

Workshop on Modelling, part 1 (WP2 - Robust 4D human driver models under uncertainty):



27-06-23



i4Driving

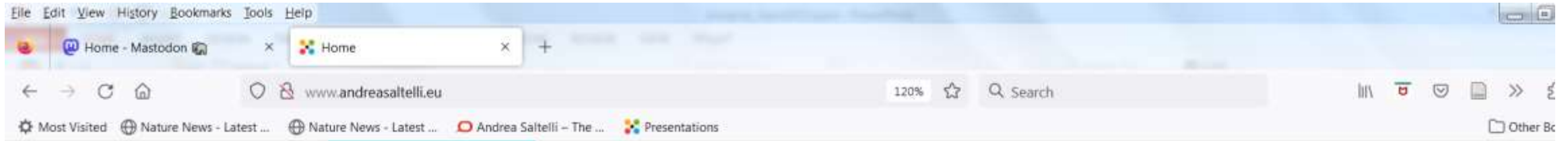
integrated 4D driver modelling under uncertainty



Sensitivity analysis made easy

Andrea Saltelli, CNR

Where to find this talk: www.andreasaltelli.eu



Andrea
Saltelli

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Coming Out Soon: The politics of modelling



Praise for the volume

"A long awaited examination of the role —and obligation —of modeling."

Nassim Nicholas Taleb, Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series *Incerto*.

"A breath of fresh air and a much needed

Mastodon Toots by

@AndreaSaltelli



AndreaSaltelli

2023/4/18 17:24

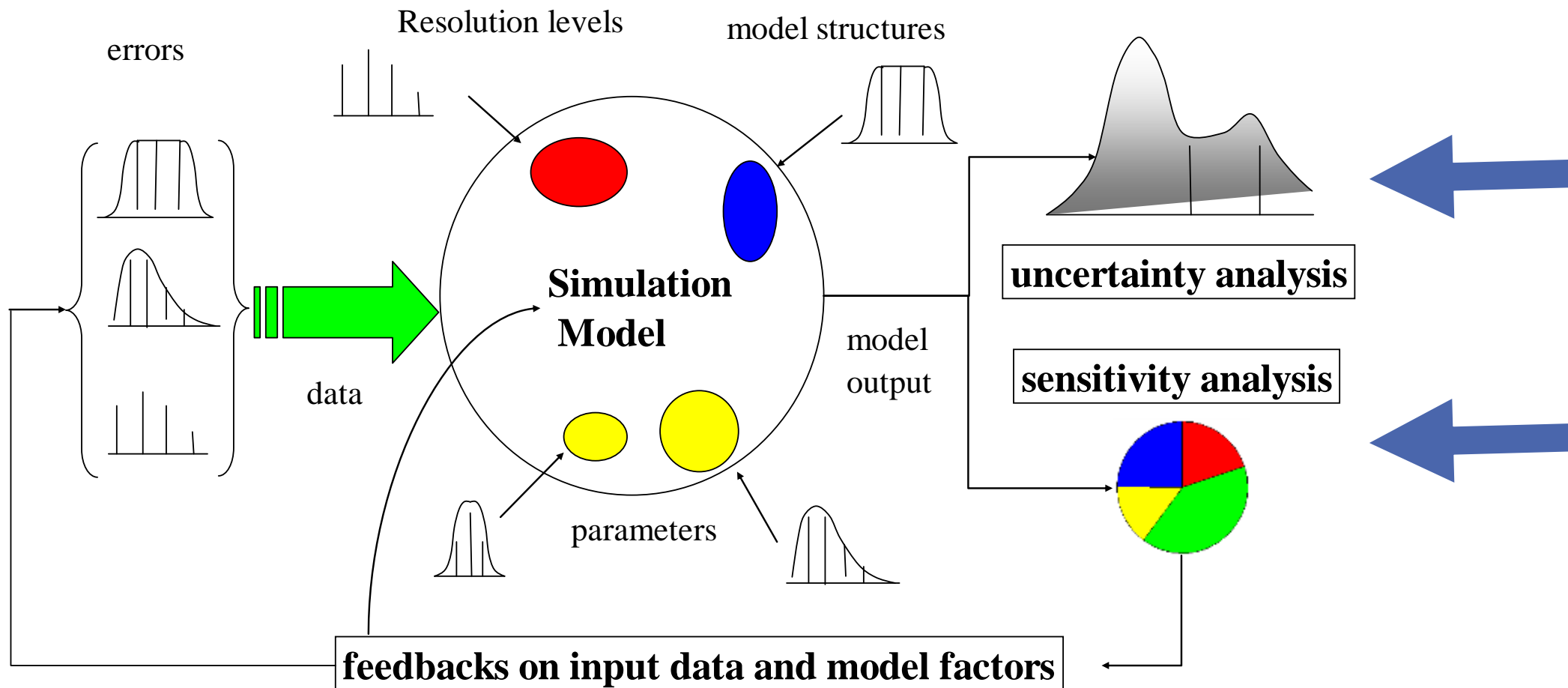
"Kaufman, who was until recently a senior climate economist in the White House, questioned the need for the government to set a single price. Regulators should stop "pretending we

lion

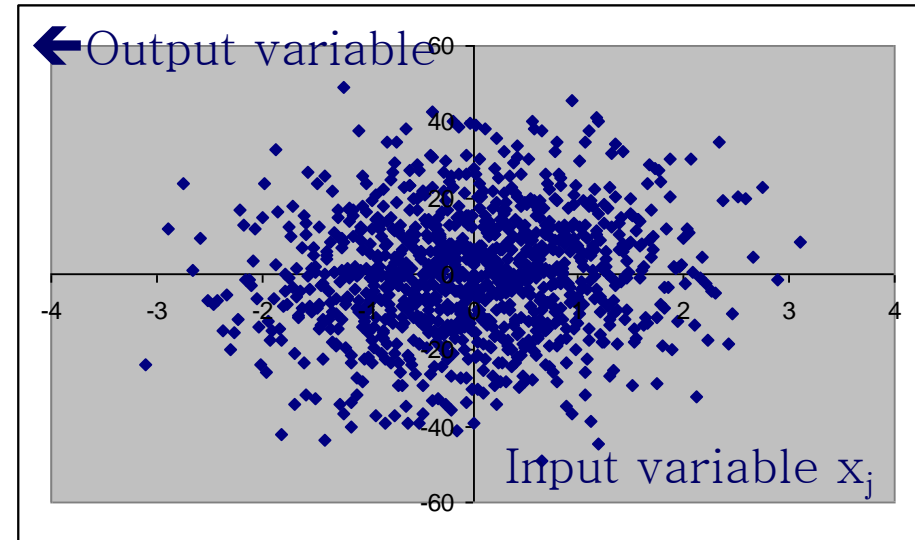
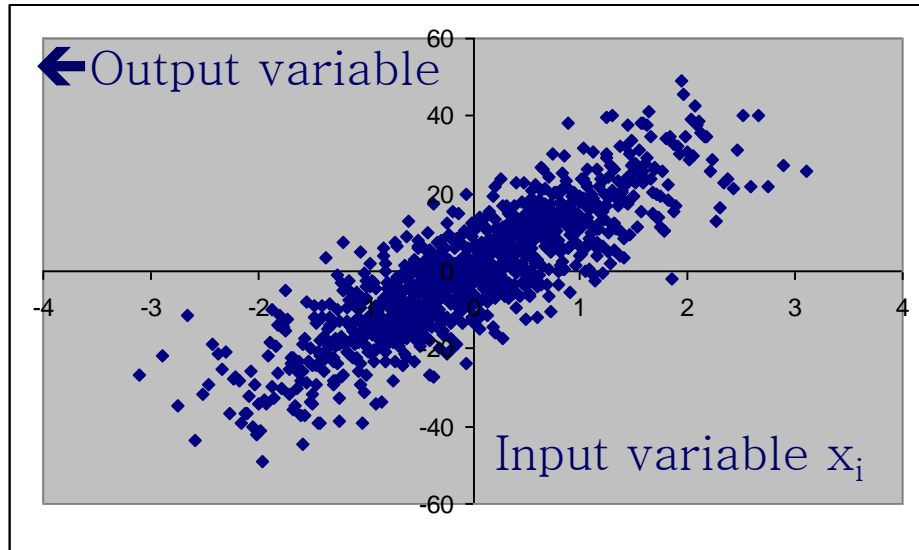
Uncertainty analysis: the study of the uncertainty in model output—see also uncertainty cascade

Sensitivity analysis: the study of the relative importance of different input factors on the model output

Sensitivity auditing : “Sensitivity auditing is a wider consideration of the effect of all types of uncertainty, including structural assumptions embedded in the model, and subjective decisions taken in the framing of the problem” (European Commission, 2021).

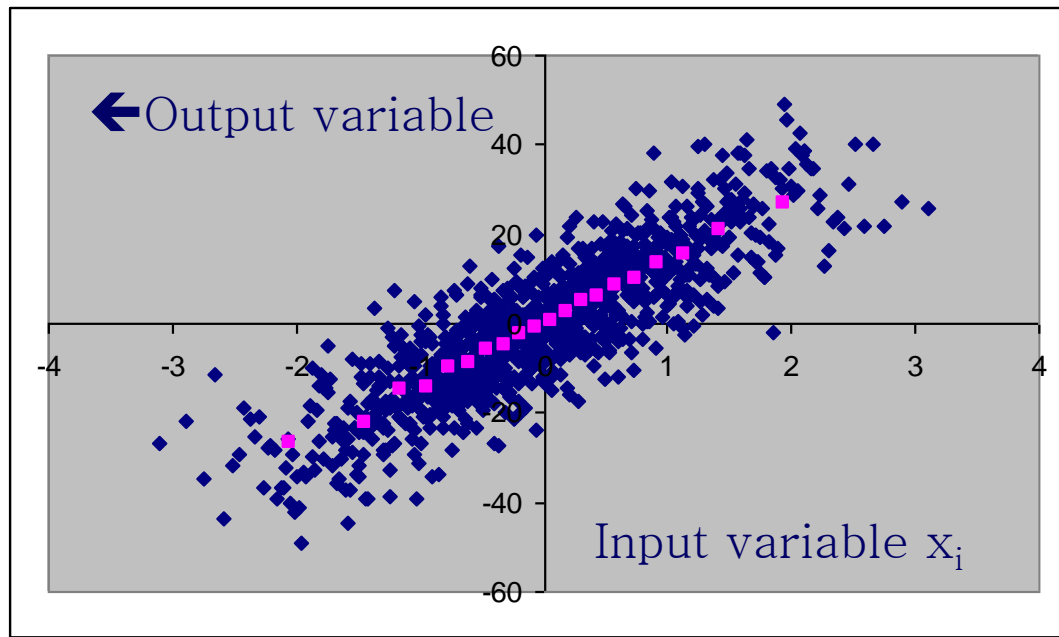


An introduction to variance based methods



Plotting the output as a function of two different input factors

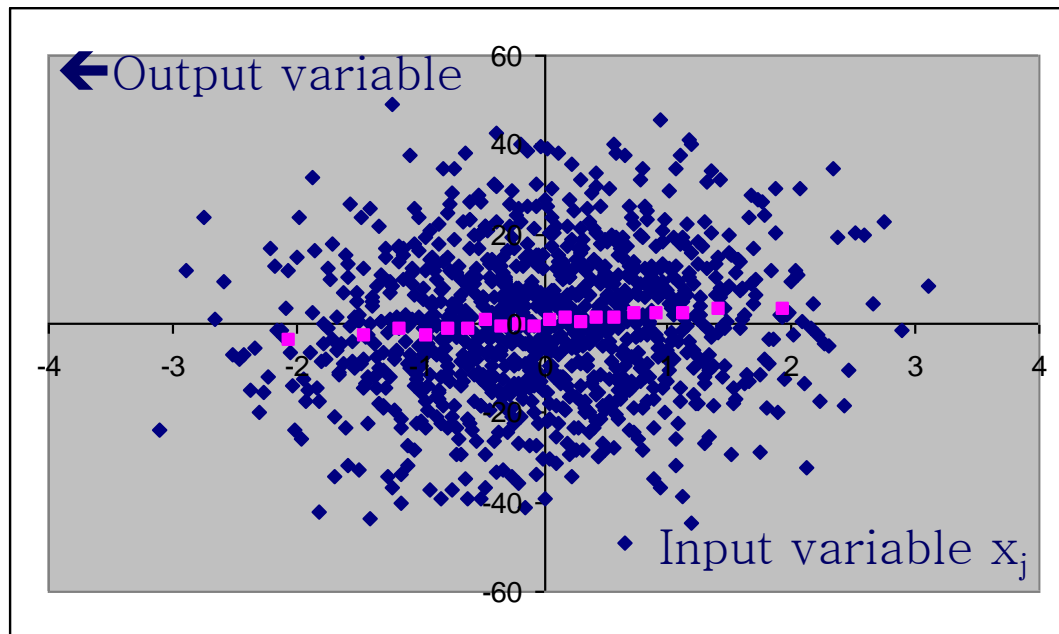
Which factor is more important?

