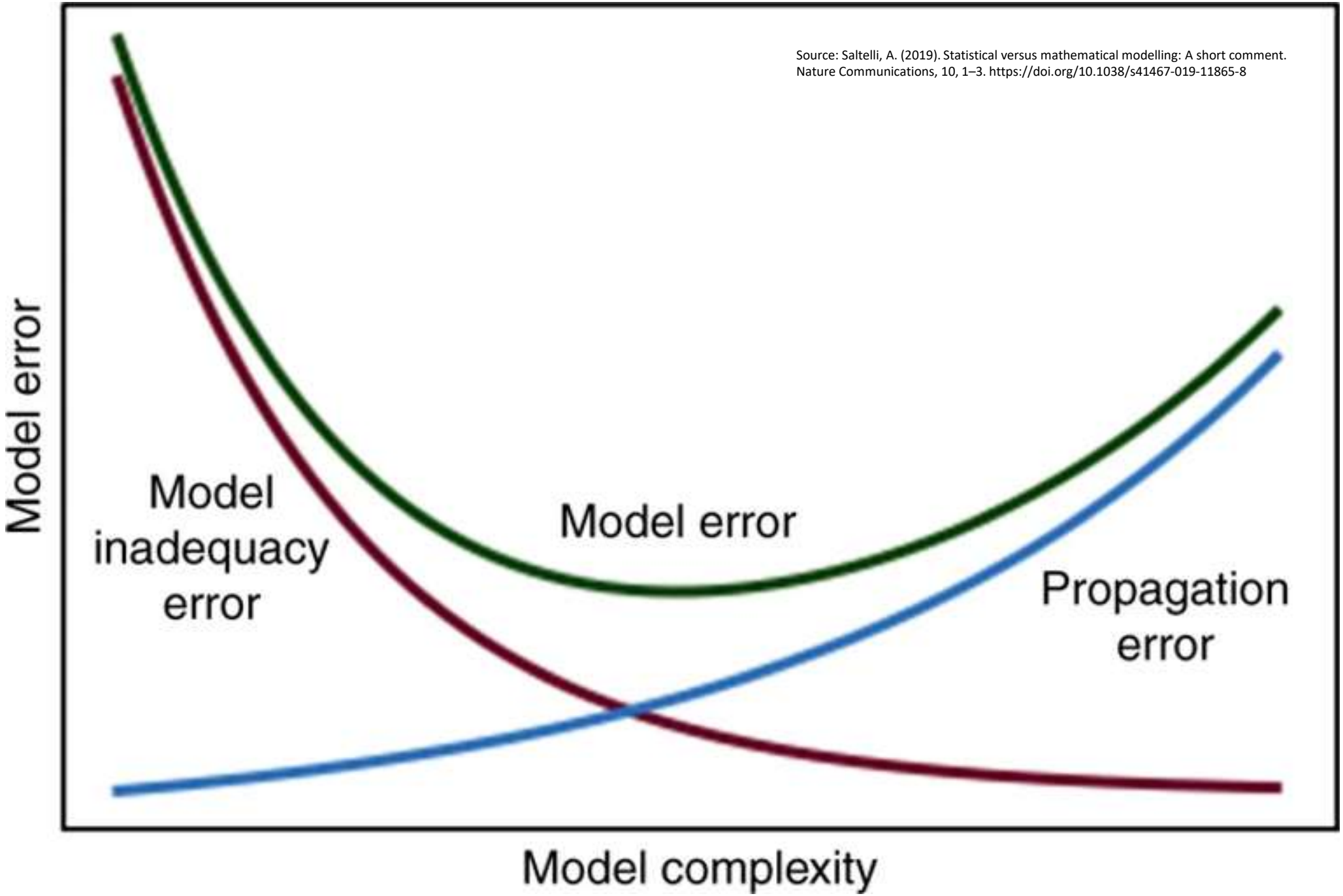
"Scientific mathematical modelling should involve constant efforts to falsify the model"

➔ Organized skepticism (as per CUDOS)

Communalism, Universalism, Disinterestedness, Organized Skepticism, from sociology of science, Robert K. Merton.

Beware the size of your model

Mind the conjecture of O'Neil



Source: Saltelli, A. (2019). Statistical versus mathematical modelling: A short comment. Nature Communications, 10, 1–3. https://doi.org/10.1038/s41467-019-11865-8

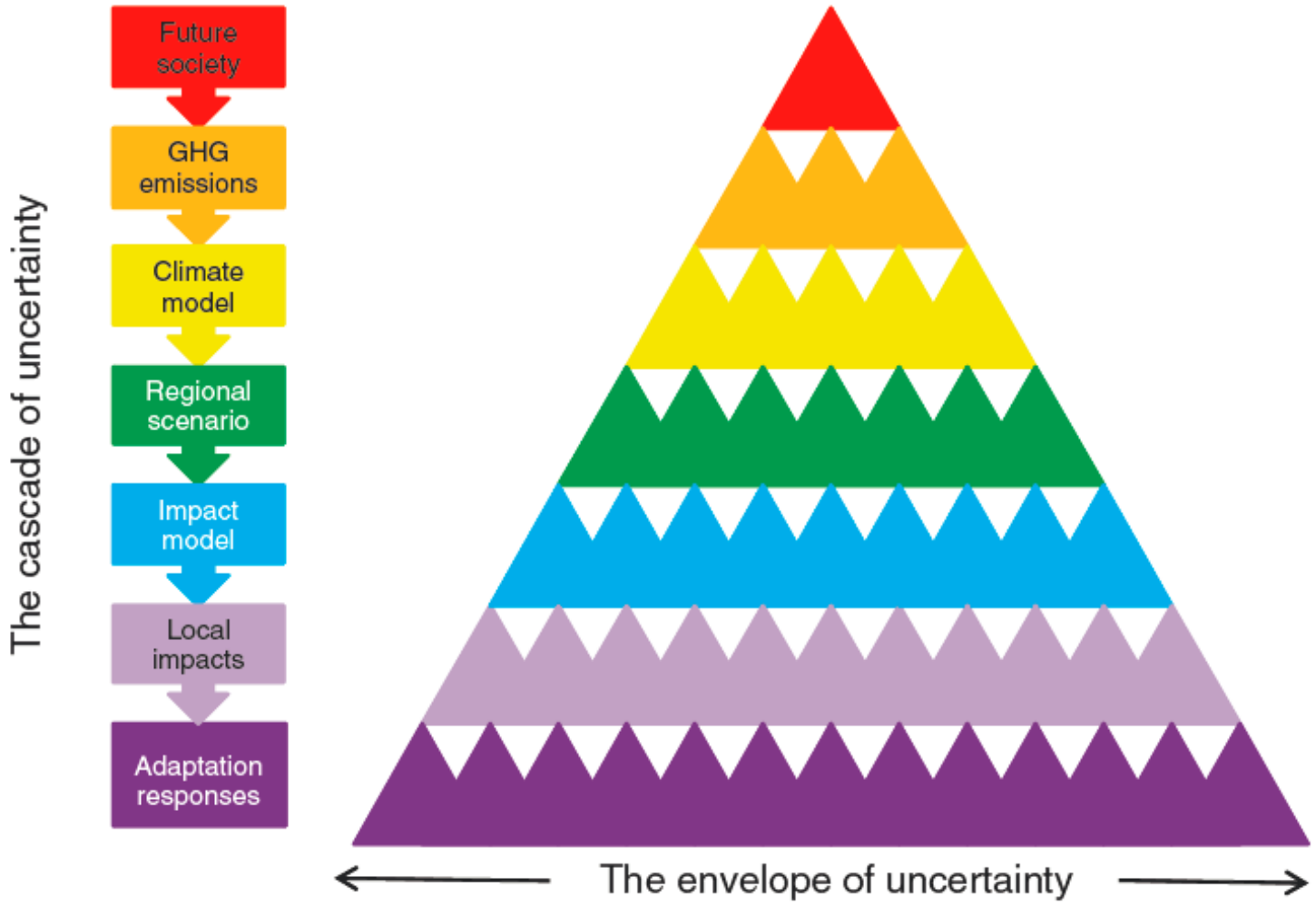
Conjecture by O'Neill, also known as Zadeh's principle of incompatibility, whereby as complexity increases "precision and significance (or relevance) become almost mutually exclusive characteristics"

In M. G. Turner and R. H. Gardner, "Introduction to Models" in Landscape Ecology in Theory and Practice, New York, NY: Springer New York, 2015, pp. 63–95.

L. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," IEEE Trans. Syst. Man. Cybern., vol. 3, no. 1, pp. 28–44, 1973.

Puy, Arnald, Pierfrancesco Beneventano, Simon A. Levin, Samuele Lo Piano, Tommaso Portaluri, and Andrea Saltelli. 2022. "Models with Higher Effective Dimensions Tend to Produce More Uncertain Estimates." Science Advances 8 (eabn9450).

Uncertainty analysis: the study of the uncertainty in model output—see also uncertainty cascade



Source: https://www.climate-lab-book.ac.uk/2014/cascade-of-uncertainty/

Sensitivity analysis: the study of the relative importance of different input factors on the model output

"Sensitivity analysis (SA) is the study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input"

Risk Analysis
AN INTERNATIONAL JOURNAL
An Official Publication of the Society for Risk Analysis

Sensitivity Analysis for Importance Assessment

Andrea Saltelli

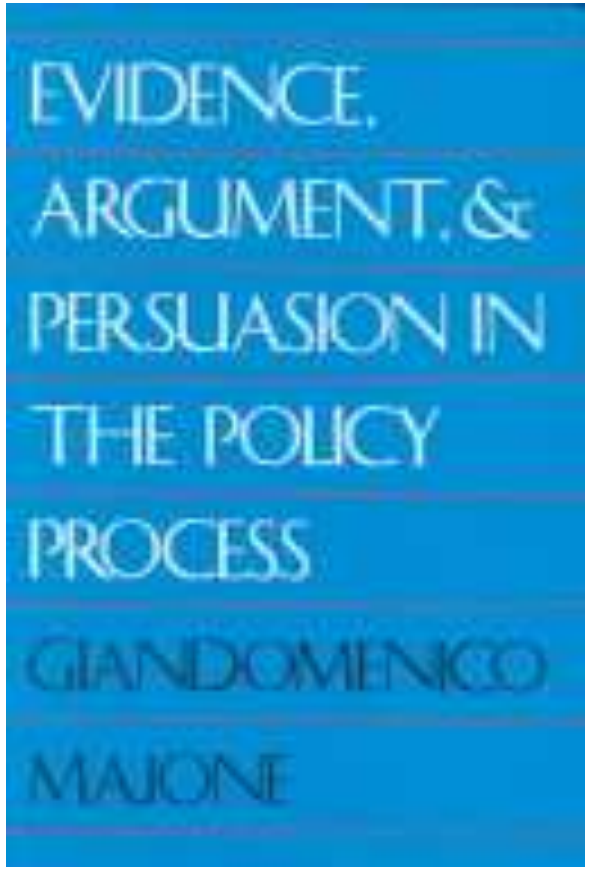
First published: 22 July 2002 | https://doi.org/10.1111/0272-4332.00040 | Citations: 896

But the real strength of the models, in my mind at least, were in sensitivity analysis (where one could examine the response of the model to parameters or structures that were not known with precision (i.e., sensitivity analysis), and in the examination of the behavior of the model components relative to that of the real system in question (i.e., validation).

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.

… By undertaking sensitivity analysis and validation, a great deal can be learned about the real system, including what you do not know. (Hall, 2020)

Hall, C. A. S. (2020). Systems Ecology and Limits to Growth: History, Models, and Present Status. In G. S. Metcalf, K. Kijima, & H. Deguchi, eds., *Handbook of Systems Sciences*, Singapore: Springer, , pp. 1–38.



"Are the results from a particular model more sensitive to changes in the model and the methods used to estimate its parameters, or to changes in the data?"

