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Sensitivity Analysis for Hydraulic Models

Jim W. Hall¹; Shawn A. Boyce²; Yueling Wang³; Richard J. Dawson⁴; Stefano Tarantola⁵; and Andrea Saltelli⁶

Abstract: Sensitivity analysis is well recognized as being an important aspect of the responsible use of hydraulic models. This paper reviews a range of methods for sensitivity analysis. Two applications, one to a simple pipe bend example and the second to an advanced Shallow Water Equation solver, illustrate the deficiencies of standardized regression coefficients in the context of functionally nonlinear models. Derivatives and other local methods of sensitivity analysis are shown to give an incomplete picture of model response over the range of variability in the model inputs. The use of global variance-based sensitivity analysis is shown to be more general in its applicability and in its capacity to reflect nonlinear processes and the effects of interactions among variables.

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Introduction

Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation (Saltelli et al. 2004). To avoid distinctions between model input variables, boundary conditions and parameters, all of the inputs to a model are collectively referred to as “input factors.” Among the reasons for using sensitivity analysis are

- To identify the factors that have the most influence on model output.
- To identify factors that may need more research to improve confidence in model output.
- To identify factors that are insignificant to model output and can be eliminated from further analysis.
- To determine if a model reproduces known influences upon the processes it is simulating.
- To identify regions in the space of inputs where the variation in model output is maximum.

- To find the optimal regions within the parameter space for use in calibration studies.
- To identify which, if any, factors or groups of factors interact with each other.
- To establish whether model predictions are robust to plausible variations in input factors or, on the other hand, are strongly dependent on fragile assumptions.

Sensitivity analysis is widely accepted as a necessary part of good modeling practice. However, the increasing complexity of computer models used in hydraulic engineering has not, in general, been accompanied by a corresponding increase in the rigour and sophistication of sensitivity analysis. The increasing use of coupled models from different disciplines, for example the coupling of hydrodynamic, structural reliability and impacts models to provide estimates of risk (Dawson et al. 2005), provides additional motivation for improved sensitivity analysis. Since these models derive from different scientific communities, model developers and users cannot be expected to have reliable intuitions about the model behavior and interactions without a systematic approach to exploring the model response to varying inputs.

Understanding and analyzing uncertainties has concerned hydraulic engineers for many years (Johnson 1996; Melching 1995; Tang and Yen 1972; Yen and Tung 1993). Uncertainty analysis involves quantification of uncertainties in model inputs and propagating them through to model predictions (Hall 2003; Hall and Solomatine 2008). Sensitivity analysis can be thought of as addressing the inverse of this problem, which is to diagnose the influence that model input factors, individually and in combination, have on the variation in the model prediction. Typically, uncertainty analysis is applied in situations where quantities in a system being analyzed are not known precisely (e.g., channel roughness) or vary in nature (e.g., river discharge). As will be demonstrated herein, sensitivity analysis is more general in that it can also usefully be applied to design variables, i.e., quantities that will be decided upon by the engineer. While it is not meaningful to apply uncertainty analysis to these variables (as they will in future be fixed, subject to some tolerance) it is useful to apply sensitivity analysis to identify which design variables have an important influence upon system performance and which are less important.

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To achieve the aims outlined above, a method of sensitivity analysis should have the following desirable properties

- The method should be able to diagnose the effect of input factors acting individually or in combination, in the latter case in order to identify the effect of interactions between factors.
- The method should test the influence of a model input factor over its entire range of variation.
- The method should be applicable, within the range of input variation, to linear and nonlinear models.
- The method should be model independent.
- The method should be computationally efficient.
- The method should be applicable to both discrete and continuously varying inputs.

In the remainder of this paper, a variety of methods for sensitivity analysis are reviewed against the criteria set out above, and evaluated by means of an application to a text book example in hydraulic engineering and then to a more complex numerical model. The implications of these examples for sensitivity analysis of functionally nonlinear models are discussed. Based on these insights, recommendations are made for the choice of appropriate sensitivity analysis methods in hydraulic engineering.

Approaches to Sensitivity Analysis

The main approaches to sensitivity analysis that are applicable in hydraulic engineering are introduced in the following. More theoretical discussion can be found in Saltelli et al. (2000), Cacuci (2003), Saltelli et al. (2005, 2008).

Derivative-Based Sensitivity Indices

Model output derivatives, with respect to input factors, are intuitive sensitivity indices. In general consider a model $Y=f(X_1, \dots, X_k)$ with k input factors. In the following capital notation (e.g., Y) is used to denote a random variable and lower case (e.g., y) to denote a fixed value of that variable. The partial derivative of Y with respect to an input factor X_i , $\partial Y/\partial X_i$, measures how sensitive the output is to a perturbation of the input. If factors are uncertain within a known or hypothesized range, then the measure

$$S_i^\sigma = \frac{\sigma_i}{\sigma_Y} \frac{\partial Y}{\partial X_i} \quad (1)$$

provides a standardized index, where σ_i , σ_Y =standard deviations of the inputs, i , and output of uncertainty analysis, respectively. The sensitivity measures $\partial Y/\partial X_i$ can be efficiently computed by algorithmic differentiation, where the computer program that implements the model is modified so that the derivatives are computed with a modicum of extra execution time (Grievank 2000). Furthermore, a variety of methods is available to compute these derivatives for large systems of differential equations, such as the Green functions method, the direct method, the decoupled direct method, the adjoint method and others (Turanyi and Rabitz 2000). A less sophisticated but much more commonplace approach is to vary each input factor in turn by a positive and negative increment around its central value, while keeping all other input factors at their central value. This “one-at-a-time” sensitivity analysis can be thought of as providing informal sensitivity indices using arbitrary finite difference estimates of the partial derivatives (Rabitz 1989).

It is clear that if X_i does not vary linearly with Y over the space

of X_i , the calculated derivatives at a single point may misrepresent the sensitivity of the output to uncertainty in the input factors, so sensitivity indices obtained from derivatives methods are referred to as local in the following. Notwithstanding these limitations, the majority of sensitivity analysis met in hydraulic engineering and indeed hydrology, where there has been more attention paid to the problems of uncertainty estimation, are local, derivative-based (Chen and Chen 2003; Cornell 1972; Horritt 2006; Indelman et al. 1996; Kabala 2001; Nash and Karney 1999; Oliver and Smettem 2005; Podsechin et al. 2006; Renault and Hemakumara 1999; Rocha et al. 2006; Swaminathan et al. 1986).

