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Adaptive Monte Carlo Approach for Sensitivity Analysis

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Abstract

An adaptive Monte Carlo strategy for computing global Sobol' sensitivity indices has been presented and discussed. The experimental scheme including an approximation tool, variance-based approaches for sensitivity analysis and Monte Carlo technique for multidimensional integration has been described and studied.

Keywords: Variance-based sensitivity analysis; Sobol' global sensitivity indices; Multidimensional integration; Adaptive Monte Carlo strategy

1. Main text

Environmental security is a very important topic for the modern society. Development of reliable and sustainable mathematical models has a significant role at this area. Specification of the most influential factors (chemical rates, boundary conditions, emission levels) on model outputs using sensitivity analysis techniques already achieves valuable information for an improvement of the model and identification of parameters that must be studied more carefully. It will lead to an increase of reliability and robustness of predictions obtained by large-scale environmental and climate models.

Variance-based methods (Sobol' approach, Fourier Amplitude Sensitivity Test) are among the most reliable approaches for providing sensitivity analysis [4]. They have certain advantages - take into account interaction effects of inputs besides of main effect and do not depend on model features as linearity, monotonicity and additivity of the relationship between input factors and model output. They also have an optimal rate (order) of computational cost for estimating all first-order and total sensitivity indices - it is proportional to sample size and number of input parameters [3].

The theoretical background and motivation of adaptive Monte Carlo procedure for sensitivity analysis have been presented. This numerical technique has been applied for numerical integration of functions with local computational difficulties. This is important since in a number of practical applications the integrand has "computational irregularities". For example, the function may be smooth, but the first derivative may increase "too fast" and may be "too large" in computational sense. The aim is to increase the reliability for evaluating Sobol' global sensitivity indices [2, 5] by reducing the variance of the corresponding Monte Carlo estimator. There are

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various adaptive Monte Carlo algorithms depending on the strategy of adaptation [1]. Our adaptive algorithm uses a posteriori information about the variance. The idea of the algorithm consists in the following: the domain of integration is separated initially into sub-domains with identical volume. Then according to the a posteriori estimates of the variance the algorithm chooses which sub-domain should be separated. The process continues until the needed accuracy is reached.

Various numerical approaches have been studied in the case of small sensitivity indices [6]. Let us note that for some large-scale non-linear models even small by value sensitivity indices may be important. The combined approach (reduction of the mean value and correlated sampling) has been chosen. We demonstrate that the corresponding results are the most reliable in comparison with other approaches.

2. References

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