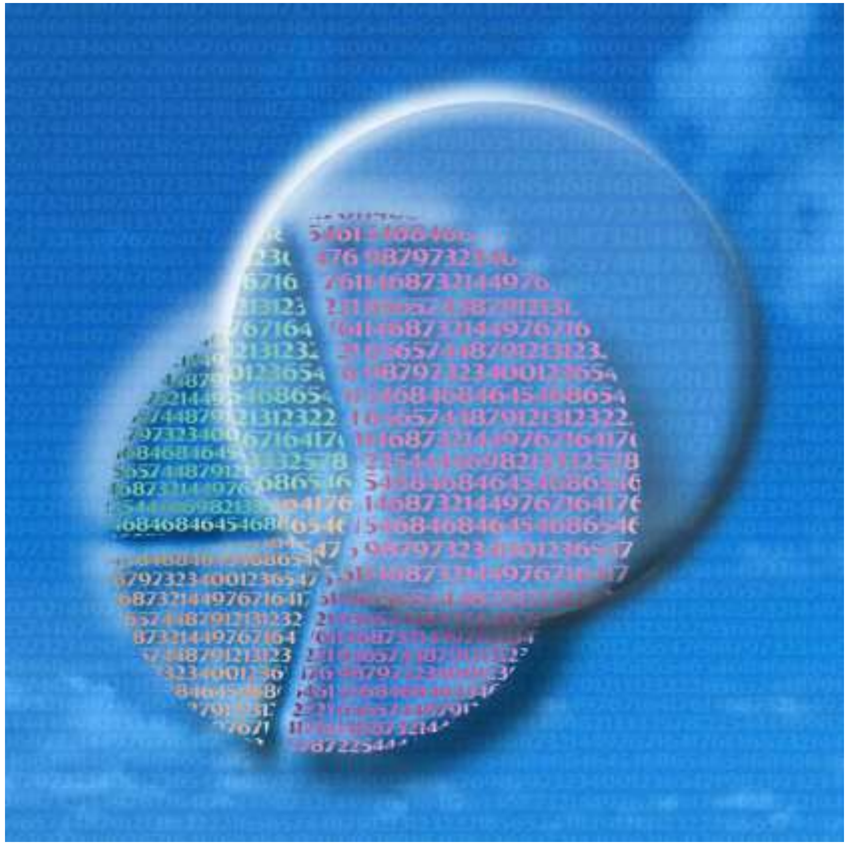

Sensitivity analysis, an introduction

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Seminar at CERFACS, December 3rd, 2019, 10.30, 42 Avenue Gaspard Coriolis, Toulouse

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Where to find this talk: www.andreasaltelli.eu

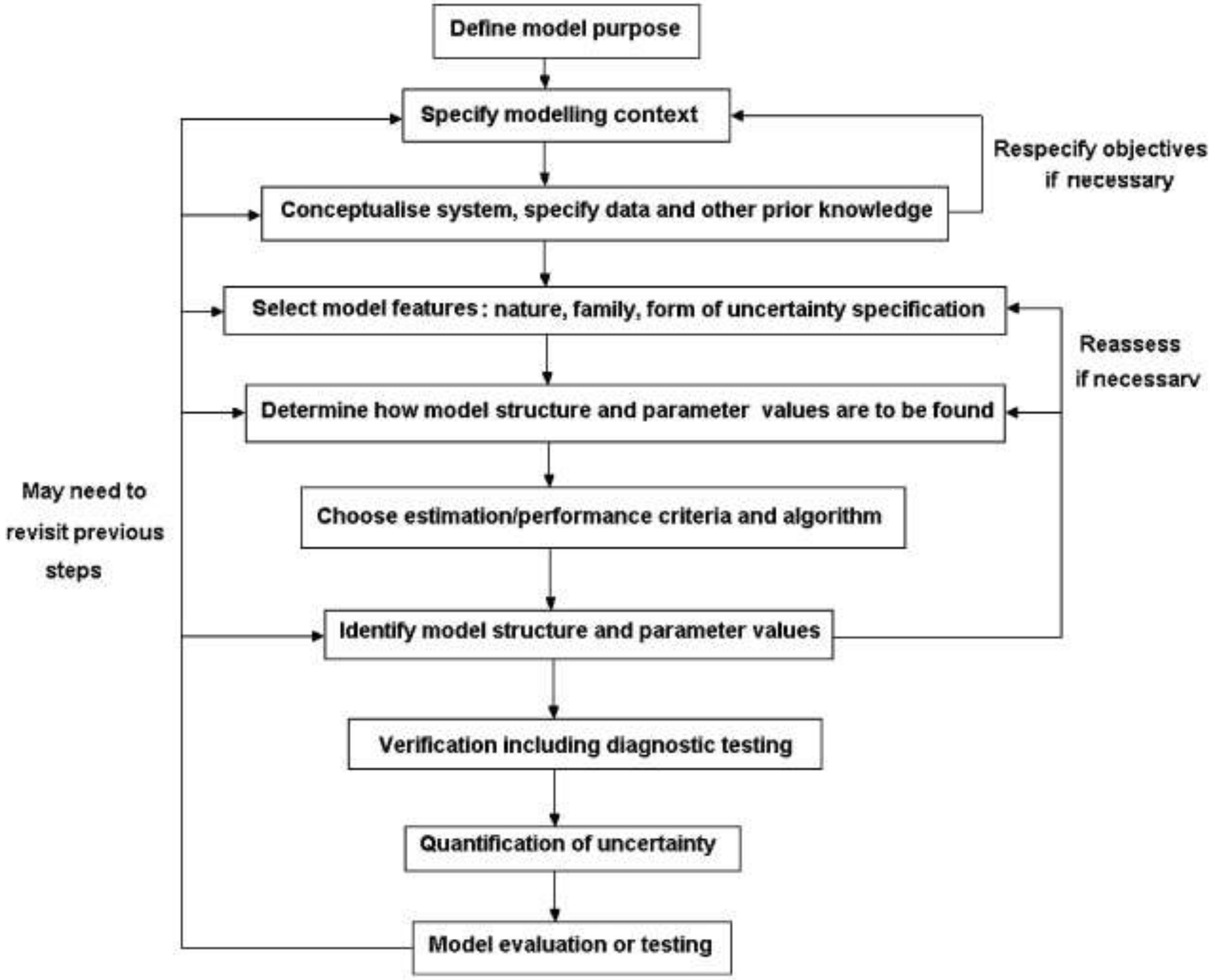
On modelling

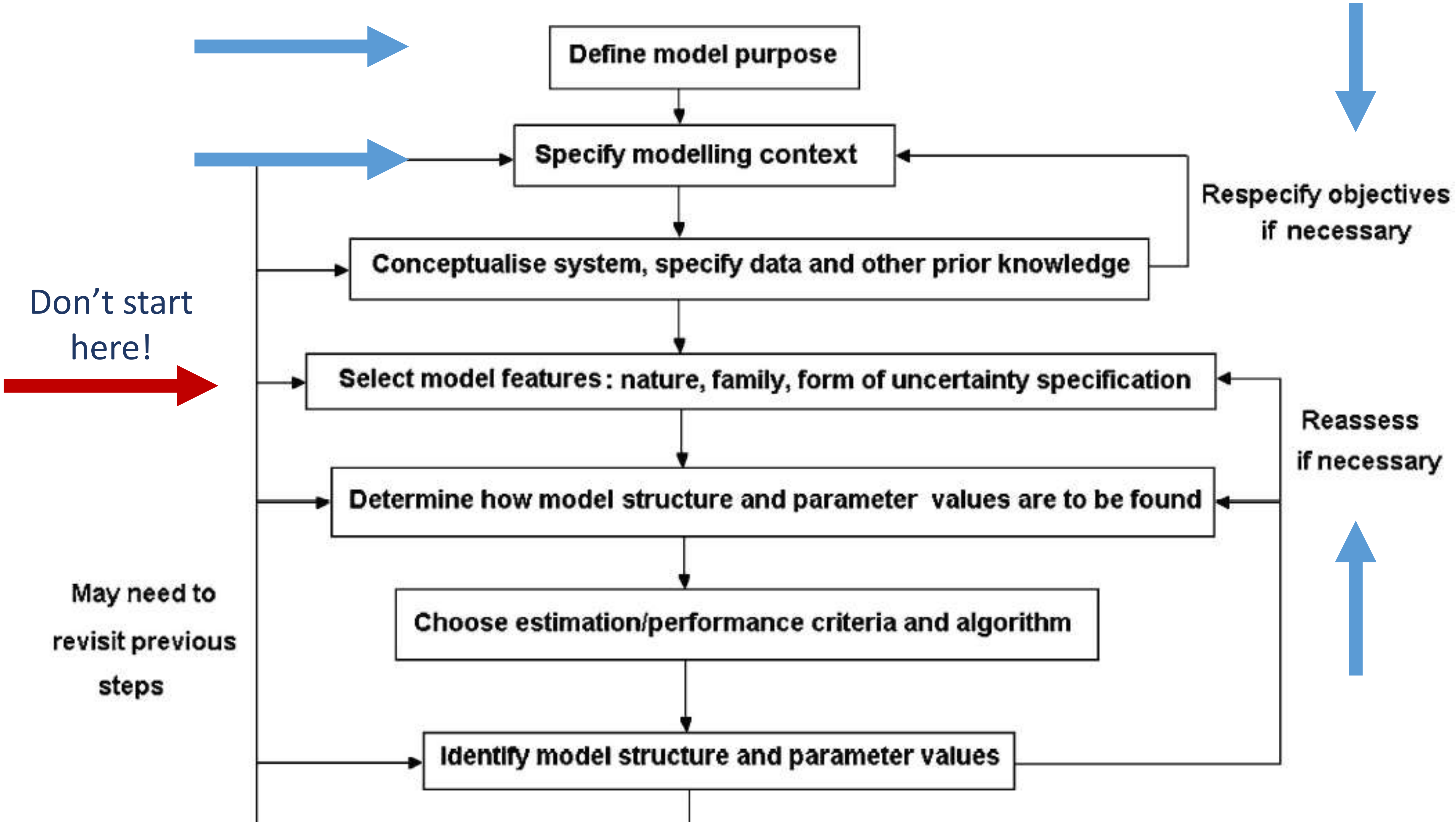
Padilla et al. call for a more structured, generalized and standardized approach to verification

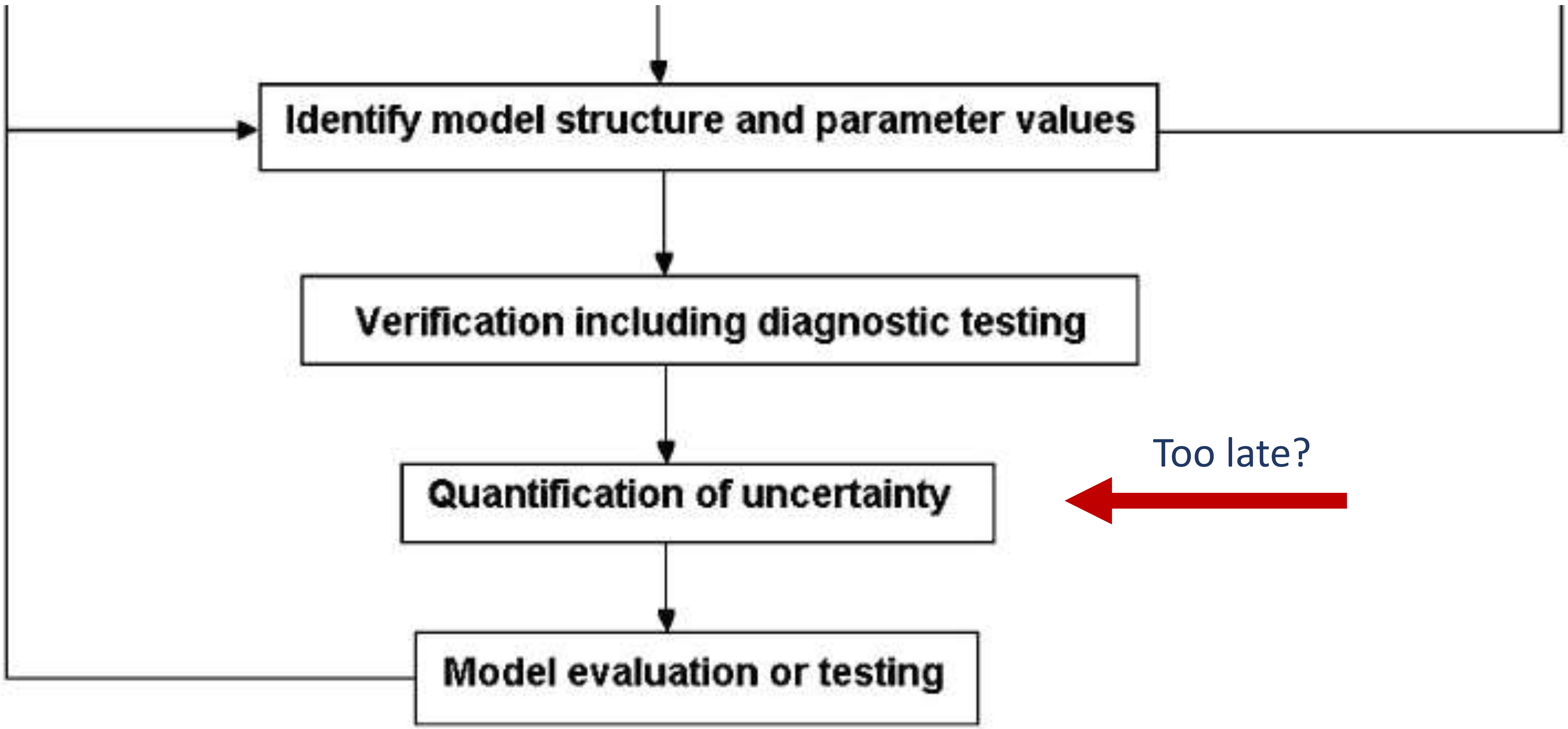
Jakeman et al. call for a 10 points participatory checklist

Padilla, J. J., Diallo, S. Y., Lynch, C. J., & Gore, R. (2018). Observations on the practice and profession of modeling and simulation: A survey approach. SIMULATION, 94(6), 493–506.

Jakeman, A. J., Letcher, R. A., & Norton, J. P. (2006). Ten iterative steps in development and evaluation of environmental models,. Environmental Modelling & Software, 21(5), 602–614.


Define model purpose
Specify modelling context
Respecify objectives
if necessary
Conceptualise system, specify data and other prior knowledge
Select model features : nature, family, form of uncertainty specification
Reassess
if necessarv
Determine how model structure and parameter values are to be found
May need to
revisit previous
steps
Choose estimation/performance criteria and algorithm
Identify model structure and parameter values
Verification including diagnostic testing
Quantification of uncertainty
Model evaluation or testing


Define model purpose
Specify modelling context
Respecify objectives
if necessary
Conceptualise system, specify data and other prior knowledge
Don't start
here!
Select model features : nature, family, form of uncertainty specification
Reassess
if necessary
Determine how model structure and parameter values are to be found
May need to
revisit previous
steps
Choose estimation/performance criteria and algorithm
Identify model structure and parameter values


Identify model structure and parameter values
Verification including diagnostic testing
Quantification of uncertainty
Model evaluation or testing
Too late?

Modelling is not a discipline

Unlike statistics, mathematical modelling is not a discipline, hence the lack of universally accepted quality standards, disciplinary fora and journals and recognized leaders

Saltelli, A., 2018, Discussion Paper: Should statistics rescue mathematical modelling? https://arxiv.org/abs/1712.06457

Model-based knowing is conditional

For John Kay modelling may need as input information which we don’t have

John Kay

J. A. Kay, “Knowing when we don’t know,” 2012, https://www.ifs.org.uk/docs/john_kay_feb2012.pdf

WebTAG: Annual Percentage Change in Car Occupancy (% pa) up to 2036

Journey Purpose	Weekday					Weekend	All Week
	7am-10am	10am-4pm	4pm-7pm	7pm-7am	Weekday Average		
Work	-0.48	-0.4	-0.62	-0.5	-0.44	-0.48	-0.45
Non - Work (commuting and other)	-0.67	-0.65	-0.53	-0.47	-0.59	-0.52	-0.56

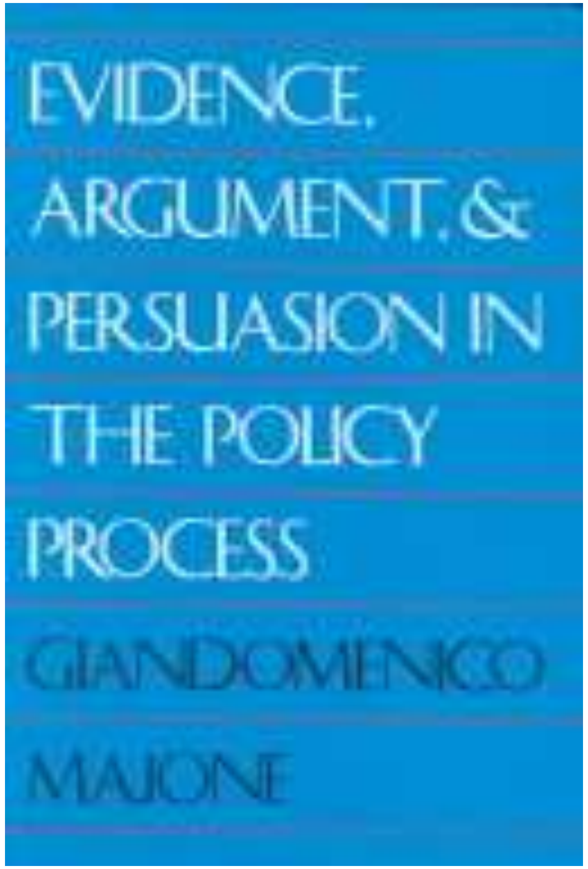
Uncertainty and sensitivity analysis

Definitions

Uncertainty analysis: Focuses on just quantifying the uncertainty in model output

Sensitivity analysis: The study of the relative importance of different input factors on the model output

Why Sensitivity analysis?



"Are the results from a particular model more sensitive to changes in the model and the methods used to estimate its parameters, or to changes in the data?"

European Commission, 2015

Office for the Management and Budget, 2006

Environmental Protection Agency, 2009

EPA, 2009, March. Guidance on the Development, Evaluation, and Application of Environmental Models. Technical Report EPA/100/K-09/003. Office of the Science Advisor, Council for Regulatory Environmental Modeling, http://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=P1003E4R.PDF, Last accessed December 2015.

EUROPEAN COMMISSION, Better regulation toolbox, appendix to the Better Regulation Guidelines, Strasbourg, 19.5.2015, SWD(2015) 111 final, COM(2015) 215 final, http://ec.europa.eu/smart-regulation/guidelines/docs/swd_br_guidelines_en.pdf.