Linear Regression

For linear models, the linear regression coefficients between input and output provide natural sensitivity indices. In the case of numerical models, this can be achieved by constructing a Monte Carlo sample of the model inputs and regressing the corresponding outputs, Y against the inputs X_i using a multiple regression analysis model of the form

$$Y = b_0 + \sum_{i=1}^k b_i X_i \quad (2)$$

where b_i =fixed regression coefficients. The linear regression coefficients will have dimensions but can be standardized so that

$$\tilde{Y} = \beta_0 + \sum_{i=1}^k \beta_i \tilde{X}_i \quad (3)$$

where $\tilde{Y}=(Y-\mu_Y)/\sigma_Y$, $\tilde{X}_i=(X_i-\mu_i)/\sigma_i$, and $\beta_i=(\sigma_Y/\sigma_i)b_i$. \tilde{Y} and \tilde{X}_i are the standardized variables, μ_Y , σ_Y and μ_i , σ_i are the means and standard deviations of the output and input factors respectively and β_i are known as standardized regression coefficients (SRCs) (Helton et al. 2006). The values of β_i can be estimated by evaluating y at each point in a Monte Carlo sample of the input variables X_i and then applying regression analysis to the sample of points.

For linear models $\beta_i^2=(S_i^\sigma)^2$ and if the model is nonlinear, SRCs are still a reflection of the contribution of the variance of each input factor to the overall output variance and are more attractive than local derivatives as they offer a measure of the effect of each given factor on Y , which is averaged over a sample of possible values, as opposed to being computed at the fixed point. SRCs are, therefore, a global sensitivity measure, their limitation being in their applicability to nonlinear models.

The sum of the squares of the SRCs represents the proportion of the model output variance explained by the regression model and gives insight into model linearity. This sum can be formulated from a Monte Carlo sample of model simulation data and is expressed as the model coefficient of determination, R_Y^2

$$R_Y^2 = 1 - \sum_{i=1}^m \frac{(y_i - \hat{y}_i)^2}{(y_i - \mu_Y)^2} \quad (4)$$

where m =number of simulations, y_i =simulation results for model realization i and \hat{y}_i =values of y provided by the regression model for input vector x_i . When the coefficient of determination is high, e.g., 0.7 or higher, then the SRCs can be used for sensitivity analysis, albeit at the price of remaining ignorant about that fraction of the model variance not explained by the SRCs.

SRCs have been used with some success as sensitivity measures in hydraulic engineering. Even though many hydraulic models are in principle nonlinear, they are often found in practice to

be nearly linear over the range of variation of the inputs, so SRCs are a convenient sensitivity measure, with the added advantage that R_Y^2 provides a diagnostic of the appropriateness of the linear assumption. For example, the regression analysis of the pit-migration model yielded an R_Y^2 of 0.981 in Yeh and Tung (1993). Jaffe and Ferrara (1984) analyzed water quality model sensitivity using ranked inputs and outputs, which can yield higher values of R_Y^2 for nonlinear but still monotonic models. Melching (2001) and Manache and Melching (2004) reported R_Y^2 values in excess of 0.9 for features of the simulated dissolved oxygen concentration from water resource simulation models (Melching and Bauwens 2001). Siebera and Uhlenbrook (2005) used a regression-based sensitivity analysis to verify the structure of a distributed catchment model. Pappenberger et al. (2006) used a regression-based technique to determine the influence of uncertainties in rating curves and boundary conditions on flood inundation predictions. SRCs therefore provide a convenient starting point for sensitivity analysis, recognizing that if the models concerned are demonstrably nonlinear over the range of input variation then the following more sophisticated methods will be required.

Fourier Amplitude Sensitivity Test

While linear regression methods have seen quite wide application in hydraulic engineering, when the coefficient of determination of a linear regression model is low then it is necessary to adopt a sensitivity method that is applicable to nonlinear models. The Fourier amplitude sensitivity test (FAST) (Cukier et al. 1973; Cukier et al. 1978; Cukier et al. 1975; Schaibly and Shuler 1973) works irrespective of the degree of linearity or additivity of the model.

In FAST, the variance $V(Y)$ of Y is decomposed using spectral analysis, so that $V = V_1 + V_2 + \dots + V_k + r$, where V_i is that part of the variance of Y that can be attributed to X_i alone and r is a residual. The ratio $S_i = V_i/V$ can be taken as a measure of the sensitivity of Y with respect to X_i . For linear models $\beta_i^2 = S_i = (S_i^\sigma)^2$. Saltelli et al. (2004) show how S_i^σ is an effective measure for linear models, β_i is an effective measure for moderately nonlinear models, for which an effective linear regression model can be built, and S_i is the model-free extension that is effective even for strongly nonlinear models.

There is a natural and intuitive interpretation of these variance-based sensitivity indices in terms of the amount by which the output variance from a model is reduced by fixing an input at a given value. Let $V(Y)$ denote the variance of Y and $V(Y|X_i = x_i^*)$ the variance that would be obtained if X_i were to be fixed to some value x_i^* , in other words the conditional variance of Y given $X_i = x_i^*$. x_i^* could be thought of as the true value of X_i determined with a measurement. However, it is not known a priori what x_i^* happens to be, but if the distribution of X_i is known it is natural to compute the average $E_{X_i}[V(Y|X_i)]$ over all possible values of x_i^* . The quantity $V(Y) - E_{X_i}[V(Y|X_i)]$ is the average amount by which the variance $V(Y)$ will be reduced if the uncertainty in X_i is removed.

A known algebraic result (Saltelli et al. 2000) is that

$$V(Y) = E_{X_i}[V(Y|X_i)] + V[E_{X_{-i}}(Y|X_i)] \quad (5)$$

where $V[E_{X_{-i}}(Y|X_i)]$ is referred to as the “variance of conditional expectation” of Y given X_i and the subscript X_{-i} denotes the vector of all factors other than X_i . The FAST based sensitivity index is simply

$$S_i = \frac{V[E_{X_{-i}}(Y|X_i)]}{V(Y)} = \frac{V_i}{V} \quad (6)$$

In classic FAST, only the main effect terms V_i are computed, and the success of a given analysis is empirically evaluated by the sum of these terms. If this is high, as rule of the thumb greater than 0.6 (Liepmann and Stephanopoulos 1985), then the analysis is successful. The V_i describe the so called “additive” part of a model and additive models are defined as those for which $\sum_i S_i = 1$. Note that in this context, the condition of “additivity” is more general than linearity, to which SRCs are restricted. Extended FAST (Saltelli et al. 1999) allows the computation of higher order terms

$$V(Y) = \sum_i V_i + \sum_{\substack{i,j \\ i < j}} V_{ij} + \sum_{\substack{ijl \\ i < j \\ j < l}} V_{ijl} + \dots + V_{123\dots k} \quad (7)$$

where

$$V_i = V[E_{X_{-i}}(Y|X_i)] \quad (8)$$

$$V_{ij} = V[E_{X_{-ij}}(Y|X_i, X_j)] - V_i - V_j \quad (9)$$

$$V_{ijl} = V[E_{X_{-ijl}}(Y|X_i, X_j, X_l)] - V_{ij} - V_{jl} - V_{il} - V_i - V_j - V_l \quad (10)$$

and so on, and their corresponding sensitivity indices, are obtained from normalization by V . Eq. (7) is the same variance decomposition as is employed in ANOVA (analysis of variance) (Box et al. 1978).

