Máster Universitario en Administración y Dirección de Empresas
Full Time MBA

Quantitative methods for decision making

Professor Andrea Saltelli

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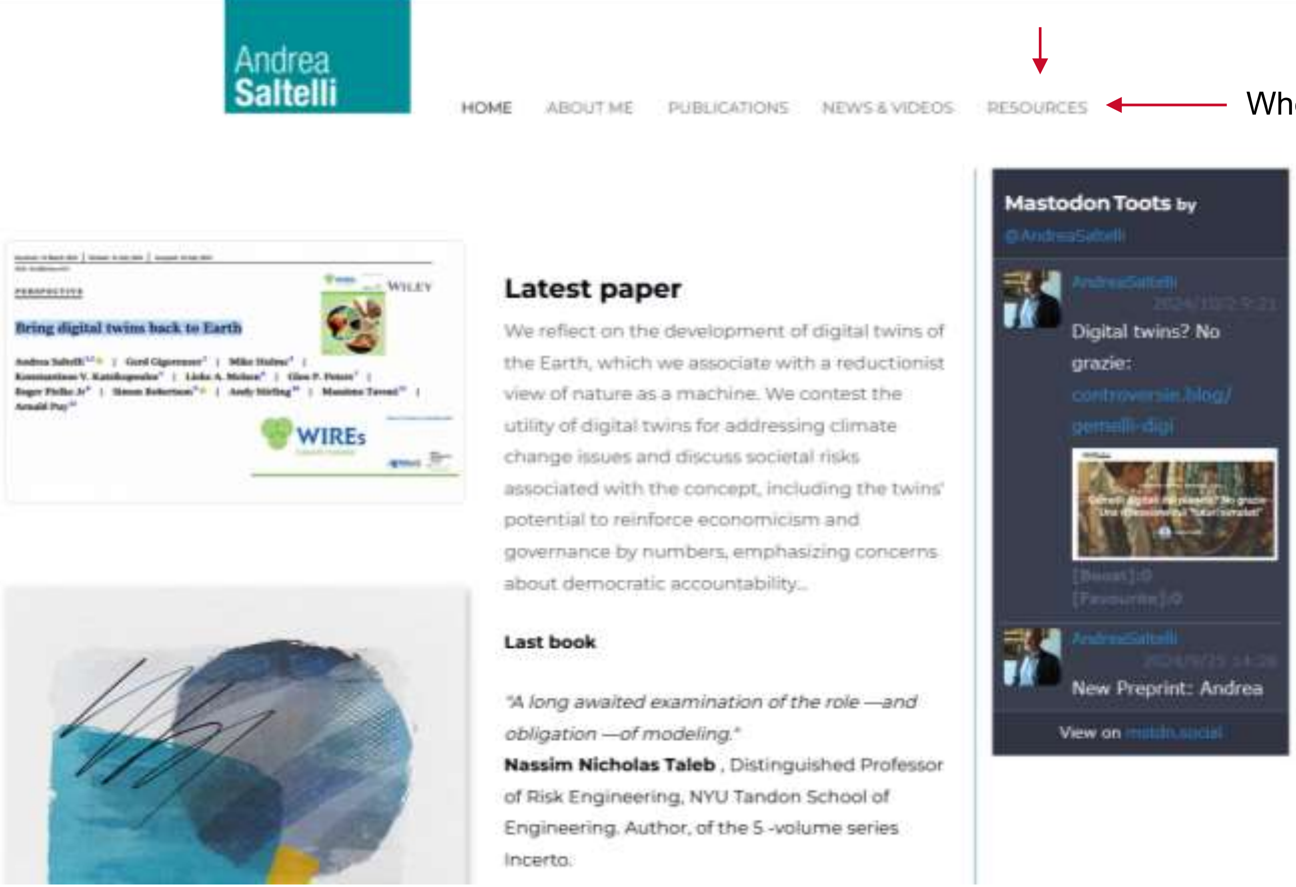

Where to find this talk

The talk is also at

https://ecampus.bsm.upf.edu/,

where you find additional reading material

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Elements of quantification for decision making with emphasis on operation research

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Homework

5) Solve Hillier online book problem 12.3-1, page 535, only question (a) Formulate a BIP model for this problem.

2.3-1.* The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix. A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs). Let the continuous decision variables x1, x2, x3, and x4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

1. No more than two of the products can be produced.

2. Either product 3 or 4 can be produced only if either product 1 or 2 is produced.

3. Either

$$5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6{,}000$$

Or

$$4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6{,}000$$

	Product			
	1	2	3	4
Start-up cost	\$50,000	\$40,000	\$70,000	\$60,000
Marginal revenue	\$70	\$60	\$90	\$80

(a) Introduce auxiliary binary variables to formulate a mixed BIP model for this problem.

Revisiting the homework of the previous lesson. Some of you preferred to maximize

$$70x_1 + 60x_2 + 90x_3 + 80x_4 - 50{,}000y_1 - 40{,}000y_2 - 70{,}000y_3 - 60{,}000y_4$$

With the additional constraints

$x_1 \le My_1$

$x_2 \le My_2$

$x_3 \le My_3$

$x_4 \le My_4$

This is OK but increases the number of constraints

Maximize

$$(70x_1 - 50{,}000)y_1 + (60x_2 - 40{,}000)y_2 + (90x_3 - 70{,}000)y_3 + (80x_4 - 60{,}000)y_4$$

Subject to:

$y_1 + y_2 + y_3 + y_4 \le 2$

$y_3 \le y_1 + y_2$

$y_4 \le y_1 + y_2$

$5x_1 + 3x_2 + 6x_3 + 4x_4 \le 6{,}000 + My_5$

$4x_1 + 6x_2 + 3x_3 + 5x_4 \le 6{,}000 + M(1 - y_5)$

with $x_j \ge 0,\ \ j = 1,2,3,4,5\ \ y_j, j = 1,2,3,4,5$ binary, and M a large number

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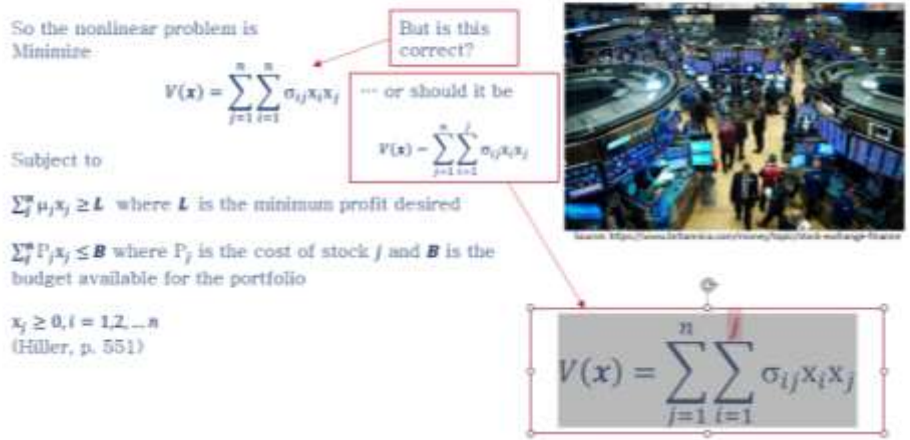