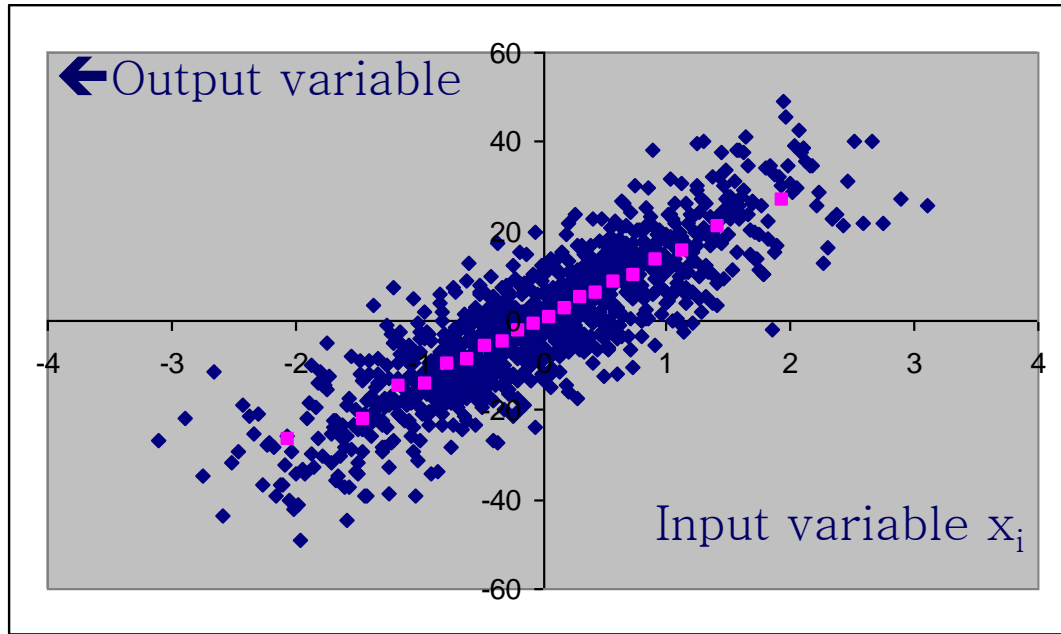


~1,000 blue points

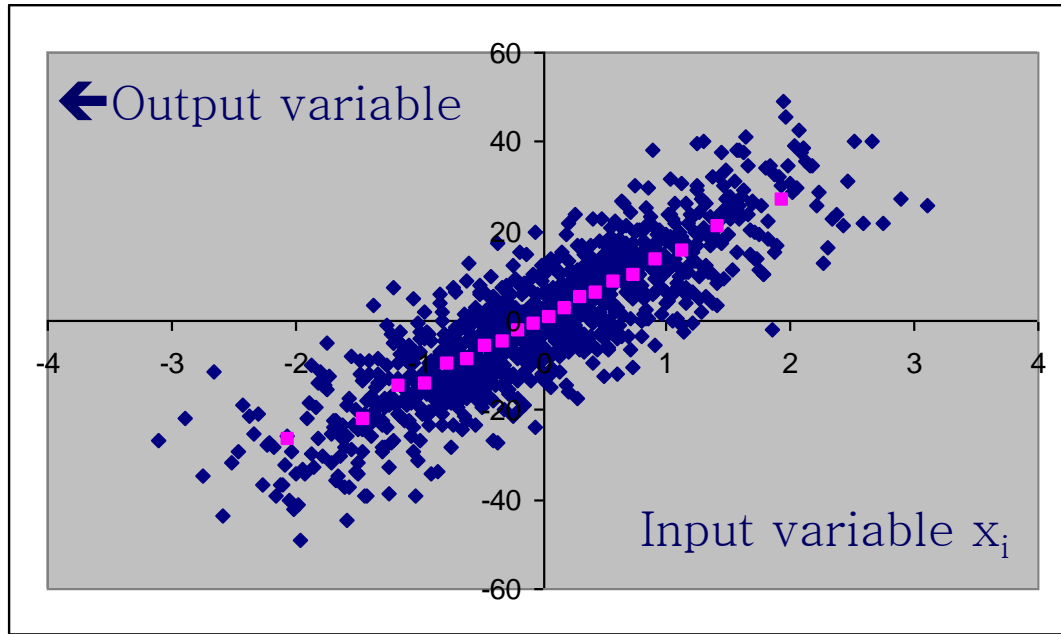
Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



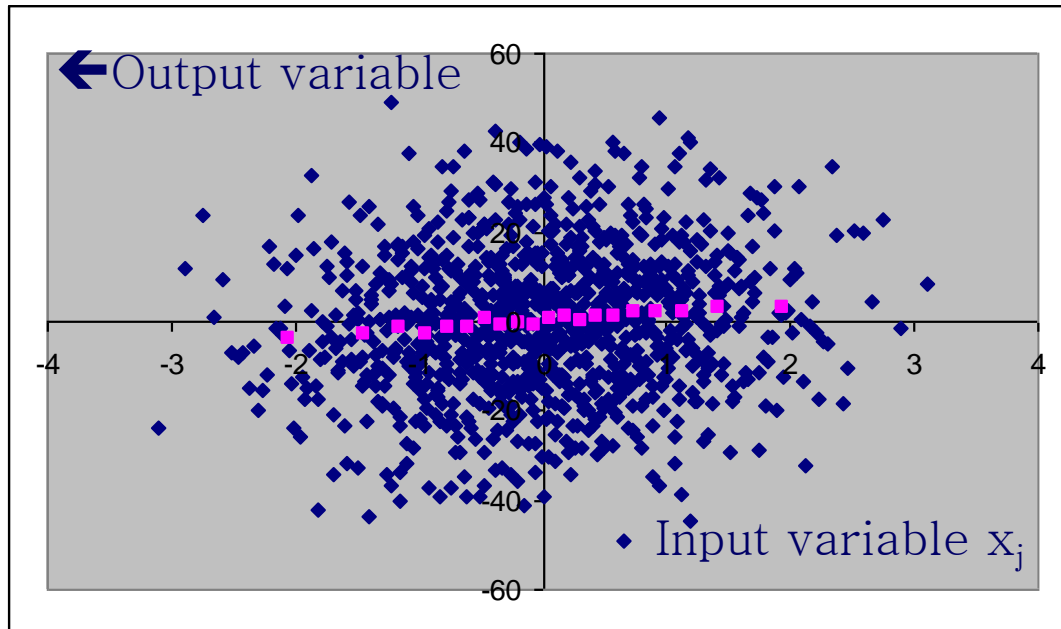
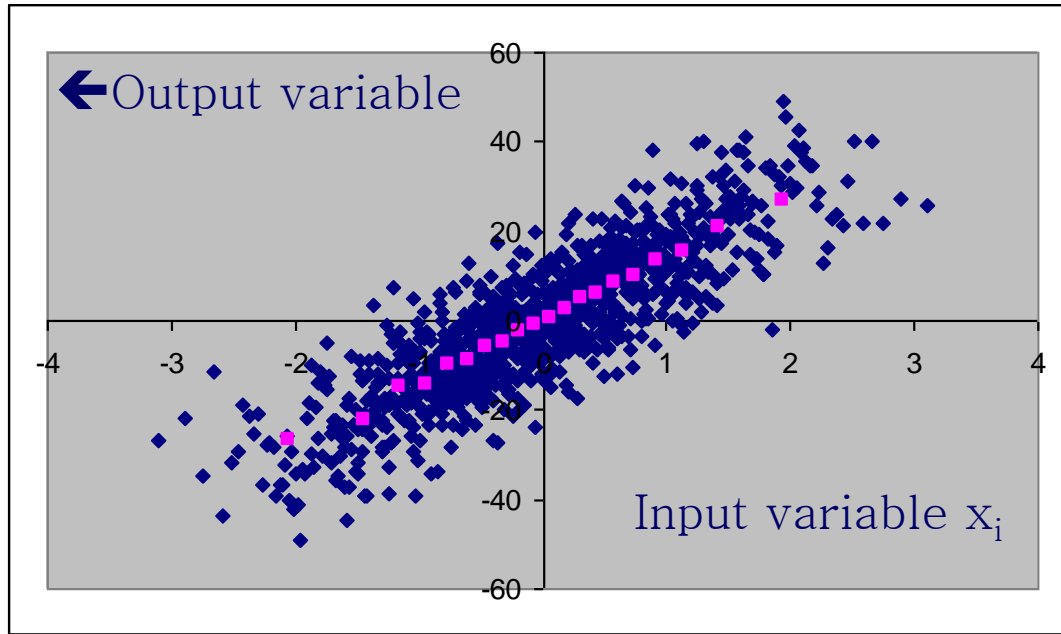


Each pink point is $\sim E_{\mathbf{X}_{\sim i}}(Y|X_i)$



Taking the variance of the pink points one obtains a sensitivity measure

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

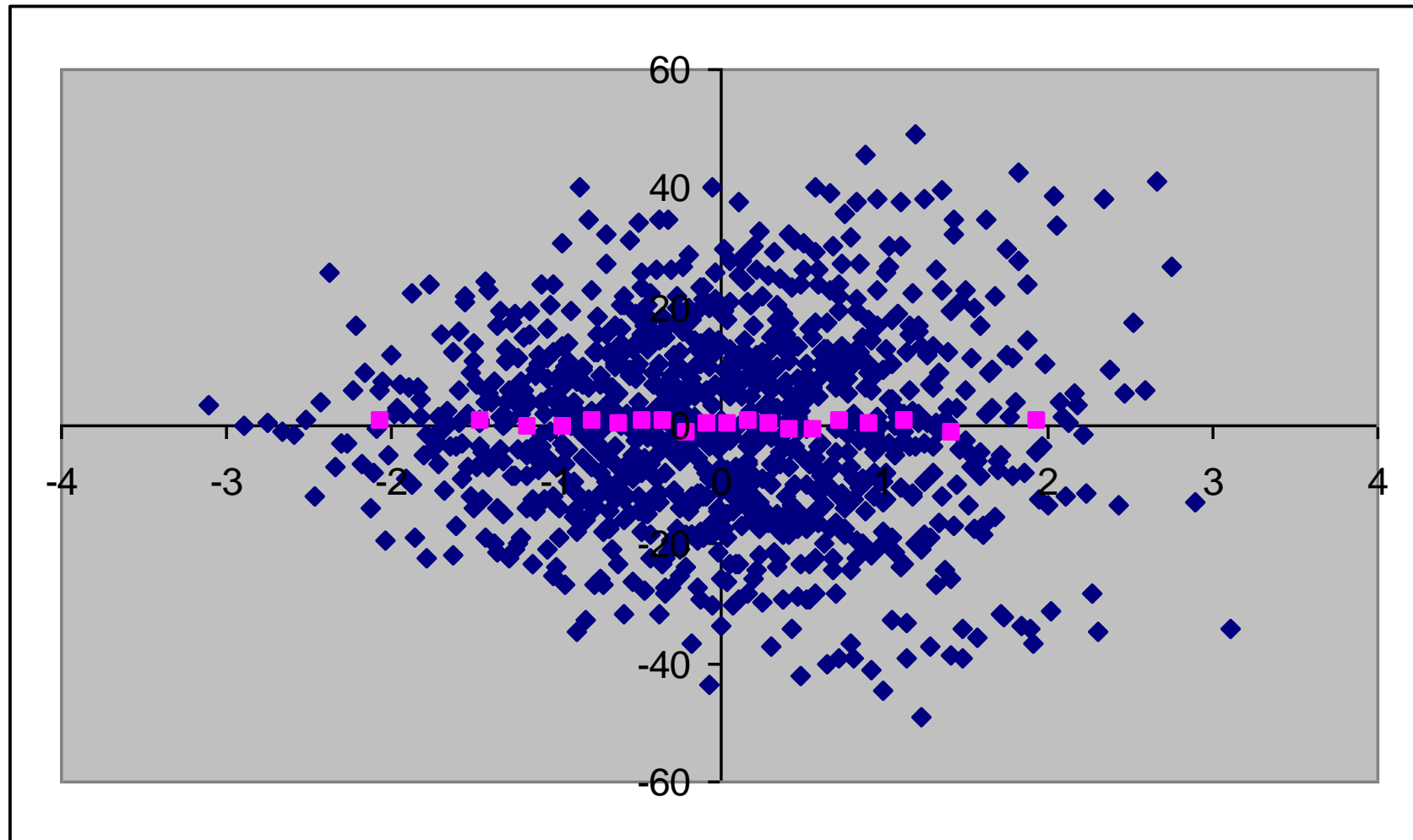


Which factor
has the highest
 $V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$?

$$S_i = \frac{V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order

Is this factor non-important?