Computing all 2^k terms in Eq. (7) is computationally prohibitive for all but functions with very small k . A more practical approach is to estimate the k total sensitivity indices, S_{Ti} , where (Homma and Saltelli 1996)

$$S_{Ti} = 1 - \frac{V[E(Y|X_{-i} = x_{-i}^*)]}{V(Y)} \quad (11)$$

The total sensitivity index therefore represents the average output variance that would remain as long as X_i stays unknown. The total sensitivity indices help in finding interactions within the model. For example, factors with small first order indices but high total sensitivity indices affect the model output Y mainly through interactions. The presence of such factors is indicative of redundancy in the model parameterization.

Unlike derivative and linear regression sensitivity analysis, there has to date been rather limited published application of these more general variance based sensitivity indices in the hydraulic engineering literature. The only three reported applications (Dawson et al. 2008; Hall et al. 2005; Pappenberger et al. 2008) have been in the field of flood inundation modeling. Ratto et al. (2007b) include the use of variance-based sensitivity indices in their discussion of rainfall-runoff modeling.

Other Methods

Specialist application situations have led to development of specific sensitivity methods. For example, the problems of reliability analysis lead to a natural family of sensitivity measures related to the local derivatives at the point on the limit state function where the structure is most likely to fail (Cawfield 2000; Vrijling 2001).

Another useful sensitivity measure, which is computationally cheaper than the variance based methods, is the measure of Morris (1991), which is useful when the number of uncertain factors is high and/or the model computational time is expensive. It belongs thus to the family of screening sensitivity analysis methods (Campolongo et al. 2000). Xu and Mynett (2006) applied the Morris method to identify the most influential parameters on a river basin management model's output.

Hydrological model calibration exercises have yielded sensitivity indices as a by-product. Where the calibration is implemented as an optimization problem the local derivatives from the optimization can give insight into the sensitivity of model response. Tang et al. (2006) applied this approach to a nonlinear parameter estimation tool. Alternatively, if a population of model parameter sets is partitioned between those that reproduce observations acceptably and those that do not, then statistical methods such as the Kolmogorov-Smirnov test can be used to analyze if a parameter significantly influences the calibration. This is a specific example of Monte Carlo Filtering, where the results of a sequence of Monte Carlo model experiments are "filtered" to remove the instances that perform unacceptably (Hornberger and Spear 1981).

Measures of entropy can be used to estimate the degree of association between model inputs and outputs (Helton et al. 2006), with the attraction that entropy is less reliant than variance-based methods on the second moment as a description of dispersion. Pappenberger et al. (2008) used Kullback-Leibler entropy as a sensitivity measure. The extension of this method to the case of imprecise probabilities is explored by Hall (2006).

The field of Bayesian statistics provides a sound decision-theoretic justification for sensitivity indices in terms of the "partial expected value of perfect information" (partial EVPI) (O'Hagan et al. 1999; Oakley and O'Hagan 2004). This requires the existence of a utility function, but if such a function does exist or can be assumed (e.g., a quadratic loss function), it allows the quantification of the economic value of reducing the uncertainty in any given input factor. That value of uncertainty reduction is a natural sensitivity index, and indeed Oakley and O'Hagan (2004) demonstrated that the first order variance-based sensitivity measures are a special case of this Bayesian measure of sensitivity. A further generalization was provided by Hall (2006), who extended both the variance based measures and the partial EVPI to the situation in which the input probability distributions are not precisely known, the consequence of which is that the various sensitivity indices are output as intervals rather than points. The use of sensitivity indices in the context of imprecise information is further explored by Ferson and Tucker (2006).

Examples

Didactic Example: Force on a Pipe Bend

A simple problem concerns the force exerted on a pipe by water flowing steadily around a horizontal bend (Fig. 1). This "model" is so simple as to allow a characterization of the system sensitivity by analytic methods, but it will be dealt with here as if it were a more complex computer model. The well known solution for F_x and F_y , the orthogonal force components on the bend, are as follows:

$$F_x = p_1 a_1 - p_2 a_2 \cos \theta + \rho q (v_1 - v_2 \cos \theta) \quad (12)$$

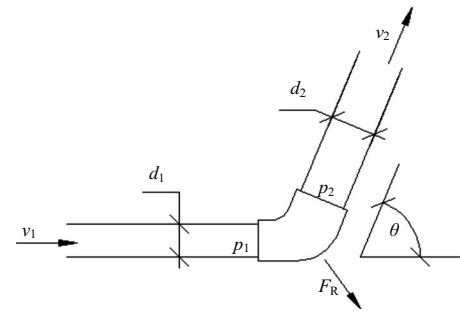


Fig. 1. Diagram of notation in pipe bend calculation

$$F_y = \rho q v_2 \sin \theta + p_2 a_2 \sin \theta \quad (13)$$

where q =discharge in the pipeline (m^3/s), v_1 and v_2 =velocities before and after the bend respectively (m/s), a_1 and a_2 =areas of the pipe before and after the bend respectively (m^2), p_1 and p_2 =pressures immediately before and after the bend (N/m^2), ρ =density of water (kg/m^3) and θ =angle of the bend. The magnitude of the resultant force is $F_R = \sqrt{F_x^2 + F_y^2}$ and $p_2 = p_1 + \rho/2(v_1^2 - v_2^2)$.