In case there are two items in the portfolio this gives $V(\boldsymbol{x}) = \sigma_{11}x_1x_1 + \sigma_{12}x_1x_2 + \sigma_{21}x_2x_1 + \sigma_{22}x_2x_2$

I think the term x_1x_2 should appear only once and hence

$$V(\boldsymbol{x}) = \sum_{j=1}^{n}\sum_{i=1}^{j} \sigma_{ij}x_ix_j$$

Remember $V(\boldsymbol{x}) = \sum_{j=1}^{n}\sum_{i=1}^{n} \sigma_{ij}x_ix_j$?

Homework

1. Both a dice and a coin are launched simultaneously in an experiment. We count a coin falling head as one and falling tail as a zero. If we call success the outcome seven (dice=six, coin=H), which is the chance of success in one experiment? Which is the chance of two successes in 4 experiments? Write down the space of the events first.

$\{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$ ➔ 12 outcomes

$$P(6H) = \frac{1}{12}$$

$$\binom{4}{2}\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)^2 = 0.035$$

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2. Solve the problem below, knowing that $\frac{\partial y}{\partial x} = 3x^2 + 2 - 4x - x^3$ and $\frac{\partial^2 y}{\partial x^2} = 6x - 4 - 3x^2$

13.4-1.* Consider the following problem:

Maximize $\quad f(x) = x^3 + 2x - 2x^2 - 0.25x^4.$

I **(a)** Apply the bisection method to (approximately) solve this problem. Use an error tolerance $\epsilon = 0.04$ and initial bounds $\underline{x} = 0$, $\bar{x} = 2.4$.

(b) Apply Newton's method, with $\epsilon = 0.001$ and $x_1 = 1.2$, to this problem.

Choose just one (bisection or Newton)

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A	B	C	D	E	F
Lower	Upper	Der Lower	Der Upper	x next	Der next
0	2.4	2	-4.144	1.2	-0.208
0	1.2	2	-0.208	0.6	0.464
0.6	1.2	0.464	-0.208	0.9	0.101
0.9	1.2	0.101	-0.208	1.05	-0.05013
0.9	1.05	0.101	-0.05013	0.975	0.025016
0.975	1.05	0.025016	-0.05013	1.0125	-0.0125
0.975	1.0125	0.025016	-0.0125	0.99375	0.00625
0.99375	1.0125	0.00625	-0.0125	1.003125	-0.00313
0.99375	1.003125	0.00625	-0.00313	0.998438	0.001563
0.998438	1.003125	0.001563	-0.00313	1.000781	-0.00078
0.998438	1.000781	0.001563	-0.00078	0.999609	0.000391
0.999609	1.000781	0.000391	-0.00078	1.000195	-0.0002
0.999609	1.000195	0.000391	-0.0002	0.999902	9.77E-05
0.999902	1.000195	9.77E-05	-0.0002	1.000049	-4.88E-05
0.999902	1.000049	9.77E-05	-4.88E-05	0.999976	2.44E-05
0.999976	1.000049	2.44E-05	-4.88E-05	1.000012	-1.22E-05

Bisection

Example cell A3:

=IF(F2<0,A2,E2)

A	B	C
x1	der 1	der 2
1.2	-0.208	-1.12
1.014286	-0.01429	-1.00061
1.000006	-5.8E-06	-1
1	0	-1
1	0	-1

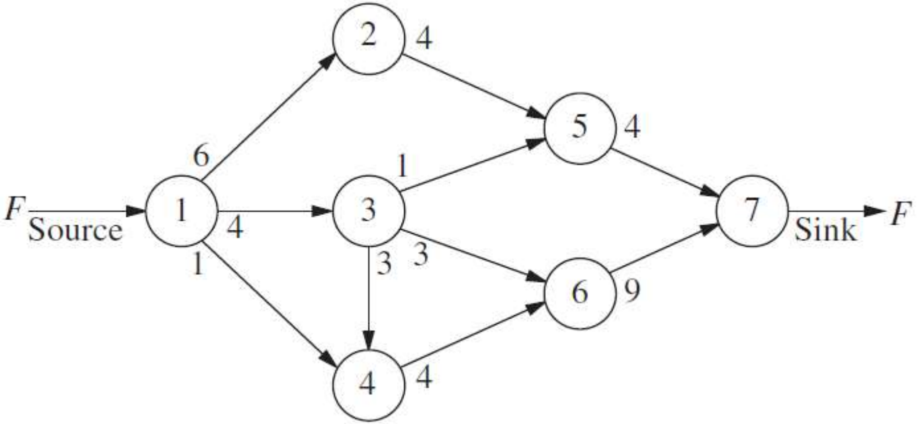
Newton

Cell A3:

=A2-(B2/C2)

Newton converges in two iterations, bisection needs six or more to obtain the same accuracy (both files in eCampus)

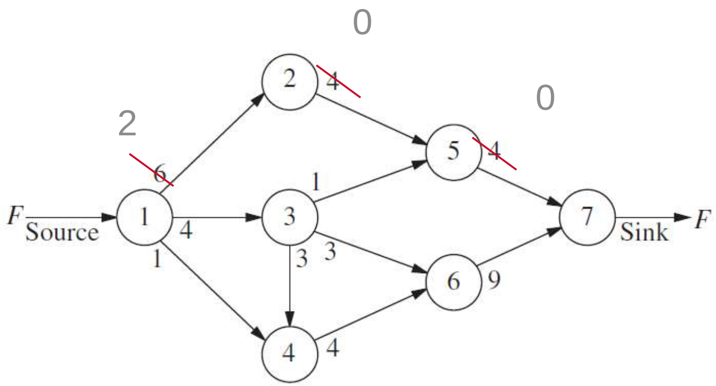
10.5-1.* For the network shown below, use the augmenting path algorithm described in Sec. 10.5 to find the flow pattern giving the *maximum flow* from the source to the sink, given that the arc capacity from node i to node j is the number nearest node i along the arc between these nodes. Show your work.