Sensitivity analysis can :

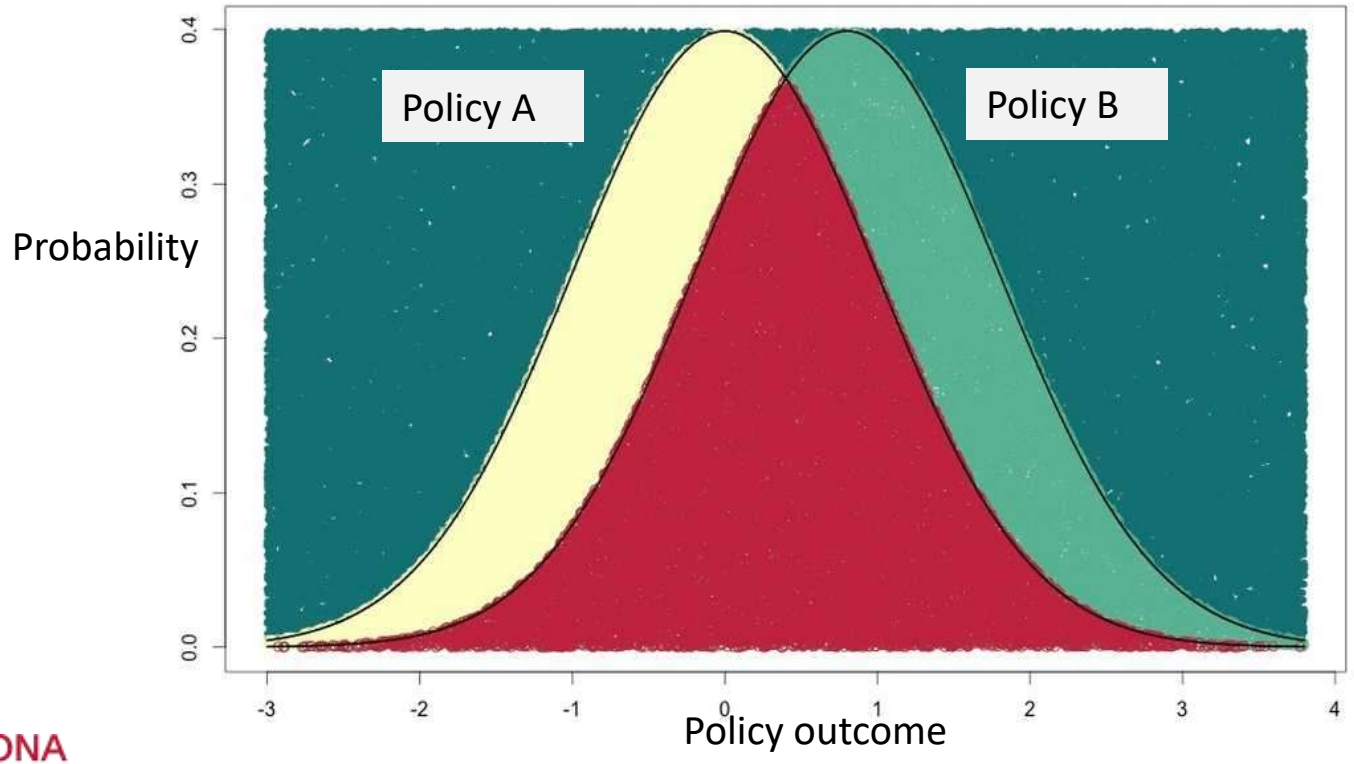
- surprise the analyst,
- uncover technical errors in the model,
- identify critical regions in the space of the inputs,
- establish priorities for research,
- simplify models
- falsify models (show that a model is false or irrelevant)
- defend against your own model being falsified
- **calibrate or prepare for calibration**

Source: The Simpson, 20th Television Animation (The Walt Disney Company)

Vincenzo later today

Sensitivity analysis can:

verify whether policy options (or marketing strategies) can be distinguished from one another given the uncertainties in the system, …



BARCELONA SCHOOL OF MANAGEMENT

What method would one choose to perform sensitivity analysis?

Source: iStock by Getty images

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots x_k)$$

$$\left.\frac{\partial y}{\partial x_i}\right|_{x_i = x_i^0} \longleftarrow \text{Local}$$

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

$$y = f(x_1, x_2, \dots x_k)$$

$$\frac{x_i^0}{y^0}\frac{\partial y}{\partial x_i}\bigg|_{x_i = x_i^0} \longleftarrow \text{Local}$$

What method would one choose to perform sensitivity analysis?

Most of the sensitivity analysis found in the literature are local or otherwise OAT (One factor At a Time)

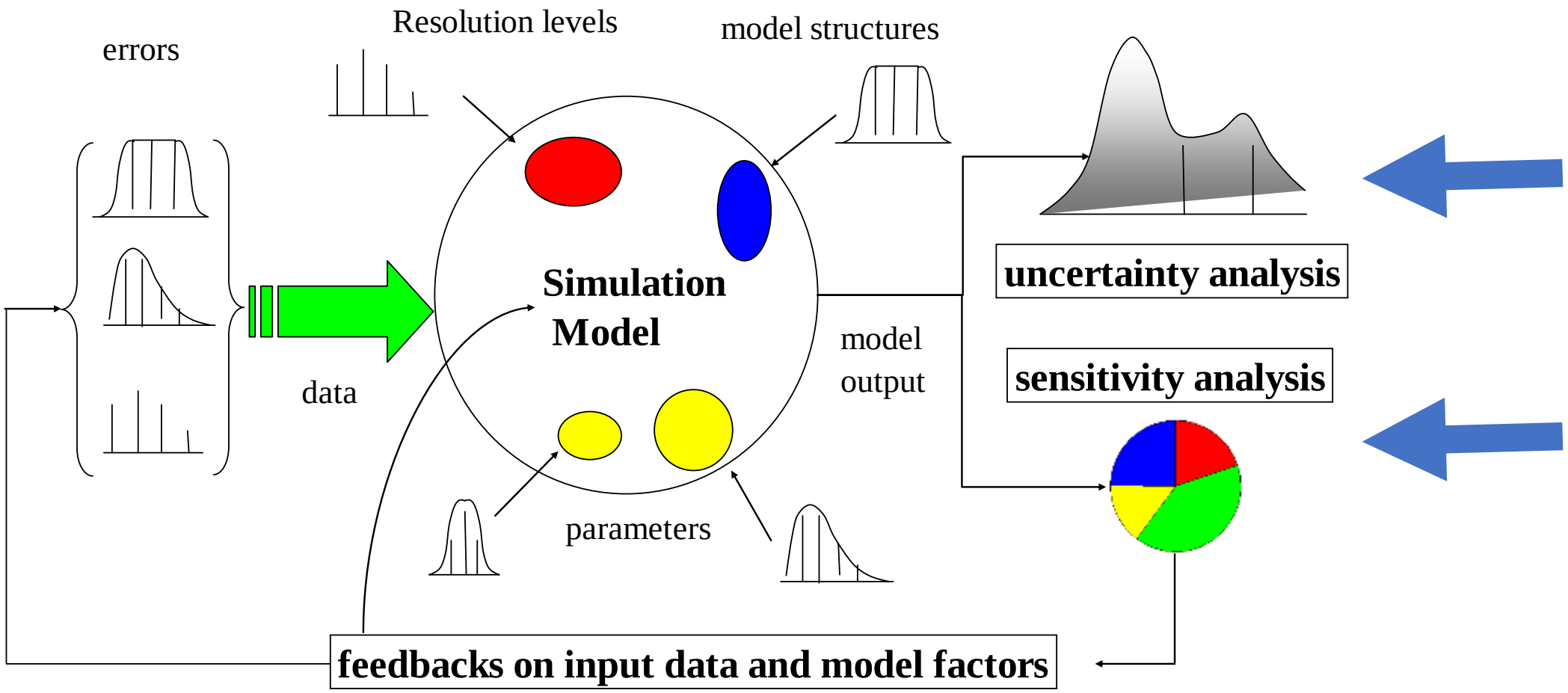
$$y = f(x_1, x_2, \dots x_k)$$

$$\left.\frac{std(x_i)}{std(y)}\frac{\partial y}{\partial x_i}\right|_{x_i = x_i^0} \longleftarrow \text{Hybrid}$$

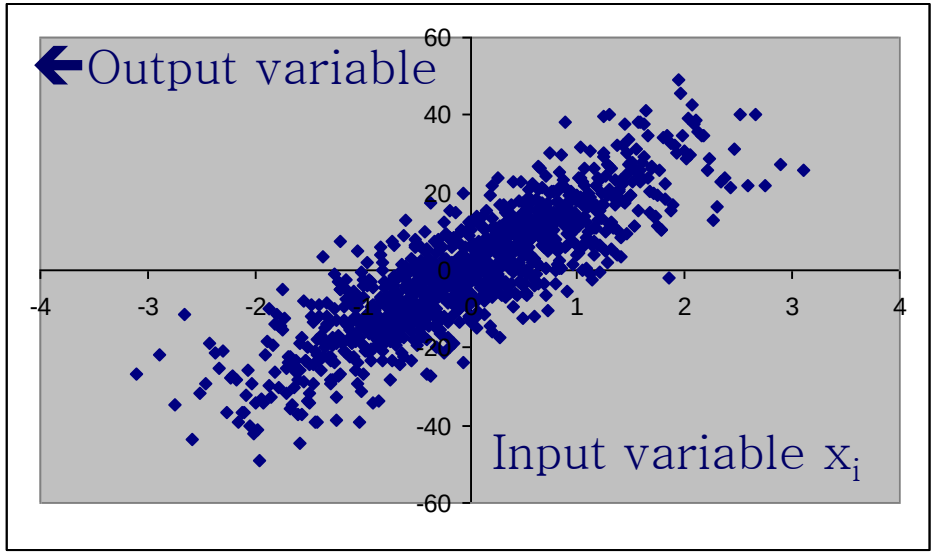
$$\left.\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i around its nominal value

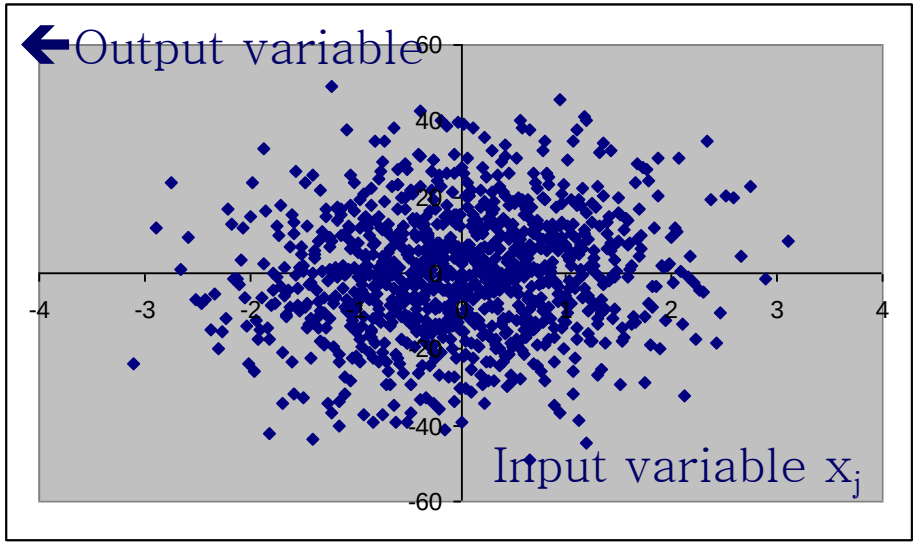
$$\left.\frac{x_i^0}{y^0}\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i by a fixed fraction of its nominal value

$$\left.\frac{std(x_i)}{std(y)}\frac{\partial y}{\partial x_i}\right|_{x_i=x_i^0}$$ ← Relative effect on y of perturbing x_i by a fixed fraction of its standard deviation


errors
Resolution levels
model structures
Simulation Model
data
model output
parameters
uncertainty analysis
sensitivity analysis
feedbacks on input data and model factors