In order to explore the sensitivity of response of this system to variation in the input factors, the input factors were assigned the distributions in Table 1. This range has deliberately been chosen to be quite wide in order to illustrate the potential for nonlinear response even in this very simple example. Fig. 2 illustrates the resultant force for a range of values of q and d_1 . Even for this simple example it is clear how the behavior of partial derivatives

Table 1. Distributions of Input Factors for the Pipe Bend Example

Variables	Distribution type	Parameters
D_1	Uniform	$D_1 \sim U(0.05, 0.15)$
D_2	Uniform	$D_2 \sim U(0.05, 0.15)$
Q	Lognormal	$\ln Q \sim N(0.01, 0.3)$
P_1	Lognormal	$\ln P_1 \sim N(11.5, 0.5)$
θ	Normal	$\theta \sim N(70^\circ, 15^\circ)$

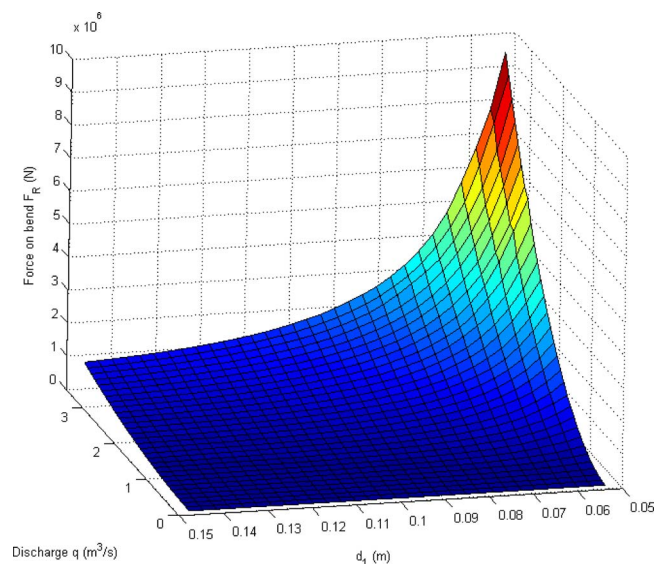


Fig. 2. Resultant force on a pipe bend as a function of discharge q and diameter d_1 ($d_2=0.1$ m, $\theta=70^\circ$, $p_1=1 \times 10^5$ N/m²)

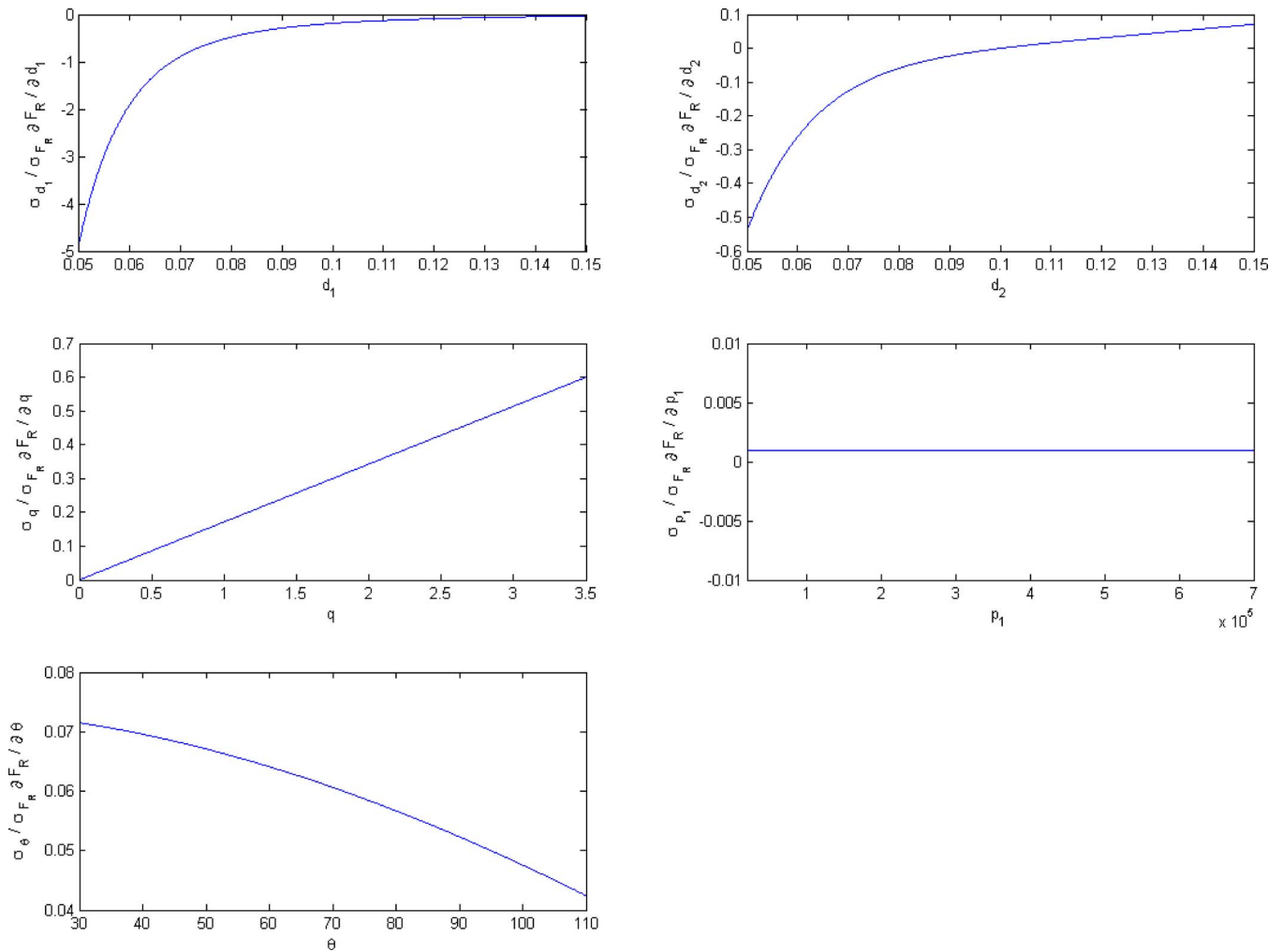


Fig. 3. Standardized derivatives (Note changing vertical scale. In each plot, variables not shown are set at their mean values.)

(Fig. 3) varies over the range of the input factors, indicating that they are only locally informative as a sensitivity measure. Fig. 3 gives only limited insight into the behavior of the partial derivatives, as it does not show the effect of interactions between variables.

Global sensitivity analysis was based on Monte Carlo sampling of the input factors and calculating the resultant force on the pipe bend. The system response can be visualized in scatter-plots (Fig. 4) (Helton 1993; Kleijnen and Helton 1999), which plot the resultant force for each member of the Monte Carlo sample as a function of the five input factors. Scatter plots are the simplest form of analysis and can reveal nonlinear relationships, parameter thresholds and, if plotted in two dimensions, variable interactions, which can aid in the understanding of model behavior (Saltelli et al. 2005). Fig. 4 at a glance indicates noticeable sensitivity to Q and nonlinear sensitivity to D_1 .