For cases where S_i is zero but the variable is still important we need to compute something else

There are terms which capture
two-way, three way, \dots interactions
among variables

All these terms are linked by a
formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i,j>i} V_{ij} + \dots + V_{123\dots k}$$

Variance decomposition (ANOVA)

The total variance can be decomposed into main effects and interaction effects up to the order k , the dimensionality of the problem (only for independent factors)

If fact interactions terms are awkward to handle: **just the second order terms** for a model with k factors are as many as $k(k-1)/2 \dots$

(10 factors=45 second order terms)

How about a single 'importance' terms for all effects?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a model $f(X_1, X_2, \dots, X_3)$

Where the variance decomposition would

$$\text{read } 1 = S_1 + S_2 + S_3 + S_{12} + S_{13} + S_{23} + S_{123}$$

We compute

$$T_1 = S_1 + S_{12} + S_{13} + S_{123}$$

$$T_2 = S_2 + S_{12} + S_{23} + S_{123}$$

$$T_3 = S_3 + S_{13} + S_{23} + S_{123}$$

$$\frac{V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)} = S_i$$

$$\frac{E_{\mathbf{X}_{\sim i}} \left(V_{X_i} (Y | \mathbf{X}_{\sim i}) \right)}{V(Y)} = T_i$$

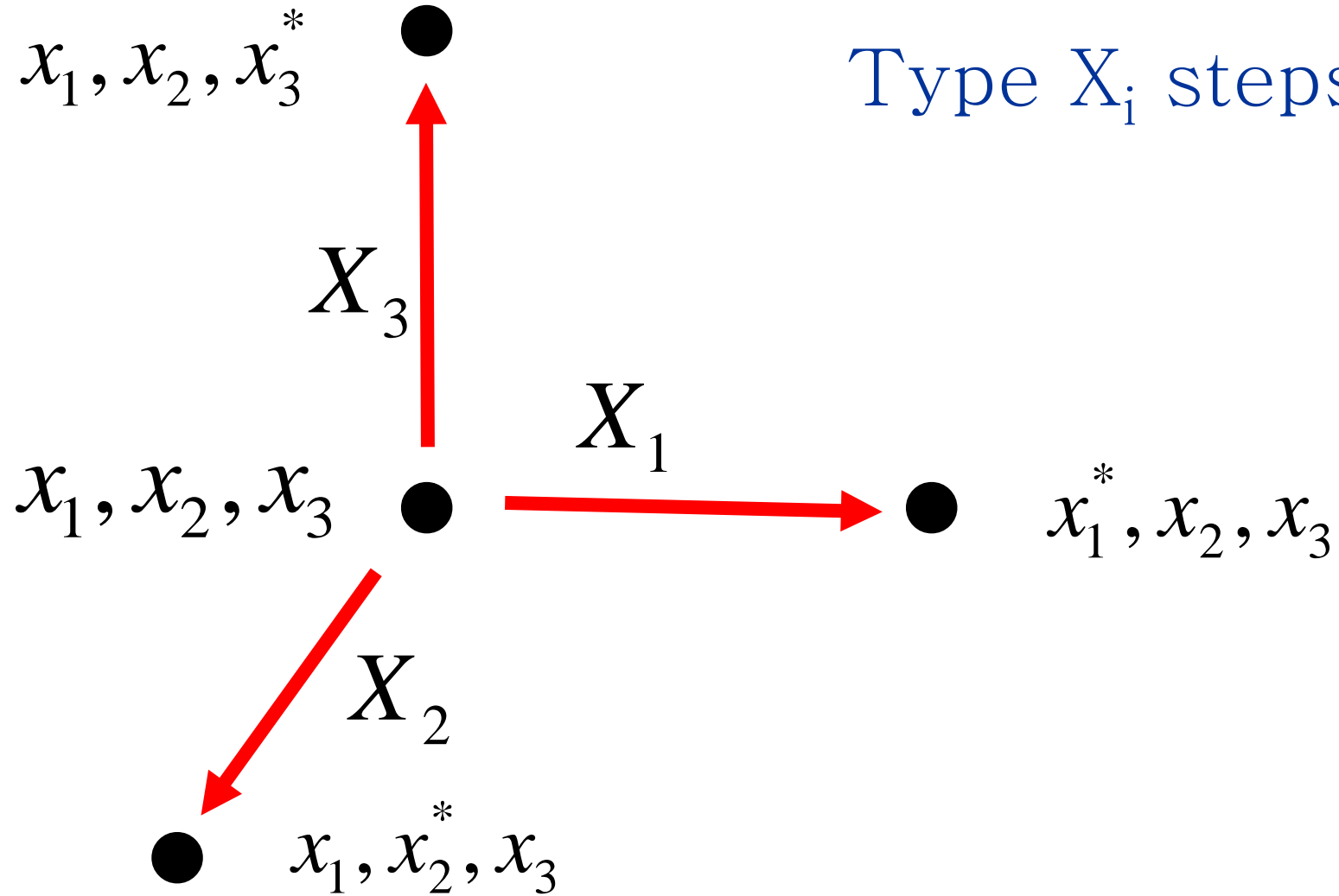
The formulae!

Estimation procedures

- No brute force. It is not needed to use a double loop, though the measures are expressed as $V(E(\cdot))$ and $E(V(\cdot))$.
- For S_i quick estimation procedures are available which are k -independent.
- For S_{Ti} estimation procedures are mostly k -dependent (unless using emulators...).

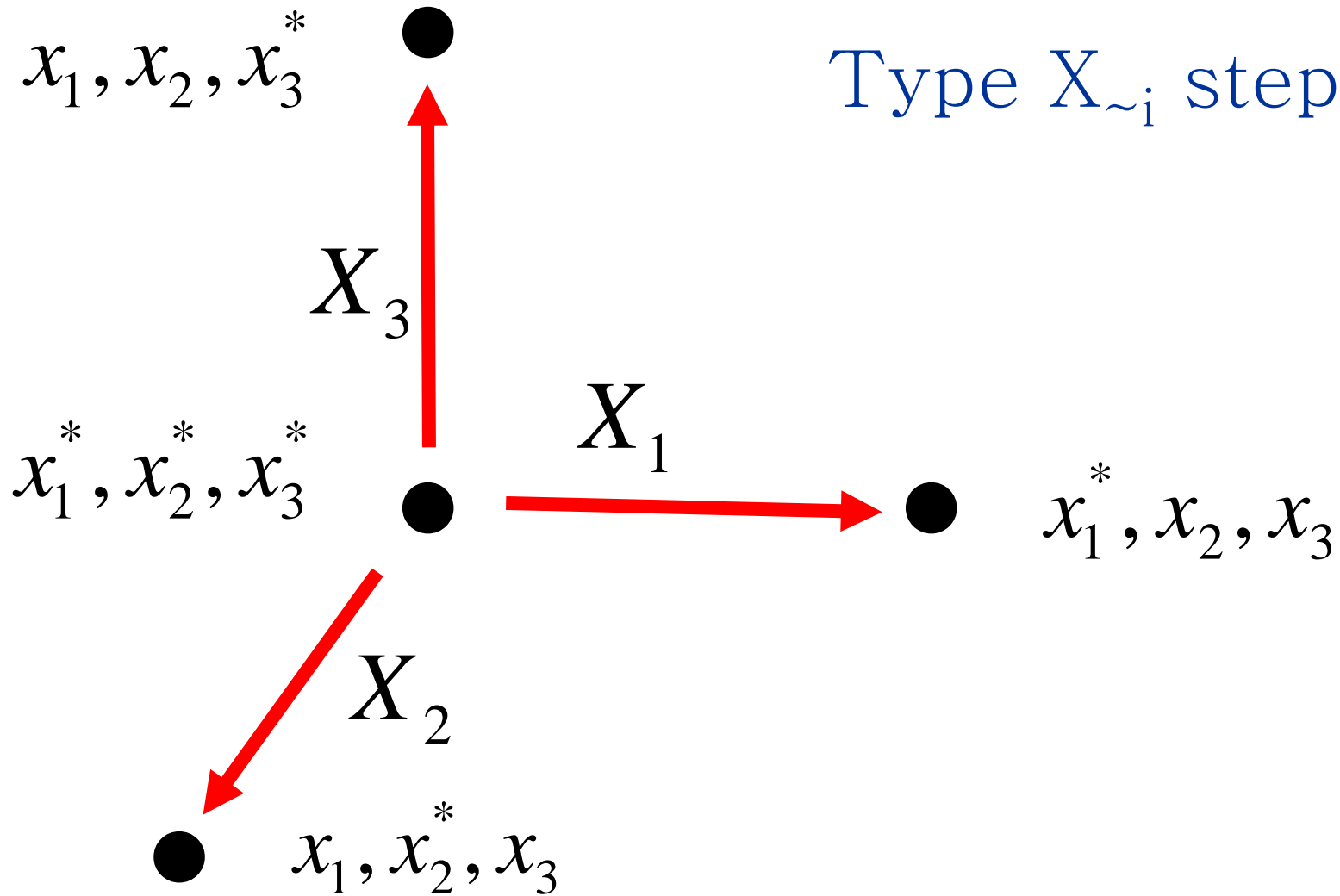
Computing S_{Ti}

Type X_i steps



Computing S_i

Type $X_{\sim i}$ steps



The measures and their 'settings' = when to use them



Journal of the American Statistical Association >

Volume 97, 2002 - Issue 459

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979

Views

286

CrossRef citations
to date

6

Altmetric

Primary Article

On the Relative Importance of Input Factors in Mathematical Models

Safety Assessment for Nuclear Waste Disposal

Andrea Saltelli & Stefano Tarantola

Pages 702-709 | Published online: 31 Dec 2011

Download citation <https://doi.org/10.1198/016214502388618447>



The measures and their ‘settings’ = when to use them

| | |
|--------------------|---|
| First order effect | Factor prioritization (orienting research) |
| Total effect | Factor fixing (model simplification) |

A short comment on statistical versus mathematical modelling

Andrea Saltelli

Model error

Model inadequacy error

Model error

Propagation error

Model complexity

led by
European Union

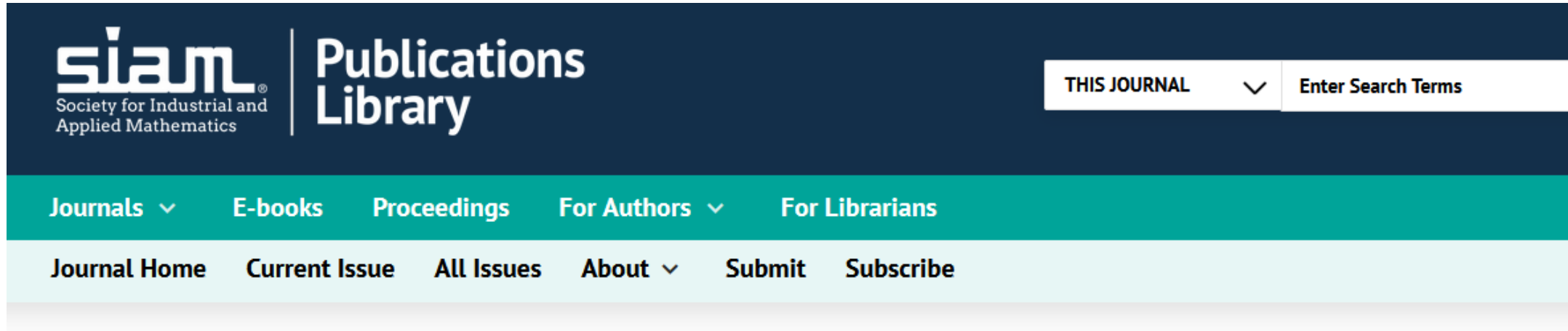
Model's effective dimension

The difficulty of a function/model is not in its number of dimensions but in the number of effective dimensions, either in the **truncation** or **superposition** sense

truncation sense = how many factors are important?

superposition sense=how high is the highest interaction?

Or you can compute the mean dimension directly



The screenshot shows the top navigation area of the SIAM Publications Library website. On the left is the SIAM logo (Society for Industrial and Applied Mathematics) and the text "Publications Library". On the right is a search bar with a dropdown menu currently set to "THIS JOURNAL" and a text input field labeled "Enter Search Terms". Below this is a teal navigation bar with links for "Journals", "E-books", "Proceedings", "For Authors", and "For Librarians". A light blue bar below contains links for "Journal Home", "Current Issue", "All Issues", "About", "Submit", and "Subscribe".

Home → SIAM/ASA Journal on Uncertainty Quantification → Vol. 9, Iss. 2 (2021) → 10.1137/20M1350236

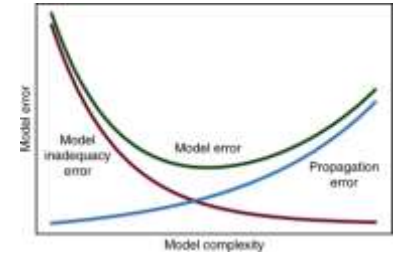
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Efficient Estimation of the ANOVA Mean Dimension, with an Application to Neural Net Classification

Christopher Hoyt and Art B. Owen

<https://doi.org/10.1137/20M1350236>



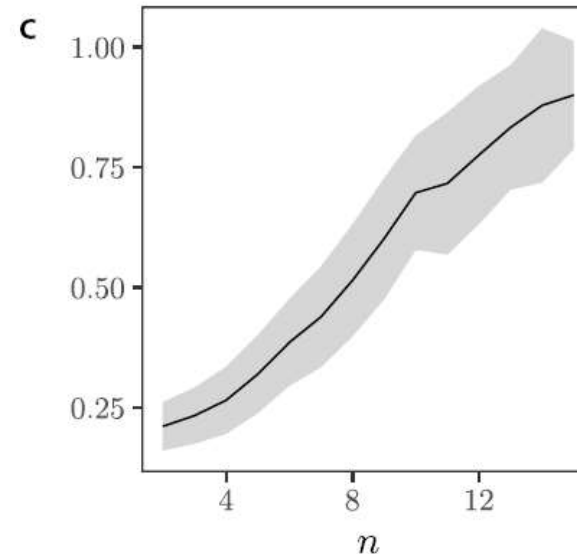
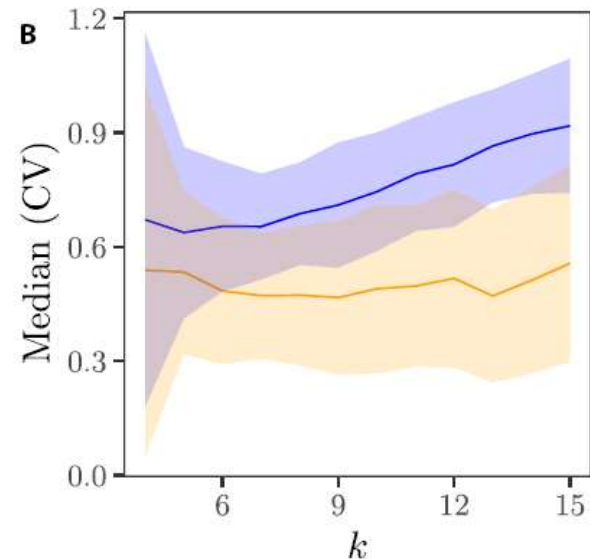
Empirical test using the SA-based concept of effective dimension

Models with higher effective dimensions tend to produce more uncertain estimates

ARNALD PUY ^{ID}, PIERFRANCESCO BENEVENTANO, SIMON A. LEVIN ^{ID}, SAMUELE LO PIANO ^{ID}, TOMMASO PORTALURI, AND ANDREA SALTELLI ^{ID}

SCIENCE ADVANCES • 19 Oct 2022 • Vol 8, Issue 42 • DOI: 10.1126/sciadv.abn9450

Interactions ■ Up to the k th order ■ Up to the n th order for $k = 15$



Plenty of code available in R, MATLAB, and Python



<https://cran.r-project.org/web/packages/sensitivity/sensitivity.pdf>

<https://cran.rstudio.com/web/packages/sensobol/index.html>



<https://www.uqlab.com/> (in MatLab, by Bruno Sudret and his team)