OMB, Proposed risk assessment bulletin, Technical report, The Office of Management and Budget's – Office of Information and Regulatory Affairs (OIRA), January 2006, https://www.whitehouse.gov/sites/default/files/omb/assets/omb/inforeg/proposed_risk_assessment_bulletin_010906.pdf, pp. 16–17, accessed December 2015.

http://ec.europa.eu/smart-regulation/

Source: IA Toolbox, p. 391

EUROPEAN COMMISSION

Better Regulation "Toolbox"

4. Sensitivity and Uncertainty Analyses

Page 391

Six steps for a global SA:

1. Select one output of interest;
2. Participatory step: discuss which input may matter;
3. Participatory step (extended peer review): define distributions;
4. Sample from the distributions;
5. Run (=evaluate) the model for the sampled values;
6. Obtain in this way bot the uncertainty of the prediction and the relative importance of variables.

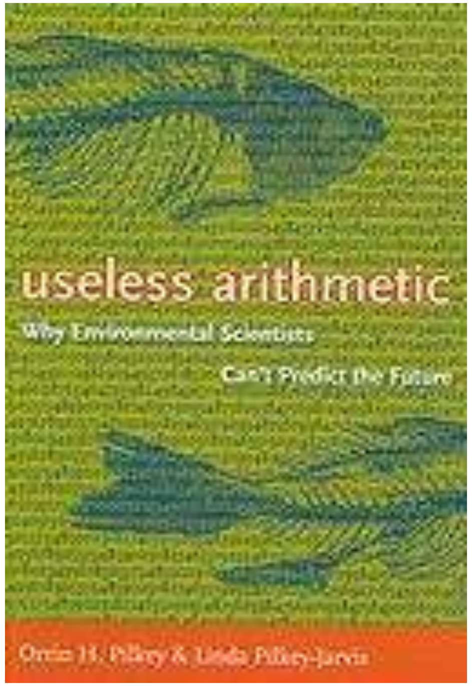
Is something wrong with this statement (p. 384 of EC guidelines)

The influence of the key variables

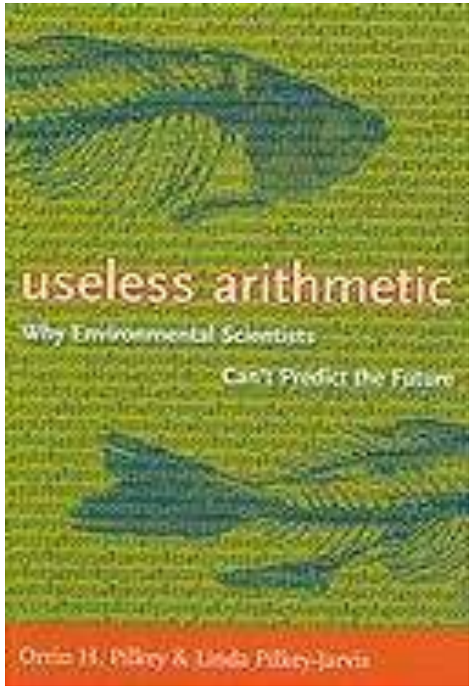
should be investigated by a sensitivity analysis.

Limits of sensitivity analysis

Orrin H. Pilkey



Useless Arithmetic: Why Environmental Scientists Can't Predict the Future by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

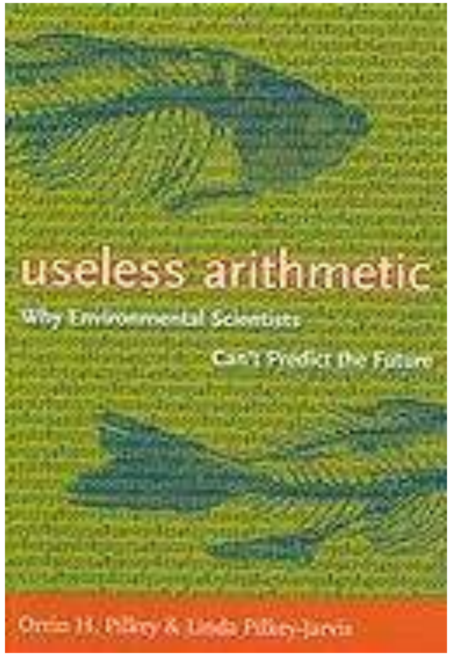


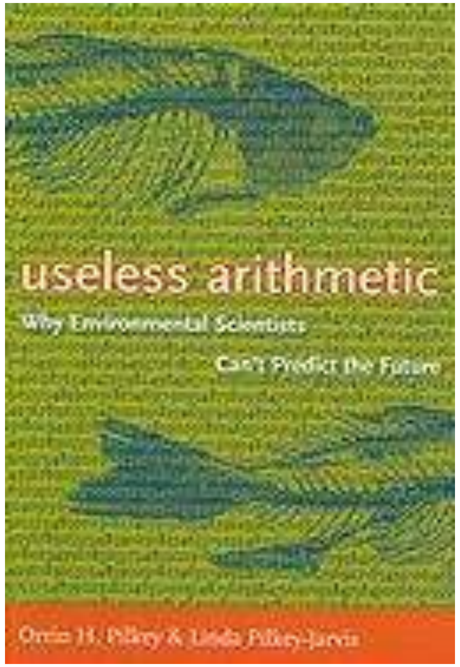
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

[…] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

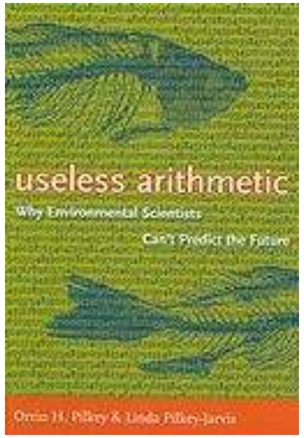
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

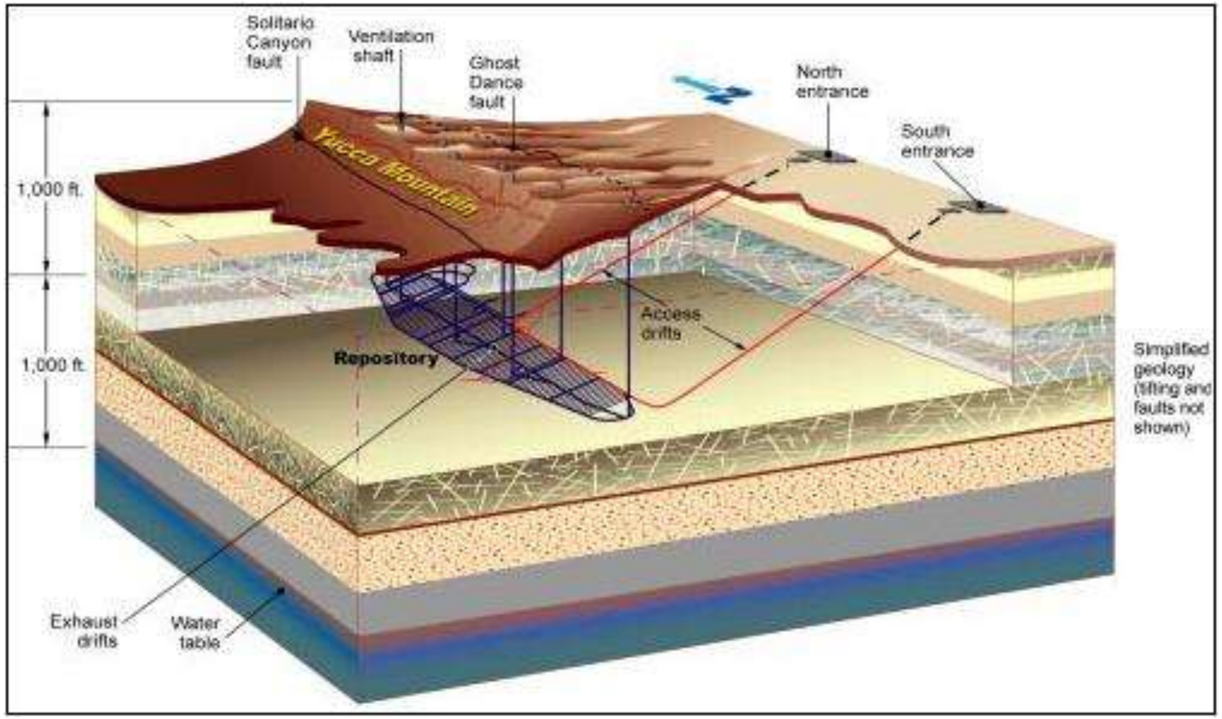
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.





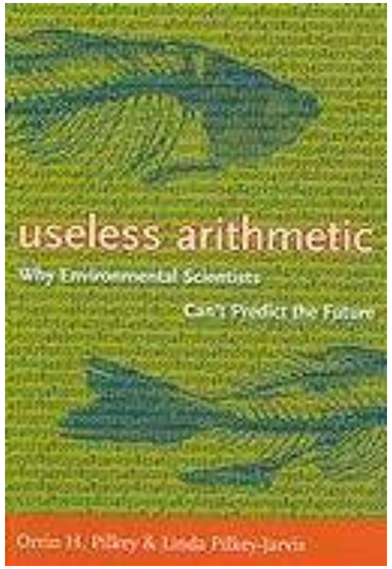
The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter

(the ^{36}Cl story)

Type III error in sensitivity: Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→… SA useless if it is instead ~ 3,000 millimetres per year.



Robert K. Merton

"Scientific mathematical modelling should involve constant efforts to falsify the model"

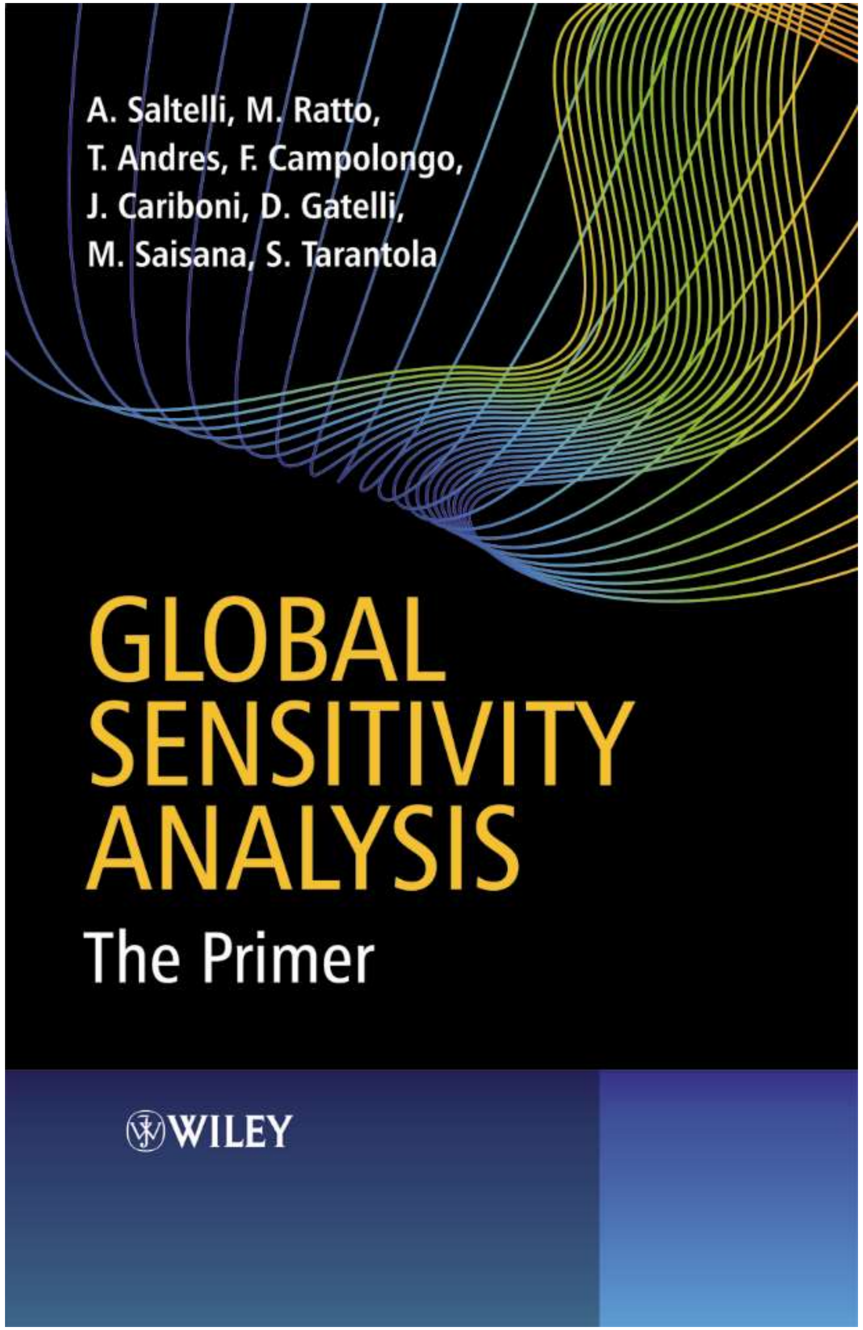
Ref. ➔ Robert K. Merton's 'Organized skepticism '

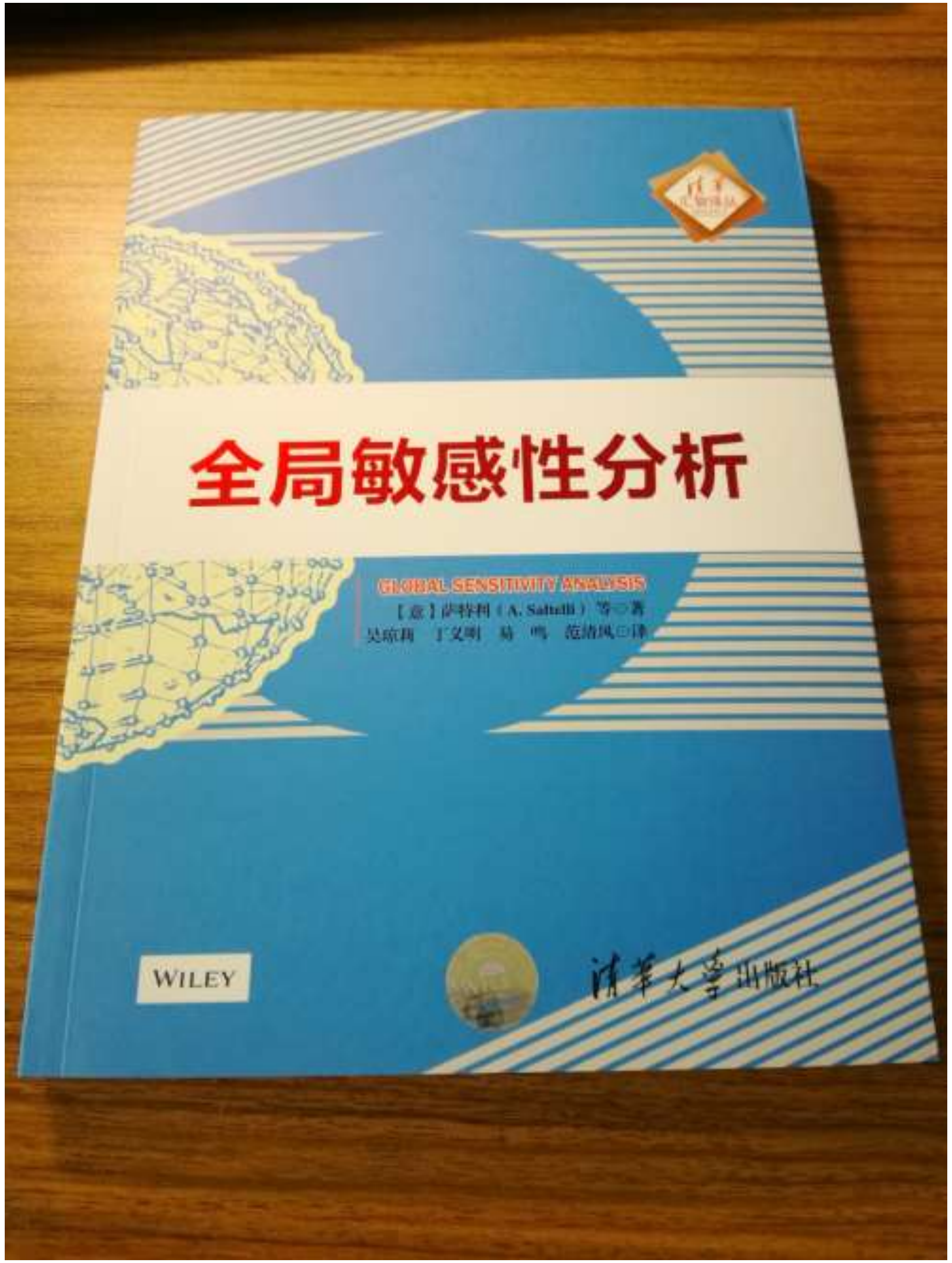
Communalism – the common ownership of scientific discoveries, according to which scientists give up intellectual property rights in exchange for recognition and esteem (Merton actually used the term Communism, but had this notion of communalism in mind, not Marxism);

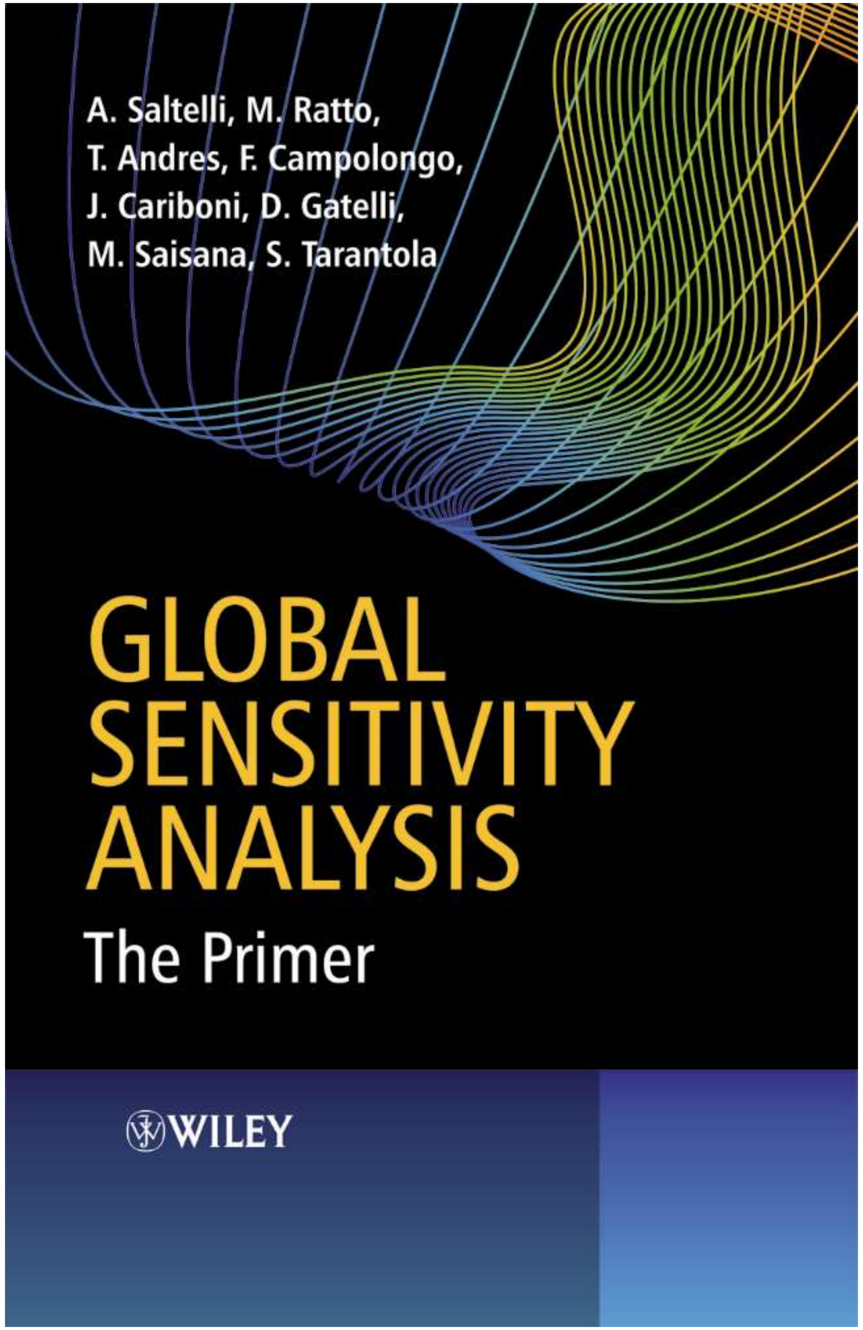
Universalism – according to which claims to truth are evaluated in terms of universal or impersonal criteria, and not on the basis of race, class, gender, religion, or nationality;

Disinterestedness – according to which scientists are rewarded for acting in ways that outwardly appear to be selfless;

Organized Skepticism – all ideas must be tested and are subject to rigorous, structured community scrutiny.