Table 2 shows that the sensitivity based on the SRCs, β_i , only captures 49% of the variance of the model output. The coefficient of determination $R^2=0.53$, indicating that the linear regression model is not satisfactory. The first order FAST based sensitivity indices, S_i , explain 62% of the total variance. This means that the additive component of the model that is not linear accounts for 13% (i.e., 62–49%) of the variability of the model output and implies that at least 38% of the variation is due to nonlinear effects as a result of higher order interactions taking place among

the uncertain variables. The same shares can be seen in Table 2 for each input factor separately. While the range of partial derivatives of the resultant force with respect to q is much less than that with respect to d_1 (Fig. 3), the first order sensitivity indices are

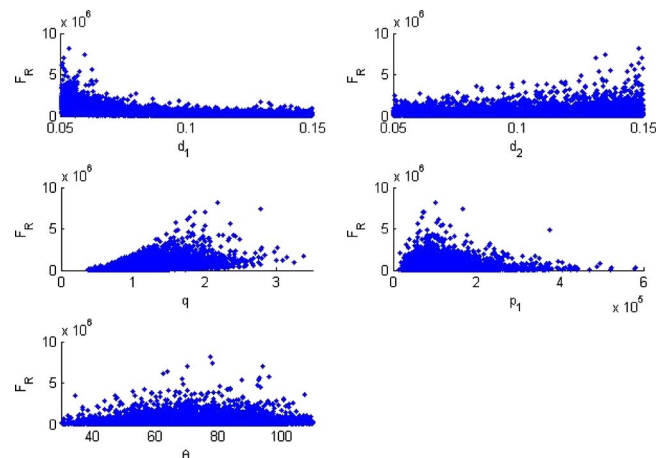


Fig. 4. Scatter plots for the resultant force F_R (N) as a function of input factors

Table 2. Sensitivity Measures for Force on Pipe Bend

Variables	β_i^2	S_i	S_{Ti}
D_1	0.263	0.381	0.765
D_2	0.002	0.014	0.188
Q	0.210	0.215	0.370
P_1	0.000	0.000	0.044
θ	0.015	0.010	0.060
Total	0.49	0.62	

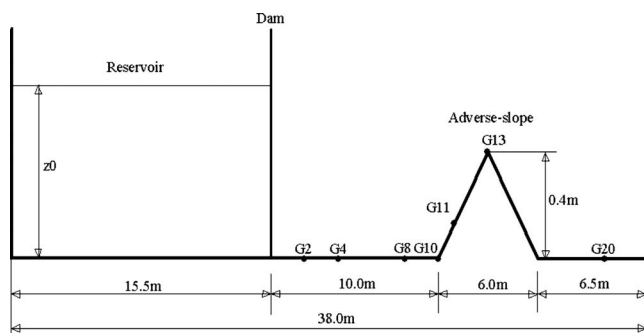
not so different. This reflects the fact that the FAST sensitivity indices are probability weighted, so the very strong sensitivity at extreme low end of the range of d_1 is not particularly influential in S_i . Nonetheless, D_1 still emerges as the most influential variable.

Based on the first order analysis, it is evident that the factors that offer the best chance of minimizing the variation of the force on the pipe bend are D_1 and Q , which together account for just under half of the variations the indices capture. However, the size of the unknown interactions suggests that a much larger reduction can be achieved if the interacting factors can be identified. The total sensitivity indices, S_{Ti} , (Table 2) are all more than 70% greater than the corresponding first order indices, demonstrating the importance of interactions between the variables.

Sensitivity Analysis of a Hydrodynamic Model of a Dam-Break Experiment

Advanced hydrodynamic simulation codes can exhibit nonlinear behavior and may be sensitive to boundary conditions. They are therefore candidates for more rigorous use of sensitivity analysis methods. Here the Shallow Water Equation solver of Liang et al. (2004, 2007, 2008) has been applied to the benchmark example of the sudden release of water in a flume including an adverse slope.

The simulation model setup corresponds to the flume experiment reported by Brufau et al. (2002) and is illustrated in Fig. 5, in which water is retained behind a gate (“dam”) which is suddenly removed vertically, allowing the water to rush from left to right over the adverse slope. The whole channel is 38 m long. The 15.5 m long reservoir is connected with the bump by a straight channel of 10 m length. The adverse-slope of the bump is 3 m long and 0.4 m high followed by a slope with 3 m length. Initially, the water in the reservoir is still with a water depth z_0 . In the dry bed case tested here, the water depth of the channel downstream of the bump is zero. The time taken to remove the gate (dam), is denoted t_r . The experiment lasted roughly 40 s. The

**Fig. 5.** Hydrodynamic model setup

gauging points, “G,” are defined by the distance (in meters) between the gauging point and the dam. Water surface elevation, z , and velocity, u , were measured at these points.

The two-dimensional Shallow Water Equation solver was set up to reproduce these experimental conditions. The regular orthogonal grid was 100×10 cells. Slip boundary conditions were imposed in the whole channel except at the right hand end of the channel, which is set to be open outflow. Typical outputs of water surface elevation, z , (which can be compared with observations) and velocity, u , at the seven gauges are illustrated in Fig. 6.

The system is sensitive to the dam removal time (T_r), the water surface elevation in the reservoir (Z_0) and the Manning’s friction coefficient (N) in the flume, so these were investigated using the distributions given in Table 3. The range for T_r and N have deliberately been chosen to be quite wide in order to illustrate the potential for nonlinear response. Sensitivity analysis was conducted using standardized regression coefficients and extended FAST. Results for two gauging points are presented in Table 4.

At gauge G4 the SRCs β_i captures 65% of the variance in the prediction of water surface elevation and 81% of the variance in the velocity. Nonetheless, the FAST total sensitivity indices S_{Ti} indicate appreciable interaction between the variables (they are 30%–50% higher than the first order indices S_i). In this case, these interactions are sufficient to alter the importance rankings obtained from the linear regression analysis (which excludes the interactions) and the FAST analysis in which the interacting terms are included via the expansion in Eq. (7).

At G10, the linear regression is clearly not well determined. The small sum of the first order FAST indices indicates strong nonlinearity and interaction between the variables. With such a low coefficient of determination, the SRCs do not provide a reliable guide to sensitivity. Under these circumstances, the FAST total order sensitivity indices, S_{Ti} , provide a global measure of sensitivity that accounts for the interactions between variables.