4+3+1+1

4 via 1,2,5,7
3 via 1,3,6,7
1 via 1,3,4,7
1 via 1,4,6,7

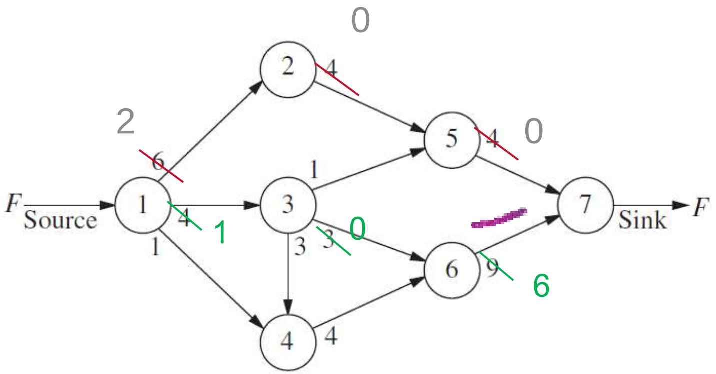
10.5-1. Arc	(1, 2)	(1, 3)	(1, 4)	(2, 5)	(3, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 7)	(6, 7)
Flow	4	4	1	4	1	0	3	2	4	5



4 via 1,2,5,7

Similarly for the next two moves:

1 via 1,3,4,7



1 via 1,4,6,7

3 via 1,3,6,7

4. Write in binary the fist 10 prime numbers

1	01
2	10
3	11
5	101
7	111
11	1011
13	1101
17	10001
19	10011
23	10111

Using the algorithm for 23:

Division by 2		Remainder
23/2	11	1
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

Reading upward:

10111

In this set of slides:

17 Decision Theory
18 Programming and Planning. PERT and CPM
19 Queueing Theory
20 Ethical considerations for OR

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17.

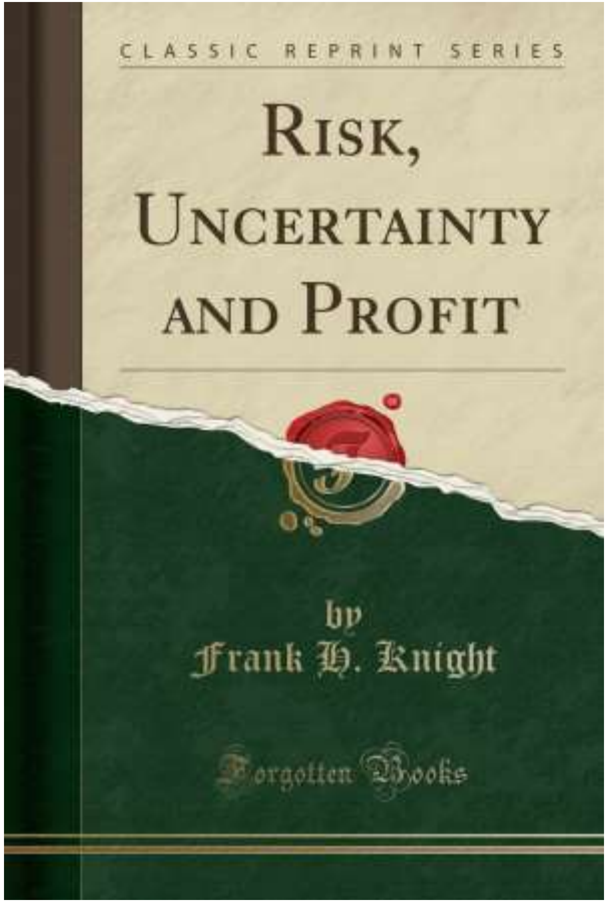
Decision Analysis

Knight. Decision making with and without experimentation. Example: drilling or selling? Bayes in full. Decision trees. Multi Criteria Decision Analysis. Linearization. Borda count, Condorcet's outranking matrix and Balinski-Laraki's majority judgment. Hillier (2014) chapter 16 plus various authors.

Frank Knight (1921) distinguished risk from uncertainty

Risk = know outcomes & probabilities; roulette game

Uncertainty = unsure about the probabilities; starting a business



Frank H. Knight
1885-1972

But the story has many more twists

The debate between Leonard Savage and Frank Ramsey on one side and John Maynard Keynes and Frank Knight on the other: **a fundamental disagreements in decision theory**

Savage and Ramsey: pioneers of subjective probability theory for rational decision making – Keynes and Knight: sceptical that all uncertainties could be quantified probabilistically due to uncertainty & ambiguities

Leonard Jimmie Savage (1917-1971)

Frank Ramsey (1903-1930)

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But the story has many more twists … Suggested reading

Savage's agents are rational and indifferent to ambiguity. Keynes and Knight suggested that known risks are preferred over unknown ones ➔ today behavioural economics

Savage and Ramsey "won" but see "info-gap theory" and "robust decision-making" (both involving better exploration and sensitivity analysis)

Frank H. Knight
1885-1972

John Maynard Keynes
1883-1946

Available on eCampus

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A prototype example of decision under uncertainty; drilling or selling?

Source: https://ecsgeothermal.com/oil-drilling-on-land/

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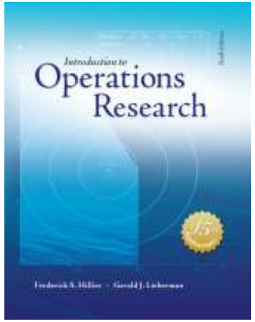
A company own land
where there could be oil

Another company offers
to purchase said land



Source: https://ecsgeothermal.com/oil-drilling-on-land/

TABLE 16.1 Prospective profits for the Goferbroke Company

Alternative \ Status of Land	Payoff: Oil	Payoff: Dry
Drill for oil	\$700,000	−\$100,000
Sell the land	\$ 90,000	\$ 90,000
Chance of status	1 in 4	3 in 4

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The table offers different payoffs associated to different decision (sell, drill) versus two possible states of nature (oil, no-oil)

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Drill for oil	\$700,000	−\$100,000
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How to act on this table? Different **alternatives** are available.