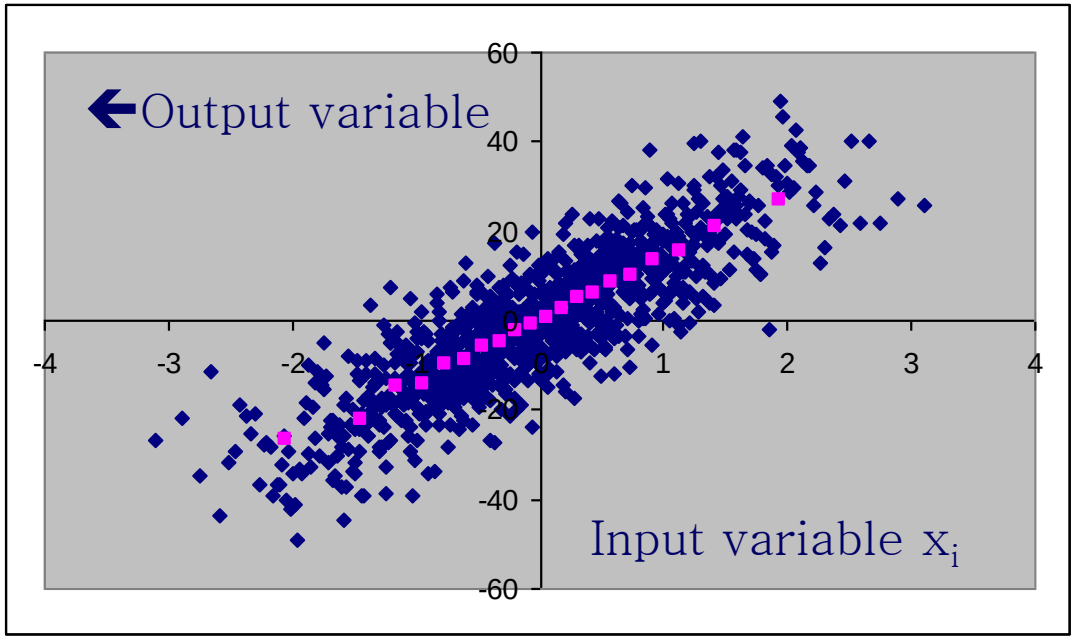
An introduction to variance based methods

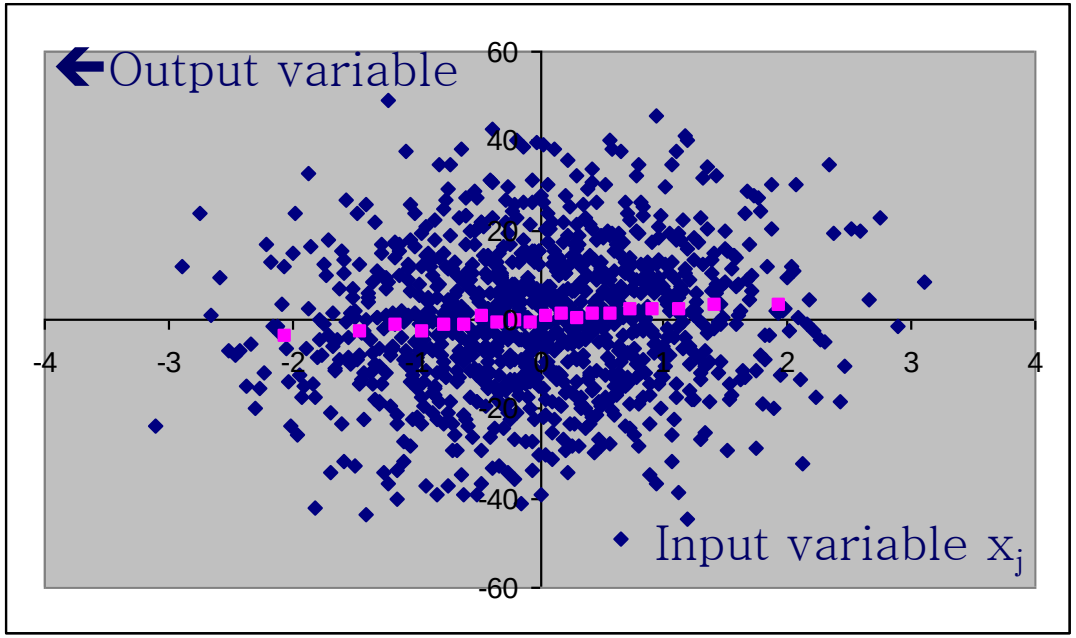




Plotting the output as a function of two different input factors

Which factor is more important?

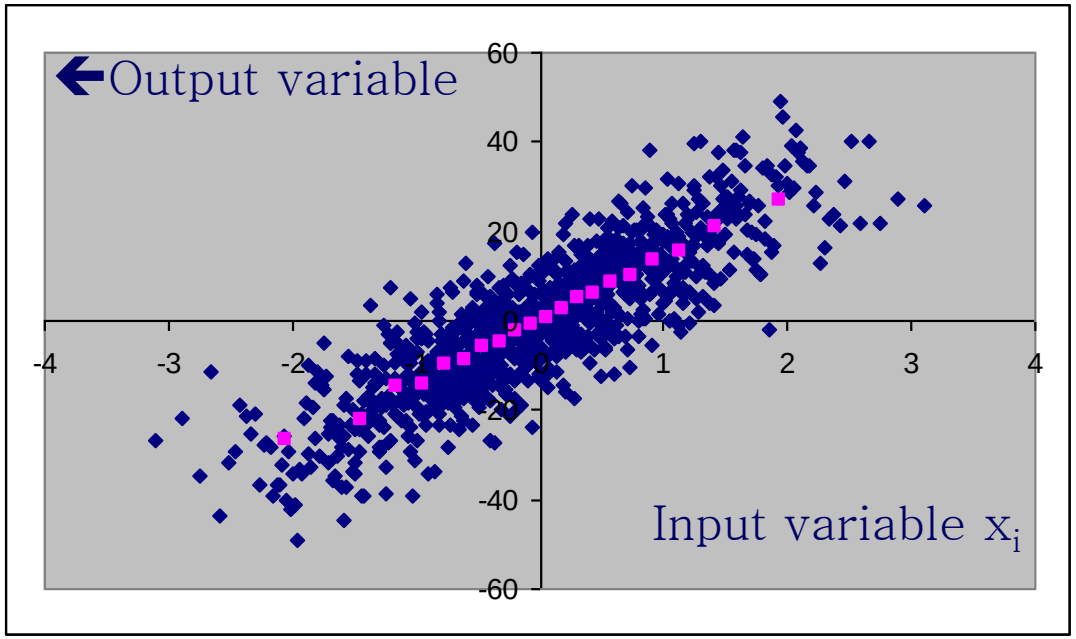




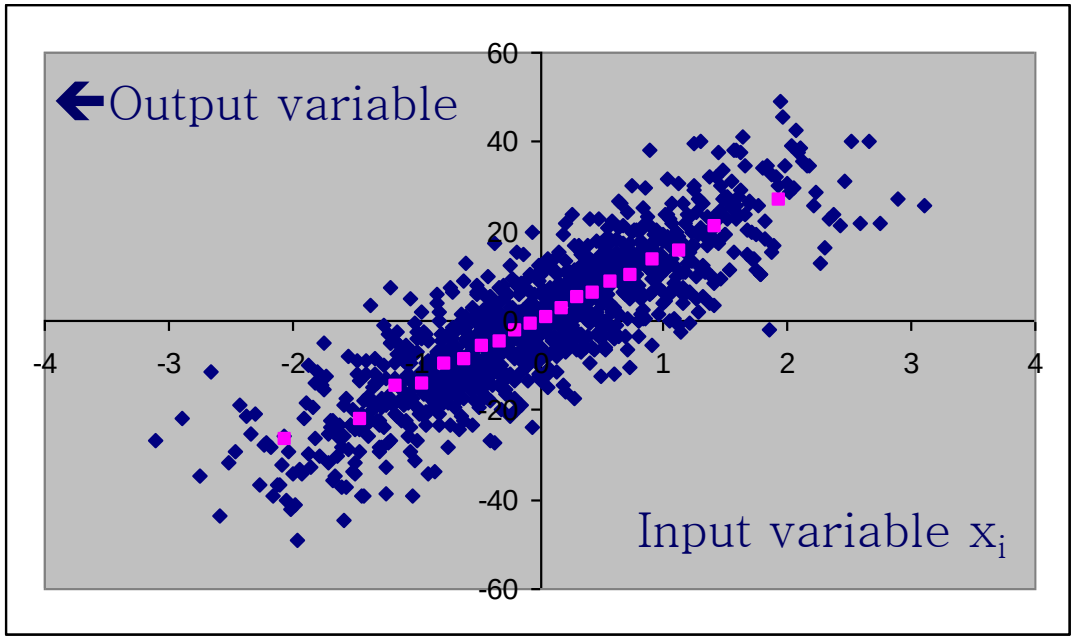
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)

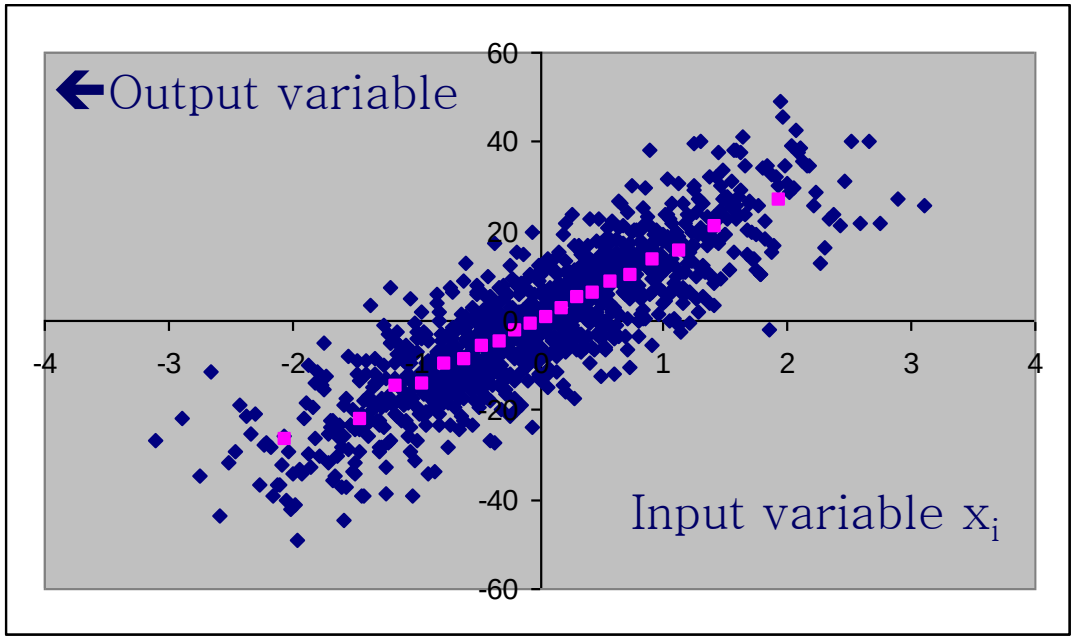


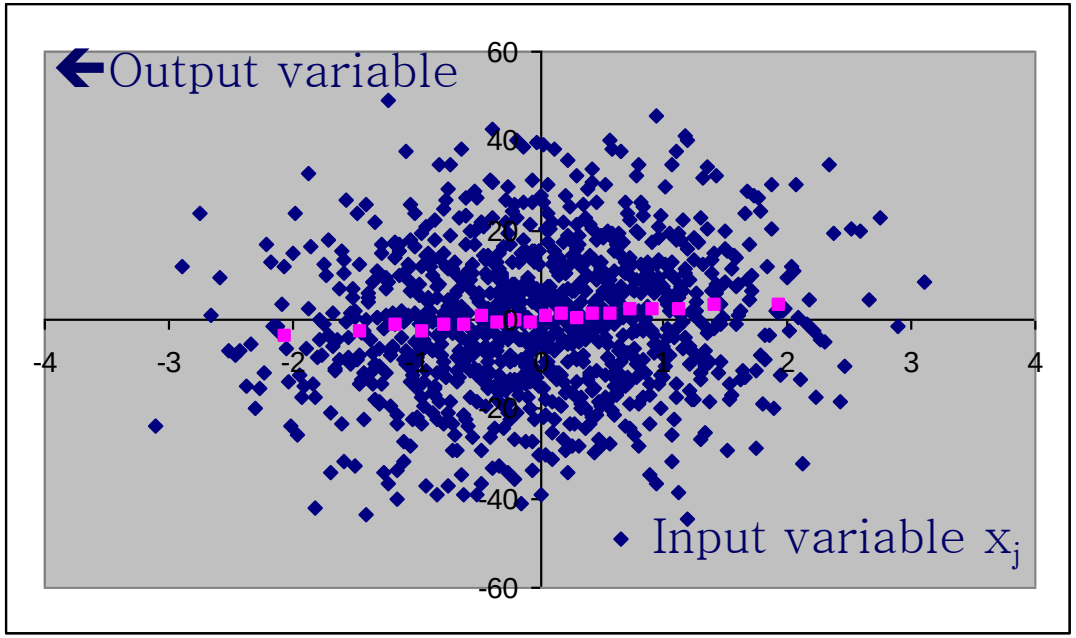
Each pink point is ~ $E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)$