To illustrate the variation in sensitivity through the experiment, the FAST first order indices for Z and U are plotted over the whole length of the flume (right of the dam) and duration of the experiment in Fig. 7. While the pattern of sensitivity illustrated in Fig. 7 is complex, it is clear that downstream of the adverse slope (downstream of G13) Manning’s n is the most important variable for determining both Z and U . Upstream of G13, the interactions between variables are more influential and the sensitivity oscillates, though from $t=15$ s to roughly $t=30$ s T_r is the most significant variable in determining both Z and U . To understand the complex patterns of sensitivity in nonlinear models like the one tested here requires computation of the full range of indices and careful scrutiny of their variation in space and time.

Choice of Methods

The choice of the most appropriate sensitivity analysis technique depends upon

- The computational cost of running the model.
- The number of input factors.
- The degree of nonlinearity of the model over the range of input factors.
- The degree of complexity of the model coding.
- The amount of analyst’s time available for sensitivity analysis.
- The setting for the analysis.

Exploratory analysis with scatter plots and computation of SRCs can provide initial insights into the degree of nonlinearity

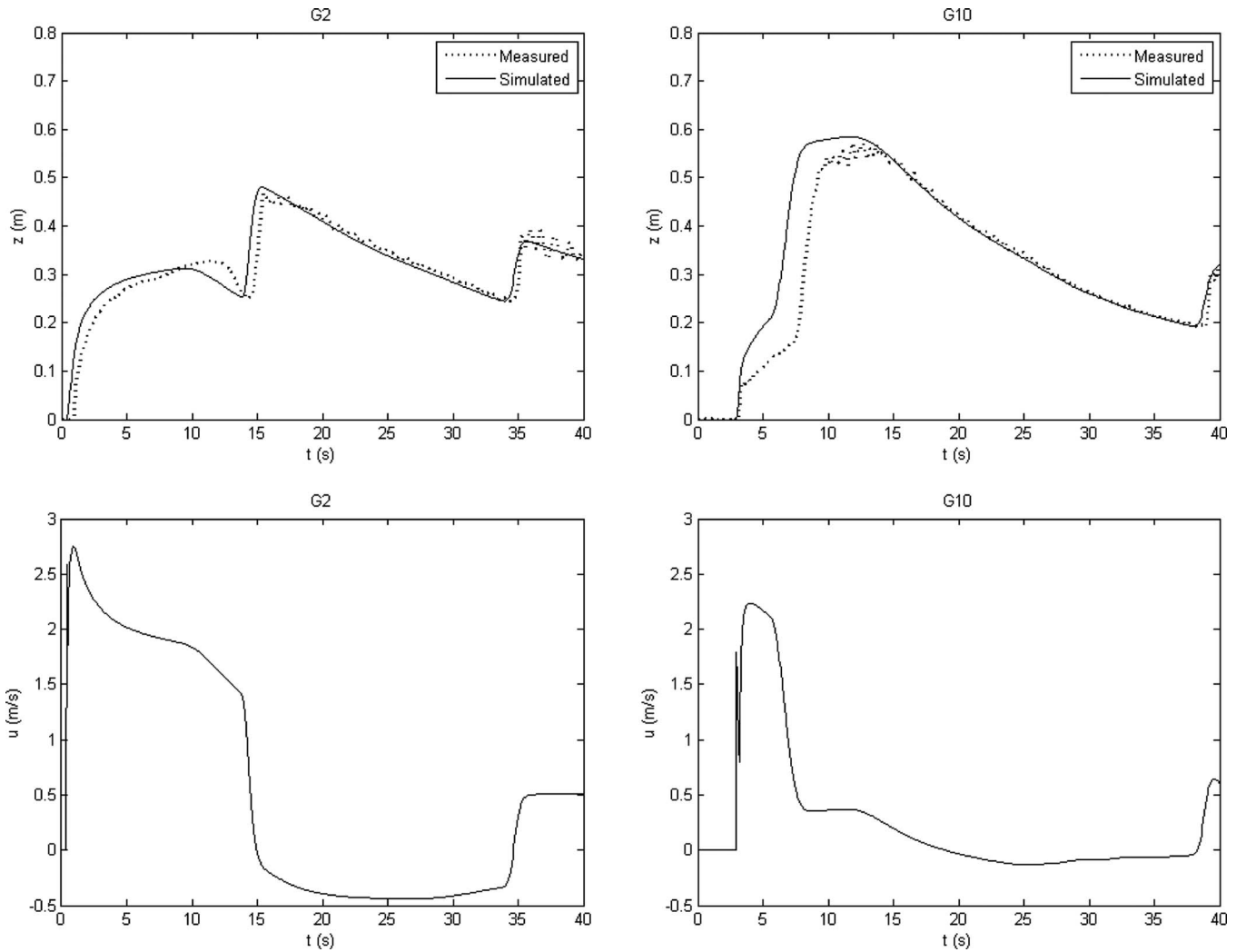


Fig. 6. Typical outputs of the Shallow Water Equation solver, where G2 and G10 are the locations shown in Fig. 5

in model response and a computationally cheaper alternative to the variance-based methods. With a single batch of m sampled points, the SRCs and their rank transformed version can be estimated for all the input factors, though, as already discussed, the SRC's are only effective for linear or quasilinear models, i.e., for $R^2 \geq 0.7$. Manache and Melching (2008) provide a thorough re-

view of regression and correlation measures and their properties and provide guidance on selecting the most robust and reliable measures for practical use.

If R^2 is high, then SRCs are a convenient sensitivity index to use. If this is not the case, then for models that require a modest amount CPU time (i.e., up to the order of 1 min per run), and with a number of input factors which does not exceed, say, 20, the variance-based techniques yield a more accurate pattern of sensitivity (Tang et al. 2006). Both the method of Sobol' (Saltelli 2002; Saltelli et al. 2000; Sobol' 1993) and the extended FAST (Saltelli et al. 1999) provide all the pairs of first order and total indices at a cost of $(k+2)m$ model runs for Sobol' and $\approx km$ model runs for the extended FAST. Typically m is between 500 and 1,000. To

Table 3. Distributions of Input Factors for Dam-Break Example

Variables	Distribution type	Parameters
Dam removal (T_r)	Lognormal	$\ln(T_r) \sim N(0.8, 0.5)$
Free surface (Z_0)	Normal	$Z \sim N(0.75, 0.03)$
Manning's (N)	Lognormal	$\ln(N) \sim N(-4.8, 0.5)$

Table 4. Comparison of FAST and SRCs for Selected Gauge Points ($t=8$ s) (Sensitivity of Water Surface Elevation, Z , and Velocity, U)