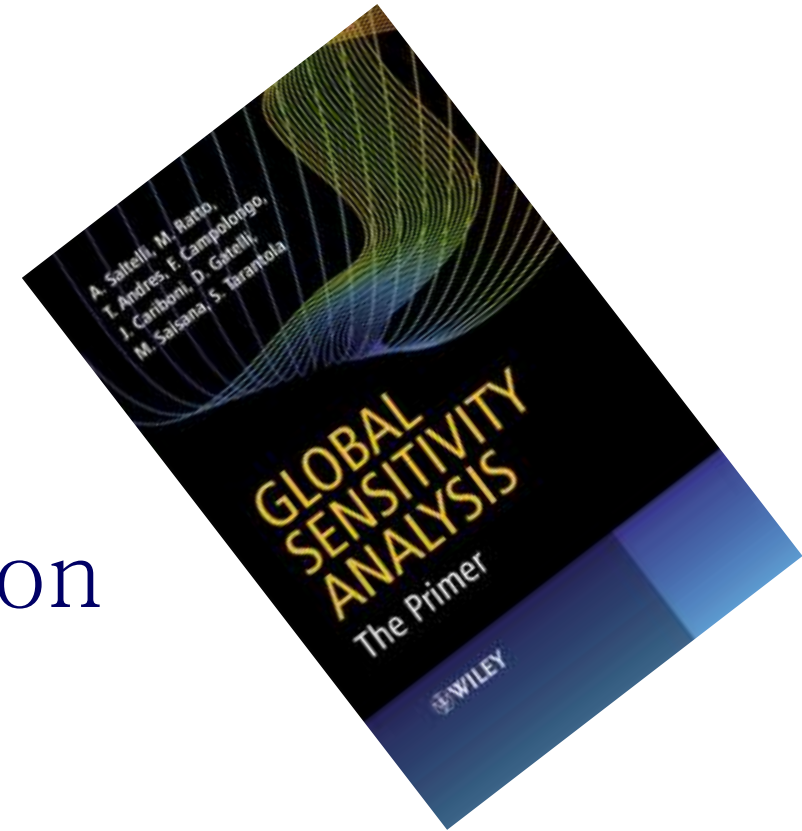


SALib <https://salib.readthedocs.io/en/latest/>

...but there is more,
such as Sensobol in R,
SALib in Python ...

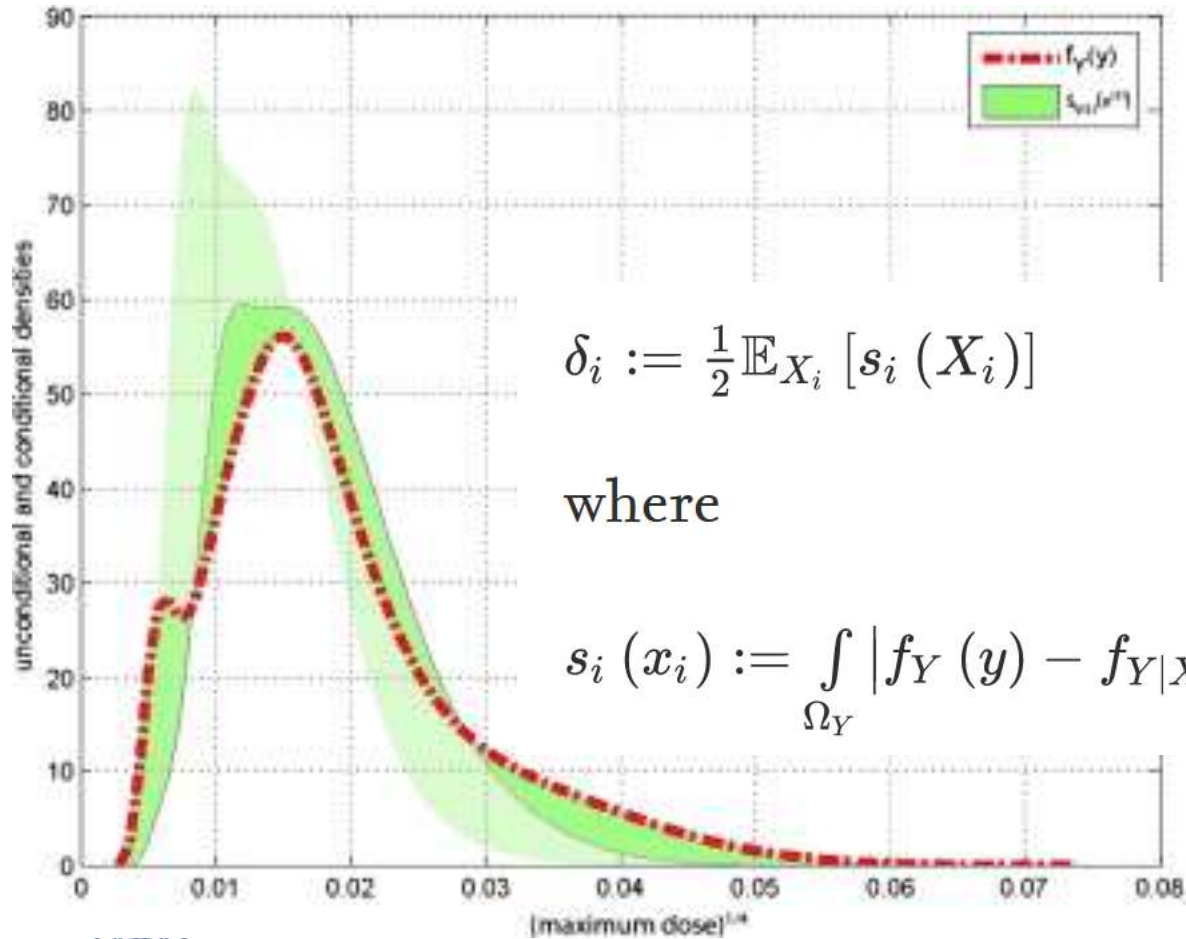
Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation (ANOVA)
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization
- give the effective dimension



Chapter 1 its
exercises

... but there are other methods that can be used for different settings, e.g. moment independent methods, Shapley coefficients, reduced spaces, VARS ...



$$\delta_i := \frac{1}{2} \mathbb{E}_{X_i} [s_i (X_i)]$$

where

$$s_i (x_i) := \int_{\Omega_Y} |f_Y (y) - f_{Y|X_i=x_i} (y)| dy$$



Environmental Modelling & Software

Volume 34, June 2012, Pages 105-115



Model emulation and moment-independent sensitivity analysis: An application to environmental modelling

E. Borgonovo ^a, W. Castaings ^{b, c}, S. Tarantola ^d  

Don't use One factor At a Time (OAT)

A geometric proof



Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

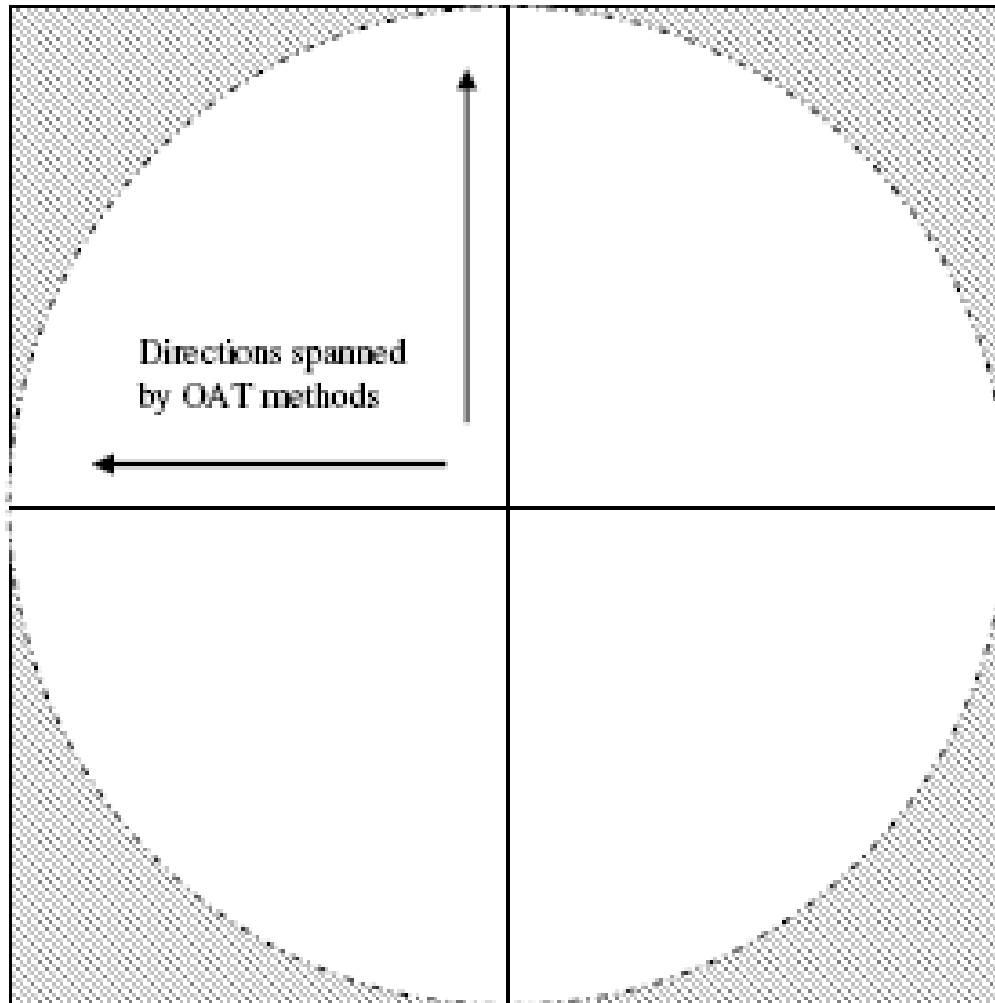


How to avoid a **perfunctory** sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy

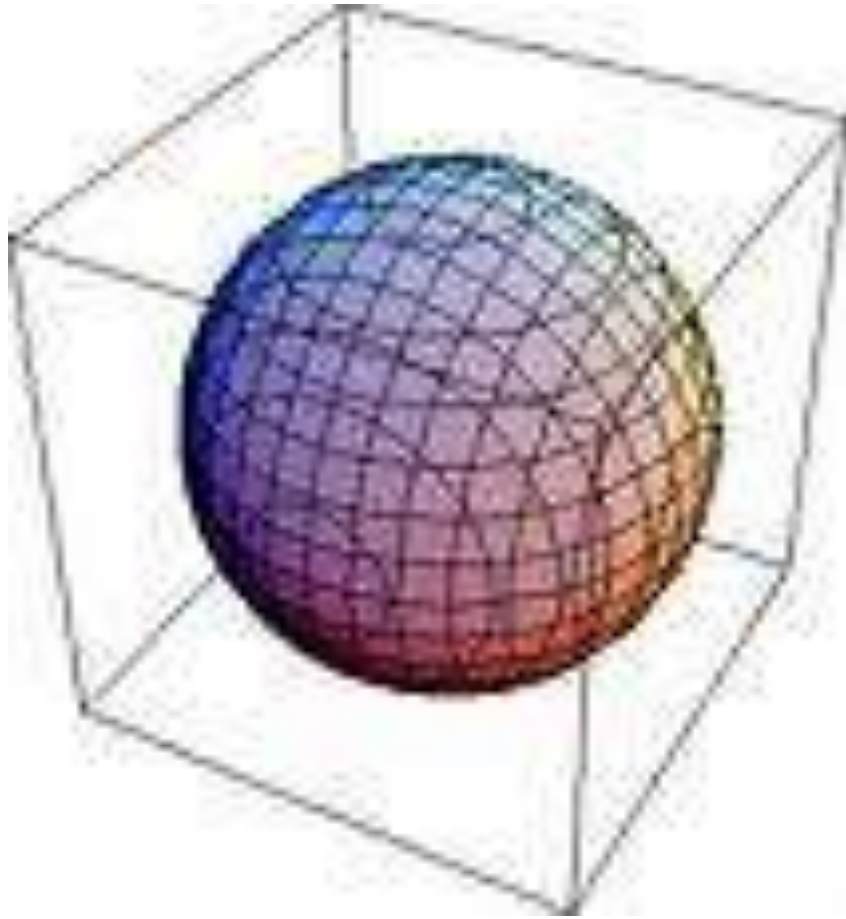
OAT in 2 dimensions



Area circle
/ area
square = ?