Taking the variance of the pink points one obtains a sensitivity measure

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$





Which factor has the highest

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y\middle|X_i\right)\right)?$$

$$S_i = \frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order

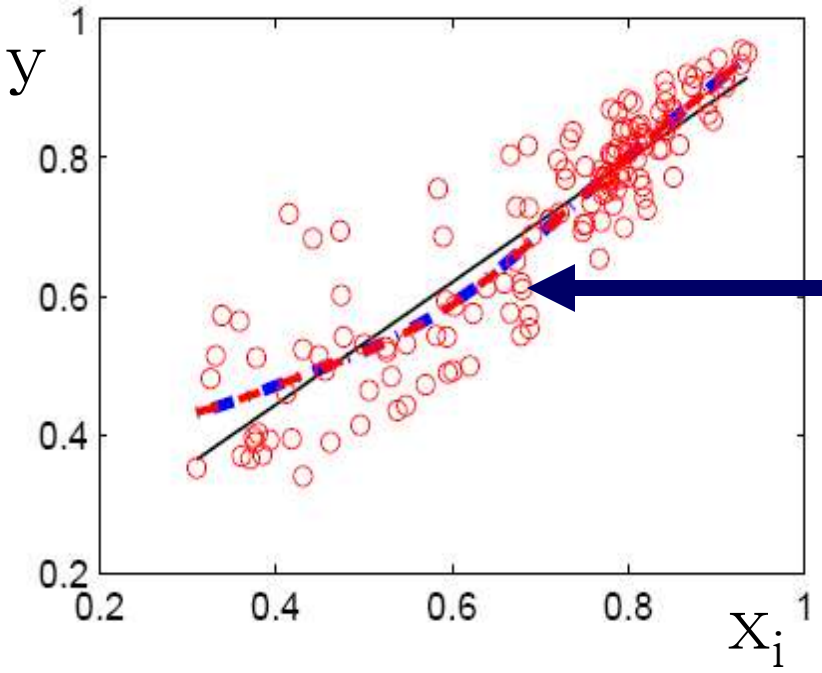
For additive models one can decompose the total variance as a sum of those partial variances

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right) \approx V(Y)$$

… which is also how additive models are defined

$$S_i = \frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order, identical in formulation to Pearson's correlation ratio



Smoothed curve:

$$\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)$$

First order sensitivity index:

$$\frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

Pearson's correlation ratio

Smoothed curve

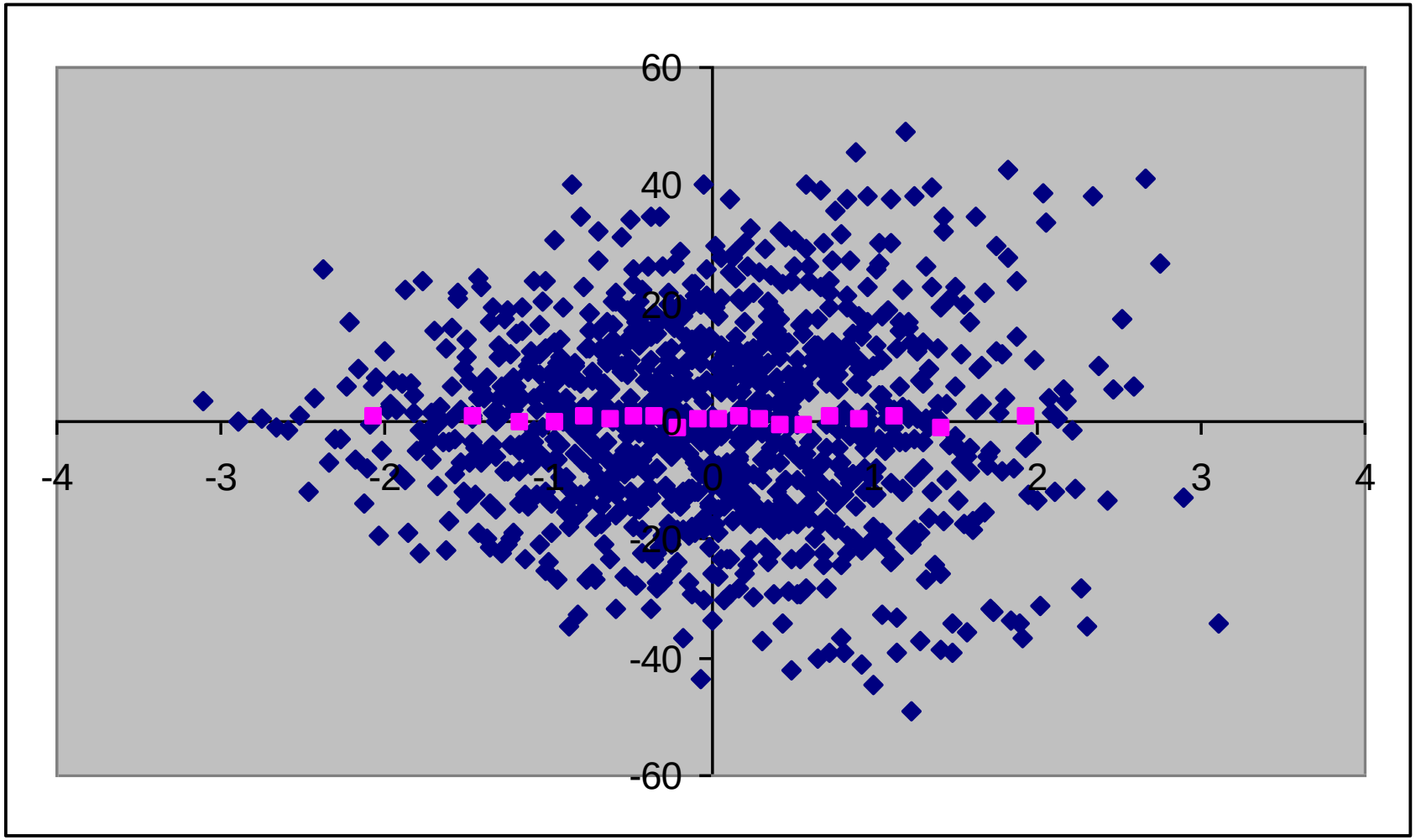
$$S_i \equiv \eta_i^2 := \frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

First order sensitivity index

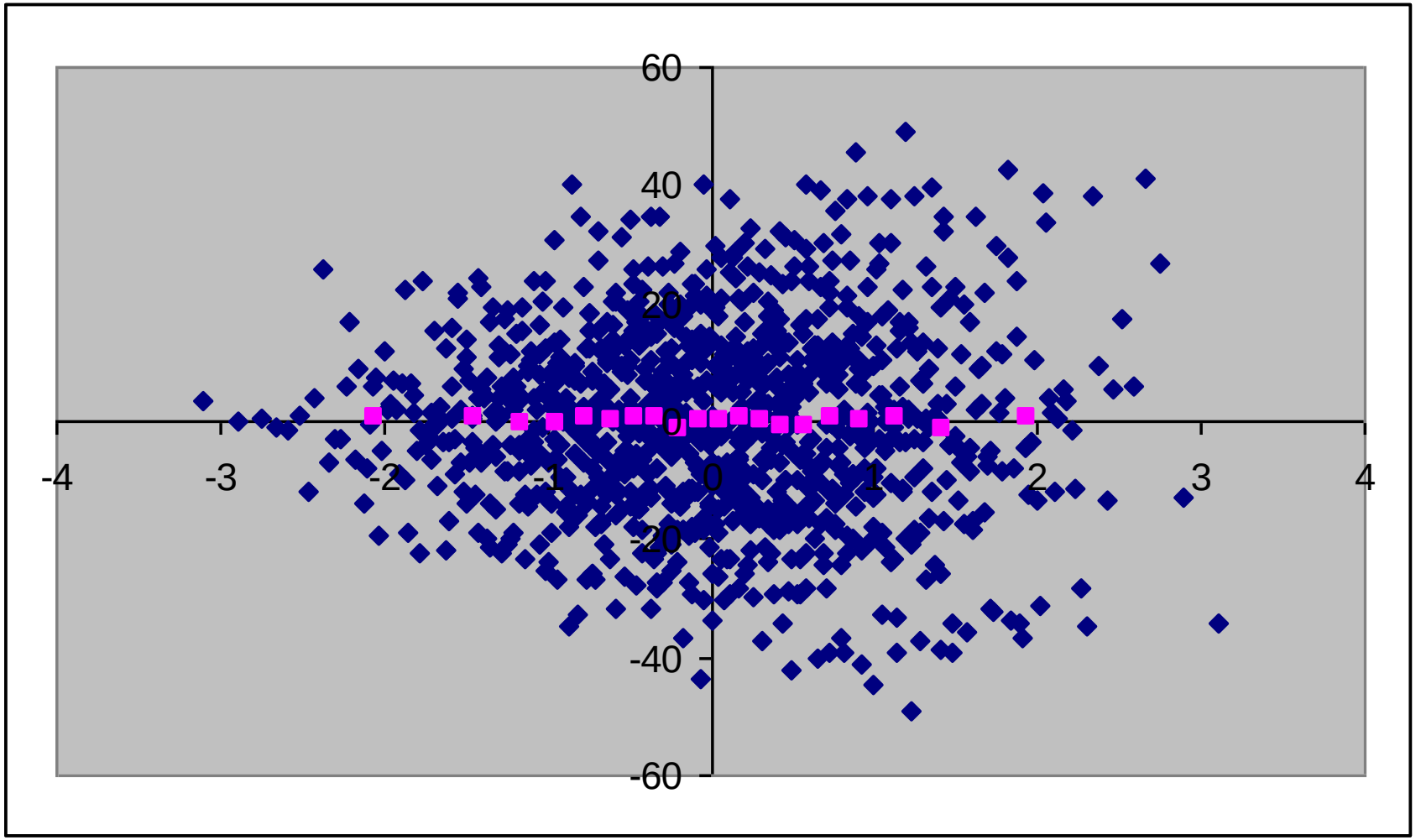
Unconditional variance

Non additive models

Is $S_i = 0$?



Is this factor non-important?



There are terms which capture two-way, three way, … interactions among variables

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_{i} V_i + \sum_{i,\, j>i} V_{ij} + \ldots + V_{123\ldots k}$$

Variance decomposition (ANOVA)

The total variance can be decomposed into main effects and interaction effects up to the order k, the dimensionality of the problem (only for independent factors)

If fact interactions terms are awkward to handle: **just** the **second** order terms for a model with k factors are as many as k(k-1)/2 …

(10 factors=45 second order terms)

How about a single ‘importance’ terms for all effects?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a 3-factor model $f(X_1, X_2, X_3)$

Where the variance decomposition would read $1 = S_1 + S_2 + S_3 + S_{12} + S_{13} + S_{23} + S_{123}$

We compute

$$T_1 = S_1 + S_{12} + S_{13} + S_{123}$$

$$T_2 = S_2 + S_{12} + S_{23} + S_{123}$$

$$T_3 = S_3 + S_{13} + S_{23} + S_{123}$$

$$S_i = \frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)}$$

Known as:

- Pearson's correlation ratio (Pearson)
- First order effect (Sobol' & disciples)
- Top marginal variance (Jansen)

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

Top marginal variance=
= the expected reduction in variance than would be achieved if factor Xi could be fixed.
Why?

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+$$
$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)=V(Y)$$

One of the exercise in the book: prove this using $V(\bullet)=E(\bullet)^2-E^2(\bullet)$

Because:

= reduction in variance if X_i could be fixed.

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+$$

$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)=V(Y)$$

What variance would be left (on average) if X_i could be fixed

For additive systems one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)\approx V(Y)$$

How to get from first order to total order

From

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$ Main effect of factor X_i

replacing X_i with $\boldsymbol{X}_{\sim i}$

Gives the main effect of non-X_i

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

But again:

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)+$$
$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)=V\left(Y\right)$$

Main effect on non$-X_i$

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

… all remaining variance must be due to X_i and its interactions; this divided the total variance gives the total effect T_i

How to compute S_i and T_i ?

If k is the number of factors we need $n(2+k)$ simulation where n is the size of a ‘base’ matrix.