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The **Maximin Payoff Criterion**

■ **TABLE 16.1 Prospective profits for the Goferbroke Company**

Alternative \ Status of Land	Payoff: Oil	Payoff: Dry
Drill for oil	$700,000	−$100,000
Sell the land	$ 90,000	$ 90,000
Chance of status	1 in 4	3 in 4

For each decision look at the worst **payoff** over all possible **states of nature** …

…and choose the one with the best outcome

Drill: Worse payoff for state of nature = dry, -100K
Sell: Worse payoff for state of nature? Indifferent (dry or oil the same payoff 90K)

Sell, as 90 is better than -100

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The **Maximum likelihood approach**

TABLE 16.1 Prospective profits for the Goferbroke Company

Alternative \ Status of Land	Payoff	
	Oil	**Dry**
Drill for oil	$700,000	−$100,000
Sell the land	$ 90,000	$ 90,000
Chance of status	1 in 4	3 in 4

Identify the most likely state of nature ← Dry, as $\frac{3}{4}$ is more than $\frac{1}{4}$ (prior probabilities)

…and choose the alternative with the best pay-off ← Sell, as 90 is better than −100

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The Maximum likelihood approach

TABLE 16.4 Application of the maximum likelihood criterion to the first Goferbroke Co. problem

	State of Nature			
Alternative	**Oil**	**Dry**		
1. Drill for oil	700	−100	−100	
2. Sell the land	90	90	90	← Maximum in this column
Prior probability	0.25	0.75		
		↑ Maximum		

Maxmin and Maximum Likelihood seem similar and give the same result but are different: the maxmin approach is pessimistic: malevolent nature, the worst will happen – as if nature were an opponent in game theory

Maximum Likelihood appears more balanced, and only need to know which state of nature is more likely (not of how much), but ignores other elements of the problem (e.g. the possible juicy payoff 700)

Bayes' rule – **the Expected Value Approach**

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Calculate the best expected payoff for each decision alternative

Payoff (Drill)=0.25∗700+ 0.75∗(−100)=100

Payoff (Sell)=0.25∗90+ 0.75∗90=90

All the available information is used

Now Drill is preferred

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Yet, remember the caveat of lesson one: expected value may suggest courses of action your wouldn't take yourself

This is the story the St. Peterburg paradox (another game!)

Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli
(1700-1782)

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

And indeed if one takes 'dig' there remain a risk of losing 100 … in what cases would one take the risk?

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Bayes' rule – the expected value approach

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

What the prior probabilities are uncertain? What if instead of 0.25 the probability of oil is instead 0.15 or 0.35?

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

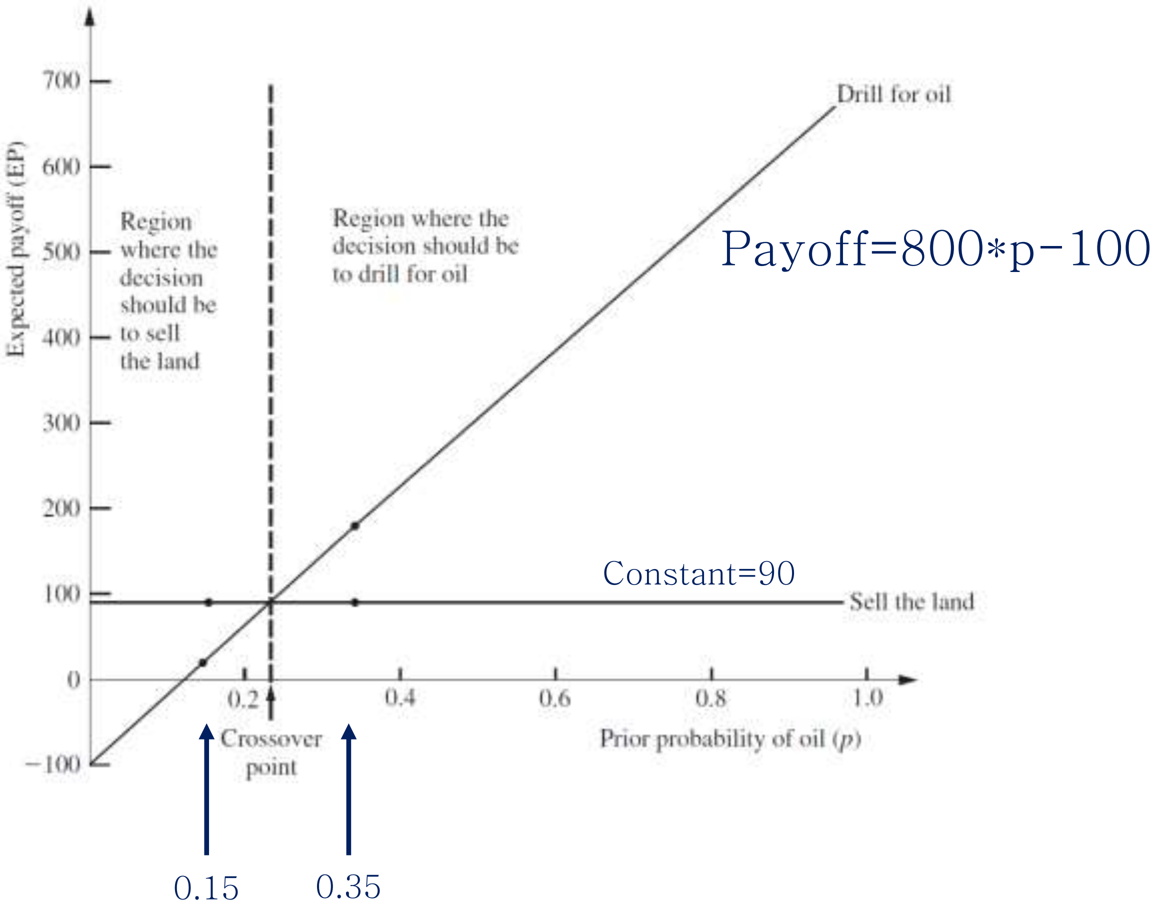
What do we do if we feel uneasy with these prior probabilities? What if instead of 0.25 the probability of oil is instead 0.15 or 0.35?

The drill payoff for a generic value p of the prior of oil

$$p*700-(1-p)*100=800*p-100$$

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

What if this is not 0.25, but for example 0.15 or 0.35?