A. Saltelli, M. Ratto,
T. Andres, F. Campolongo,
J. Cariboni, D. Gatelli,
M. Saisana, S. Tarantola
GLOBAL
SENSITIVITY
ANALYSIS
The Primer
WILEY

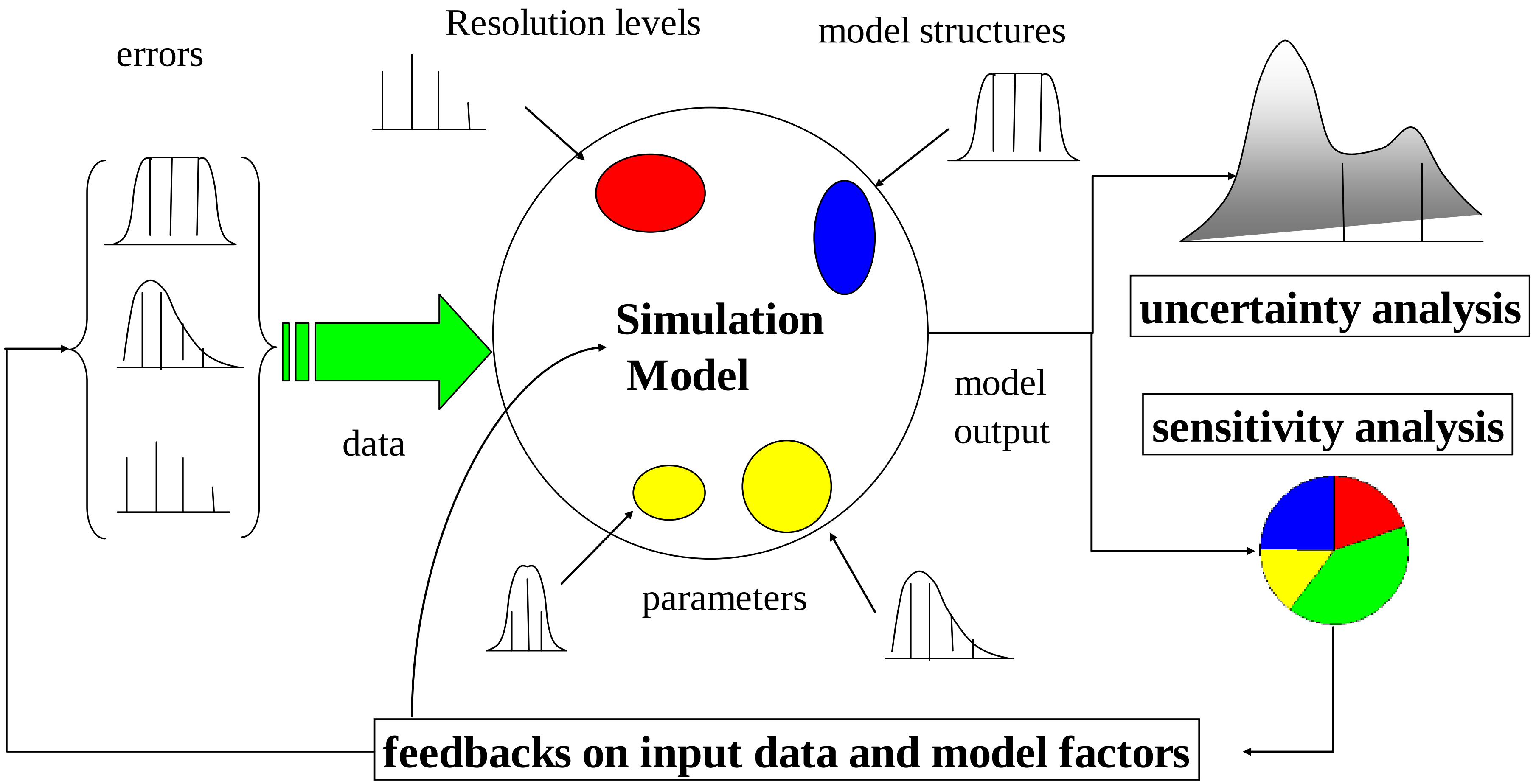

全局敏感性分析
GLOBAL SENSITIVITY ANALYSIS
【意】萨特利（A. Saltelli）等 著
WILEY
清华大学出版社



Hacked! available for free at

http://www.andreasaltelli.eu

An engineer's vision of UA, SA

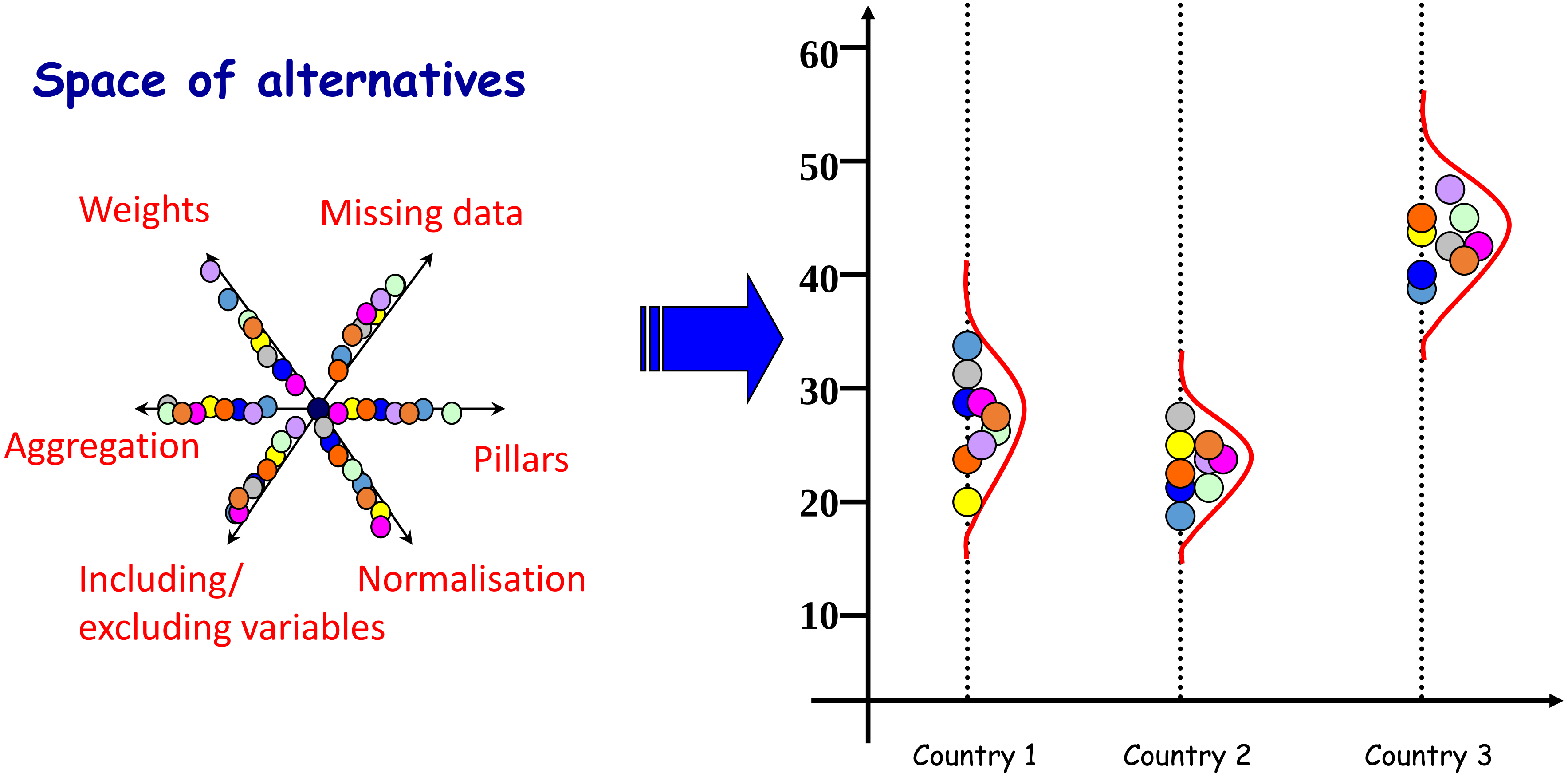


One can sample more than just factors

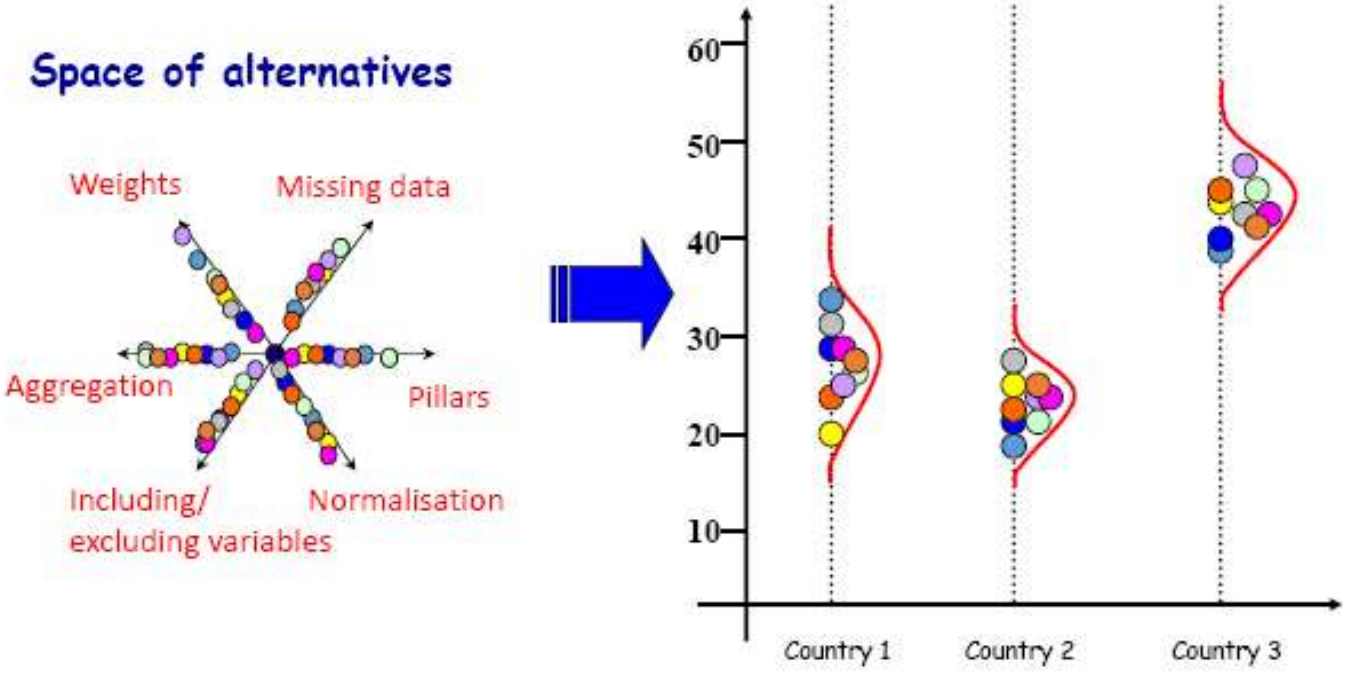
One can sample modelling assumptions, alternative data sets, resolution levels, scenarios …

Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

Space of alternatives

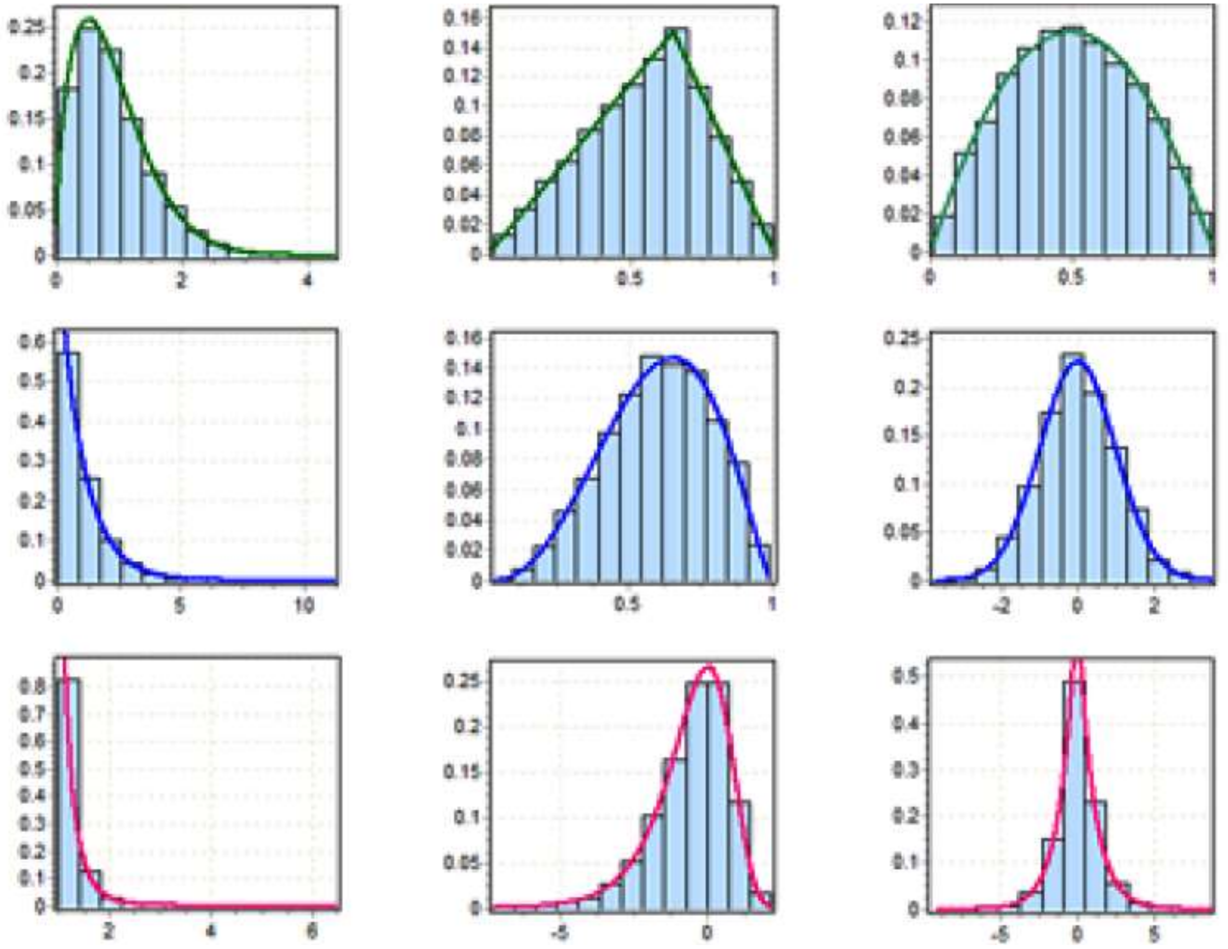


Was this an uncertainty or a sensitivity analysis?