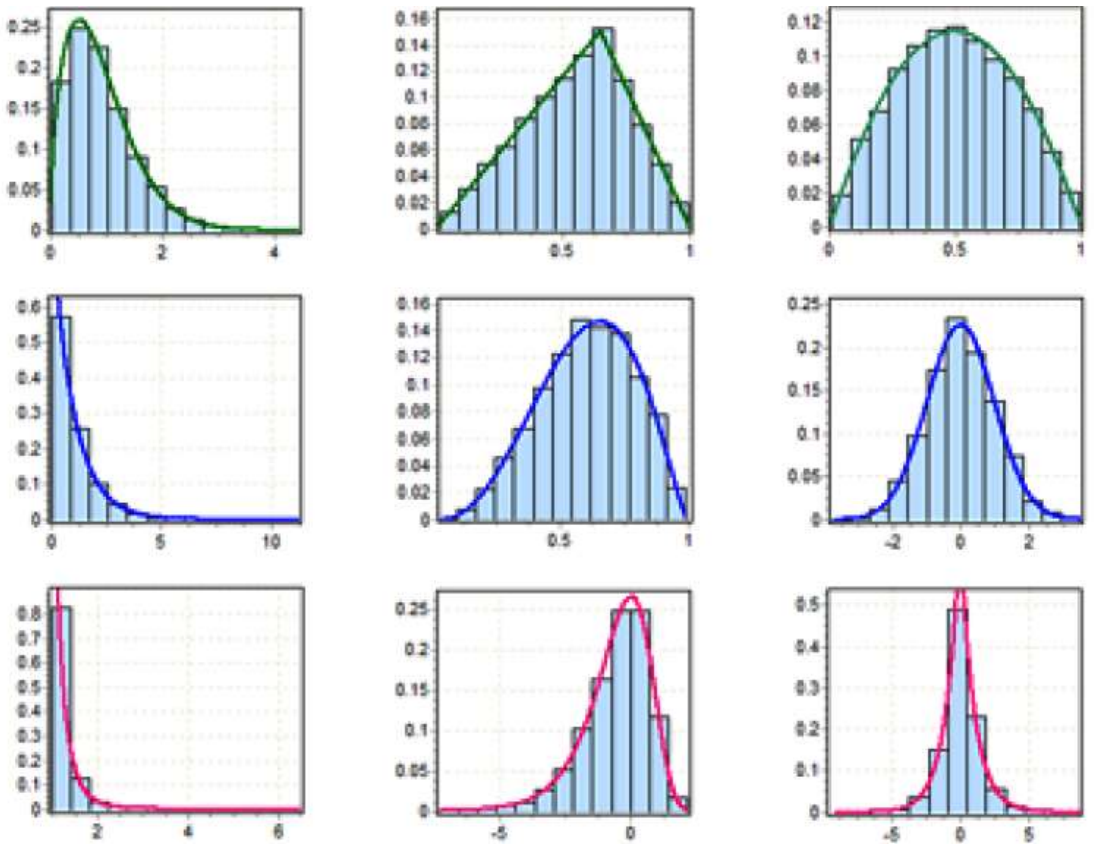
What is needed is a design (as in statistical DOE theory)

Building a Monte Carlo analysis

$$\begin{matrix} x_{11} & x_{12}\ldots & x_{1k} \\ x_{21} & x_{22}\ldots & x_{2k} \\ \ldots & \ldots & \ldots \\ x_{N1} & x_{N2} & x_{Nk1} \end{matrix}$$

Input matrix: each column is a sample of size N from the distribution of a factor

Each row is a sample trial of size k to generate a value of y



Examples of distributions of input factors

$$\mathbf{M}_1 = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2k} \\ \dots & & & \\ x_{n1} & x_{n2} & \dots & x_{nk} \end{pmatrix}$$

Base matrix M_1

$$\mathbf{M}_2 = \begin{pmatrix} x'_{11} & x'_{12} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x'_{2k} \\ \dots & & & \\ x'_{n1} & x'_{n2} & \dots & x'_{nk} \end{pmatrix}$$

Base matrix M_2

$$\mathbf{N}_j = \begin{pmatrix} x'_{11} & x'_{12} & \dots & x_{1j} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x_{2j} & \dots & x'_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x'_{n1} & x'_{n2} & \dots & x_{nj} & \dots & x'_{nk} \end{pmatrix}$$

k of these are needed

Matrix quasi-M_2 for factor j

'quasi' because all columns are from M_2 but column j from M_1

$$\mathbf{M}_1 = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2k} \\ \dots & & & \\ x_{n1} & x_{n2} & \dots & x_{nk} \end{pmatrix}$$

Base matrix M_1

$$\mathbf{M}_2 = \begin{pmatrix} x'_{11} & x'_{12} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x'_{2k} \\ \dots & & & \\ x'_{n1} & x'_{n2} & \dots & x'_{nk} \end{pmatrix}$$

Base matrix M_2

S_j

T_j

$$\mathbf{N}_j = \begin{pmatrix} x'_{11} & x'_{12} & \dots & x_{1j} & \dots & x'_{1k} \\ x'_{21} & x'_{22} & \dots & x_{2j} & \dots & x'_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x'_{n1} & x'_{n2} & \dots & x_{nj} & \dots & x'_{nk} \end{pmatrix}$$

k of these are needed

Matrix quasi-M_2 for factor j

The measures and their ‘settings’ = when to use them

Journal of the American Statistical Association ›
Volume 97, 2002 - Issue 459

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Primary Article

On the Relative Importance of Input Factors in Mathematical Models

Safety Assessment for Nuclear Waste Disposal

Andrea Saltelli & Stefano Tarantola

Pages 702-709 | Published online: 31 Dec 2011

Download citation https://doi.org/10.1198/016214502388618447

The measures and their ‘settings’ = when to use them

First order effect	Factor prioritization (orienting research)
Total effect	Factor fixing (model simplification)

ELSEVIER

Computer Physics Communications

Volume 145, Issue 2, 15 May 2002, Pages 280-297

Making best use of model evaluations to compute sensitivity indices

Andrea Saltelli ✉ ⊕

Computing the indices efficiently

Higher order Sobol' indices

Get access >

Art B. Owen ✉, Josef Dick, Su Chen

Information and Inference: A Journal of the IMA, Volume 3, Issue 1, March 2014, Pages 59–81, https://doi.org/10.1093/imaiai/iau001

Published: 01 March 2014 **Article history** ▾

Plenty of code available in R, MATLAB, and Phyton

https://cran.r-project.org/web/packages/sensitivity/sensitivity.pdf

https://cran.rstudio.com/web/packages/sensobol/index.html

https://www.uqlab.com/ (in MatLab, by Bruno Sudret and his team)

SALib https://salib.readthedocs.io/en/latest/

…but there is more, in R, Phython, Giulia …

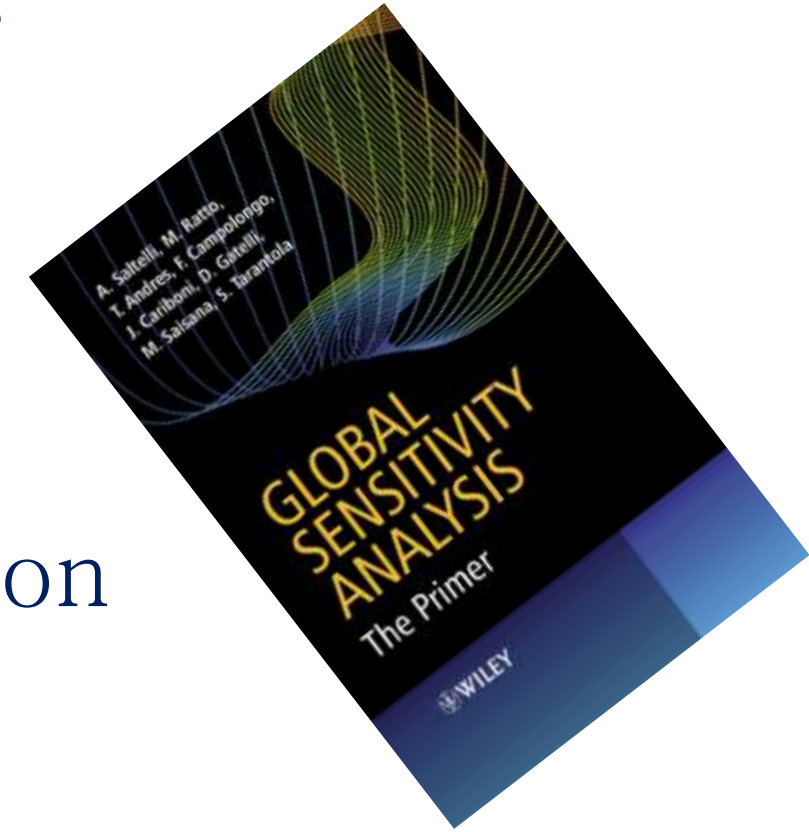
Journal of Statistical Software

sensobol: An R Package to Compute Variance-Based Sensitivity Indices

Arnald Puy, **Samuele Lo Piano**, **Andrea Saltelli**, **Simon A. Levin**

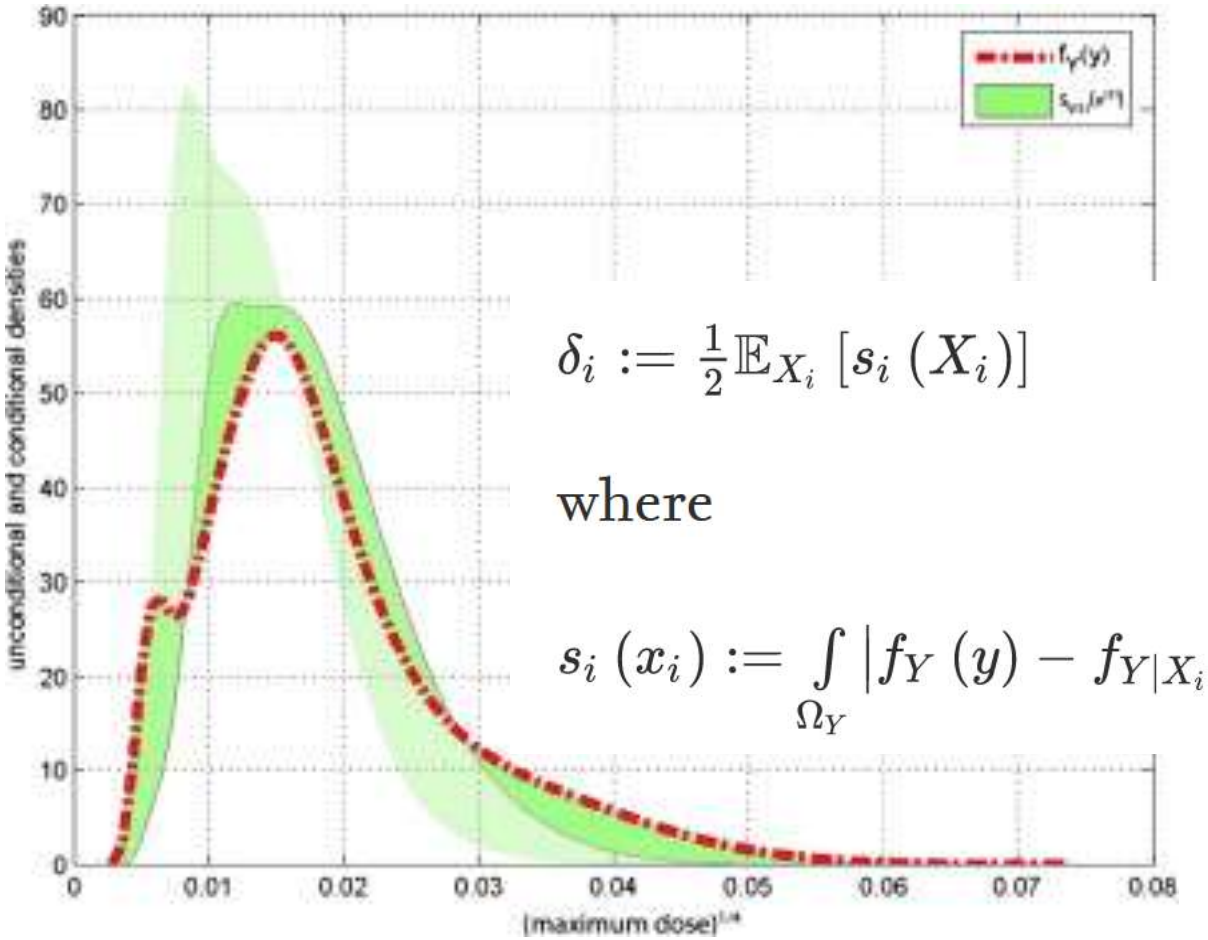
Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation (ANOVA)
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization
- give the effective dimension