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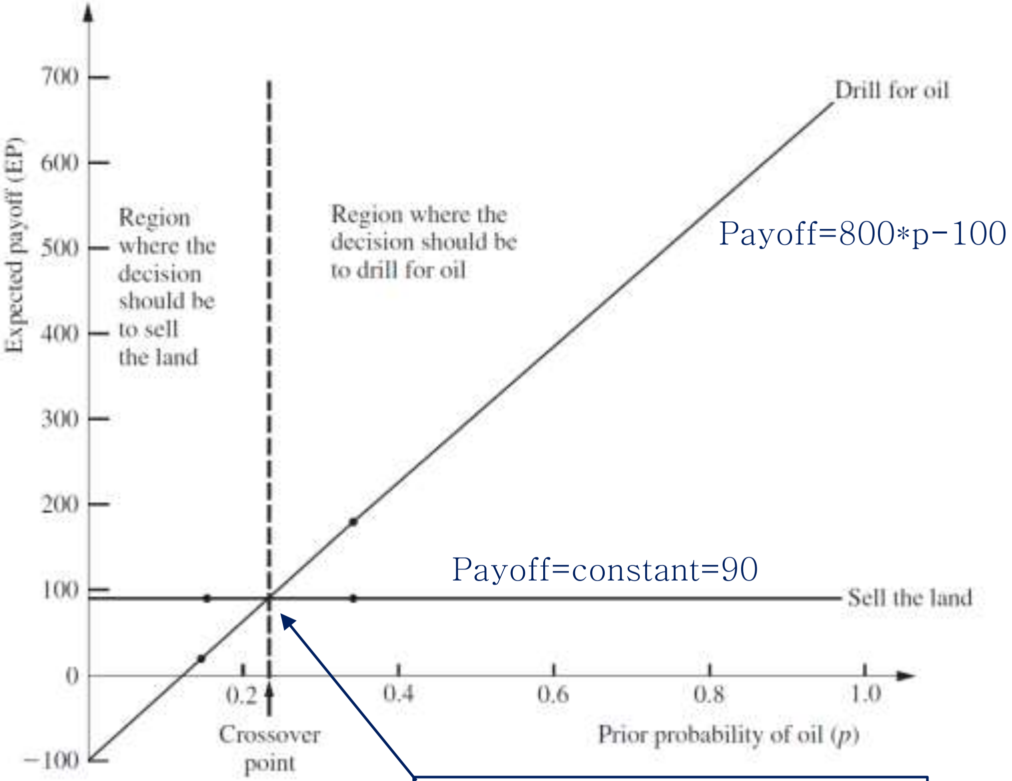
	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Exercise: compute cross over coordinates

Source: https://simpsons.fandom.com/wiki/Bart_Gets_Famous



(EP, *p*)=(190/800, 90)=(~2.4, 90)

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	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Exercise: compute cross over coordinates

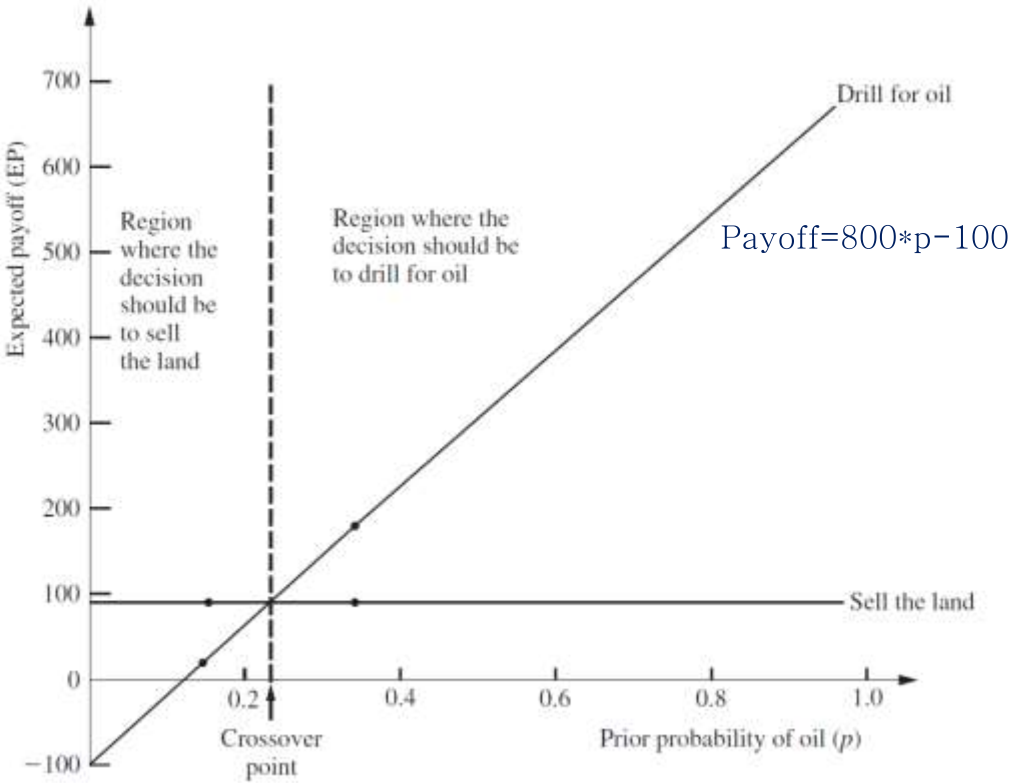
Intersection of

$$y = 800x - 100$$

and

$$y = 90$$

$$x = 190/800 = .2375$$



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	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Conclusion:

if p < .2375 then sell

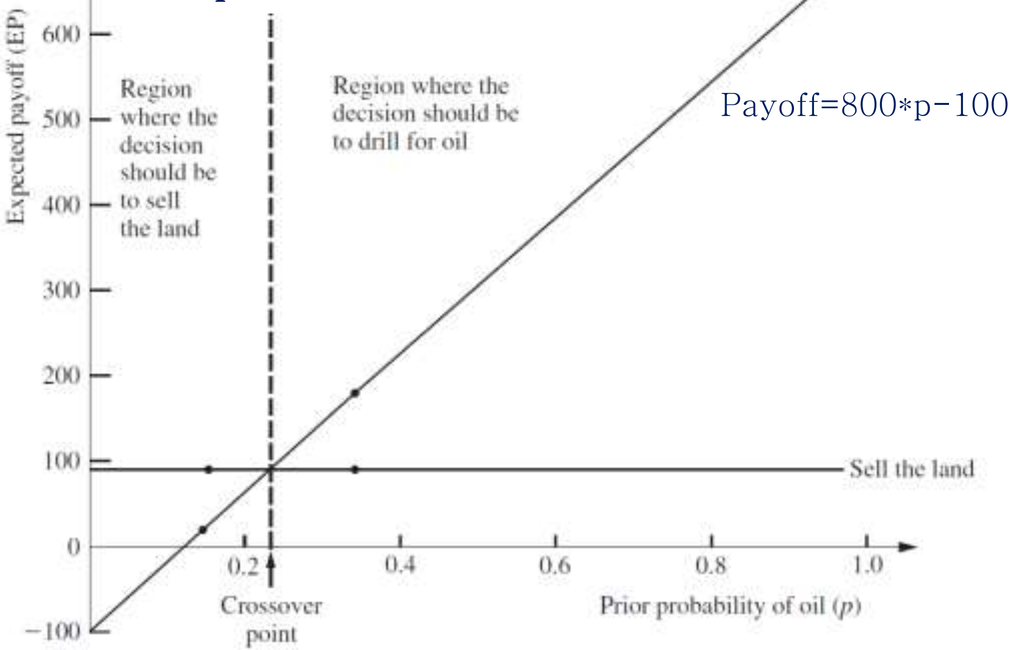
if p > .2375 then drill

Is this a good sensitivity analysis?