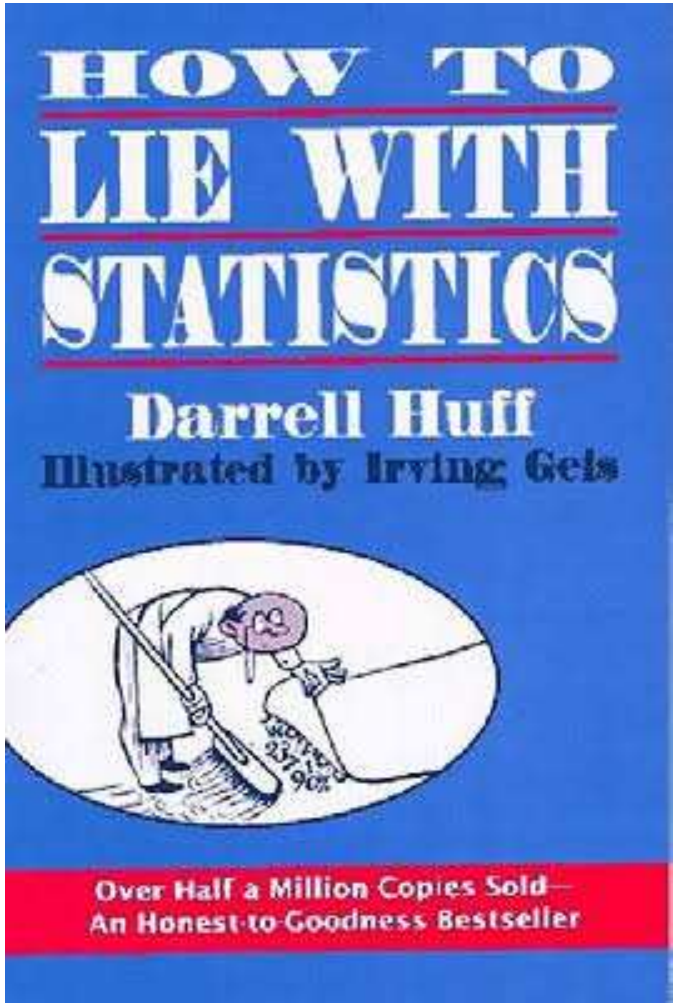
$$\begin{array}{cccc} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{Nk} \end{array}$$

Each column is a sample from the distribution of a factor
Each row is a sample trial to generate a value of y



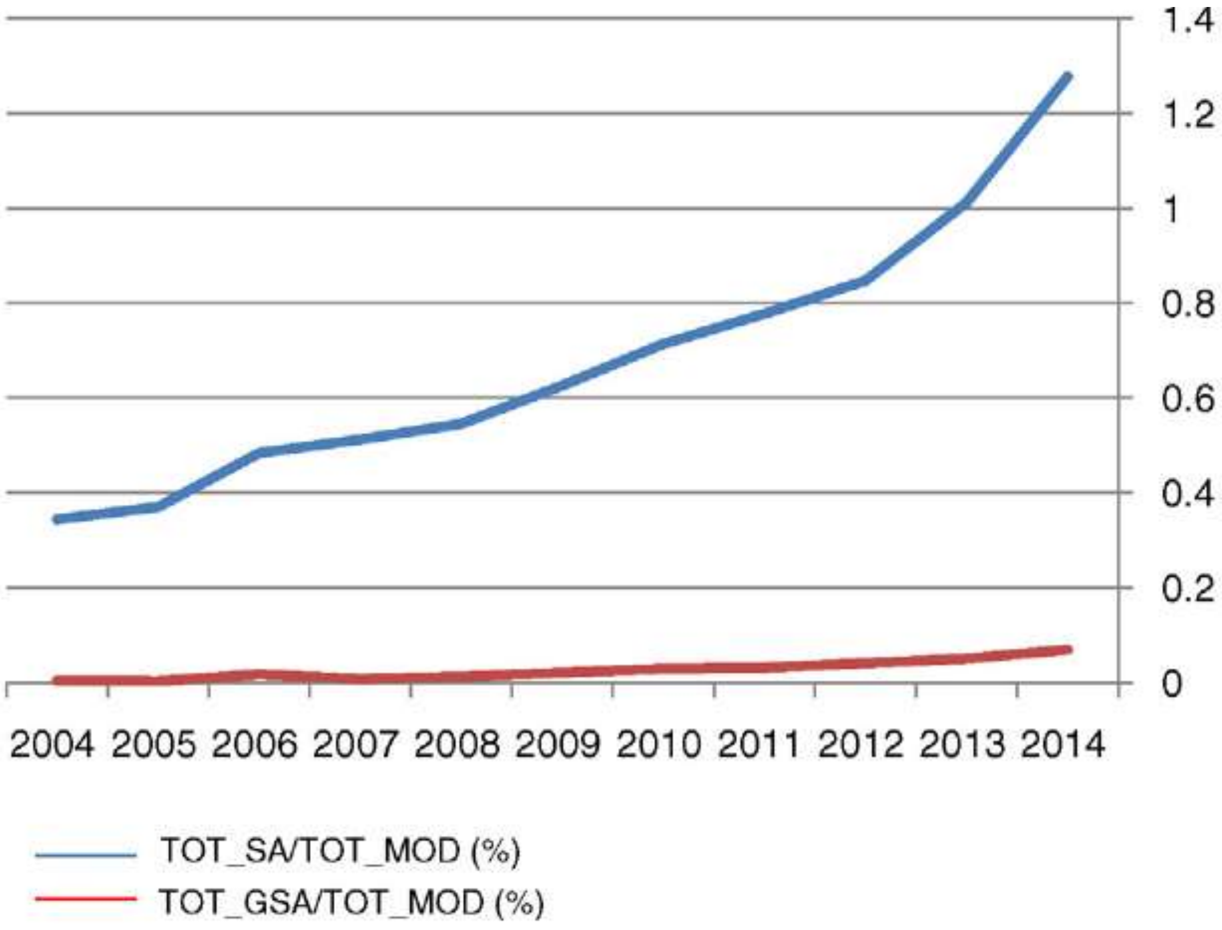
Examples of distributions of input factors

Can one lie with sensitivity analysis as one can lie with statistics?



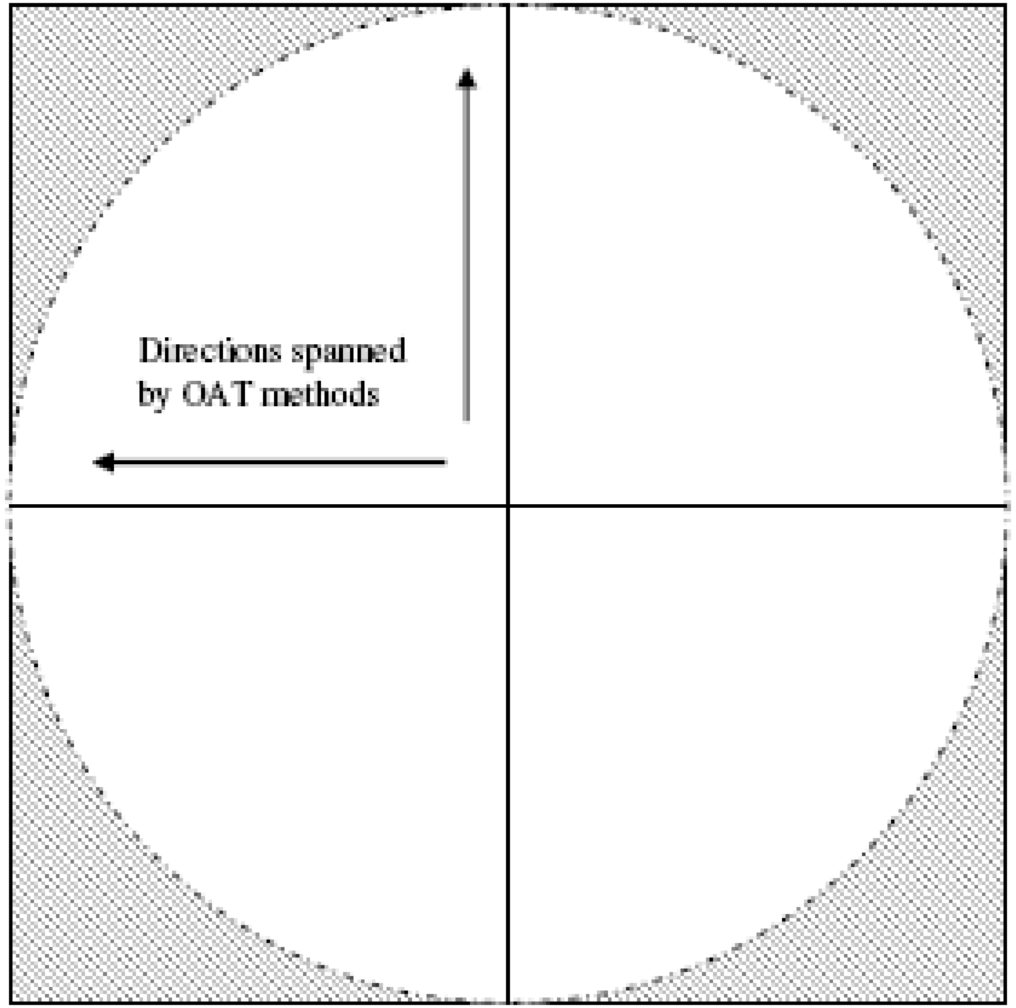
Saltelli, A., Annoni P., 2010, How to avoid a perfunctory sensitivity analysis, Environmental Modeling and Software, 25, 1508–1517.

In 2014 out of 1000 papers in modelling 12 have a sensitivity analysis and < 1 a global SA; most SA still move one factor at a time



Ferretti, F., Saltelli A., Tarantola, S., 2016, Trends in Sensitivity Analysis practice in the last decade, Science of the Total Environment, http://dx.doi.org/10.1016/j.scitotenv.2016.02.133

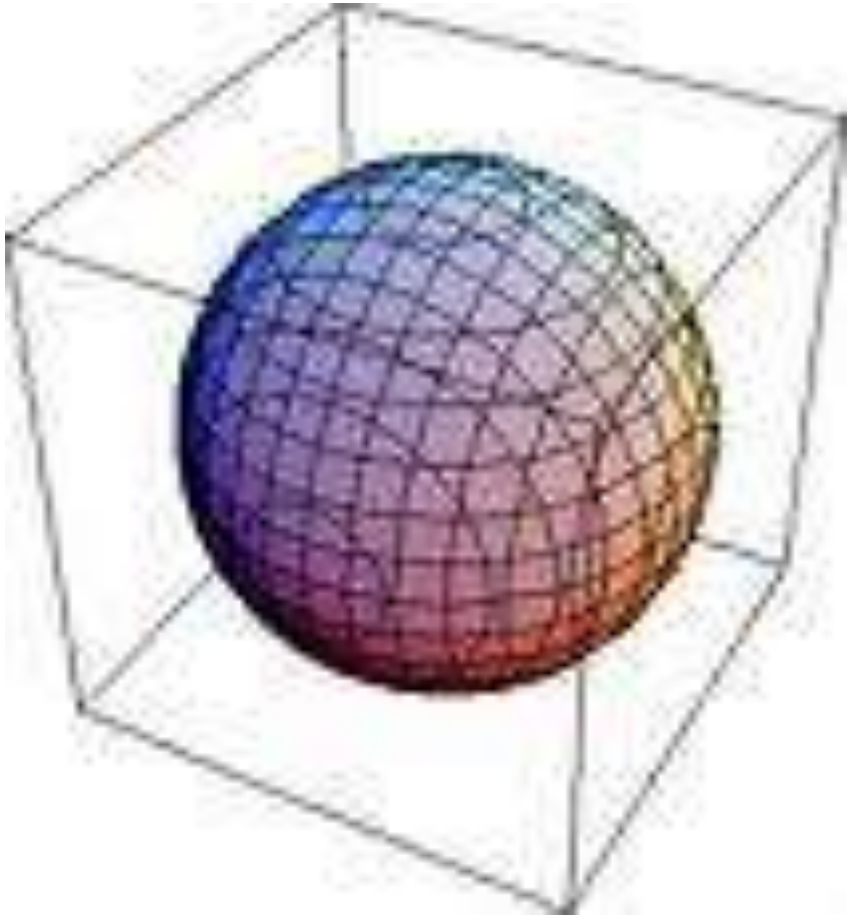
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions

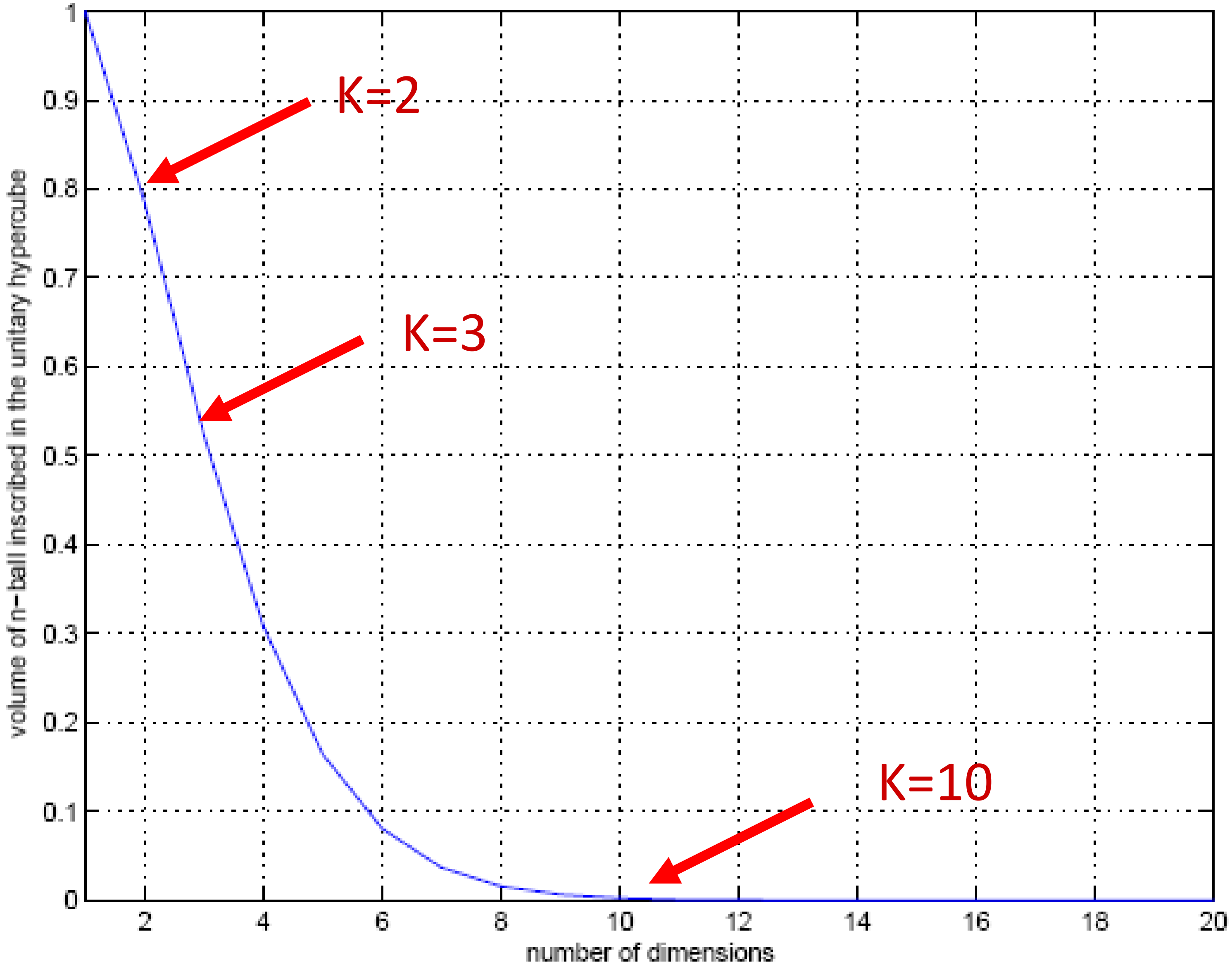


Volume sphere / volume cube =?

~ 1/2

OAT in 10 dimensions; Volume hypersphere / volume ten dimensional hypercube =? ~ 0.0025

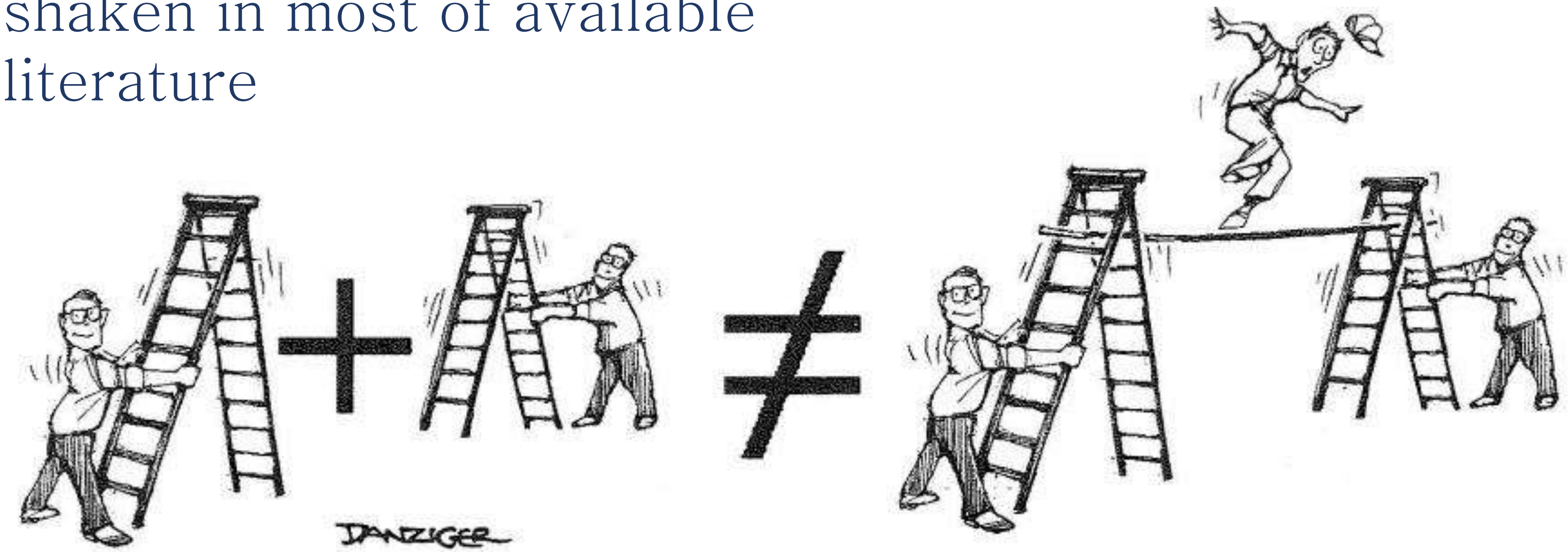
OAT in k dimensions

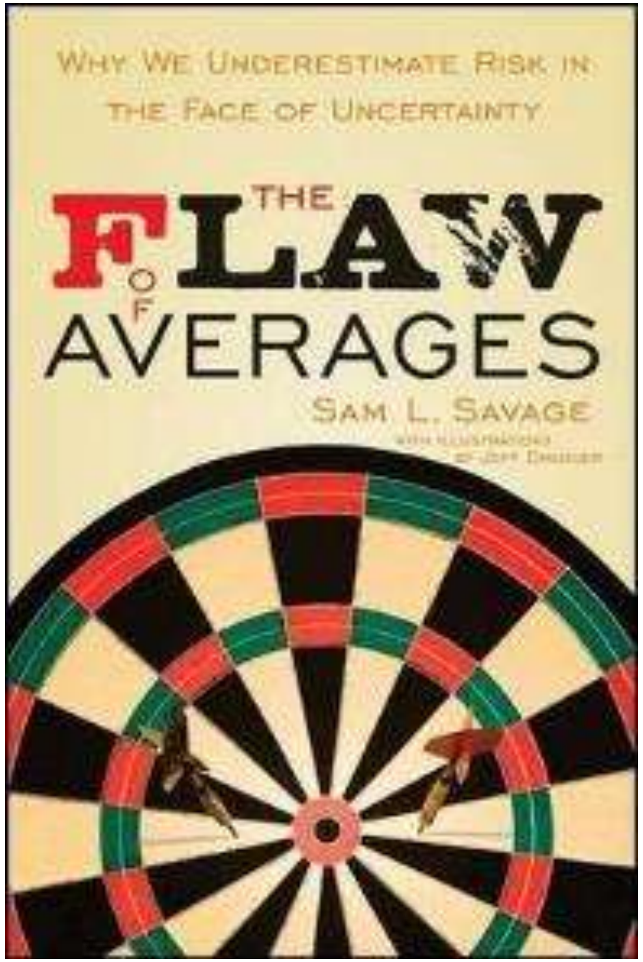


How would you test the scaffolding?