~ 3/4

OAT in 3 dimensions



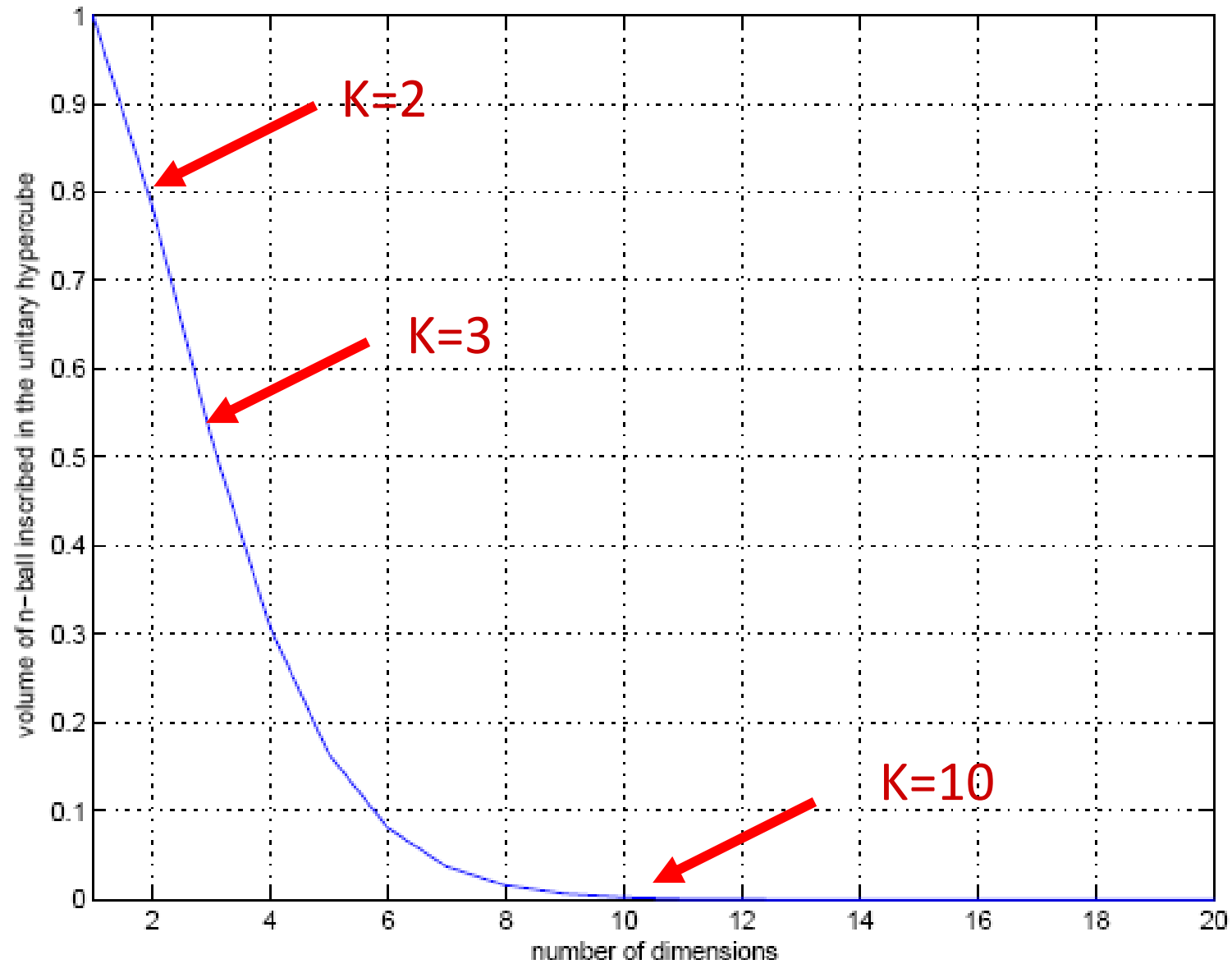
Volume sphere /
volume cube = ?

~ 1/2

OAT in 10 dimensions; Volume
hypersphere / volume ten dimensional
hypercube =? ~ 0.0025



OAT in k dimensions



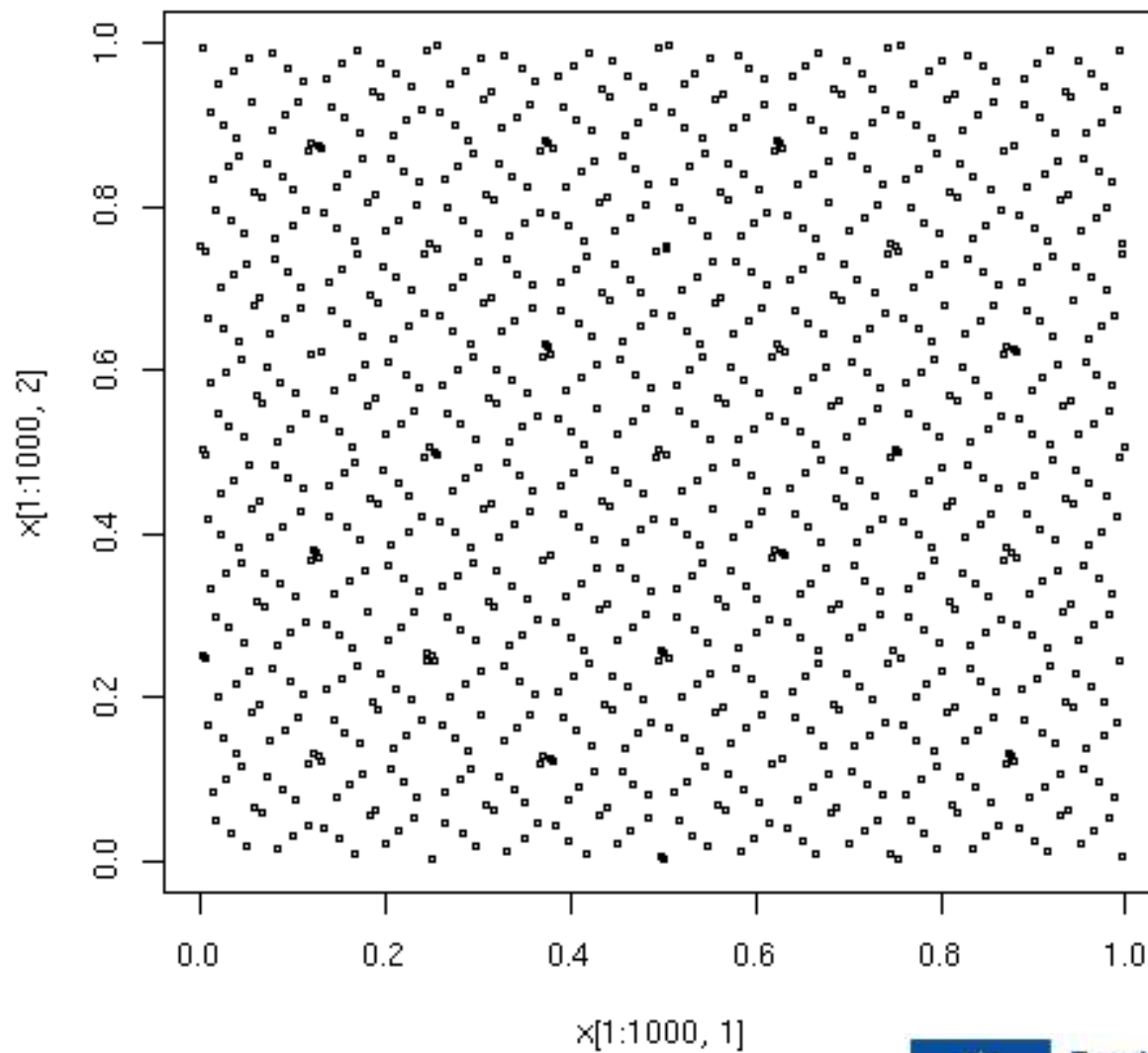
OAT does not capture interactions

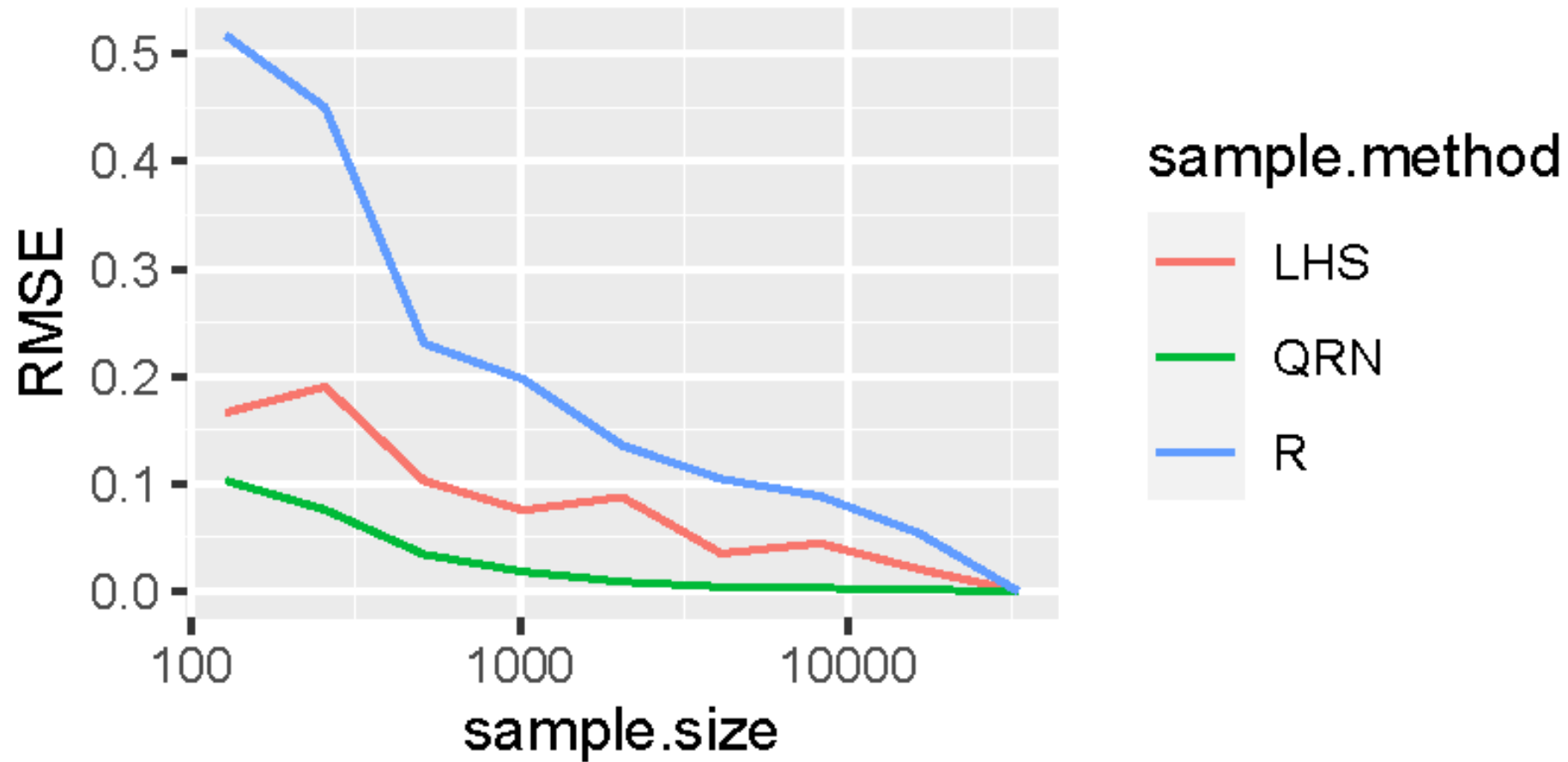
➔ The resulting analysis is non conservative

Quasi random sequences



Ilya M. Sobol'





Root mean square error with different designs.

Sensitivity analysis made easy



Cornell University

arXiv > stat > arXiv:2206.13470

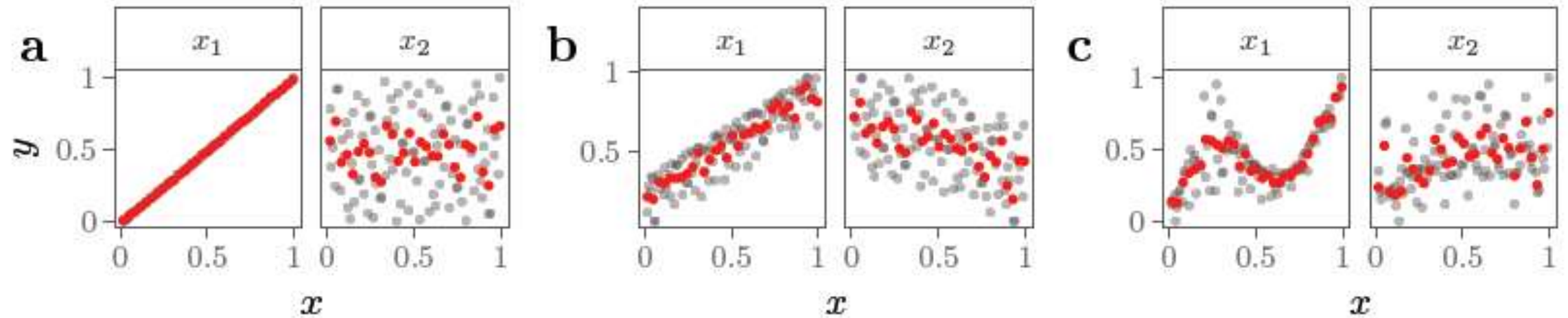
Statistics > Applications

[Submitted on 27 Jun 2022 (v1), last revised 17 Mar 2023 (this version, v2)]