p*700−(1−p)*100=800*p−100

p was varied

700 and 100 were not



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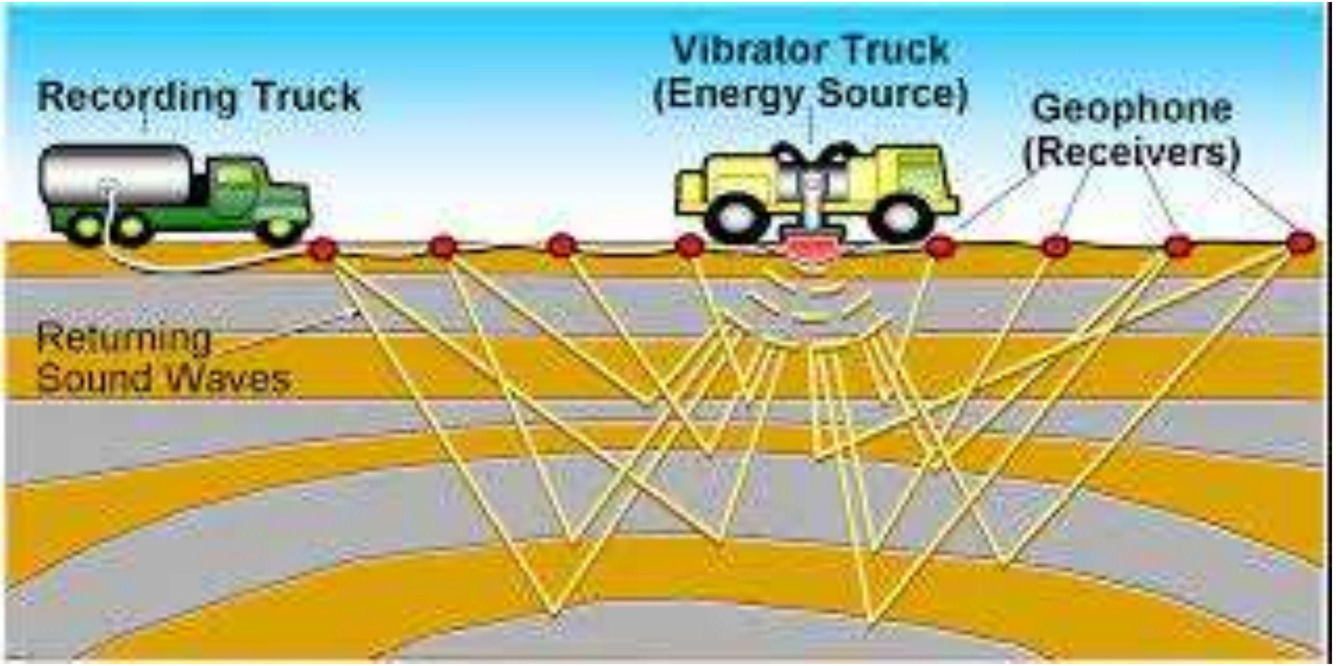
Decision making with experimentation



Source: https://gov.nu.ca/sites/default/files/2017_seismic_eng.pdf

Perhaps before deciding whether to sel
some prospection study should be done
as seismic surveying

This would come to a cost, so even in this case, before the survey, it would be wise to crunch some numbers

The cost of the seismic survey is $30,000.

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Experience says that:
USS: **U**nfavorable **S**eismic **S**oundings ➔ oil is fairly unlikely.
FSS: **F**avorable **S**eismic **S**oundings ➔ oil is fairly likely.

Again experience translates this into
(these are **data** of the problem; **they are given to us**)

$$p(USS|Oil) = 0.4 \text{ and } p(FSS|Oil) = 1 - 0.4 = 0.6$$

$$p(USS|Dry) = 0.8 \text{ and } p(FSS|Dry) = 1 - 0.8 = 0.2$$

In plain English:

Geologists tell us that **if the oil is there**, the test will be negative (no oil) 40% of the times and positive (yes there is oil) 60% of the times

… and that **if the oil is not there** the test will be negative 80% of the times and positive 20% of the times

ChatGPT

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As it is written, the famous theorem ‘looks’ symmetric in A and B $\cdots$

$$p(A|B)p(B) = p(B|A)p(A) = p(A \cap B)$$

Conditional

Prior

Joint

In fact the way it is used in practice is rather asymmetric, and aims to update A based on B being true, B being for example an experiment and A a theory

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

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A = All swans are white, $p(A) \sim 1$ THE THEORY

B = A black swan will be observed AN EXPERIMENT

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} = \frac{0*1}{p(B)} = 0 \text{ for any value of } p(B) \neq 0 \text{ since } p(B|A) = 0$$

ChatGPT

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Remember from Lesson #1; Theory could be “This disease”, “This culprit”, and Experiment could be “A fever” “A DNA sample”

Inferring the causes from observed events is “the way of the historian, the policemen, and a doctor, who suggest a diagnosis based on symptoms” (Desrosières 1993)

Arthur Conan Doyle was a medical doctor …

… much of what we watch on television is about inferring causes from symptoms or clues

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When B is the outcome of an experiment and A is a state of nature (here oil) $p(A|B)$ becomes the probability that given the outcome B (for example a favourable outcome of the survey FSS) then we indeed have A – the oil;

We do not know $p(A|B)$ but we do know from experience $p(B|A)$, the probability that if there is oil the test will be favourable

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

We don’t know

We know from experience

This is where the asymmetry emerges

we do not know $p(A|B)$ but we do know $p(B|A)$, in this case the probability that if there is oil the test will be favourable

We also know $P(Oil)$, as this is the old prior, the probability of oil being there before the survey

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

We know the prior

we do not know $p(A|B)$ but we do know $p(B|A)$, in this case the probability that if there is oil the test will be favourable

We also know $P(Oil)$, as this is the old prior, the probability of oil being there before the survey

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

We lack this but ⋯

We only lack $P(FSS)$, the unconditional probability of favourable drilling. Can we derive it **before the drilling**?

Deriving $P(FSS)$. This is a delicate point. The unconditional probability of favourable drilling is the total probability of this outcome in all cases, e.g. both oil and no-oil

Which are the priors?

Which are the conditional?