How coupled ladders are shaken in most of available literature

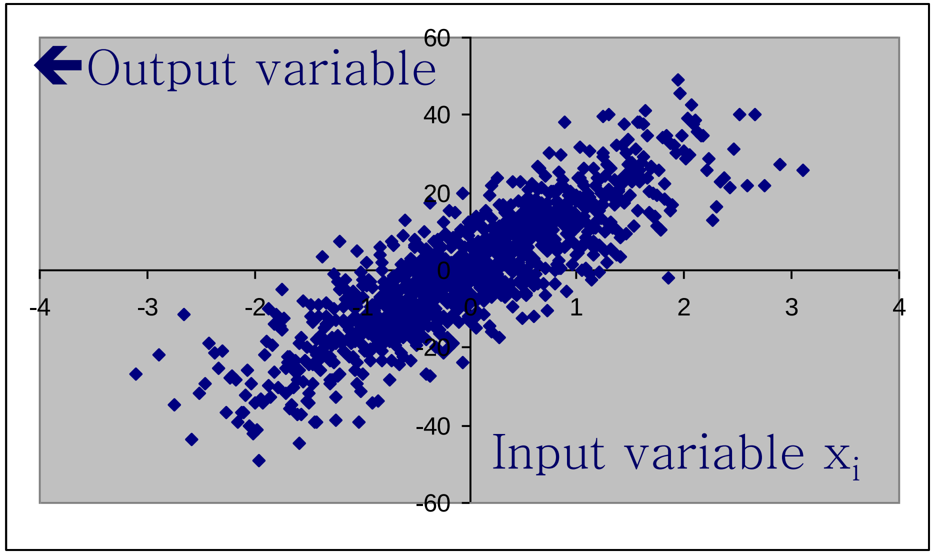
How to shake coupled ladders

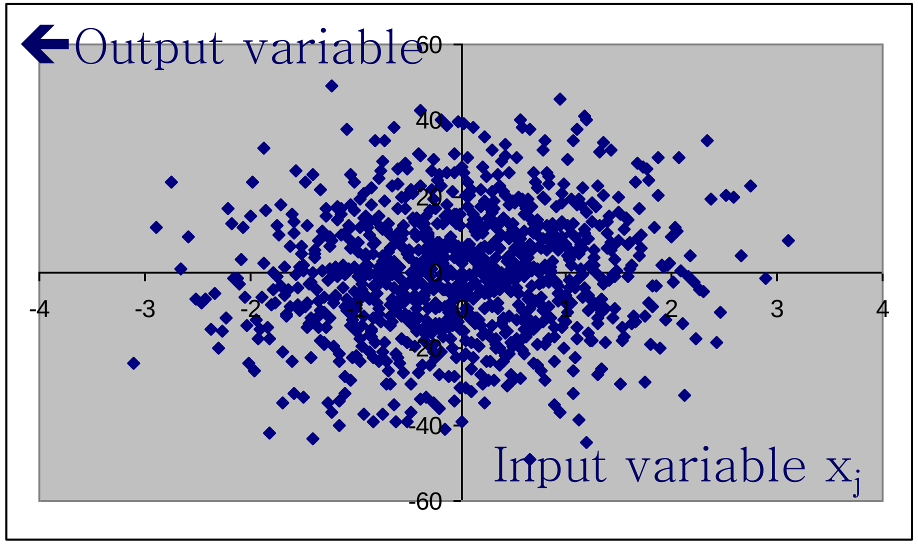




Once a sensitivity analysis is done via OAT there is no guarantee that either uncertainty analysis (UA) or sensitivity analysis (SA) will be any good:

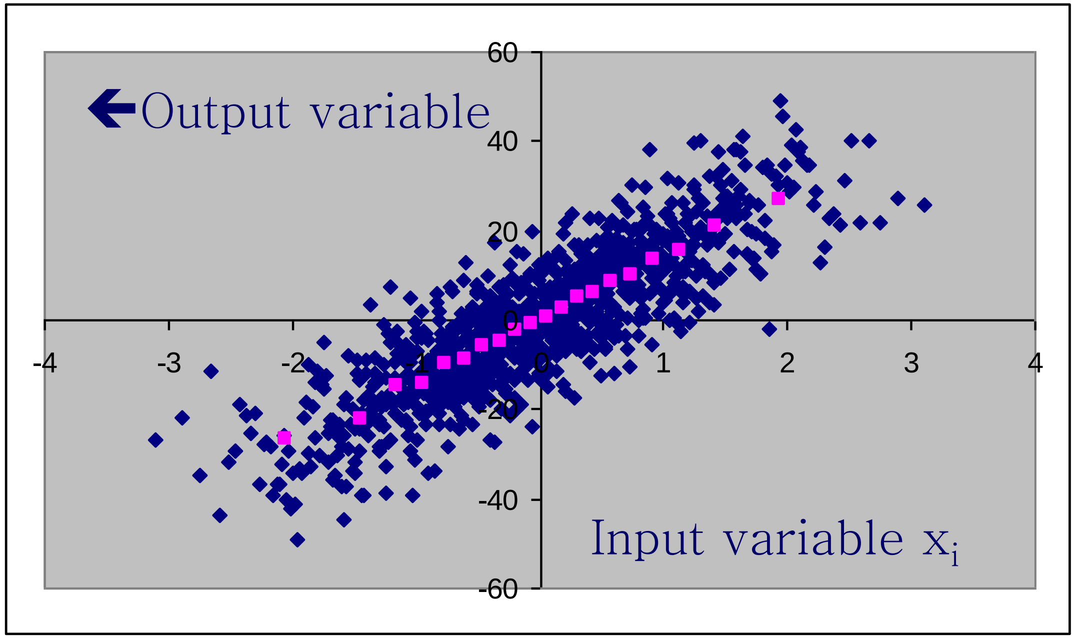
- ➔ UA will be non conservative
- ➔ SA may miss important factors

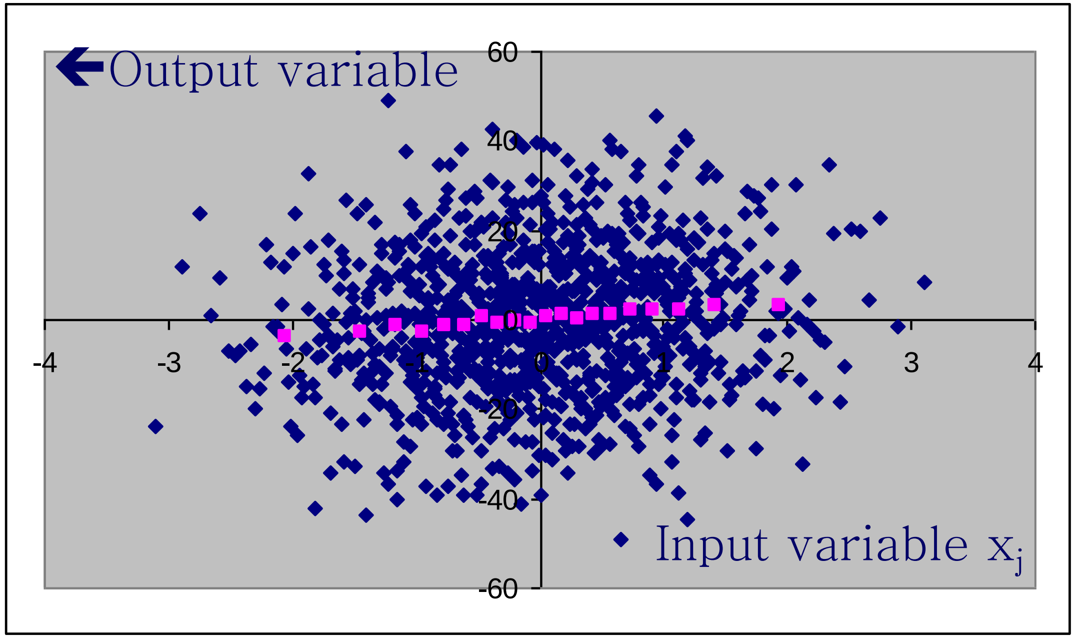




Which factor is more important?

Why?

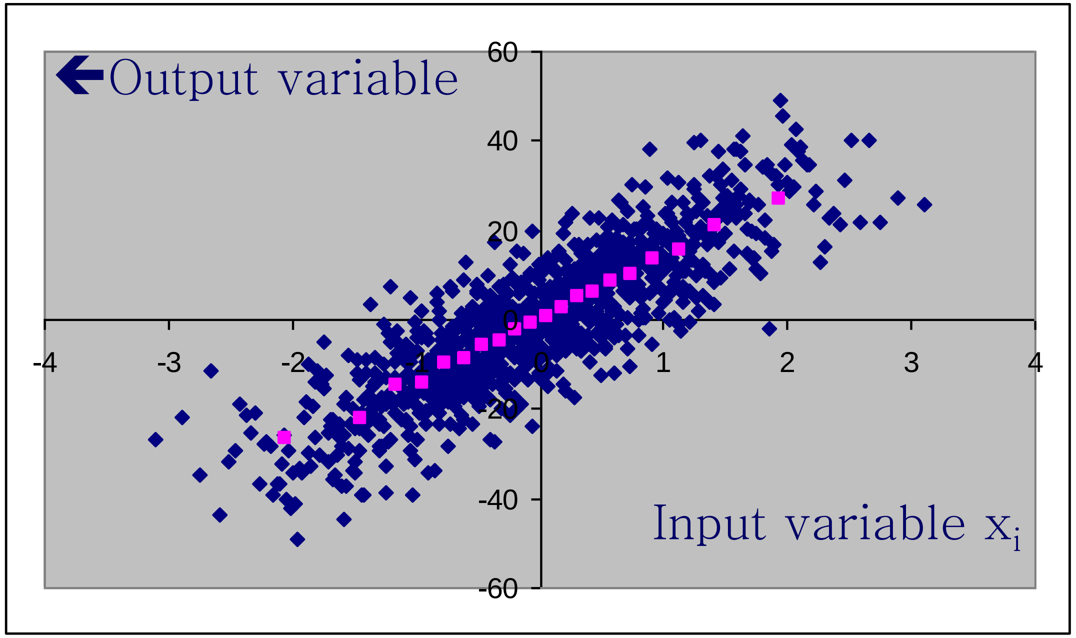




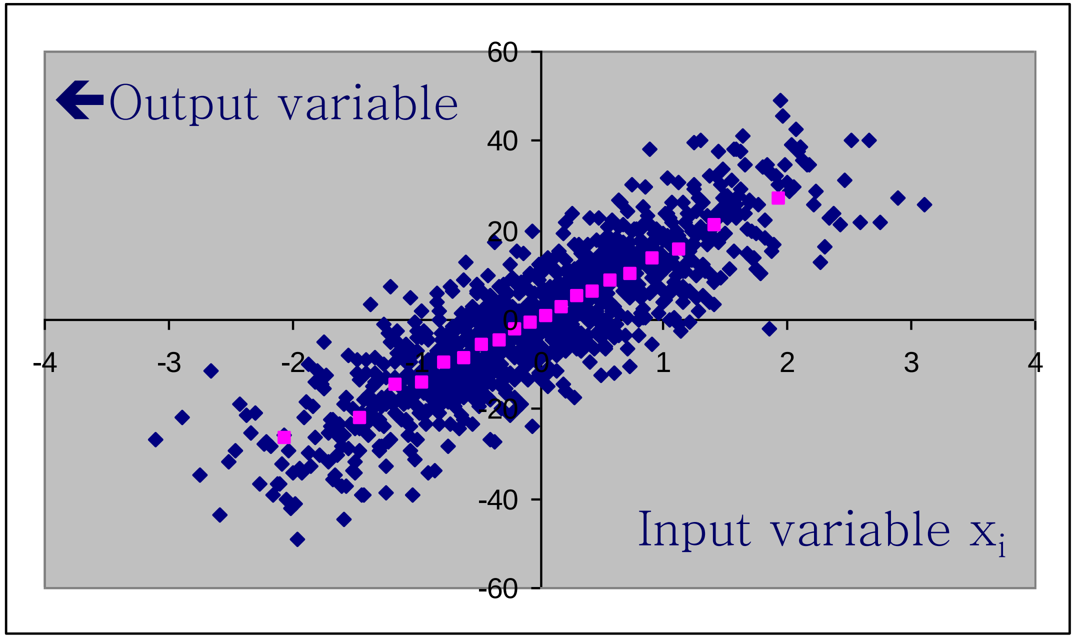
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)

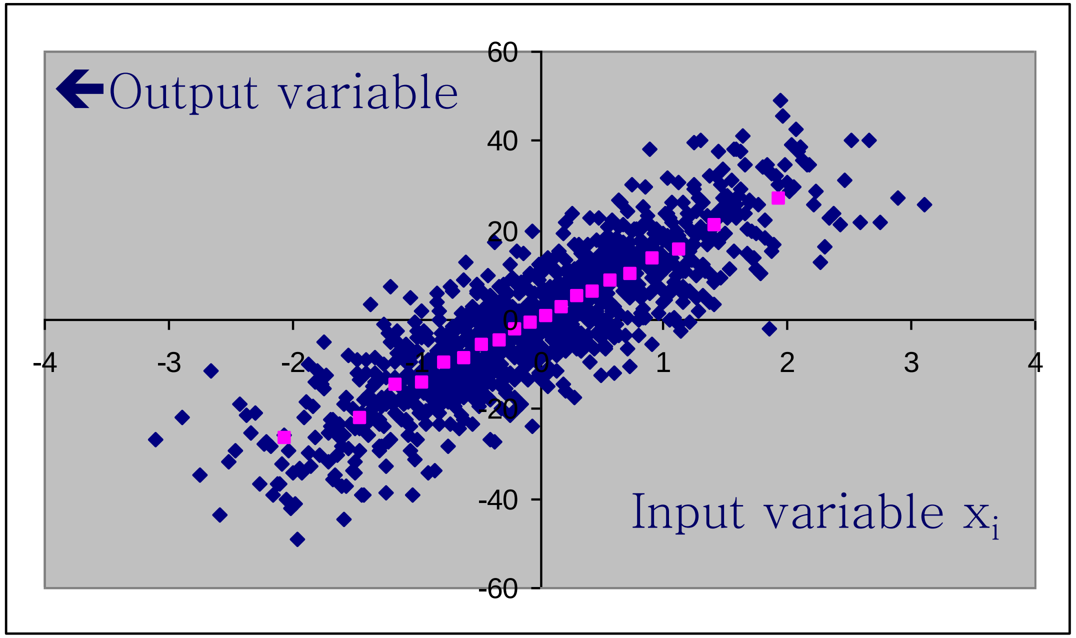


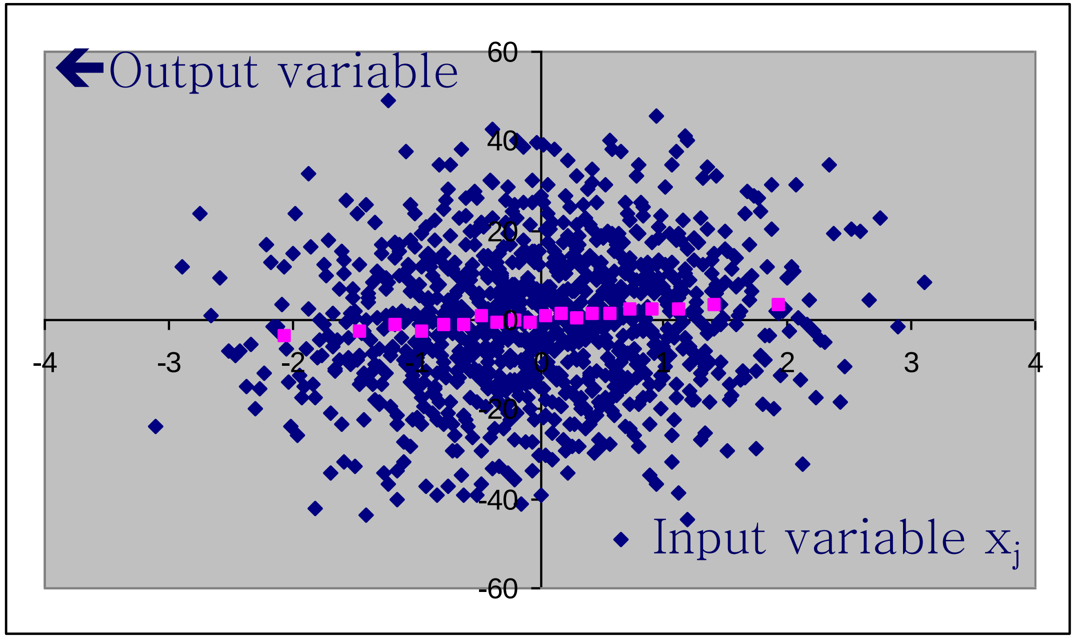
Each pink point is ~ $E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)$



Take the variance of the pink points and you have a sensitivity measure

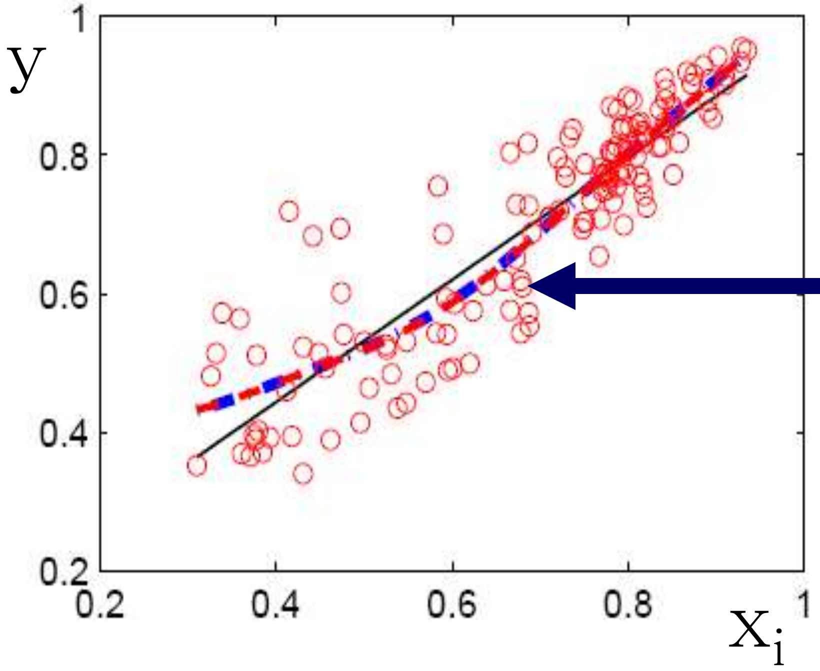
$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)$$





Which factor has the highest $V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)$?

$$S_i \equiv \frac{V\left(E\left(Y \middle| X_i\right)\right)}{V_Y}$$



Smoothed curve:

$$\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)$$

First order sensitivity index:

$$\frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

Pearson's correlation ratio

Smoothed curve

$$S_i \equiv \eta_i^2 := \frac{V_{x_i}\left(E_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{V\left(y\right)}$$

First order sensitivity index

Unconditional variance

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)$$

First order effect, or top marginal variance = the expected reduction in variance that would be achieved if factor Xi could be fixed.

Why?