Gauge	G4: Z			G4: U			G10: Z			G10: U		
	β_i^2	S_i	S_{Ti}	β_i^2	S_i	S_{Ti}	β_i^2	S_i	S_{Ti}	β_i^2	S_i	S_{Ti}
T_r	0.37	0.19	0.27	0.15	0.36	0.52	0.01	0.05	0.70	0.01	0.06	0.68
Z_0	0.07	0.11	0.16	0.12	0.09	0.17	0.03	0.03	0.59	0.02	0.03	0.53
N	0.21	0.52	0.62	0.55	0.31	0.45	0.26	0.16	0.89	0.25	0.19	0.90
Total	0.65	0.83	—	0.81	0.77	—	0.29	0.24	—	0.27	0.28	—

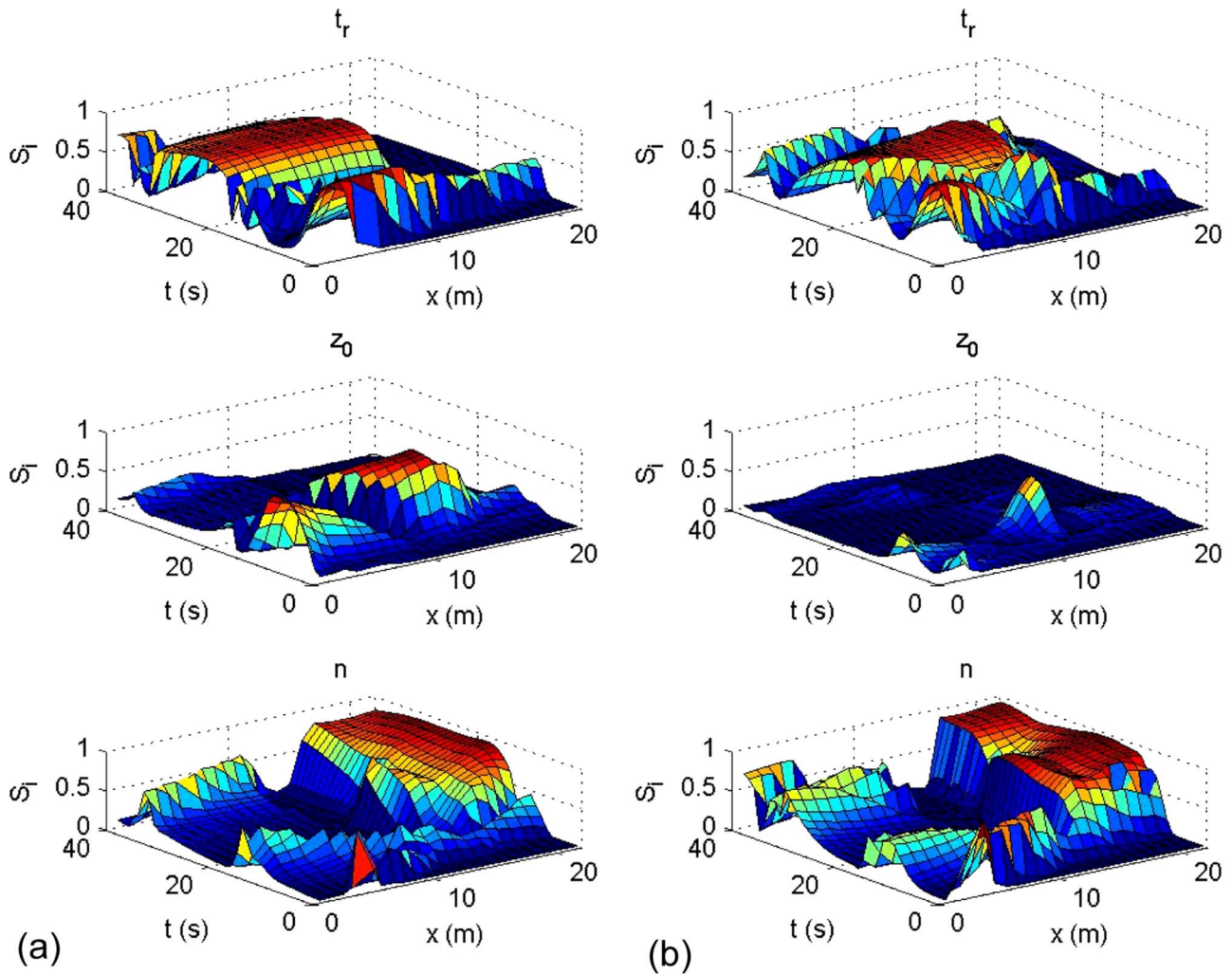


Fig. 7. First-order FAST sensitivity indices plotted against distance down the flume (x) and time during the experiment (t) [(a) surface elevation, Z ; (b) velocity, U]

give an order of magnitude of the computational requirements, for a model with 10 factors and 0.5 min of CPU time per run, a good characterization of the system via S_i and S_{T_i} can be obtained at the cost of ~ 50 hours of CPU time.

With the method of Sobol', in addition to the first order and total indices computed with $(k+2)m$ model runs, all the interaction terms of order $k-2$ can be obtained at no extra cost. At the additional cost of km model runs, double estimates of all of the first, second, $(k-2)$ -th order and total indices can be obtained. Finally, any other interaction term between the third and the $(k-3)$ -th can be estimated at the further additional cost of m model runs each (Saltelli 2002).

When the input factors are correlated, an ad hoc computational scheme must be adopted. An efficient and unbiased estimation procedure is available for first order indices and is based on replicated Latin hypercube sampling (Hamm et al. 2006; McKay et al. 1979), in which the cost to estimate all the first order indices is mr model runs, where r is the number of replicates needed (usually around 100), and the cost is independent of the number of factors.

For higher order indices as well as total indices in the case of correlated input one has to apply a brute force approach whereby the operators V and E [Eq. (5)] are written in explicit form i.e., as the variance of a mean, involving a double computing loop. The

computational cost is, thus, mr model runs for each index.

When the CPU time increases (say, up to ten minutes per run), or the number of factors increases (say, up to 100), the method of Morris (1991) offers the best result. The number of sampled points required is $m_{\text{Morris}}=r(k+1)$, where r is generally set to between 4 and 8 and k is the number of input factors. To make an example, with 80 factors and 5 min CPU time per run, all the model outputs can be ready in 27 h if $r=4$ is taken.

When the number of input factors and/or the CPU time is so large as to even preclude the use of the method of Morris (1991), then supersaturated fractional factorial designs, where factors are iteratively perturbed in batches, can be used (Campolongo et al. 2000; Iman and Hora 1990). However, these methods do not provide an effective exploration of the space of the inputs, as they mostly operate at very few factor levels and require strong assumptions on the model behavior.