Chapter 1 and its exercises

… but there are other methods that can be used for different settings, e.g. moment independents methods, Shapley coefficients, reduced spaces, VARS …



$$\delta_i := \frac{1}{2}\mathbb{E}_{X_i}\left[s_i\left(X_i\right)\right]$$

where

$$s_i\left(x_i\right) := \int_{\Omega_Y} \left| f_Y\left(y\right) - f_{Y|X_i=x_i}\left(y\right) \right| dy$$

ELSEVIER

Environmental Modelling & Software

Volume 34, June 2012, Pages 105-115

Model emulation and moment-independent sensitivity analysis: An application to environmental modelling

E. Borgonovo [a], W. Castaings [b, c], S. Tarantola [d]

Don’t use One factor At a Time (OAT)

A geometric proof

Environmental Modelling & Software 25 (2010) 1508–1517

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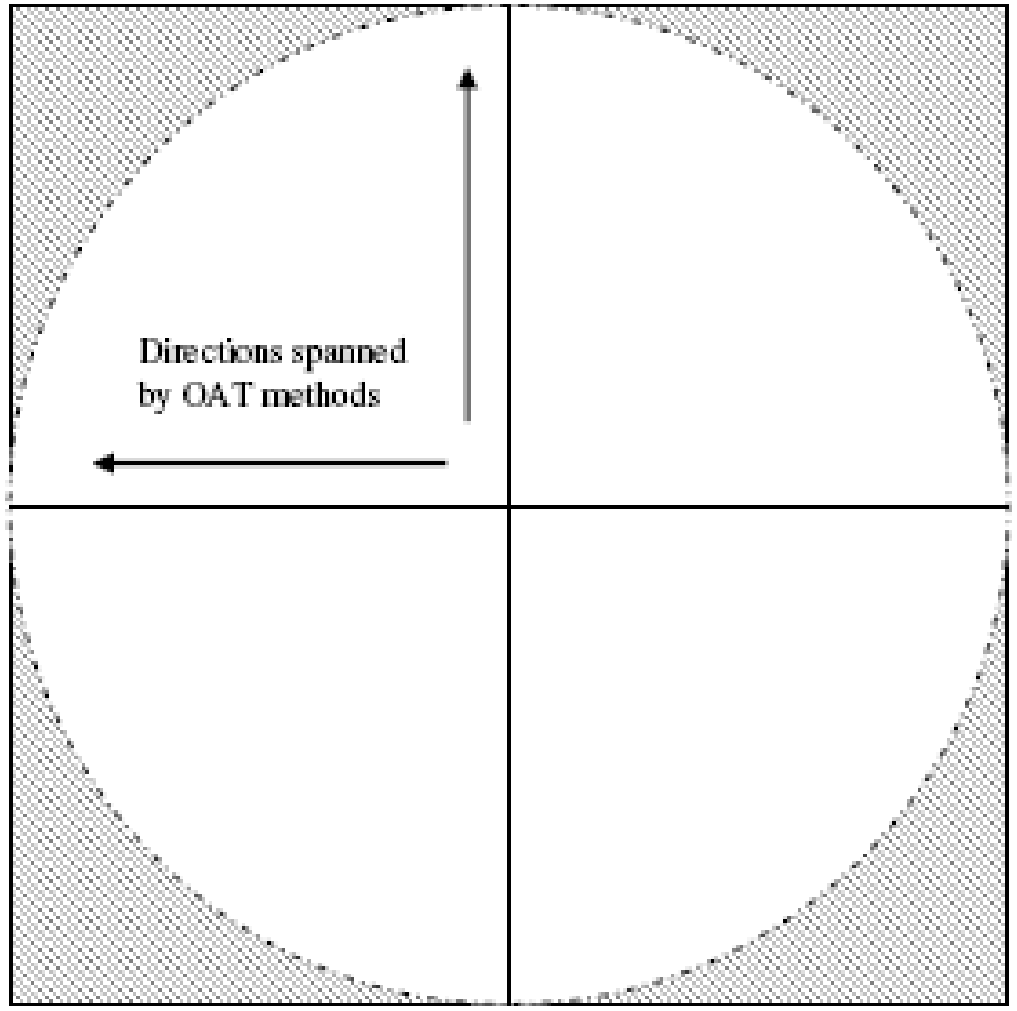
journal homepage: www.elsevier.com/locate/envsoft

How to avoid a perfunctory sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy

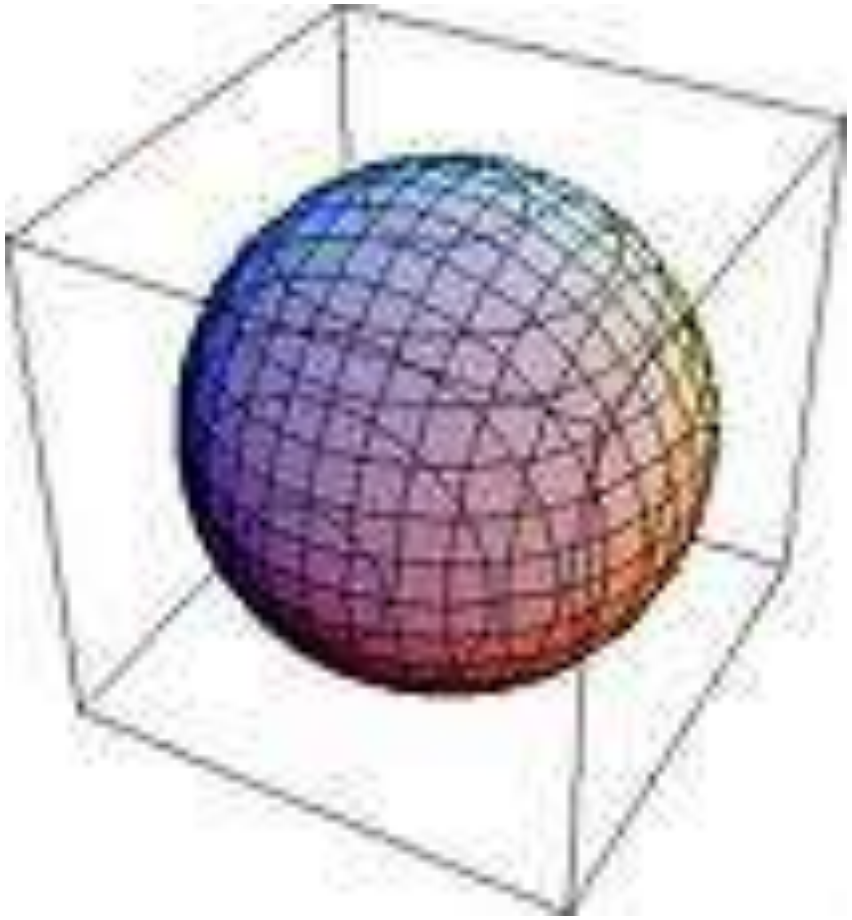
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions

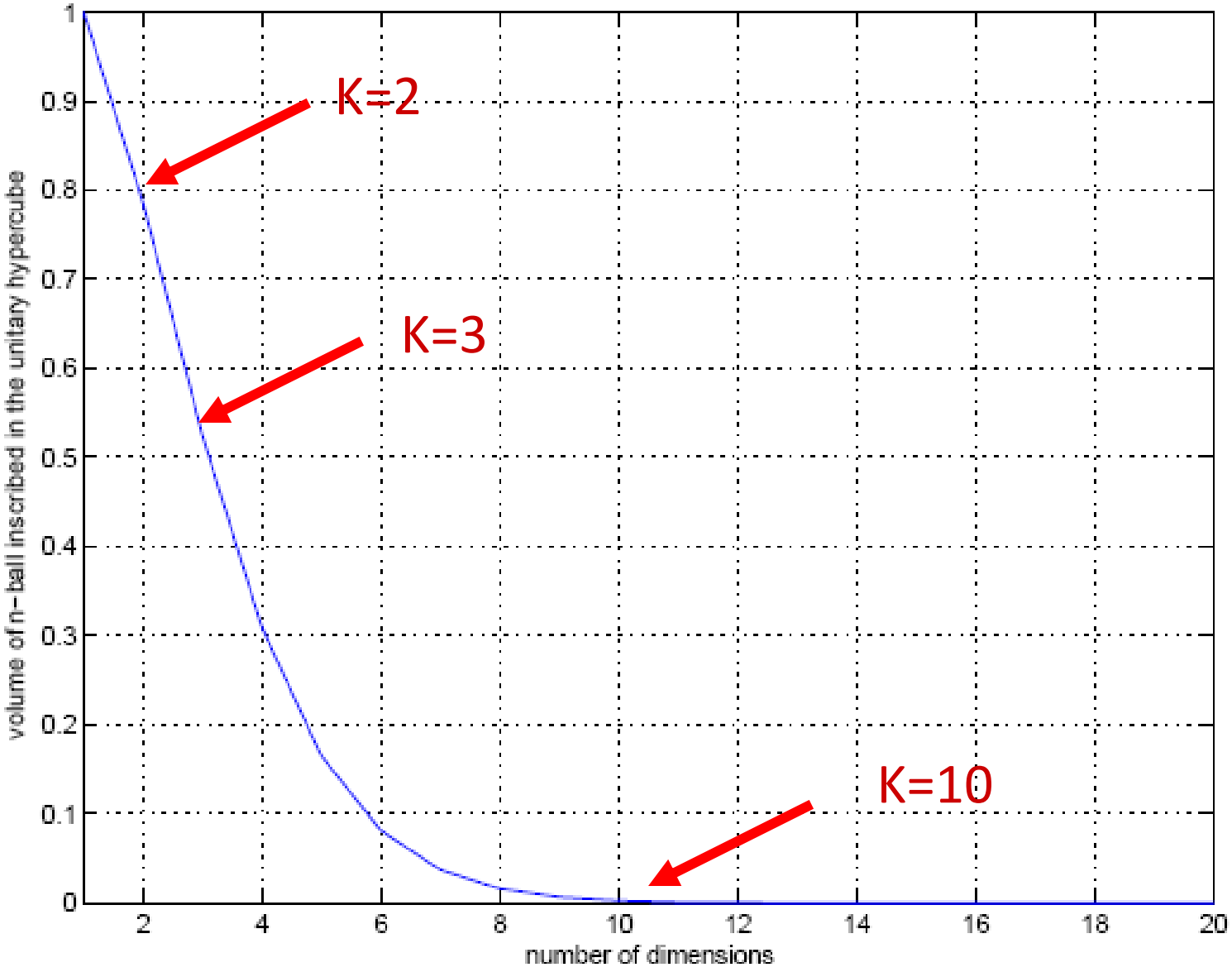


Volume sphere / volume cube =?

~ 1/2

OAT in 10 dimensions; Volume hypersphere / volume ten dimensional hypercube =? ~ 0.0025