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)+$$
$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)=V(Y)$$

Easy to prove using $V(Y)=E(Y^2)-E^2(Y)$

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) + \\ + \boxed{E_{X_i}\left(\underline{V_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)}\right)} = V(Y)$$

This is the variance when a factor Xi is fixed …

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) + \boxed{E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)} = V(Y)$$

This is what variance would be left (on average) if Xi could be fixed…

… then this …

$$\boxed{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)} + E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) = V(Y)$$

… must be the expected reduction in variance that would be achieved if factor Xi could be fixed

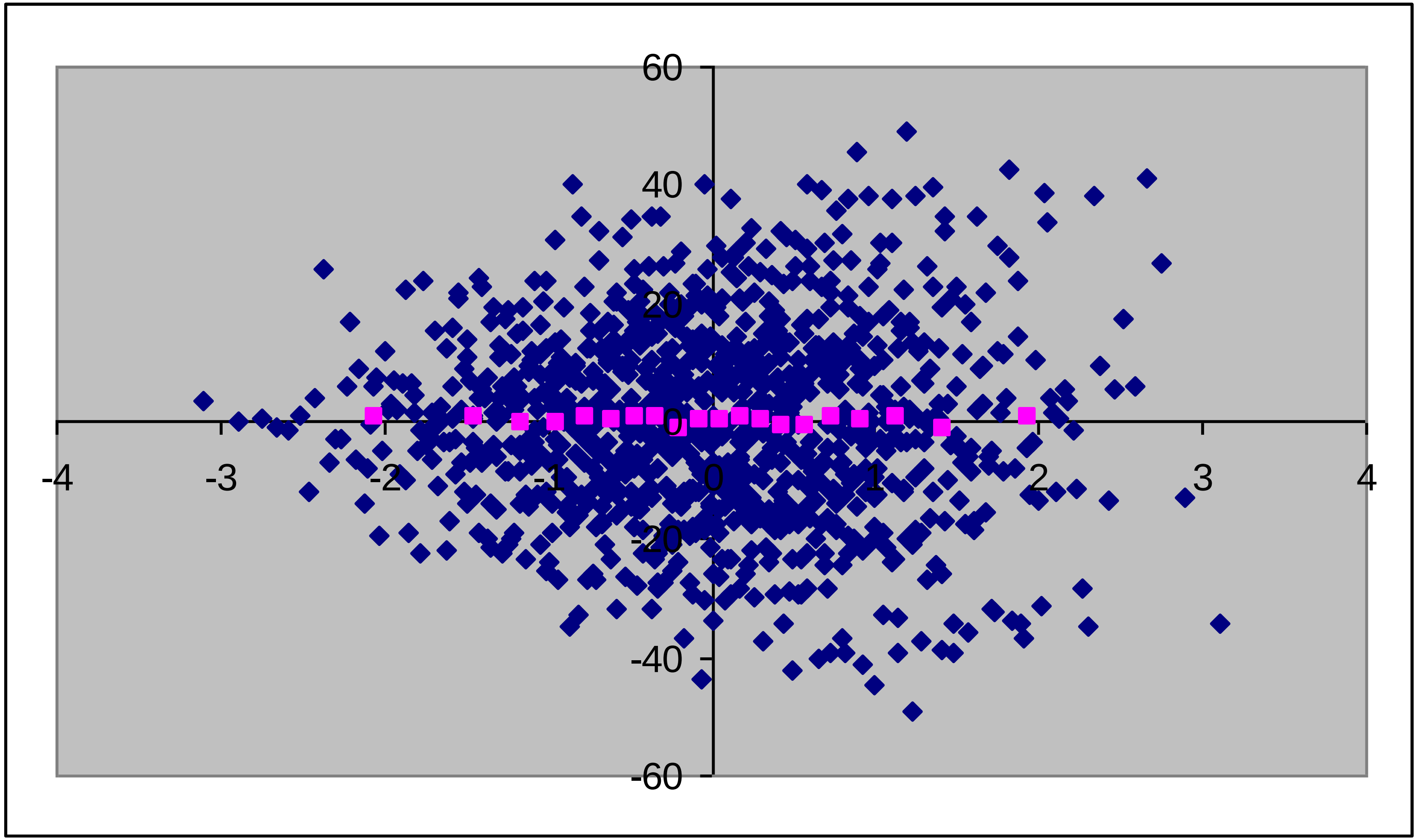
For additive models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) \approx V(Y)$$

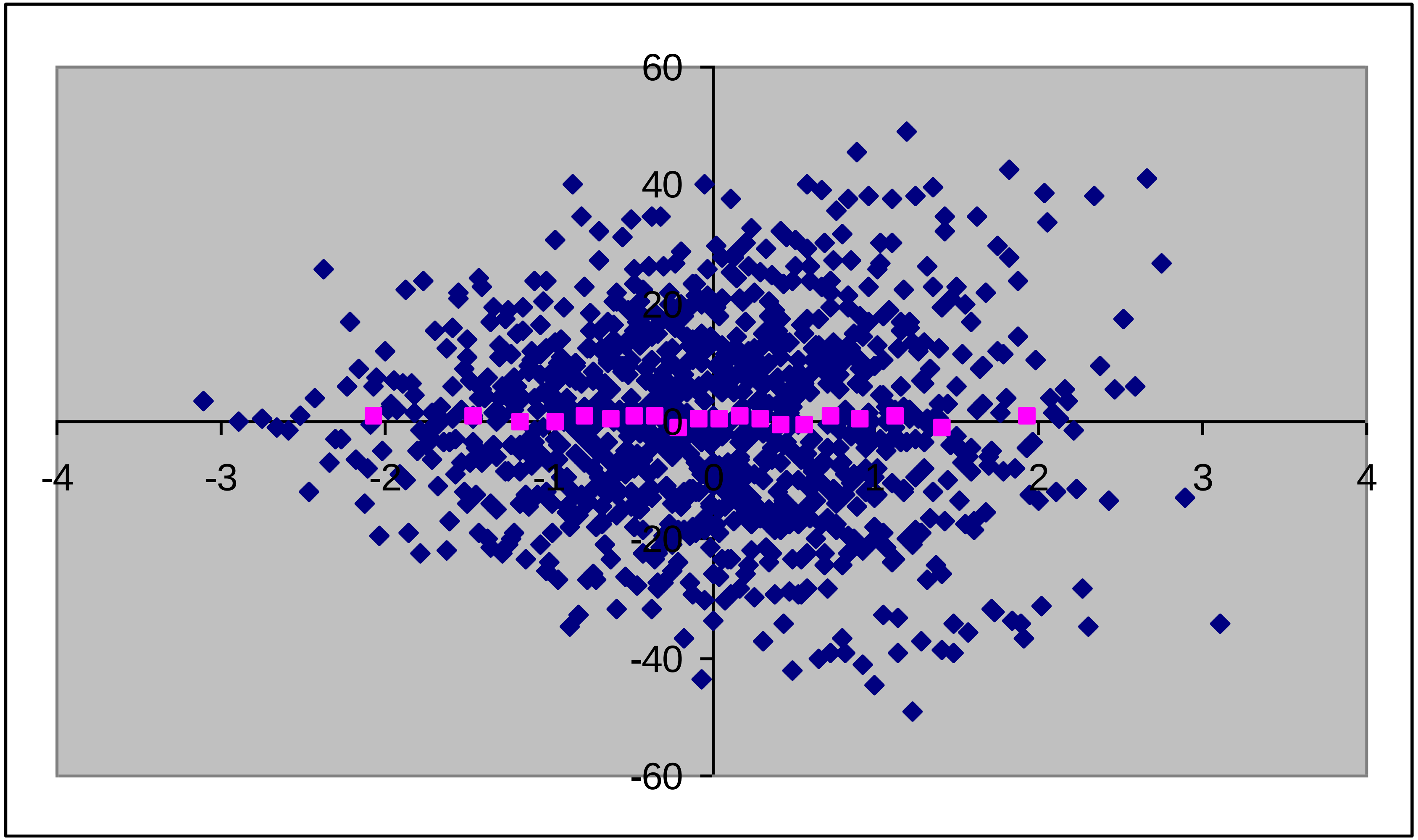
… which is also how additive models are defined

Non additive models

Is $S_i = 0$?



Is this factor non-important?



There are terms which capture two-way, three way, … interactions among variables.

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i,j>i} V_{ij} + \ldots + V_{123\ldots k}$$

Variance decomposition (ANOVA)

When the factors are independent the total variance can be decomposed into main effects and interaction effects up to the order k, the dimensionality of the problem.

If fact interactions terms are awkward to handle: second order terms for a model with k factors are as many as $k(k-1)/2$ …

Wouldn’t it be handy to have just a single ‘importance’ terms for all effects, inclusive of first order and interactions?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a model $Y=f(X_1,X_2,X_3)$

Instead of

$$V=V_1+V_2+V_3+$$
$$+V_{12}+V_{13}+V_{23}+$$
$$+V_{123}$$

Or – divided by V

$$1=S_1+S_2+S_3+$$
$$+S_{12}+S_{13}+S_{23}+$$
$$+S_{123}$$

We have:

$S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$

(and analogue formulae for S_{T2}, S_{T3}) which can be computed without knowing S_1, S_{12}, S_{13}, S_{123}

S_{T1} is called a total effect sensitivity index

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y \middle| \mathbf{X}_{\sim i}\right)\right)$$

Total effect, or bottom marginal variance=

= the expected variance that would be left if all factors but Xi could be fixed (self evident definition)

$$S_{Ti} \equiv \frac{E\left(V\left(Y \middle| \mathbf{X}_{\sim i}\right)\right)}{V_Y}$$

What is the shortcoming of S_{Ti}?

Coding Si and S_{Ti} yourself?

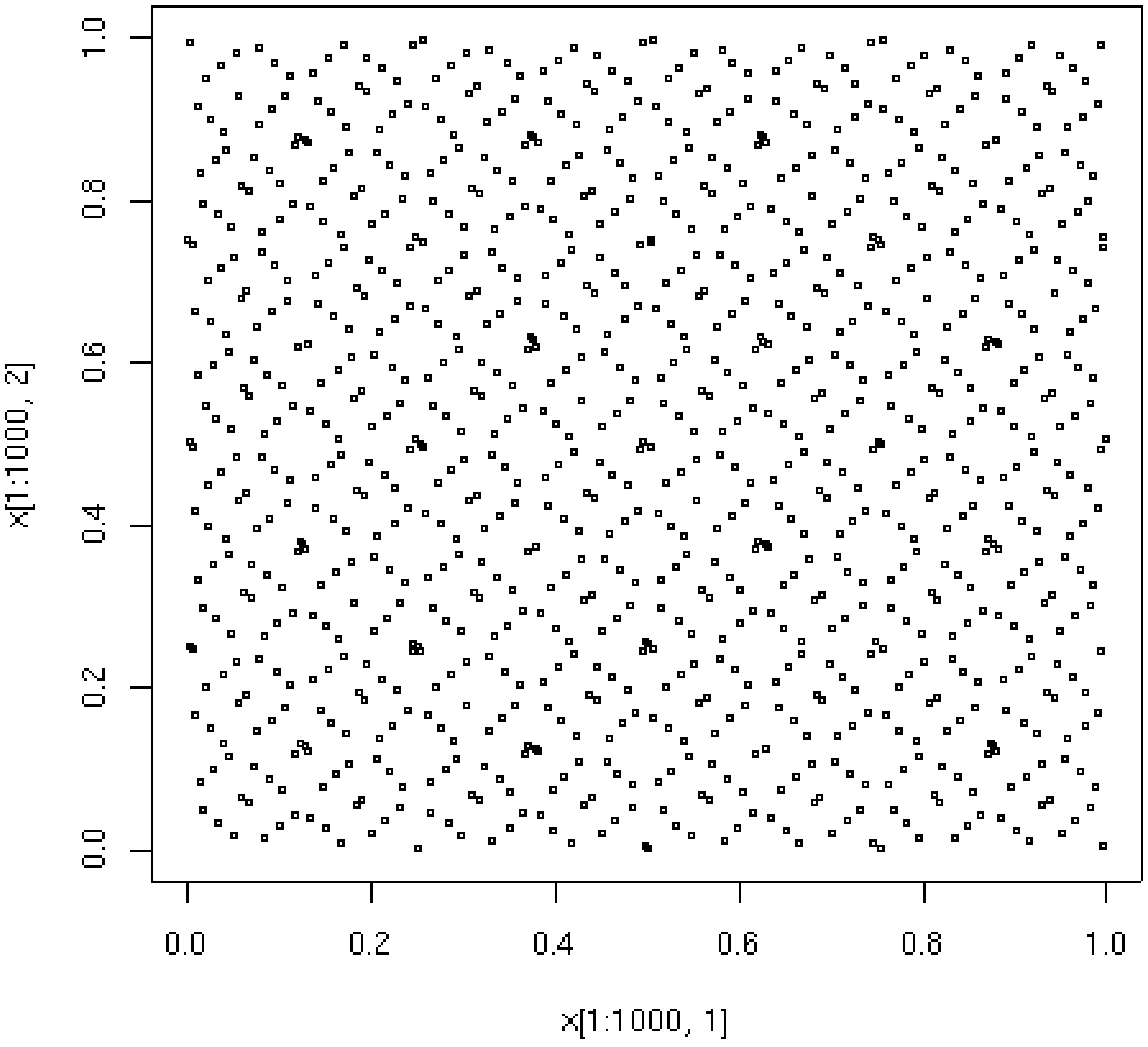
Use this work:

[Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M., Tarantola, S., 2010, Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index, Computer Physics Communications, 181, 259–270.]

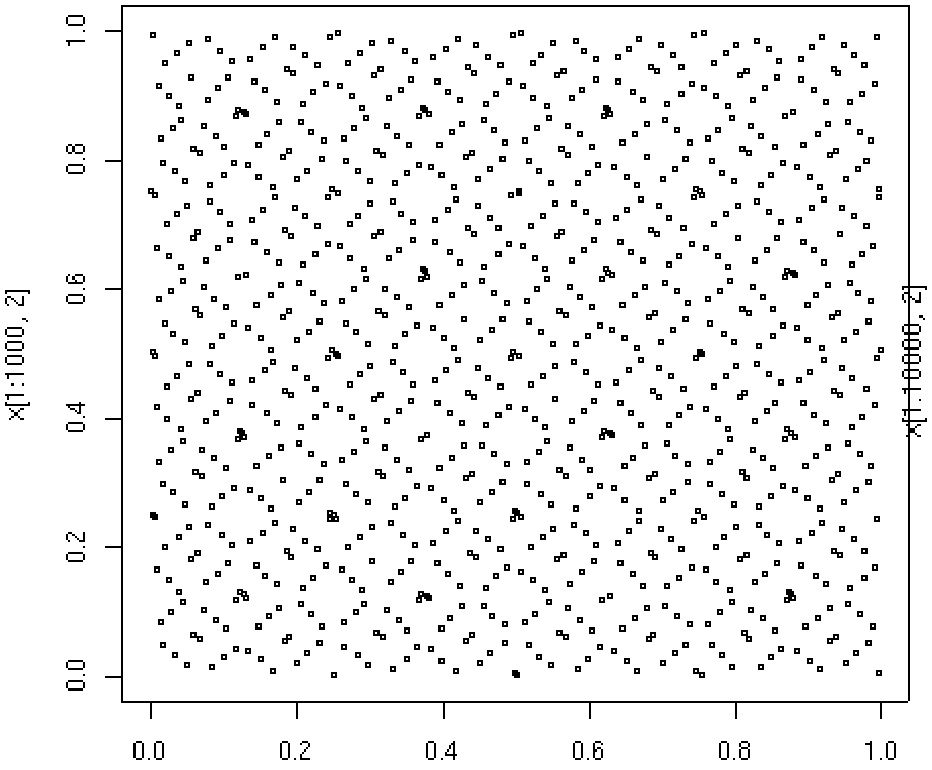
http://www.andreasaltelli.eu/file/repository/PUBLISHED_PAPER.pdf

How to generate the random sample?

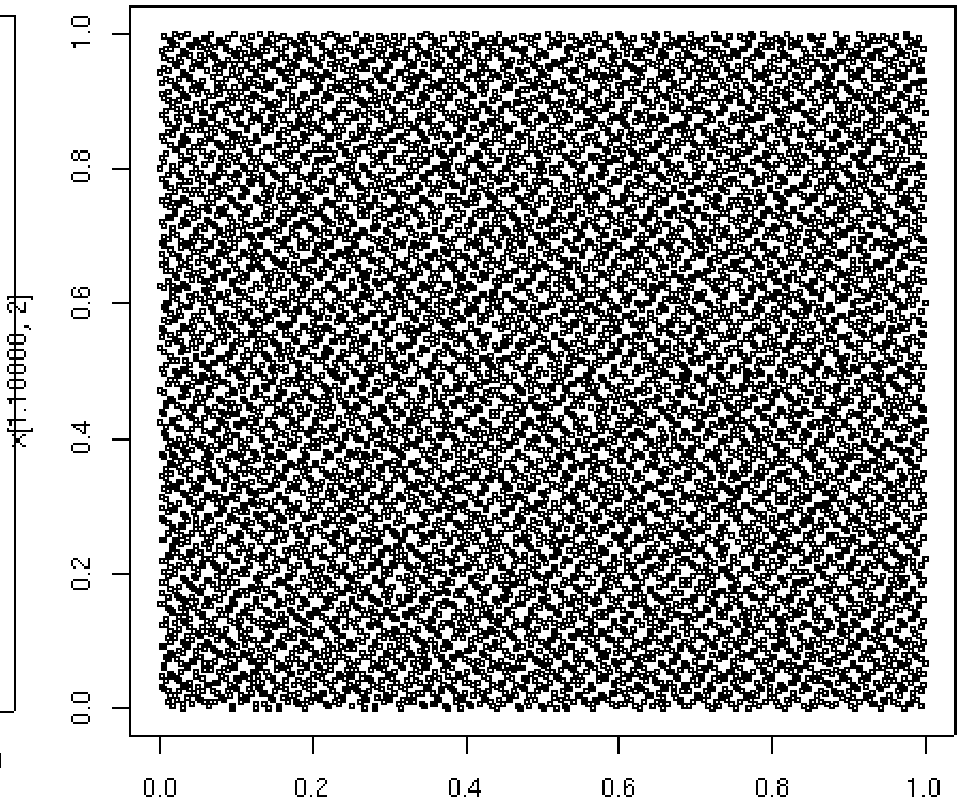
Quasi random sequences developed by I.M. Sobol’