Discrepancy measures for sensitivity analysis

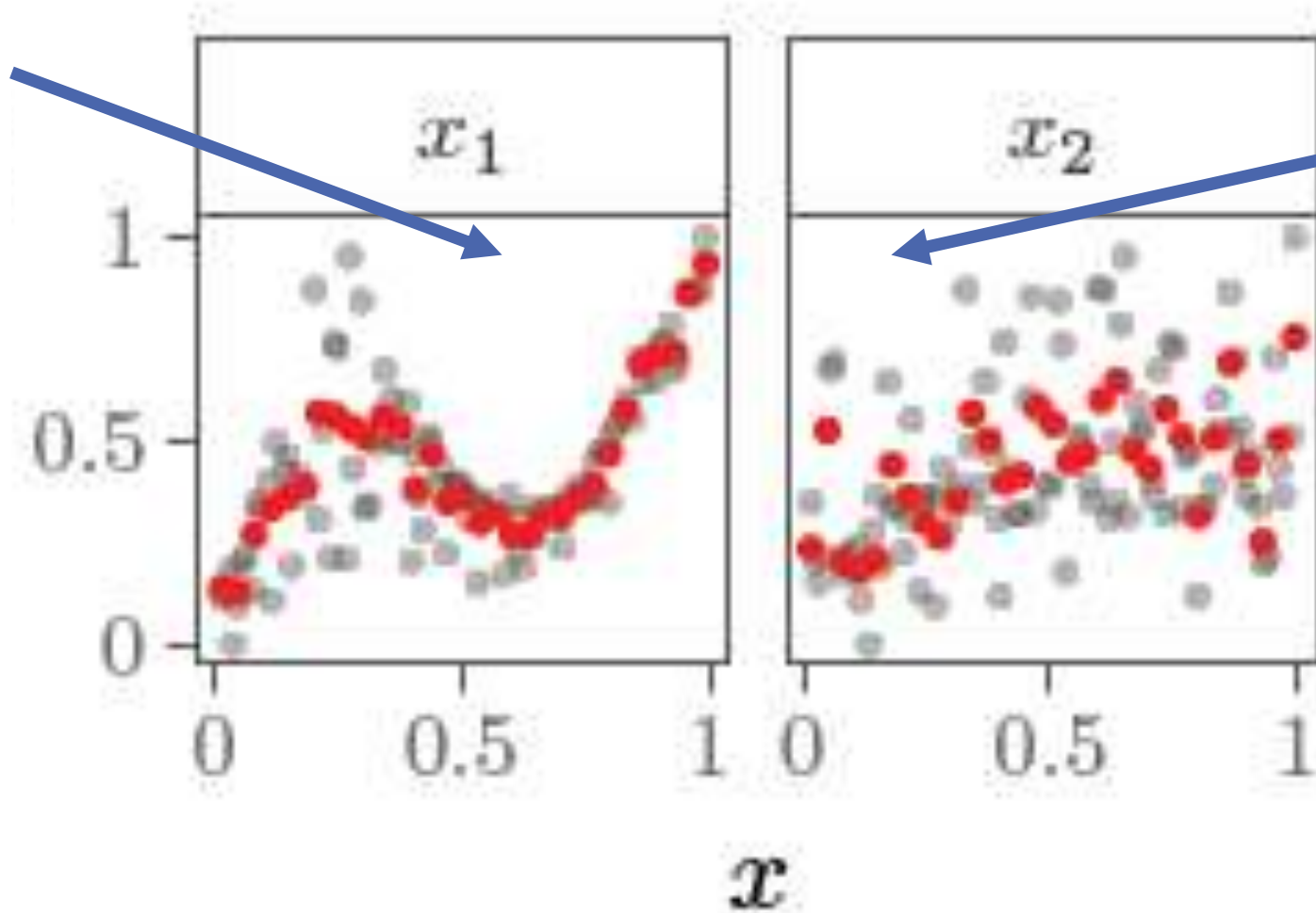
Arnald Puy, Pamphile T. Roy, Andrea Saltelli

Do we need to compute indices?
Can we do without statistics and calculus



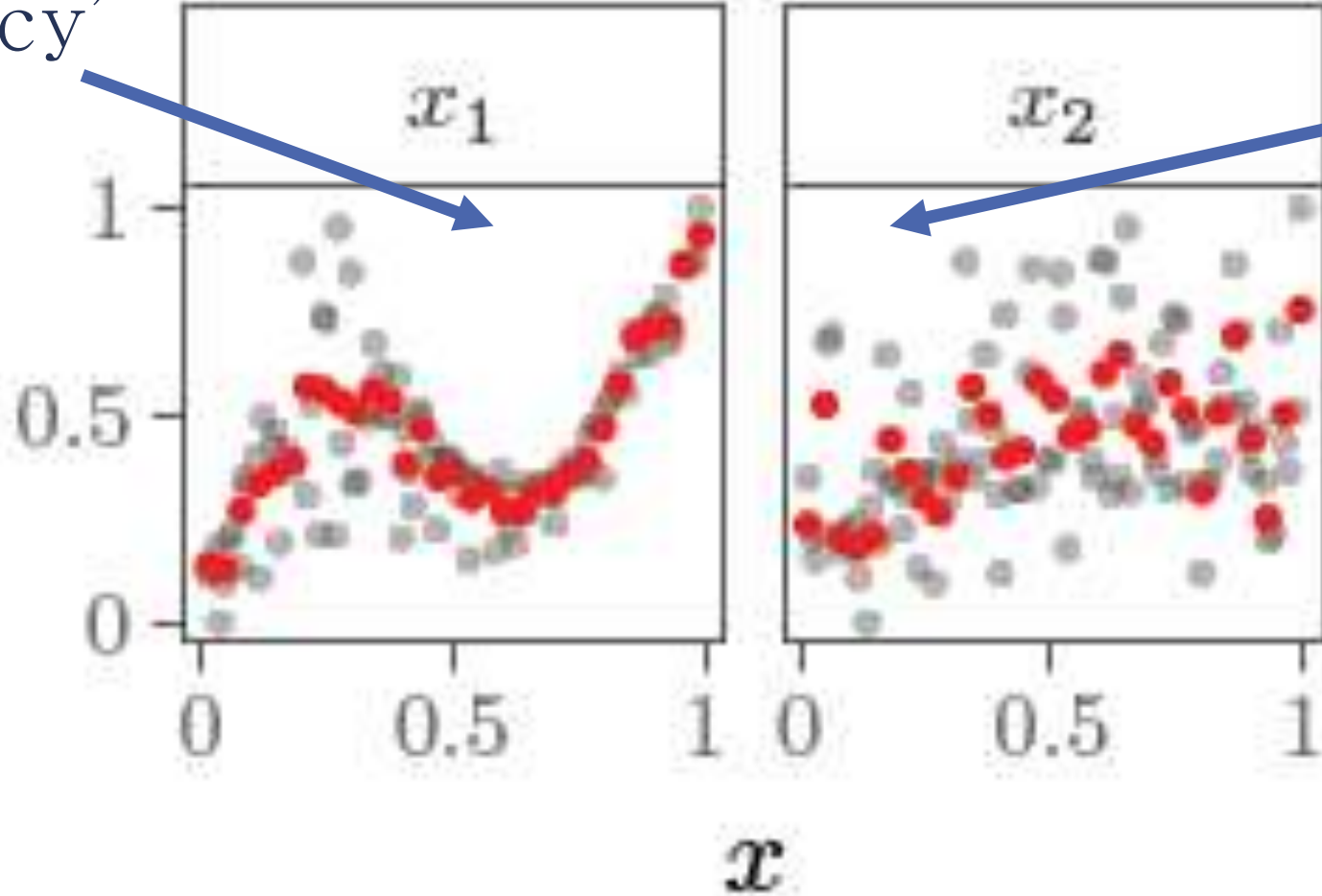
‘Stupid’ histograms in the
 X_i, Y plane, both in $[0,1]$, for different $Y=f(X_i)$

Bigger
'holes'



Smaller
'holes'

Bigger
'discrepancy'



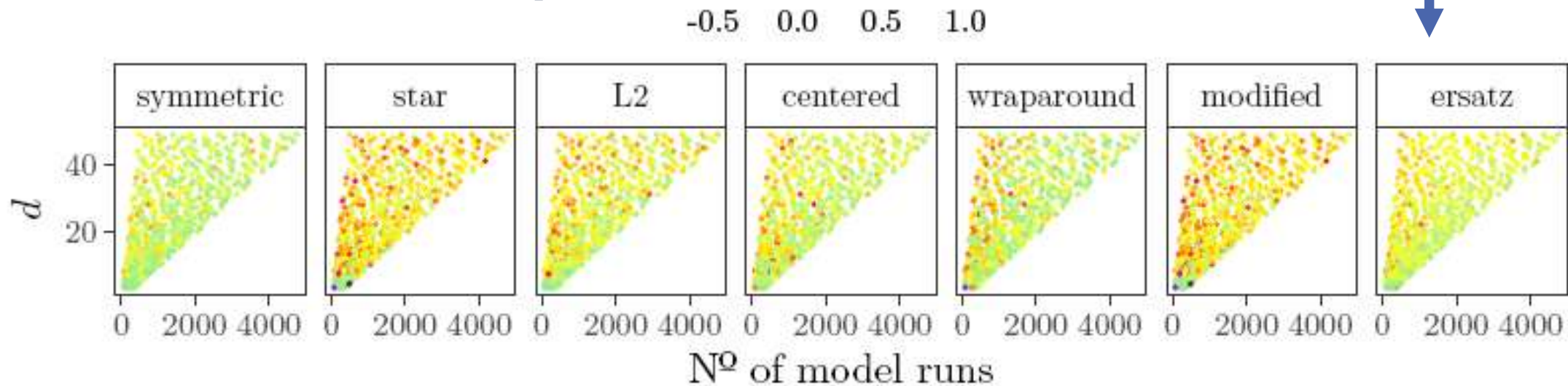
Smaller
'discrepancy'

Existing discrepancies (star, L2, wraparound...) are expensive to compute; how about an 'ersatz' discrepancy?

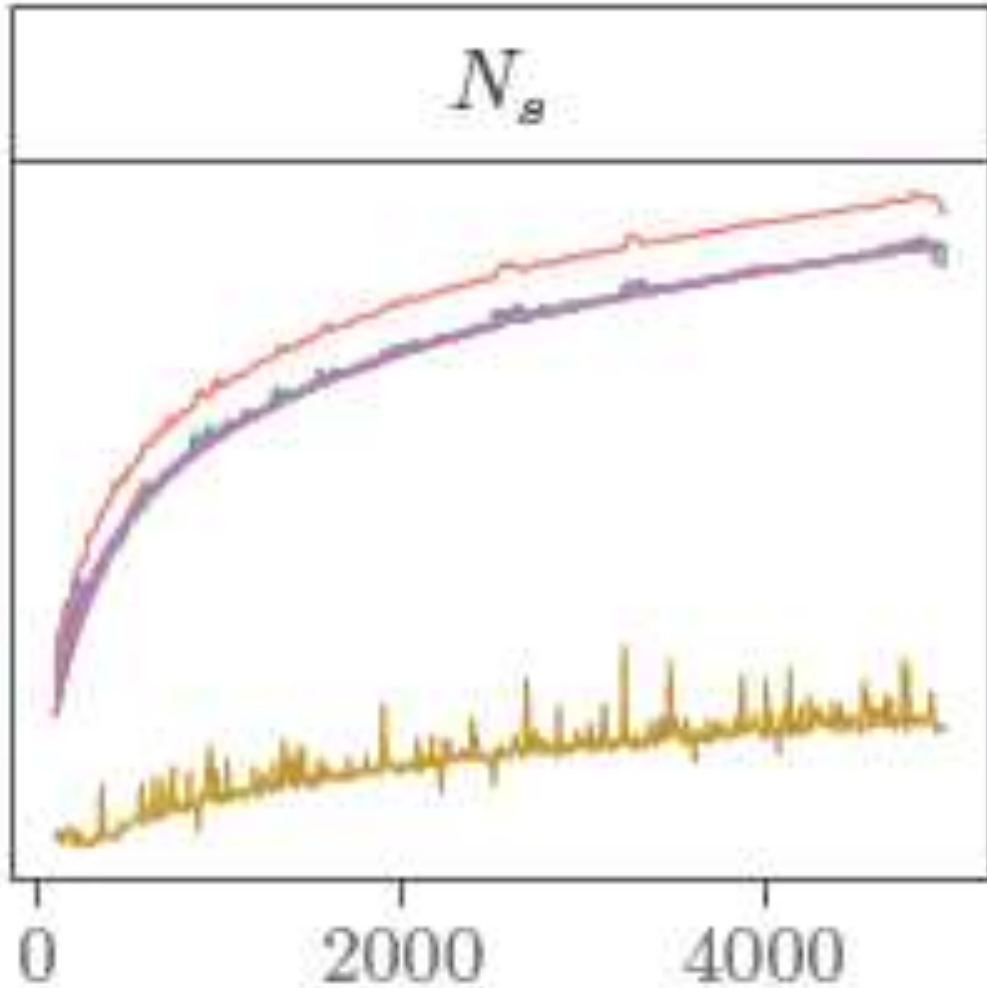
It works!



Agreement with T_i



How about an 'ersatz' discrepancy?



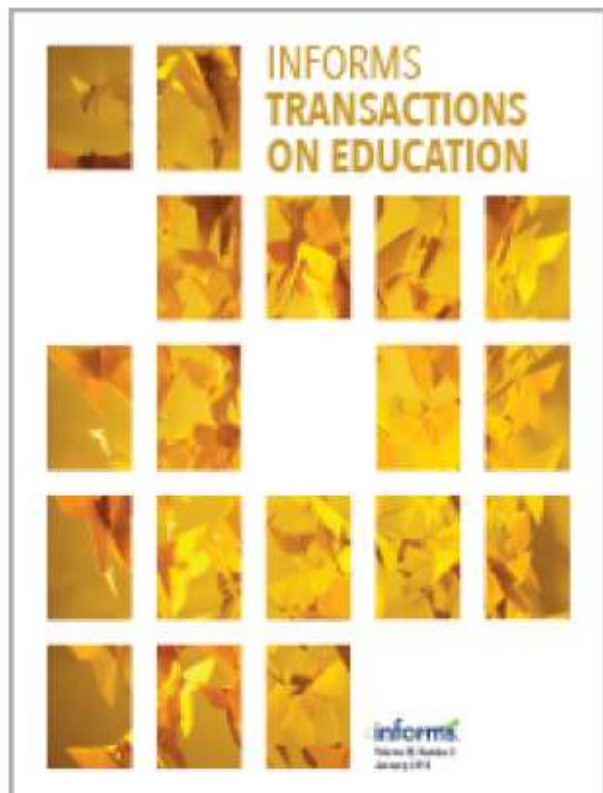
Discrepancy

- centered
- ersatz
- L2
- modified
- star
- symmetric
- wraparound

Is fast!



