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## Application of Global Sensitivity Analysis to Nuclear Reactor Calculations

F. Puente Espel<sup>a,\*</sup>, S. Tarantola<sup>b</sup>, S. Ghayeb<sup>a</sup>, K. Ivanov<sup>a</sup>

<sup>a</sup> The Pennsylvania State University, 334 Reber Bldg., University Park, PA, 16802, USA

<sup>b</sup> Joint Research Centre of the European Commission, Ispra, Italy

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### Abstract

The studies presented in this paper, describe the application of global sensitivity analysis to the modeling of nuclear reactor physics for better model understanding. Specifically, we investigate how much criticality conditions are affected by uncertainties in various inputs, including nuclear cross-sections, at different energies, from several isotopes in the fuel, the absorber and the moderator. The sensitivity analysis uses the Sobol' and Jansen formulas, which allow us to estimate, for each uncertain input, its main effect, its total effect (i.e. the overall effect, which includes all the interactions, at any order, with all the other uncertain inputs), and all two-way interactions among all possible pairs of uncertain inputs. The sensitivity analysis consists of a number of model simulations, which are performed using Monte Carlo code MCNP5.

*Keywords:* MCNP5; Sensitivity Analysis

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### 1. Application of Global Sensitivity Analysis to Nuclear Reactor Applications

The present paper applies global sensitivity analysis to the modeling of nuclear reactor applications using the Monte Carlo method.

The uncertainty within the nuclear data files is difficult to assess. The uncertainty data, expressed in percentage with respect to the nominal value of a given cross section for a given energy, is available for few isotopes and not all the nuclear data files include uncertainties. The development of uncertainty nuclear data is in progress in the major Nuclear Data Libraries (NDLs).

The reactions that have cross section values larger than 100 barns were identified among the available uncertainty data, considering that the uncertainty effect of less probable reactions have no effect on  $k_{inf}$ . It is assumed that the cross sections values are normally distributed with a mean value given by the nominal value (generally used in the computations) and a standard deviation that is given by the product of the nominal cross section and the uncertainty value expressed in percentage points. Ideally, the uncertainties of the cross sections should be modified within the nuclear data file. For this purpose, a computer code was written to apply the sensitivity studies to the MCNP5 [1] continuous energy cross-section libraries [3]. For this study, the ENDF/B-VII.0 [4] nuclear data files are used. This script is able to go directly in to the continuous energy cross section libraries and modify them by applying the uncertainties to the specific cross sections for the desire energy value. The script is capable of modifying total, capture, scattering, absorption and fission cross sections, as well as other type of reaction rates and parameters

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\* Corresponding author. Tel.: +1-814-865-0040; fax: +1-814-865-8499.

E-mail address: [fup104@psu.edu](mailto:fup104@psu.edu).

like the neutron yield per fission. Since continuous energy cross sections have point values instead of group values like the multi-group cross sections, a specific energy point can be identified.

In order to apply global sensitivity analysis to the modeling of nuclear reactor applications using the Monte Carlo method, the cross section values of several important isotopes of different regions were modified. Fuel, absorber and moderator are the three main regions selected for the presented global sensitivity analysis. For each of these regions, the most important isotopes, for which the uncertainty values exist, are selected and adjusted.

The cross section values of the three regions are modified simultaneously. A 20-column sample matrix with several thousand rows (different cases) was generated to represent such uncertainties. Each row represents a case with 20 modified cross section values (one for each column) that are used to simulate continuous energy MCNP5 criticality calculation. For the remaining isotopes the continuous energy nominal cross-section values were used (no uncertainties were attached). At the end of each run/case, the infinity multiplication factor,  $k_{inf}$ , is extracted and the process is repeated until all the runs have been calculated.

The methodology applied in this study for the sensitivity analysis is originally from Sobol and Jansen formulas [6], but it has been ameliorated by Saltelli [7]. Sobol's method is variance-based, which means that the variance of the model output can be decomposed into terms of increasing dimension, called partial variances, that represent the contribution of each single input (but even pairs, triplets, etc) to the overall uncertainty of the model output. This method allows the simultaneous exploration of the space of the uncertain inputs, which is carried out via Monte Carlo or quasi Monte Carlo sampling. Statistical estimators of partial variances are available to quantify the sensitivities of all the inputs and of groups of inputs through multi-dimensional integrals.

In terms of model simulations, the computational cost required to estimate the sensitivities of higher-order interactions between inputs can be very high. In order to prevent this, the concept of total sensitivity index has been introduced by [8], which requires much less computational effort. The total sensitivity index provides an indication of the overall effect of a given input, taking into account all possible interaction of that input with all the others. We refer to the above mentioned literature for the technical details of the methods. In this paper we estimate both first order and total sensitivity indices.

The results (i.e., both the first order and the total sensitivity indices), showing that the uncertainties of specific cross sections are the major responsible for the uncertainty of  $k_{inf}$ , are calculated.

The highest indices were observed for the elastic scattering of H-1 at thermal energy. This result indicates that the uncertainties of this cross section have the major impact on the uncertainty of  $k_{inf}$ . It reflects how much the output changes with a variation of an input. The second highest sensitivity indices were found for the elastic scattering of H-1 at a middle energy point (8 MeV). Then, the next higher indices were for the fission cross sections (U-235, Pu-239 and Pu-241, for which the highest is for U-235, as expected) and the capture cross sections. Quite the opposite, the sensitivity indices for the H-1 elastic scattering at the two highest energy groups indicate that these two cross sections have no effect on  $k_{inf}$ .

The bootstrap analysis [9] (with their estimated 90% confidence intervals) is very similar for both studies. Even that the number of cases/runs was increased considerably, from 2816 to 5110. As the number of calculations increase, the amplitude of the confidence intervals will become smaller, as expected. The small variation on the bound's amplitude illustrate the certain convergence has been reached. The results are stable and no unexpected changes should appear.

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