An LP_τ sequence

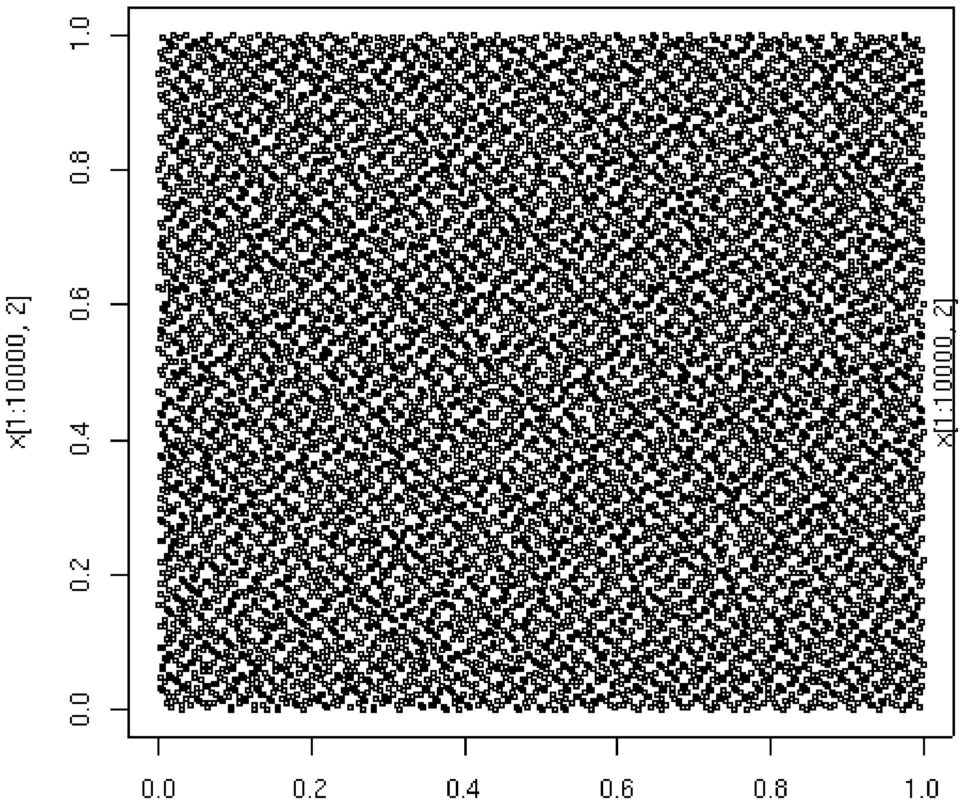


X1,X2 plane, 1000 Sobol' points

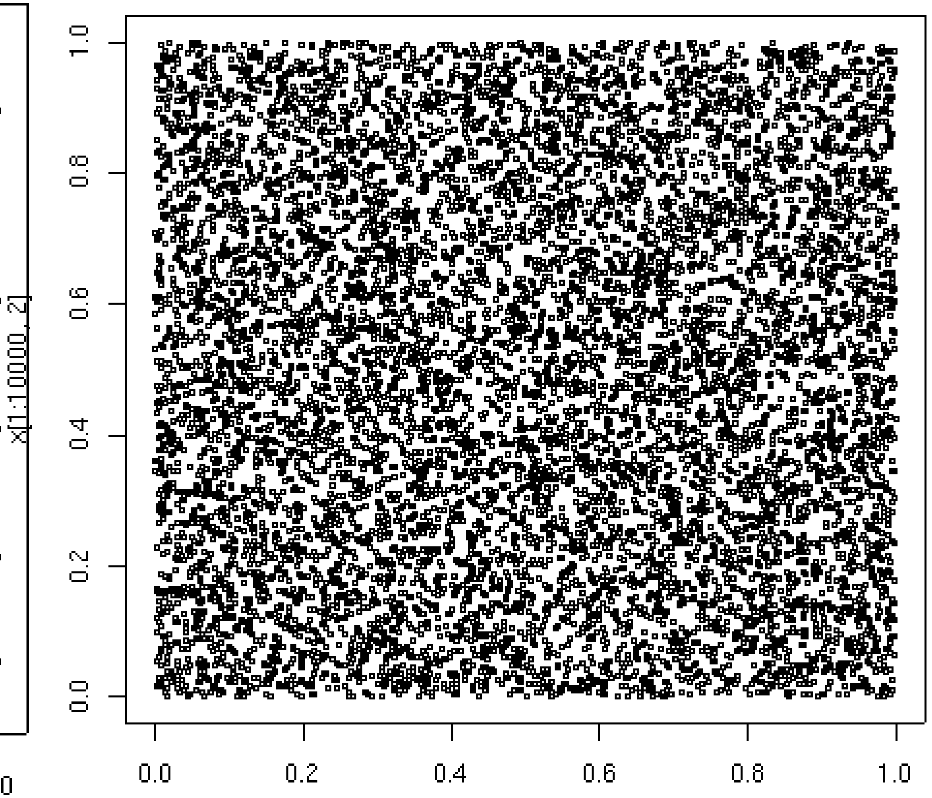


X1,X2 plane, 10000 Sobol' points

Sobol' sequences of quasi-random points



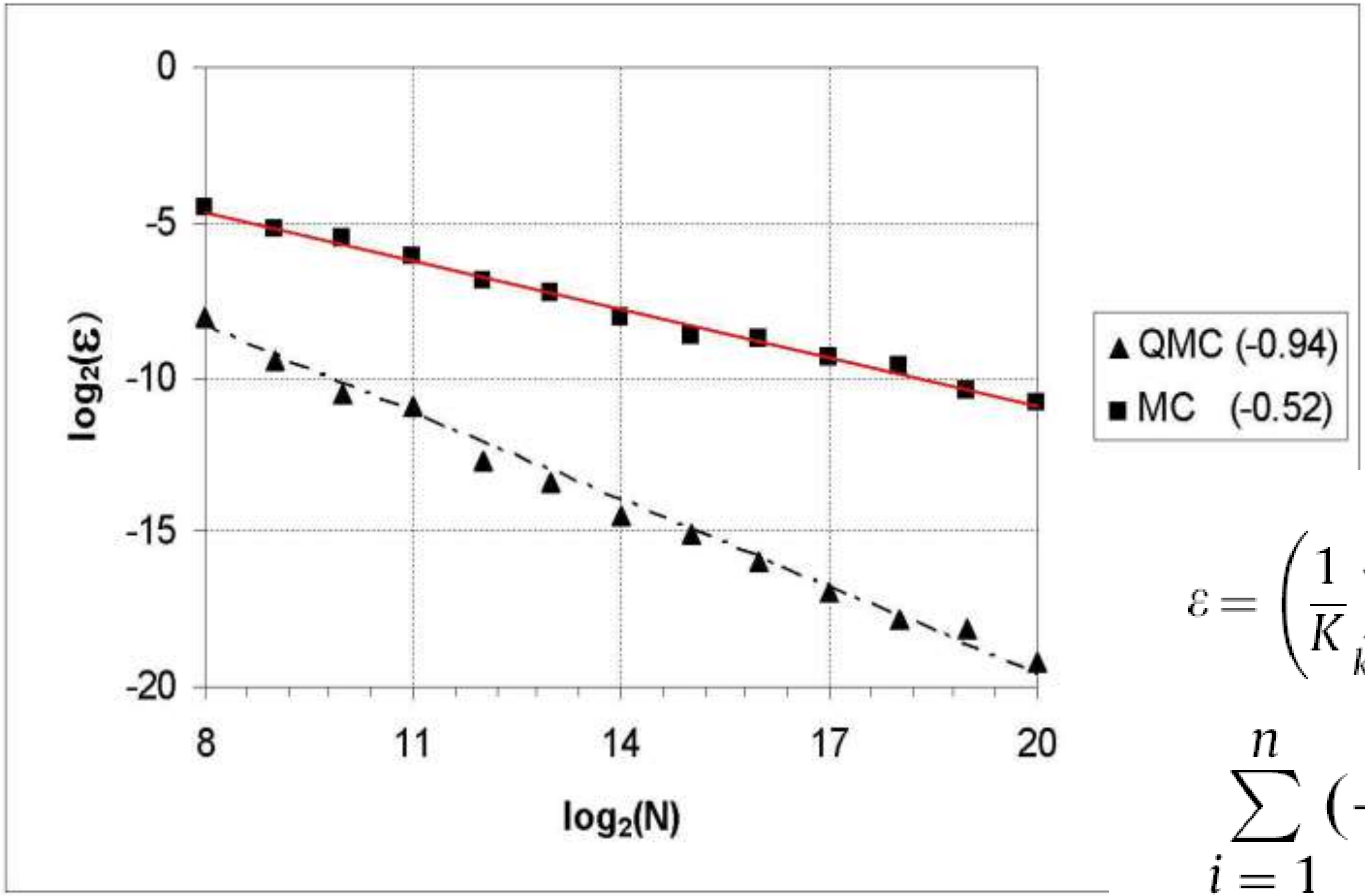
X1,X2 plane, 10000 Sobol' points



X1,X2 plane, 10000 random points

Sobol' sequences of quasi-random points against random points

Why quasi-random



Sergei Kucherenko,
Imperial College London

$$\varepsilon = \left(\frac{1}{K} \sum_{k=1}^{K} (I[f] - I_k[f])^2 \right)^{1/2}$$

$$\sum_{i=1}^{n} (-1)^i \prod_{j=1}^{i} x_j$$

Root mean square error over K=50 different trials. The error refers to the numeric-versus-analytic value the integral of the function (for n=360) over its dominion.

Source: Kucherenko S., Feil B., Shah N., Mauntz W. The identification of model effective dimensions using global sensitivity analysis Reliability Engineering and System Safety 96 (2011) 440–449.

Secrets of sensitivity analysis

Why should one ever run a model just once?

First secret: The most important question is the question.

Or: sensitivity analysis is not “run” on a model but on a model once applied to a question

Second secret: Sensitivity analysis should not be used to hide assumptions
[it often is]

Third secret: If sensitivity analysis shows that a question cannot be answered by the model one should find another question or model

[Often the love for one's own model prevails]

Badly kept secret:

There is always one more bug!

(Lubarsky's Law of Cybernetic Entomology)

And of course please don’t run a sensitivity analysis where each factors has a 5% uncertainty

Why?

An example:

Sensitivity analysis: the case of the Stern review

Global Environmental Change 20 (2010) 298–302

Contents lists available at ScienceDirect

Global Environmental Change

journal homepage: www.elsevier.com/locate/gloenvcha

Sensitivity analysis didn't help. A practitioner's critique of the Stern review

Andrea Saltelli *, Beatrice D'Hombres

Joint Research Centre, Institute for the Protection and Security of the Citizen, Ispra, Italy

The case of Stern's Review – Technical Annex to postscript

Nicholas Stern, London School of Economics

William Nordhaus, University of Yale

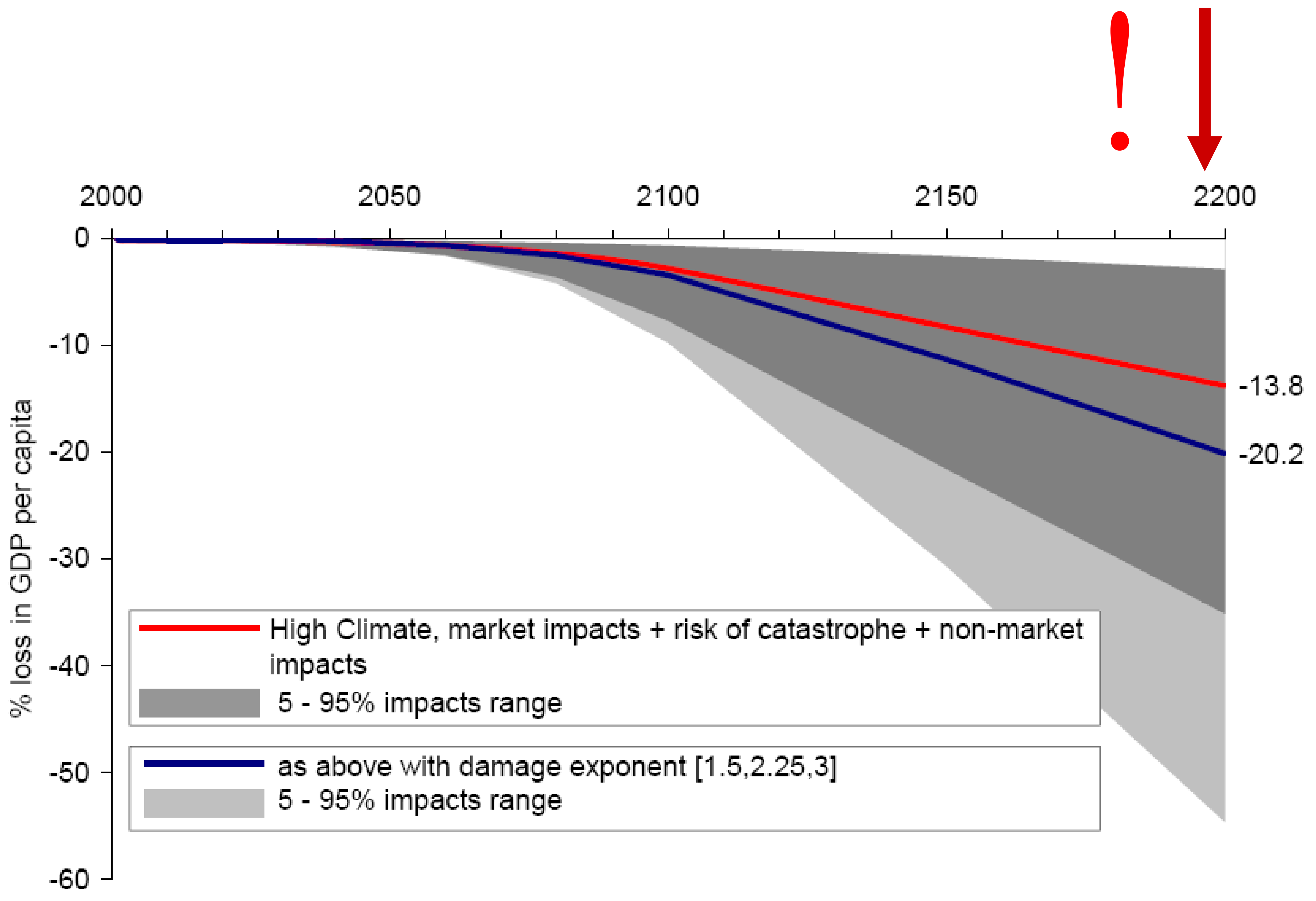
Stern, N., Stern Review on the Economics of Climate Change. UK Government Economic Service, London, www.sternreview.org.uk.
Nordhaus W., Critical Assumptions in the Stern Review on Climate Change, SCIENCE, 317, 201-202, (2007).

The Stern – Nordhaus exchange on *SCIENCE*

1) Nordhaus falsifies Stern based on ‘wrong’ range of discount rate

2) Stern’s complements its review with a postscript: a sensitivity analysis of the cost benefit analysis

3) Stern thus says: My analysis shows robustness’

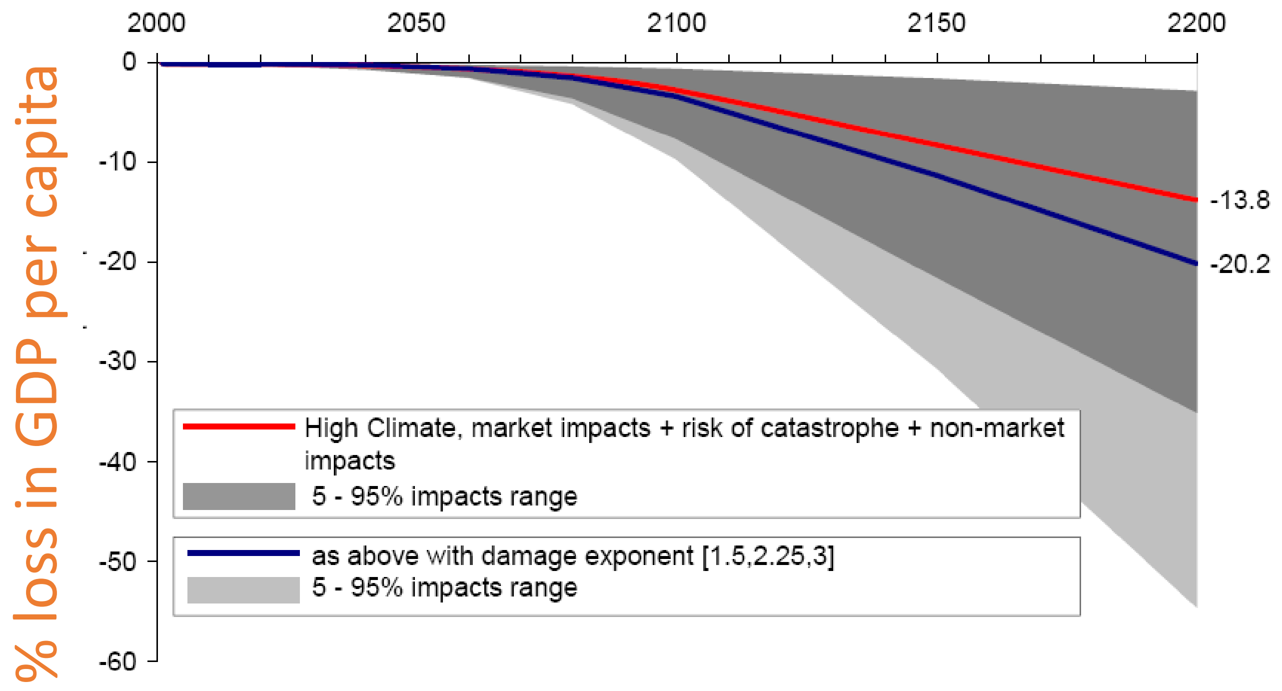

2000
2050
2100
2150
2200
0
-10
-20
-30
-40
-50
-60
% loss in GDP per capita
-13.8
-20.2
High Climate, market impacts + risk of catastrophe + non-market impacts
5 - 95% impacts range
as above with damage exponent [1.5,2.25,3]
5 - 95% impacts range