OAT in k dimensions

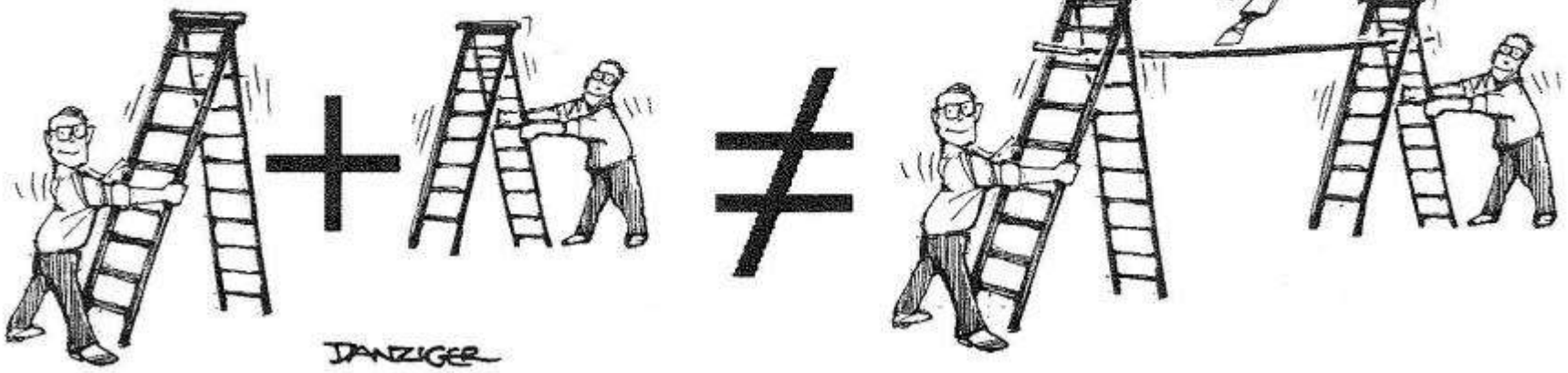


OAT does not capture interactions

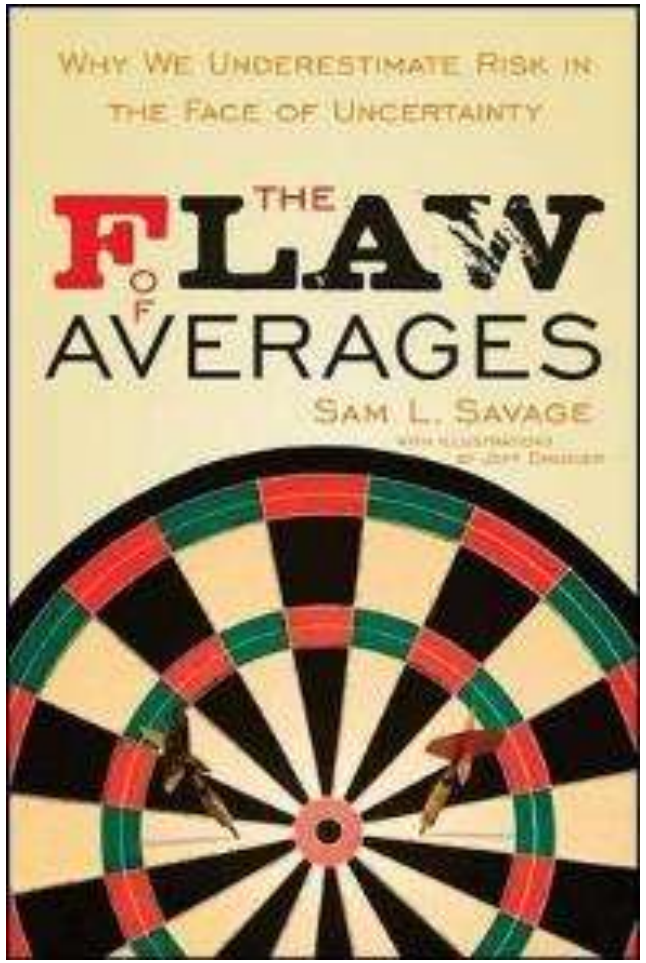
➔ The resulting analysis is non conservative

How would you test the scaffolding?

How coupled ladders are shaken in most of available literature

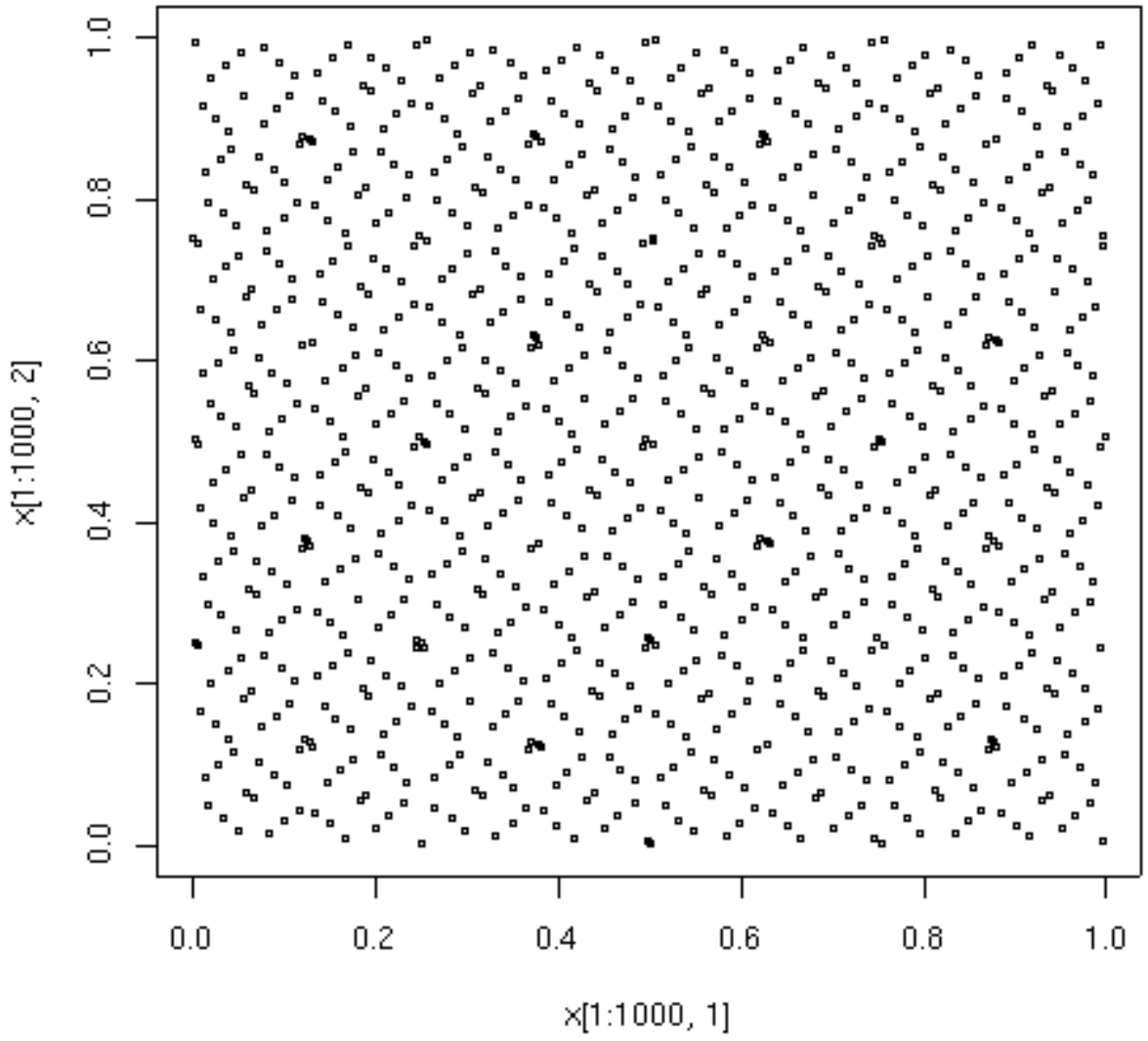


How to shake coupled ladders



Quasi random sequences

Ilya M. Sobol’



Cornell University

the

arXiv.org > stat > arXiv:1505.02350

Search...

Help | Adva

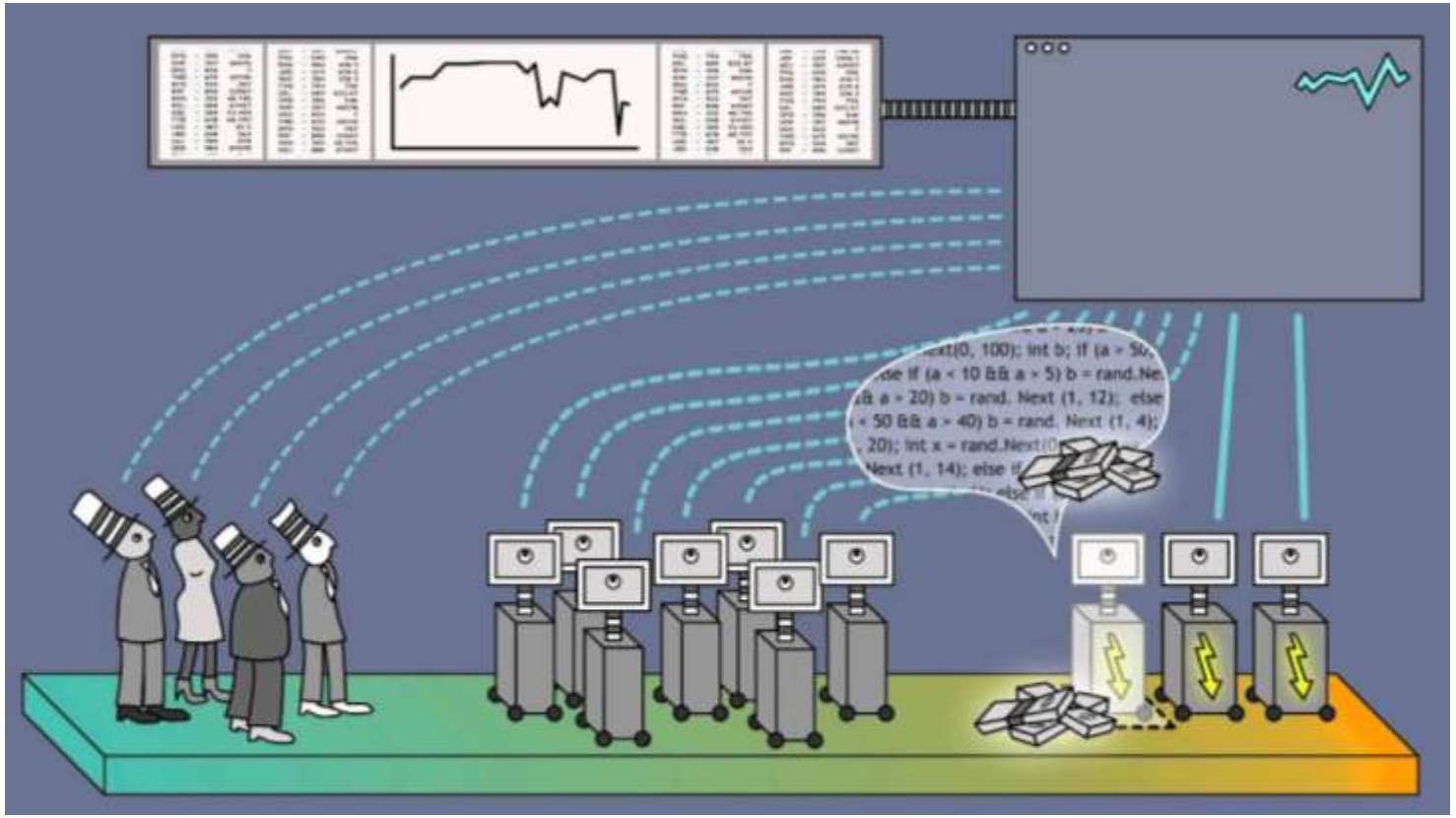
Statistics > Applications

[Submitted on 10 May 2015]

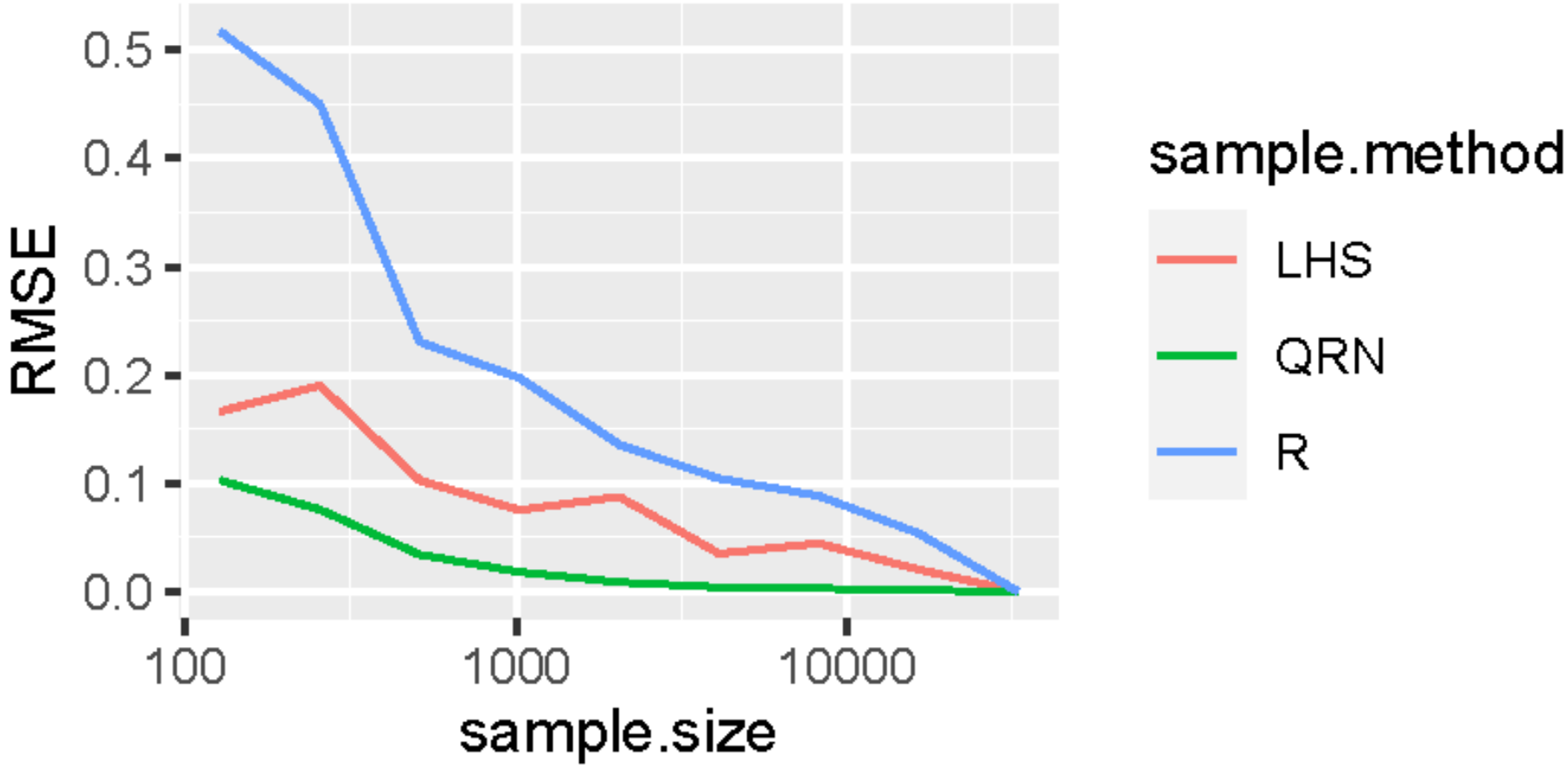
Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli

