

Sixth International Conference on Sensitivity Analysis of Model Output

Distribution-based global sensitivity analysis using polynomial chaos expansions

 Y. Caniou^{1,2}, B. Sudret^{1,2}
¹ Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries, BP 10448, F-63000 Clermont-Ferrand

² Phimeca Engineering, Centre d'Affaires du Zénith, 34 rue de Sarliève, 63800 Couron d'Auvergne

Abstract

Uncertainty in the model input parameters are to be taken into account in order to assess the robustness of the model response. Sensitivity analysis based on the variance decomposition (e.g. Sobol' indices) is relatively expensive because of the conditional moments estimation. In this work, an alternative method for quantifying the uncertainty of the response due to an input variable without any reference to the response moments is applied. In order to minimize the cost of each model evaluation, a metamodel called polynomial chaos expansion is substituted to the initial model. The process is applied to numerical test cases. The results are discussed and compared with the reference obtained with variance-based methods.

Keywords : Global sensitivity analysis ; importance measure ; correlated input variables ; polynomial chaos expansion.

1. Main text

Global sensitivity analysis aims at studying how uncertainty in the model input variables can influence the uncertainty in the model output. Methods based on variance decomposition are the most famous techniques for the sensitivity analysis [1]. Considering a model $Y = M(X_1, \dots, X_N)$ with N uncertain input variables, variance-based methods allow one to decompose the total variance V of Y into partial variances, which leads to sensitivity indices such as Sobol' first order and total indices after some normalization. The computation of the conditional moment $V[E(Y | X_i)]$ is required. The numerical cost of a variance-based method is then the number of evaluations of the model M , which can be over a million. When M is not an analytical function but a numerical model such as a complex simulation code, each evaluation can last from a few seconds to a few hours and it becomes obviously impossible to run it so many times.

Metamodeling techniques enable to build an accurate representation M of the model M via only a limited number of model evaluations. In this paper we will focus on a technique called *polynomial chaos expansions*. The approach relies upon the approximation of the random response of a model M into a suitable finite-dimensional basis

$\psi_j(X), j = 0, \dots, P-1$ as follows:

$$Y \approx M(X) = \sum_{j=0}^{P-1} \alpha_j \psi_j(X) \quad (1)$$

where the α_j 's are the coefficients to be computed. Experience shows that especially when considering a high degree P , most of the α_j 's are 0. To avoid the computation of all the originally possibly null coefficients, the adaptive sparse polynomial chaos approximations based on Least Angle Regression is used [2]. This technique uses the LAR algorithm [3] that calculates the most significant coefficients until the target accuracy is reached. Then the metamodel M may be evaluated at low cost for suitable post-processing such as sensitivity analysis [4].

In this paper, we investigate a new uncertainty importance measure originally introduced in [5]. This global sensitivity indicator assesses the influence of input uncertainty on the *entire output distribution* without any reference to a specific moment of the output. Global sensitivity analysis with classical variance-based methods implies the independence hypothesis of the input parameters. This new importance measure is defined in such a way that its definition is properly posed even in the presence of correlation among the input parameters. The indicator can therefore be defined also when the input parameters X_1, \dots, X_N are dependent.

We propose to apply this method on numerical test cases that are well known in the literature on sensitivity analysis and to compare the new indicators with others indices. The method is finally applied on financial and actuarial models, for which we look after the parameters that most influence the output distribution.

2. References

- [1] A. Saltelli, S. Tarantola, F. Campolongo, M. Ratto, Sensitivity analysis in practice, John Wiley & Sons, Ltd, 2004.
- [2] G. Blatman, Adaptive sparse polynomial chaos expansions for uncertainty propagation and sensitivity Analysis, PhD thesis, Université Blaise Pascal - Clermont II, October 2009.
- [3] B. Efron, T. Hastie, I. Johnstone, and R. Tibshirani, Least angle regression, *Annals of Statistics*, 32:407–499, 2004.
- [4] B. Sudret, Global sensitivity analysis using polynomial chaos expansions, *Reliability Engineering and System Safety*, 93:964–979, 2008.
- [5] E. Borgonovo, A new uncertainty importance measure, *Reliability Engineering and System Safety*, 92:771–784, 2007.