… but foremost Stern says:

changing assumptions → important effect

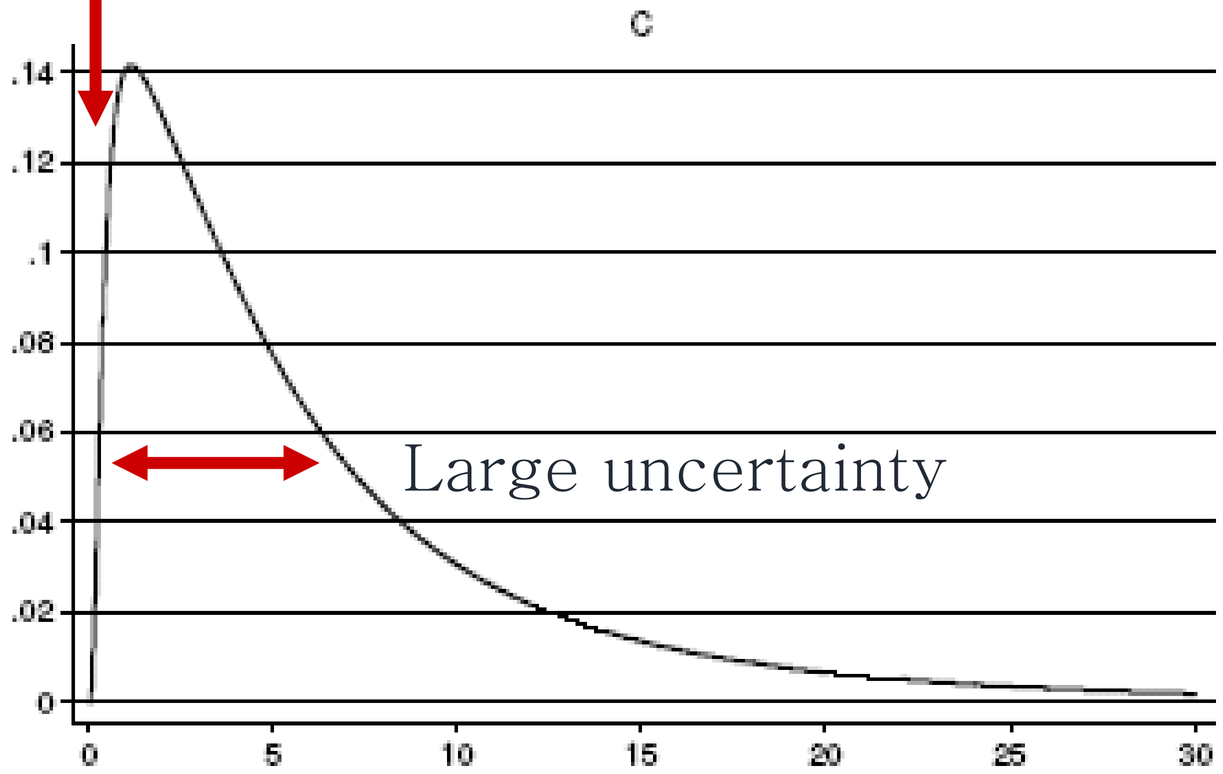
when instead he should admit that:

changing assumptions → all changes a lot



How was it done? A reverse engineering of the analysis

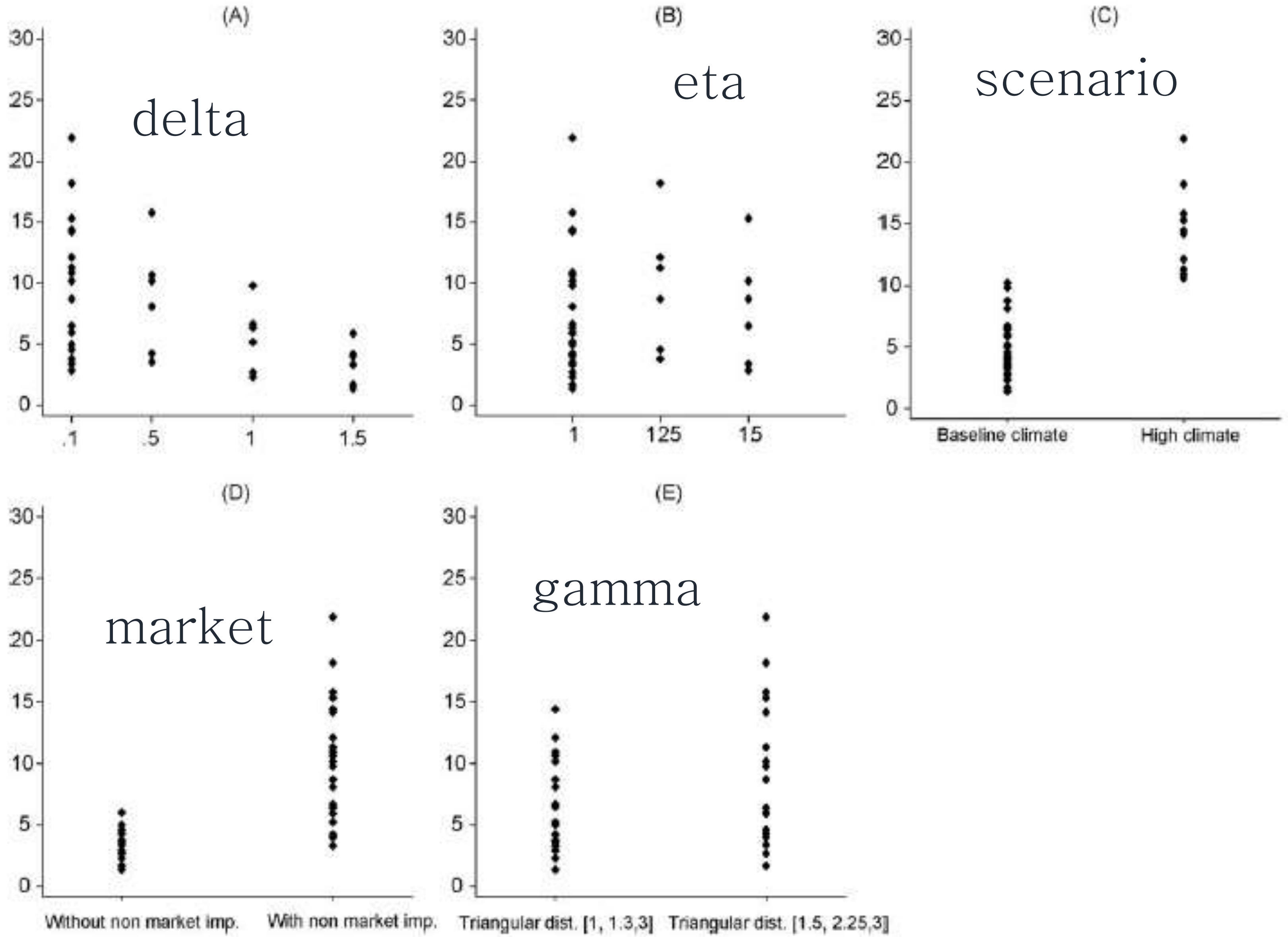
Missing points



Large uncertainty

% loss in GDP per capita

Sensitivity analysis here (by reverse engineering)



END

@andreasaltelli

EC impact assessment guidelines: sensitivity analysis & auditing

http://ec.europa.eu/smart-regulation/guidelines/docs/br_toolbox_en.pdf

Blurring lines:

"what qualities are specific to rankings, or indicators, or models, or algorithms?"

E. Popp Berman and D. Hirschman, The Sociology of Quantification: Where Are We Now?, Contemp. Sociol., vol. in press, 2017.

“[in climate modelling] it looks very little like our idealized image of science, in which pure theory is tested with pure data. [impossible to] eliminate the model-dependency of data or the data-ladenness of models”

Paul N. Edwards, 1999, Global climate science, uncertainty and politics: Data-laden models, model-filtered data.

“[For] philosophers Frederick Suppe and Stephen Norton the blurry model/data relationship pervades all science”

Paul N. Edwards, 1999, Global climate science, uncertainty and politics:

Data-laden models, model-filtered data.

More than a technical uncertainty and sensitivity analysis?

1. Uncertainty and sensitivity analysis (never execute the model once)

2. Sensitivity auditing and quantitative storytelling (investigate frames and motivations)

Saltelli, A., Guimarães Pereira, Â., Van der Sluijs, J.P. and Funtowicz, S., 2013, 'What do I make of your latinorum? Sensitivity auditing of mathematical modelling', Int. J. Foresight and Innovation Policy, (9), 2/3/4, 213–234.

Saltelli, A., Does Modelling need a reformation? Ideas for a new grammar of modelling, available at https://arxiv.org/abs/1712.06457

3. Replace 'model to predict and control the future' with 'model to help mapping ignorance about the future' …

… in the process exploiting and making explicit the metaphors embedded in the model

J. R. Ravetz, "Models as metaphors," in Public participation in sustainability science : a handbook, and W. A. B. Kasemir, J. Jäger, C. Jaeger, Gardner Matthew T., Clark William C., Ed. Cambridge University Press, 2003, available at http://www.nusap.net/download.php?op=getit&lid=11

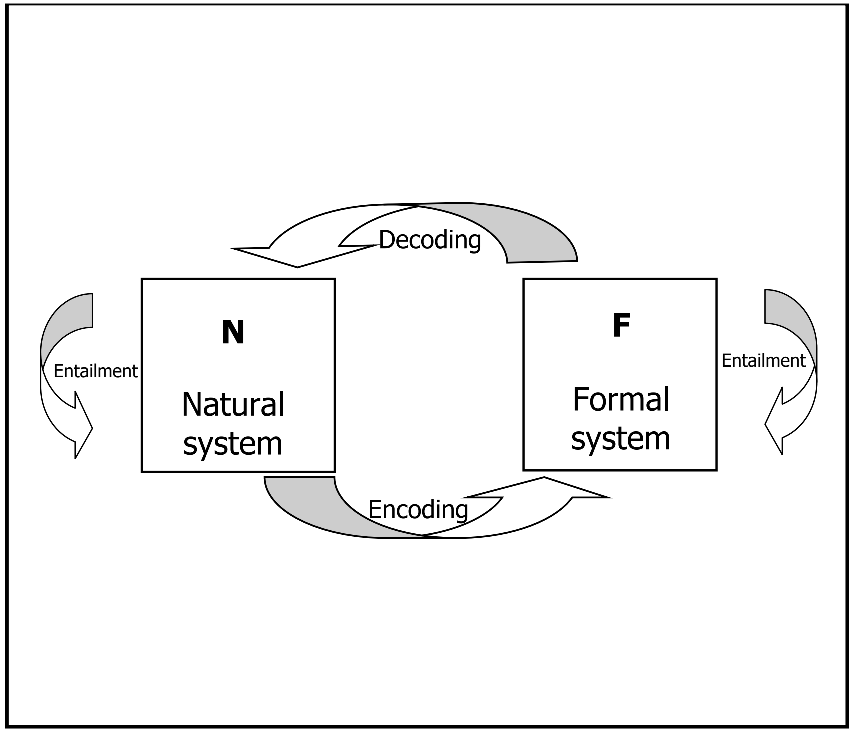
Padilla et al. call for **a more structured, generalized and standardized approach to verification**

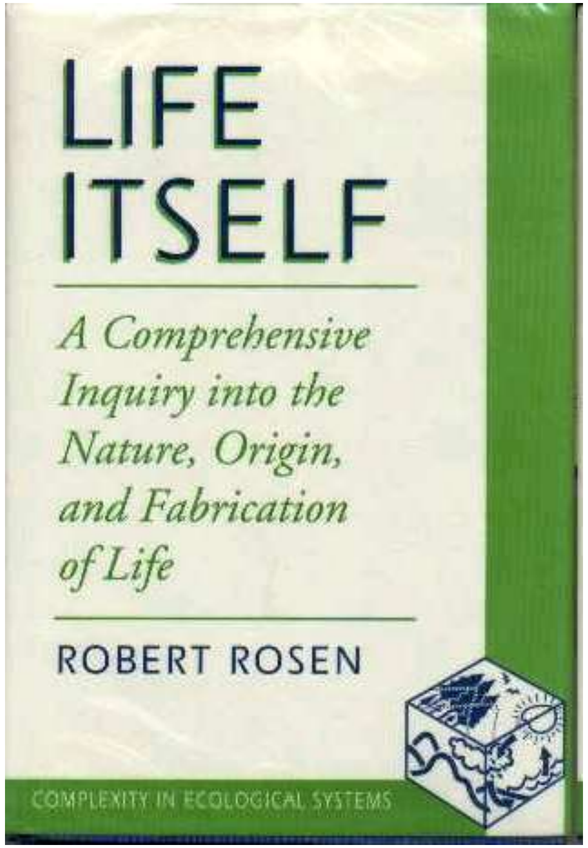
Jakeman et al. call for a **10 points participatory checklist** including **NUSAP** and **J. R. Ravetz's process based approach**

For NUSAP: Funtowicz, S.O., Ravetz, J.R., 1990. Uncertainty and Quality in Science and Policy. Kluwer, Dordrecht.

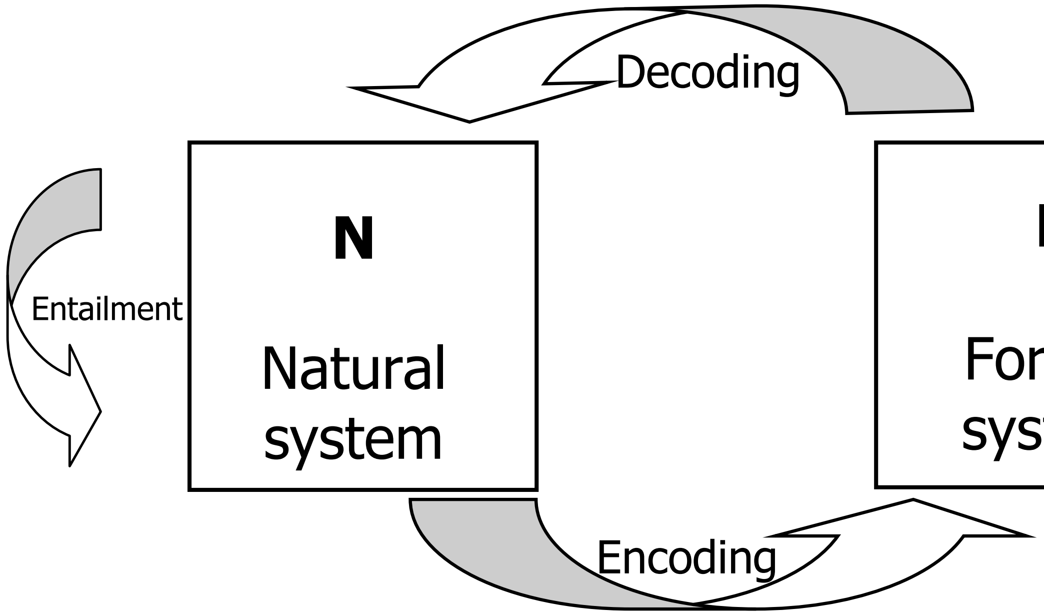
J. R. Ravetz, "Integrated Environmental Assessment Forum, developing guidelines for 'good practice', Project ULYSSES", 1997, http://www.jvds.nl/ulysses/eWP97-1.pdf

Modelling as a craft rather than as a science for Robert Rosen





R. Rosen, Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life. Columbia University Press, 1991.



What is a model ?

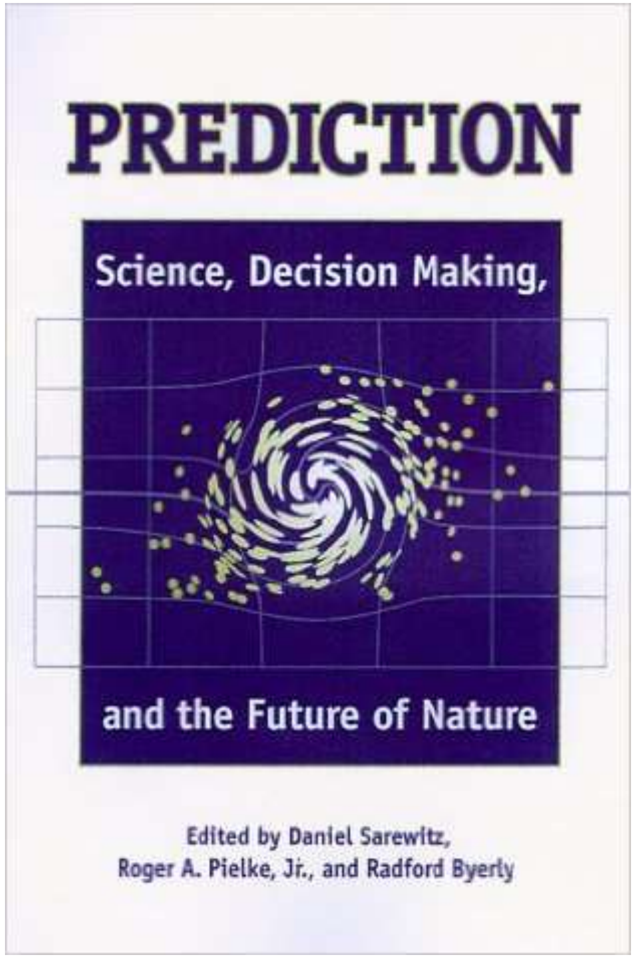
Robert Rosen

“models are most useful when they are used to challenge existing formulations, rather than to validate or verify them”

Naomi
Oreskes

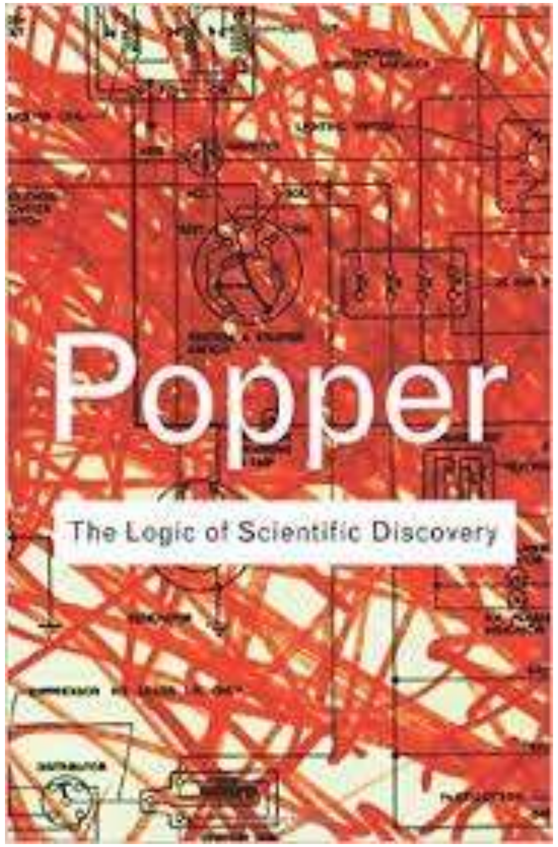
N. Oreskes, K. Shrader-Frechette, and K. Belitz, “Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences,” Science, 263, no. 5147, 1994.

Models are not physical laws



Oreskes, N., 2000, Why predict? Historical perspectives on prediction in Earth Science, in Prediction, Science, Decision Making and the future of Nature, Sarewitz et al., Eds., Island Press, Washington DC

"[⋯] to be of value in theory testing, the predictions involved must be capable of refuting the theory that generated them"
(N. Oreskes)



“In many cases, these temporal predictions **are treated with the same respect** that the hypothetic-deductive model of science accords to logical predictions. But this respect is largely misplaced”

“[⋯] models are complex amalgam of theoretical and phenomenological laws (and the governing equations and algorithms that represent them), empirical input parameters, and a model conceptualization [⋯] When a model generates a prediction, of what precisely is the prediction a test? The laws? The input data? The conceptualization? Any part (or several parts) of the model might be in error, and there is no simple way to determine which one it is”

Paul Romer’s Mathiness = use of mathematics to veil normative stances

Erik Reinert: scholastic tendencies in the mathematization of economics

P. M. Romer, “Mathiness in the Theory of Economic Growth,” Am. Econ. Rev., vol. 105, no. 5, pp. 89–93, May 2015.

E. S. Reinert, “Full circle: economics from scholasticism through innovation and back into mathematical scholasticism,” J. Econ. Stud., vol. 27, no. 4/5, pp. 364–376, Aug. 2000.

The rules of sensitivity auditing

1. Check against rhetorical use of mathematical modelling;
2. Adopt an “assumption hunting” attitude; focus on unearthing possibly implicit assumptions;
3. Check if uncertainty been instrumentally inflated or deflated.

4. Find sensitive assumptions before these find you; do your SA before publishing;

5. Aim for transparency; Show all the data;

6. Do the right sums, not just the sums right; frames; ➔ quantitative storytelling

7. Perform a proper global sensitivity analysis.