How about:

$$P(FSS) = p(FSS|Oil)P(Oil) + p(FSS|Dry)P(Dry)$$

…but we know all these; two priors and two conditional distribution that we know from experience

This is indeed the total, and hence unconditional, probability of FSS – that is to say all possible ways in which FSS can come about

Putting these two together:

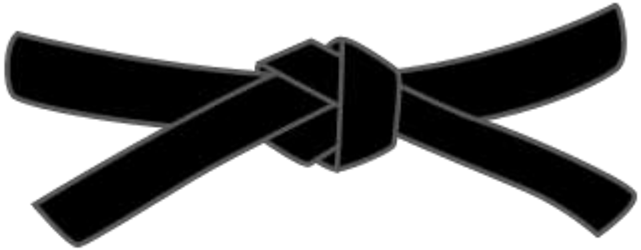
$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

$$P(FSS) = p(FSS|Oil)P(Oil) + p(FSS|Dry)P(Dry)$$

we get

$$p(Oil|FSS) = \frac{p(FSS|Oil)P(Oil)}{p(FSS|Oil)P(Oil) + p(FSS|Dry)P(Dry)}$$

You have just done your first Bayesian updating



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Plugging the numbers

$$p(Oil|FSS) = \frac{\overset{0.6}{p(FSS|Oil)}\,\overset{0.25}{P(Oil)}}{\underset{0.6}{p(FSS|Oil)}\,\underset{0.25}{P(Oil)} + \underset{0.20}{p(FSS|Dry)}\,\underset{0.75}{P(Dry)}}$$

=0.15/(.15+.15)=0.5

And this gives $p(Oil|FSS) = \frac{1}{2}$

Hence since

$$p(Oil|FSS) = \frac{1}{2}$$

then

$$p(Dry|FSS) = \frac{1}{2}$$

And following a similar path for the negative survey outcome USS

$$p(Oil|USS) = \frac{1}{7} = .14$$

$$p(Dry|USS) = \frac{6}{7} = .86$$

Indeed the survey is a game changer when compared to the prior probabilities $P(Oil)$=0.25 and $P(Dry) = 0.75$

➔ Probably nobody would be a taker for drill if USS is true

➔ One half is much better than one in four if FSS is true

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Recalling that

$$p(USS|Oil) = 0.4$$
$$p(USS|Dry) = 0.8$$

And

$$p(Oil) = 0.25$$
$$p(Dry) = 0.75$$

Compute

$$p(USS)$$

Source: https://simpsons.fandom.com/wiki/Bart_Gets_Famous

$$p(USS) = p(USS|Oil)\, p(Oil) + p(USS|Dry)\, p(Dry)$$
$$= 0.4 * 0.25 + 0.8 * 0.75 = 0.7$$

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$\cdots$ we now know that $p(USS) = 0.7$, $p(USS|Oil) = 0.4$

and that

$$p(Oil|USS)P(USS) = p(USS|Oil)P(Oil)$$

Compute $p(Oil|USS)$

$$p(Oil|USS) = \frac{p(USS|Oil)P(Oil)}{P(USS)} = \frac{0.4 * 0.25}{0.7} = .143$$

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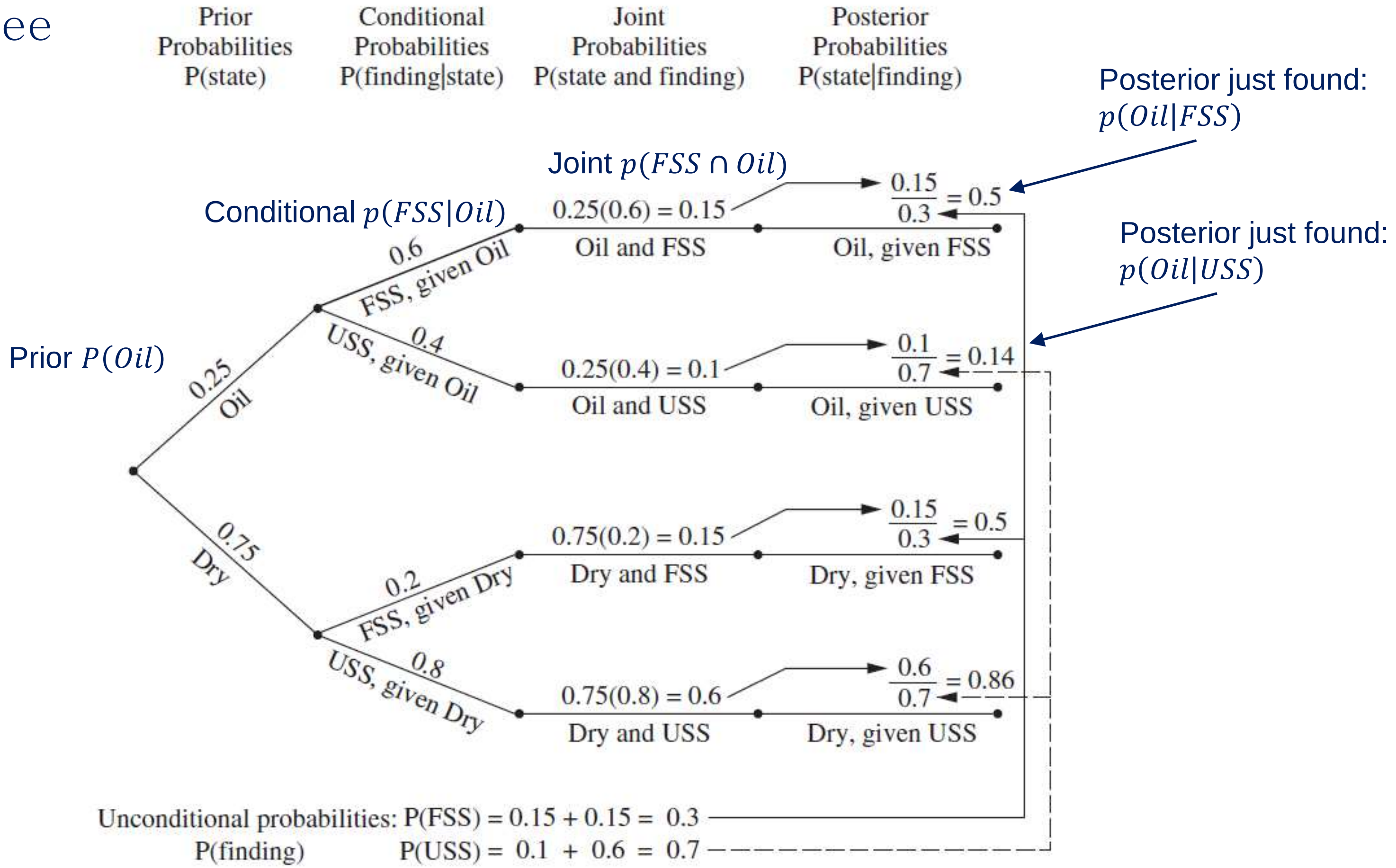
All in a tree