Another way to bypass statistics and calculus



INFORMS Transactions on Education

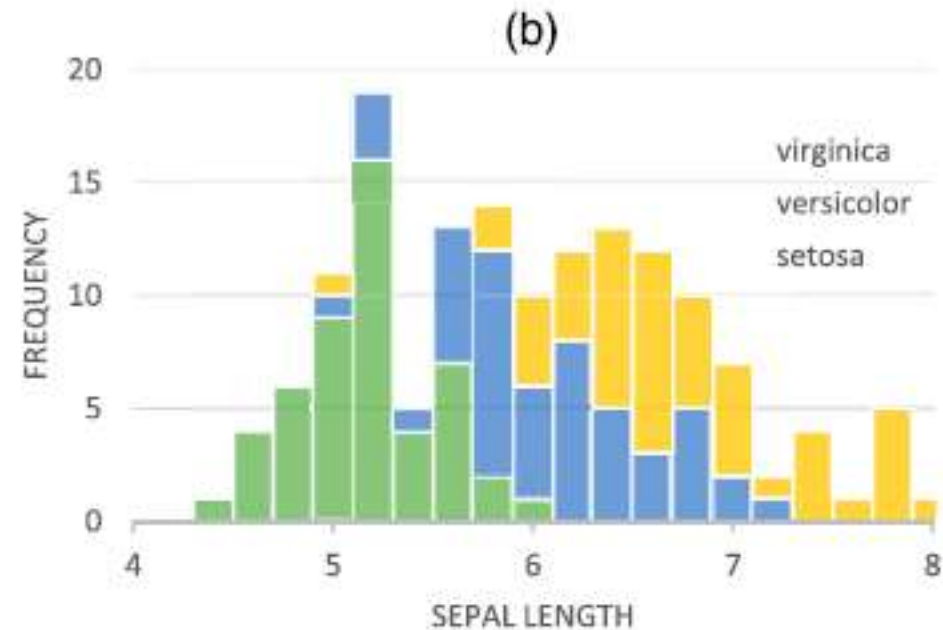
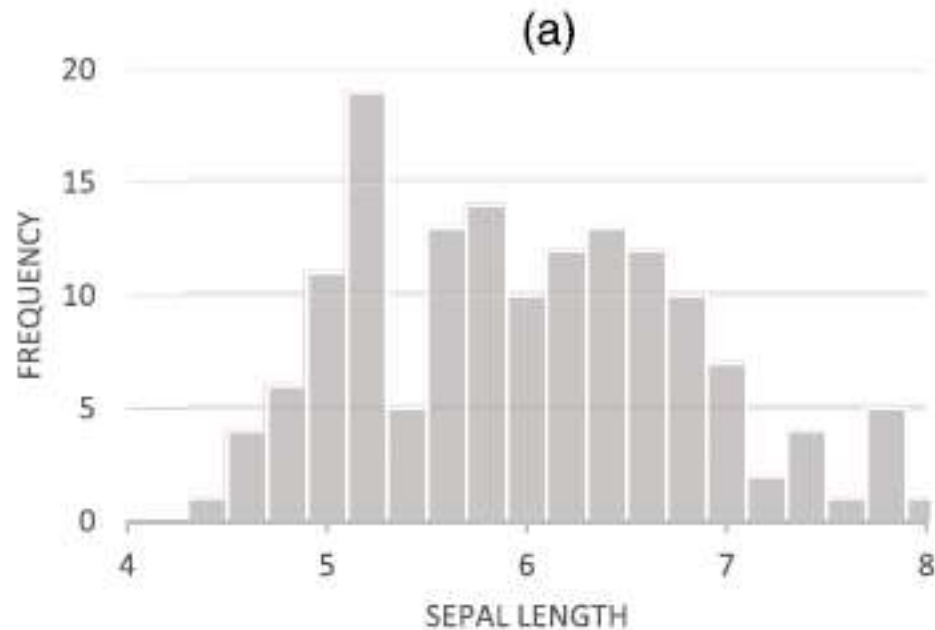
Publication details, including instructions for authors and subscription information:

<http://pubsonline.informs.org>

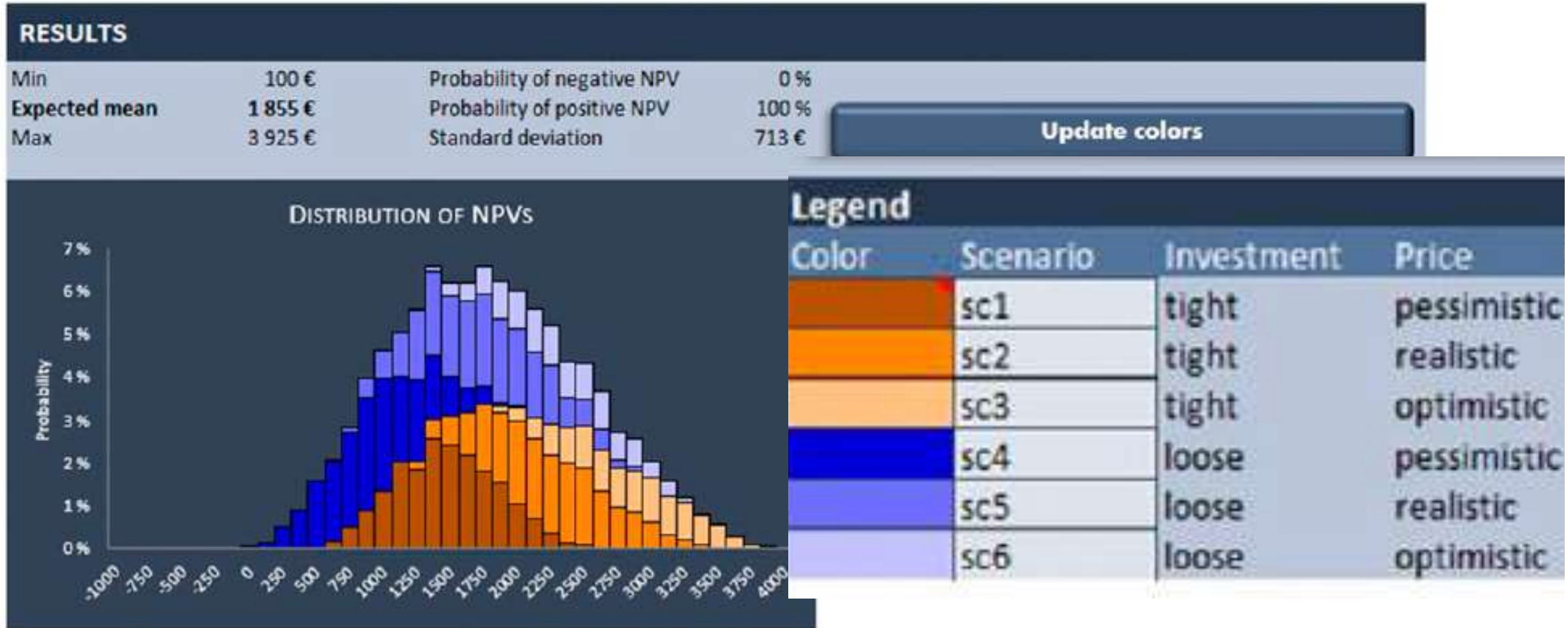
Monte Carlo Enhancement via Simulation Decomposition: A “Must-Have” Inclusion for Many Disciplines

Mariia Kozlova, Julian Scott Yeomans

Colouring the output histogram can give sensitivity insights ...



... without computing sensitivity indices



Don't run the model just once

There is much to learn by running the model a few times, especially during model building

Lubarsky's Law of Cybernetic Entomology: there is always one more bug!



Model routinely used to produce point estimates may become non-conservative when the uncertainty is plugged in

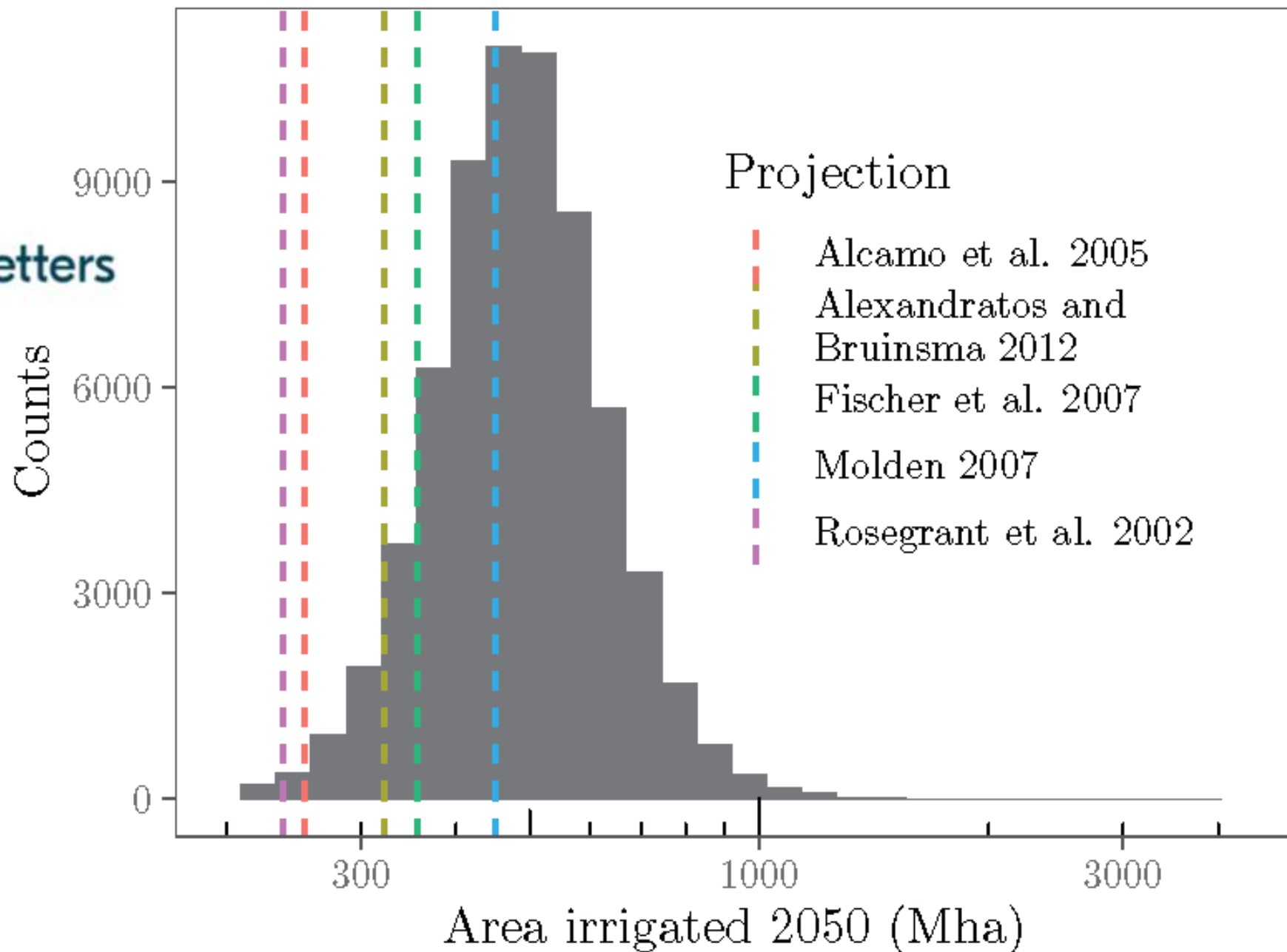
Current Models Underestimate Future Irrigated Areas

Geophysical Research Letters

A. Puy ✉, S. Lo Piano, A. Saltelli

Citation:

Puy, A., Lo Piano, S., & Saltelli, A. (2020). Current models underestimate future irrigated areas. *Geophysical Research Letters*, 47, e2020GL087360. <https://doi.org/10.1029/2020GL087360>



Solution? Modelling of
the modelling process by
taking ‘all paths in the
garden’

Don't sample just parameters and boundary conditions

Explore thoroughly the space of the
assumptions

One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios ...

Why bother?

Fishing expeditions, forking paths ...



Fishing Boats at Sea, Vincent van Gogh, Pushkin Museum, Moscow, Russia



Jorge Luis Borges
(1899–1986)



Taking different
narratives within the
same novel like Ts'ui Pên

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no “fishing expedition” or “p-hacking” and the research hypothesis was posited ahead of time*

Andrew Gelman[†] and Eric Loken[‡]

14 Nov 2013

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no “fishing expedition” or “p-hacking” and the research hypothesis was posited ahead of time*

Andrew Gelman[†] and Eric Loken[‡]

14 Nov 2013

Why this matters?