Automatic differentiation techniques can also be used when CPU time is very large. Derivatives are inherently local sensitivity indices. In addition, they require intervention of the analyst in the computer code that implements the model. However, for expensive models, these methods may provide some insight into the importance of input factors. If higher order derivatives are computed, these give information about multifactor curvature effects e.g., second order term of the type $\partial^2 Y / \partial X_i \partial X_j$ gives information

about a possible interaction effect between X_i and X_j .

An alternative to the direct use of sampling-based sensitivity methods is to construct a computationally efficient emulator of the numerical model in question and conduct sensitivity analysis using the emulator function (Storlie et al. 2008). Emulator methods based on the use of Gaussian processes can yield variance-based sensitivity indices directly (Oakley and O'Hagan 2004). Gaussian process emulators have proved to be very efficient, even for complex nonlinear models (Hankin 2005). However, they are based upon an assumption of smoothness over a continuous input space, so they are not universally appropriate, and can be hard to identify in high-dimensional problems. In high dimensional problems a variety of kernel regression (Storlie and Helton 2006), high-dimensional model representation (HDMR) (Li et al. 2002) and filtering methods (Ratto et al. 2007a) have been developed for constructing emulators over the most influential factors.

Sensitivity analysis is also driven by the setting. When the purpose of the analysis is to prioritize factors, the first order sensitivity indices S_i have a strong motivation for use. If the objective is to fix noninfluential factors, then the total sensitivity indices S_{Ti} , or the measure of Morris (1991), are more appropriate. If a particular region in the space of the output (e.g., above or below a given threshold) is of interest, then Monte Carlo filtering can complement the measures just mentioned. In all of these settings, the computation of derivatives, especially if achieved with a modicum of extra computing, is advisable for a general understanding of the model.

Sensitivity analysis can also be extended to address issues of model choice. For example there may be alternative representations of a particular process within a simulation model. These alternative process modules can be indexed by an integer "switching" variable, which is then included in the uncertainty analysis, usually with a discrete uniform distribution over the set of possible modules (Tarantola et al. 2002). The sensitivity index of the switching variable can be compared with the sensitivity to parameter uncertainties. This type of analysis is particularly attractive in the context of complex coupled systems of models, where there may be a number of permutations of model choice and the influence of those choices on predictions are not necessarily intuitive.

Conclusions

Sensitivity analysis is an essential aspect of responsible model use, particularly at a time when models are becoming more complex and are being coupled in order to address multidisciplinary problems. Decision makers who make use of hydraulic model results can legitimately request thorough analysis of the sensitivity of the results to plausible variations in the model inputs. This information can be used to target data acquisition and engineering design decisions more effectively by identifying the parameters that exert the greatest influence on system performance.

This paper has sought to introduce members of the hydraulic engineering and research community to methods of sensitivity analysis with which they may not be familiar. It has been demonstrated how the incautious use of local derivatives or regression coefficients can lead to misleading conclusions that do not represent the full range of model behavior for nonlinear models. Scatter and surface plots provide a useful visual impression of sensitivity but for a limited number of input factors. Variance-based methods can deal with interacting input factors and provide a metrics of sensitivity averaged over the range of input response. However, the use of variance-based methods requires probability

distributions for the input factors. Some sense of the range of variability of the input factors is to be expected for model applications, but the meaning of input variance is less clear in the context of a numerical model being tested prior to application. In that case the use of uniform distributions over the range of applicability is perhaps the most sensible approach.

The use of first order and total variance sensitivity indices has been demonstrated to diagnose the effect of input factors acting individually or in combination, in the latter case in order to identify the effect of interactions between variables. A further attraction of the use of variance-based methods is that they are model independent and can be applied with no modification to the model code.

The computational limitations of sensitivity analysis have been discussed. In situations with computationally very expensive models or large numbers of inputs a sequential approach to sensitivity analysis, which proceeds through a process of screening and hierarchical decomposition of groups of variables is required. Where model response is reasonably smooth but very expensive to compute, the use of emulator functions can yield large computational savings.

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Notation

The following symbols are used in this paper:

- b_i = regression coefficient;
- D_1, D_2 = random variables for diameters;
- d_1, d_2 = specific pipe diameter values;
- E = expectation operator;
- E_{X_i} = expectation with respect to input factor X_i ;
- $E_{X_{-i}}$ = expectation with respect to all input factors other than X_i ;
- F_x, F_y = orthogonal pipe force components;
- F_R = magnitude of resultant force;
- G2, G4, G8, G10, G11, G13, and G20 = gauges in flume;
- g = acceleration due to gravity;
- i, j, l = counters;
- k = number of input factors;
- $N(\cdot, \cdot)$ = specification of normal distribution function;
- N = random variable for Manning's n ;
- n = specific Manning's n values;
- m = number of realizations in a statistical simulation;
- P_1, P_2 = random variables for pressures;
- p_1, p_2 = specific pressures values;
- Q = random variable for discharge in pipe;
- q = specific values of discharge in pipe;
- R_Y^2 = coefficient of determination;
- r = residual;
- S_i = first order variance-based sensitivity index for factor X_i ;
- S_i^r = nondimensionalized derivatives-based sensitivity index;

σ_i^s = sigma standardized derivatives-based sensitivity index;
 S_{Ti} = total order variance-based sensitivity index for factor X_i ;
 T_r = random variable for dam removal time;
 t = time;
 t_r = specific value of dam removal time;
 U = random variable for velocity;
 $U(\cdot, \cdot)$ = specification of uniform distribution function;
 u = specific value of velocity in flume;
 V = variance operator;
 V_i = variance component from factor i ;
 X_i = model input factor, i (random variable);
 \tilde{X}_1 = standardized variable X_i ;
 \mathbf{X}_{-i} = the vector of all factors other than X_i ;
 x = distance down flume;
 x_i = specific value of model input factor, i ;
 x_i^* = known value of model input factor X_i ;
 \bar{x}_i = is the nominal value of factor X_i ;
 Y = model output (random variable);
 \tilde{Y} = standardized variable Y ;
 y = specific value of model output;
 \bar{y} = value taken by Y when all input factors are at their nominal value;
 \hat{y}_i = prediction of y_i from a regression model;
 Z = random variable for surface elevation in flume;
 Z_0 = random variable for surface elevation in reservoir;
 z = specific value of surface elevation in flume;
 z_0 = specific value of surface elevation in reservoir;
 β_i = standardized regression coefficient;
 θ = pipe bend angle;
 μ_i = mean of factor X_i ;
 μ_Y = mean of Y ;
 σ_i = standard deviation of factor X_i ; and
 σ_Y = standard deviation of Y .

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