Sobol' LP-TAU are used in high frequency trading



Source: https://www.youtube.com/watch?v=z4nCTdQlH8w



Root mean square error with different designs

Don’t run the model just once

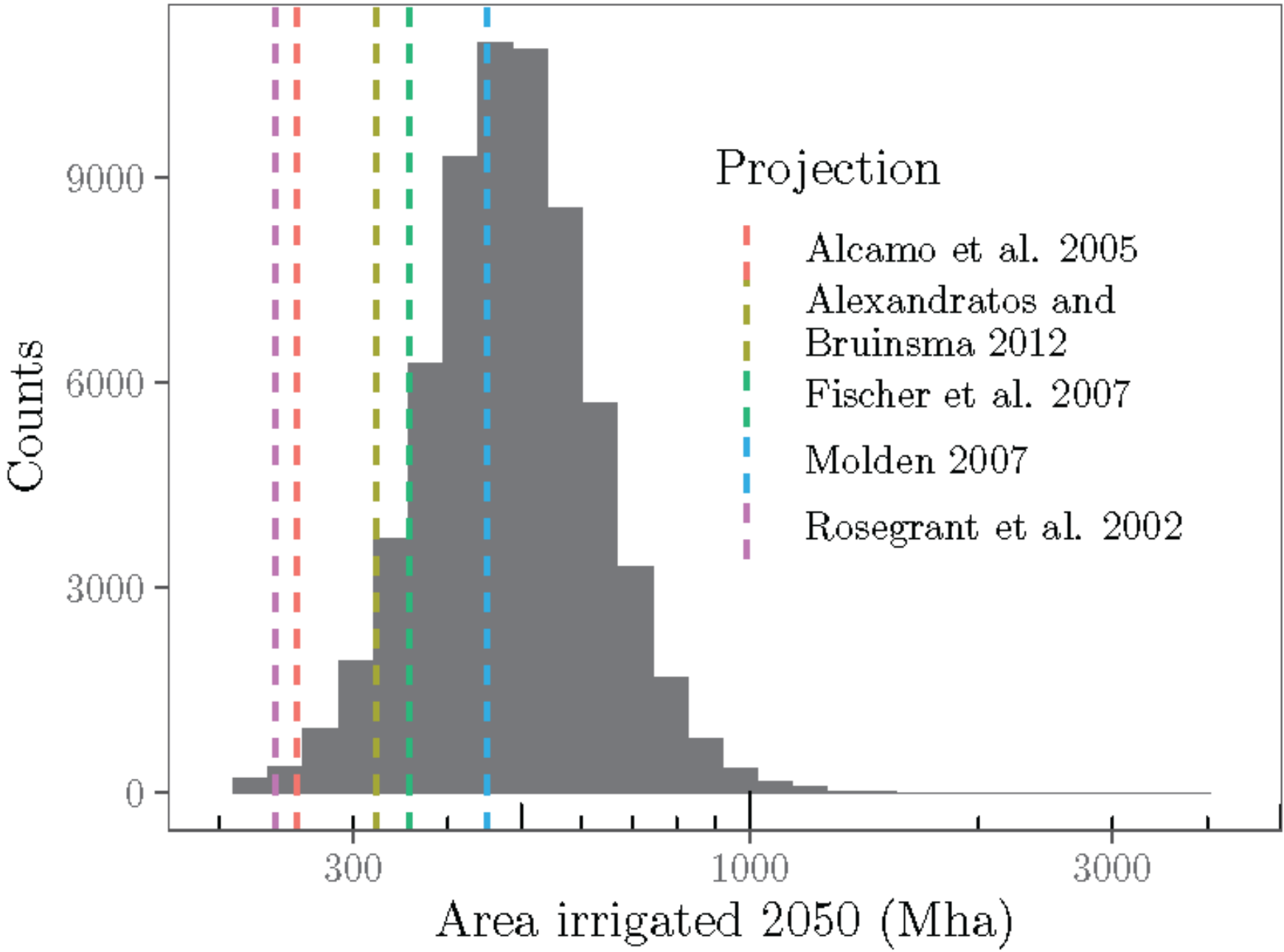
There is much to learn by running the model a few times, especially during model building

Lubarsky's Law of Cybernetic Entomology: there is always one more bug!

Model routinely used to produce point estimates may becomes non conservative when the uncertainty is plugged in

Current Models Underestimate Future Irrigated Areas

- How much land will need to be irrigated by the year 2050?
- Here the dashed lines represent deterministic model predictions from different models and datasets (from FAO & others organizations);
- An uncertainty analysis (grey histogram) reveals that the models are non-conservative: the need might be much larger



Citation:
Puy, A., Lo Piano, S., & Saltelli, A. (2020). Current models underestimate future irrigated areas. *Geophysical Research Letters*, 47, e2020GL087360. https://doi.org/10.1029/2020GL087360

Don’t sample just parameters and boundary conditions

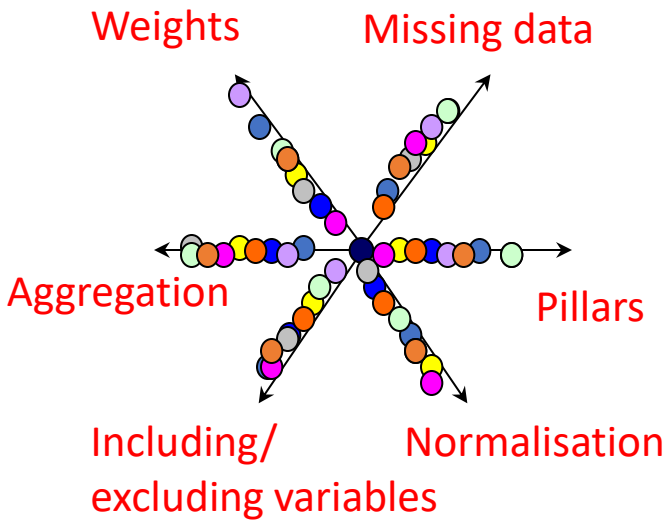
Explore thoroughly the space of the assumptions

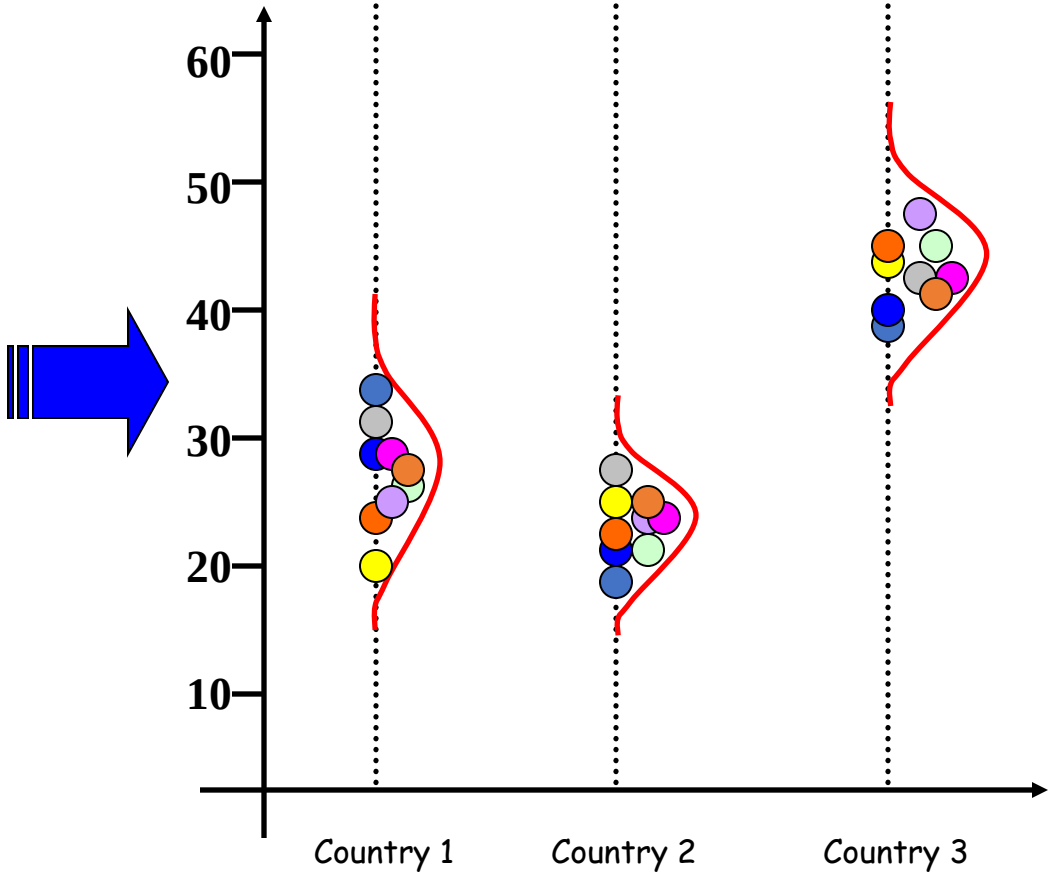
One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios …

Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

Space of alternatives





NEVER vary all factors of the same amount

Be it 5%, 10%, or 20%

New WHO estimates: Up to 190 000 people could die of COVID-19 in Africa if not controlled

07 May 2020

Brazzaville – Eighty-three thousand to 190 000 people in Africa could die of COVID-19 and 29 million to 44 million could get infected in the first year of the pandemic if containment measures fail, a new study by the World Health Organization (WHO) Regional Office for Africa finds. The research, which is based on prediction modelling, looks at 47 countries in the

Speculative scenario in which ten uncertain input probabilities are increased by an arbitrary 10% — as if they were truly equally uncertain — with no theoretical or empirical basis for such a choice

World Health Organization
REGIONAL OFFICE FOR Africa

In a numerical experiment relating to a real-life application the range of uncertainty of each input is crucial input to the analysis, and often the most expensive to get

Application to ML

Example use SA to ascertain that an algorithm does not make implicit use of protected attributes

More reading for the Machine Learning students

Vannucci, G., Siciliano, R., & Saltelli, A. (2024). Enhancing Variable Importance in Random Forests: A Novel Application of Global Sensitivity Analysis (arXiv:2407.14194). arXiv. http://arxiv.org/abs/2407.14194.

Bénesse, C., Gamboa, F., Loubes, J.-M., & Boissin, T. (2022). Fairness seen as Global Sensitivity Analysis. *Machine Learning*. https://doi.org/10.1007/s10994-022-06202-y

PROTECTED ATTRIBUTES:

- Age
- Disability
- National Origin
- Race/color
- Religion
- Sex
- (From the US Equal Opportunity Employment Commission)

The End

Announcements:

- New Special Issue on Risk Analysis on **sensitivity analysis** : https://www.sra.org/wp-content/uploads/2025/05/Call-for-Papers-Risk-Analysis-Sensitivity-Analysis.pdf, deadline October 2025

- Summer school on sensitivity analysis (one week) free registration beginning June 2025 Venice, check pages of Joint Research Centre