PNAS

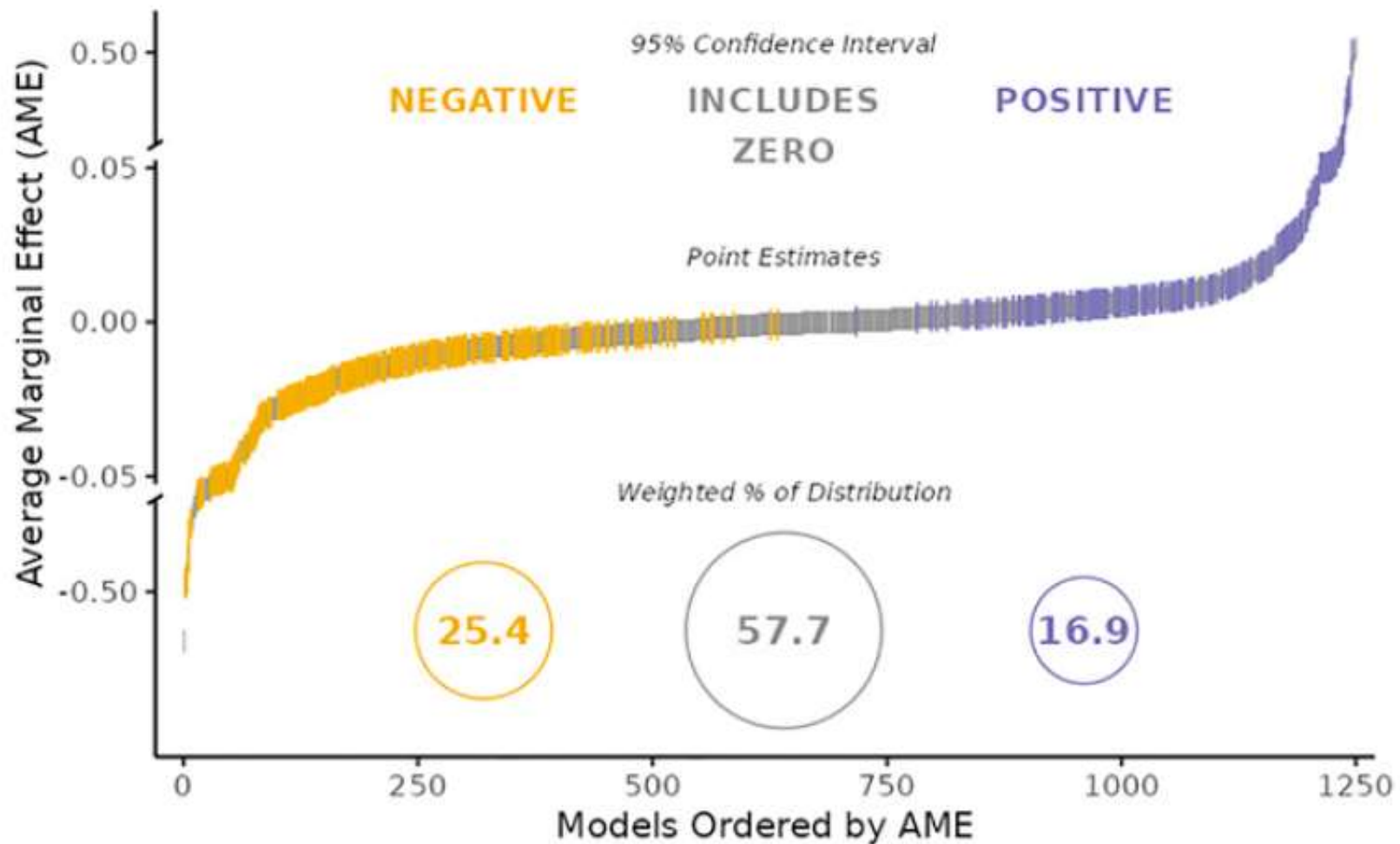
RESEARCH ARTICLE

SOCIAL SCIENCES

 OPEN

Observing many researchers using the same data and hypothesis reveals a hidden universe of uncertainty

Edited by Douglas Massey, Princeton University, Princeton, NJ; received March 6, 2022; accepted August 22, 2022



“Will different researchers [73 teams] converge on similar findings when analyzing the same data?”

“...teams’ results varied greatly, ranging from large negative to large positive effects”
(Breznau et al. 2022)

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no “fishing expedition” or “p-hacking” and the research hypothesis was posited ahead of time*

Andrew Gelman[†] and Eric Loken[‡]

14 Nov 2013

Modellers might (even in good faith) engage in “fishing expeditions”

When they do, they may delude themselves with having nailed an effect ...

...as there are many things that may go wrong (or generate forks in the path)

Since having one's model replicated by 73 teams is impractical, one needs to check the inference, for example with sensitivity auditing and its modelling of the modelling process

nature

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COMMENT | 24 June 2020

Five ways to ensure that models serve society: a manifesto





Futures

Volume 144, December 2022, 103041



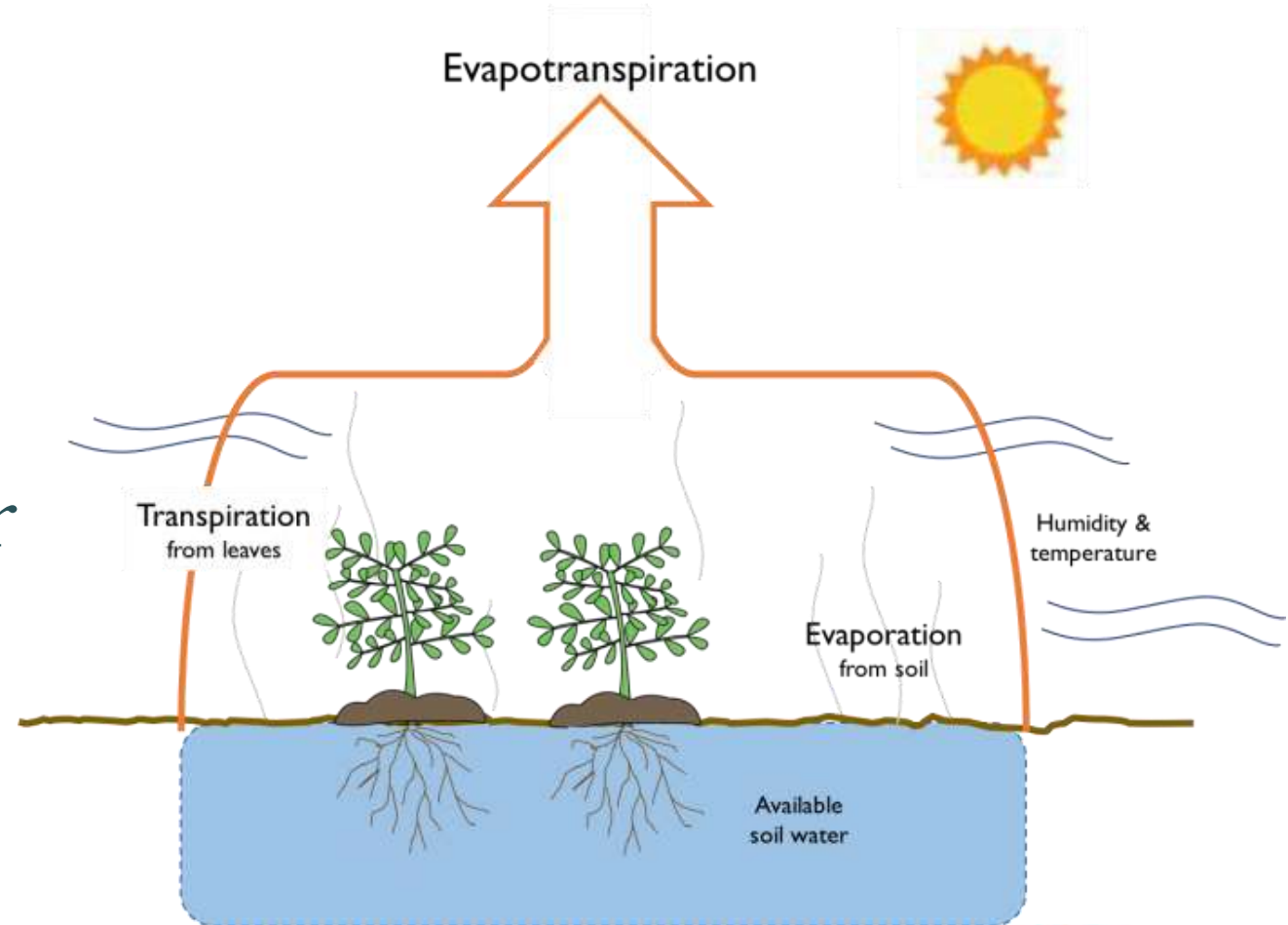
Unpacking the modelling process via sensitivity auditing

[Samuele Lo Piano](#)^a  , [Razi Sheikholeslami](#)^b, [Arnald Puy](#)^{c d e}, [Andrea Saltelli](#)^f

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If several formulas / conceptualizations are available for a given phenomenon, use them all

An example: different equations are available for evapotranspiration



Source: https://serc.carleton.edu/integrate/teaching_materials/food_supply/student_materials/1091

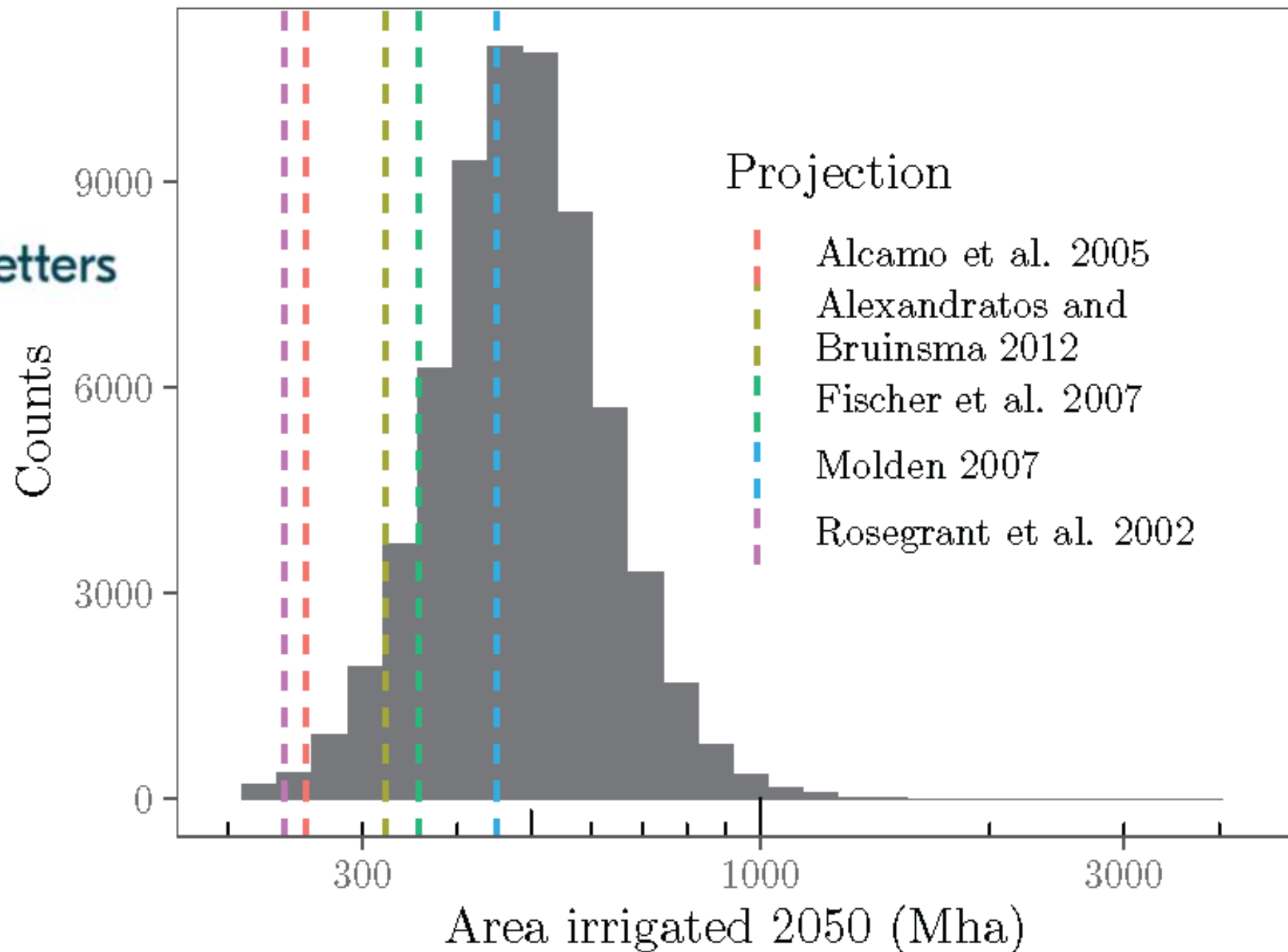
Current Models Underestimate Future Irrigated Areas

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Are we done?

We are not! Completing sensitivity auditing requires a of reflexive (sociological) investigation where the i4Driving community (developers and users) is the object of the analysis



Task 8.5: **Social sciences and humanities** are an integral part of the project ... all the activities ... have a point of reference in the cultural, epistemological, intellectual, linguistic and social idiosyncrasies ...



The idea that just because the computers is in charge we solve traffic is fanciful

Jack Stilgoe,
i4Driving
Advisory Board



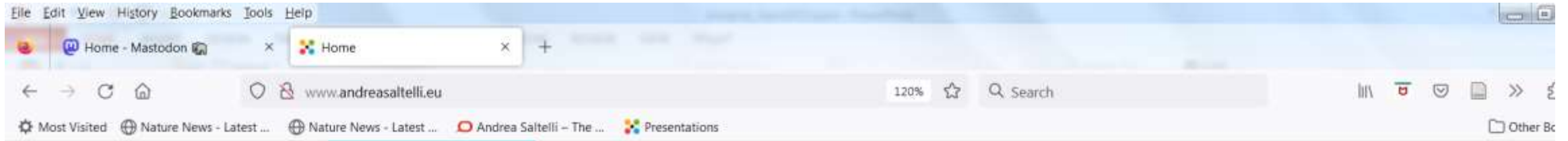
Dr Jack Stilgoe on self-driving cars and ethics in science

Listen to Dr Jack Stilgoe from UCL Science & Technology Studies talk to our hosts about his research into self-driving cars and artificial intelligence

<https://www.ucl.ac.uk/mathematical-physical-sciences/about-faculty/hypot-enthuse-podcast/dr-jack-stilgoe-self-driving-cars-and-ethics-in-science>

Coming next

More material at www.andreasaltelli.eu



Andrea
Saltelli

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Coming Out Soon: The politics of modelling



Praise for the volume

"A long awaited examination of the role —and obligation —of modeling."

Nassim Nicholas Taleb, Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series *Incerto*.

"A breath of fresh air and a much needed

Mastodon Toots by

@AndreaSaltelli



AndreaSaltelli

2023/4/18 17:24

"Kaufman, who was until recently a senior climate economist in the White House, questioned the need for the government to set a single price. Regulators should stop "pretending we

ion