Prior Probabilities P(state)
Conditional Probabilities P(finding|state)
Joint Probabilities P(state and finding)
Posterior Probabilities P(state|finding)
Posterior just found: $p(Oil|FSS)$
Posterior just found: $p(Oil|USS)$
Joint $p(FSS \cap Oil)$
Conditional $p(FSS|Oil)$
Prior $P(Oil)$
0.25 Oil
0.75 Dry
0.6 FSS, given Oil
0.4 USS, given Oil
0.2 FSS, given Dry
0.8 USS, given Dry
0.25(0.6) = 0.15
Oil and FSS
0.25(0.4) = 0.1
Oil and USS
0.75(0.2) = 0.15
Dry and FSS
0.75(0.8) = 0.6
Dry and USS
$\frac{0.15}{0.3} = 0.5$
Oil, given FSS
$\frac{0.1}{0.7} = 0.14$
Oil, given USS
$\frac{0.15}{0.3} = 0.5$
Dry, given FSS
$\frac{0.6}{0.7} = 0.86$
Dry, given USS
Unconditional probabilities: P(FSS) = 0.15 + 0.15 = 0.3
P(finding) P(USS) = 0.1 + 0.6 = 0.7

We now need to use all these

$$p(Oil|FSS) = \frac{1}{2} = 0.5$$

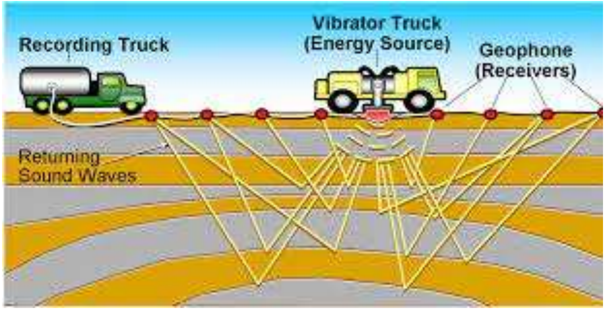
$$p(Dry|FSS) = \frac{1}{2} = 0.5$$

$$p(Oil|USS) = \frac{1}{7} = 0.14$$

$$p(Dry|USS) = \frac{6}{7} = 0.86$$

to take a decision, about drill, sell, and survey

Source: https://ecsgeothermal.com/oil-drilling-on-land/



Source: https://gov.nu.ca/sites/default/files/2017_seismic_eng.pdf

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This is now straightforward:

Payoffs if unfavourable survey (USS):

$$E(Payoff - Drill|USS) = \frac{1}{7}(700) + \frac{6}{7}(-100) - 30 = -15.7$$

$$E(Payoff - Sell|USS) = \frac{1}{7}(90) + \frac{6}{7}(90) - 30 = 60$$

$$p(Oil|FSS) = \frac{1}{2}$$

$$p(Dry|FSS) = \frac{1}{2}$$

$$p(Oil|USS) = \frac{1}{7}$$

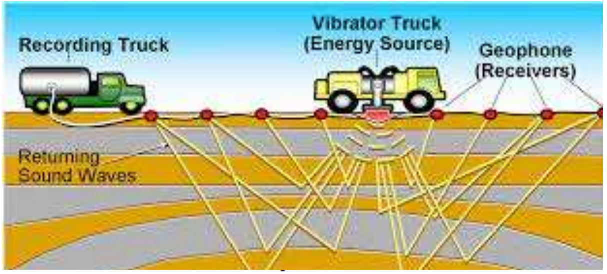
$$p(Dry|USS) = \frac{6}{7}$$

Source: https://ecsgeothermal.com/oil-drilling-on-land/

Payoffs if favourable survey (FSS):

$$E(Payoff - Drill|FSS) = \frac{1}{2}(700) + \frac{1}{2}(-100) - 30 = 270$$

$$E(Payoff - Sell|FSS) = \frac{1}{2}(90) + \frac{1}{2}(90) - 30 = 60$$



We got an idea of what to do after drill … but let us take the example further to get something more …

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

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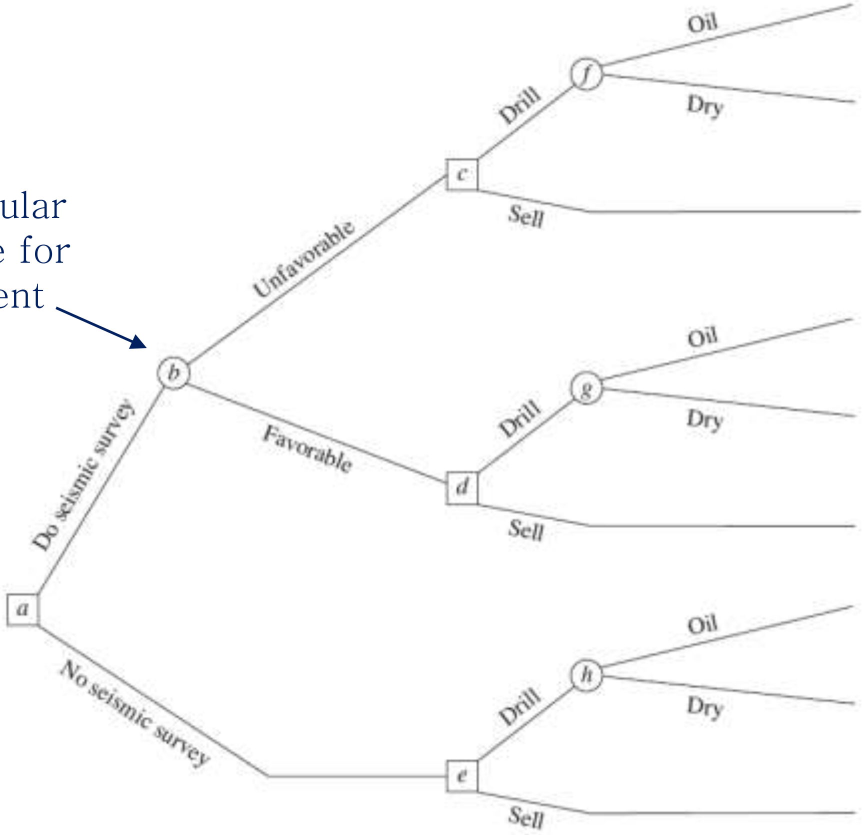
Decision tree for the same problem

Circular node for event

Square node for decision



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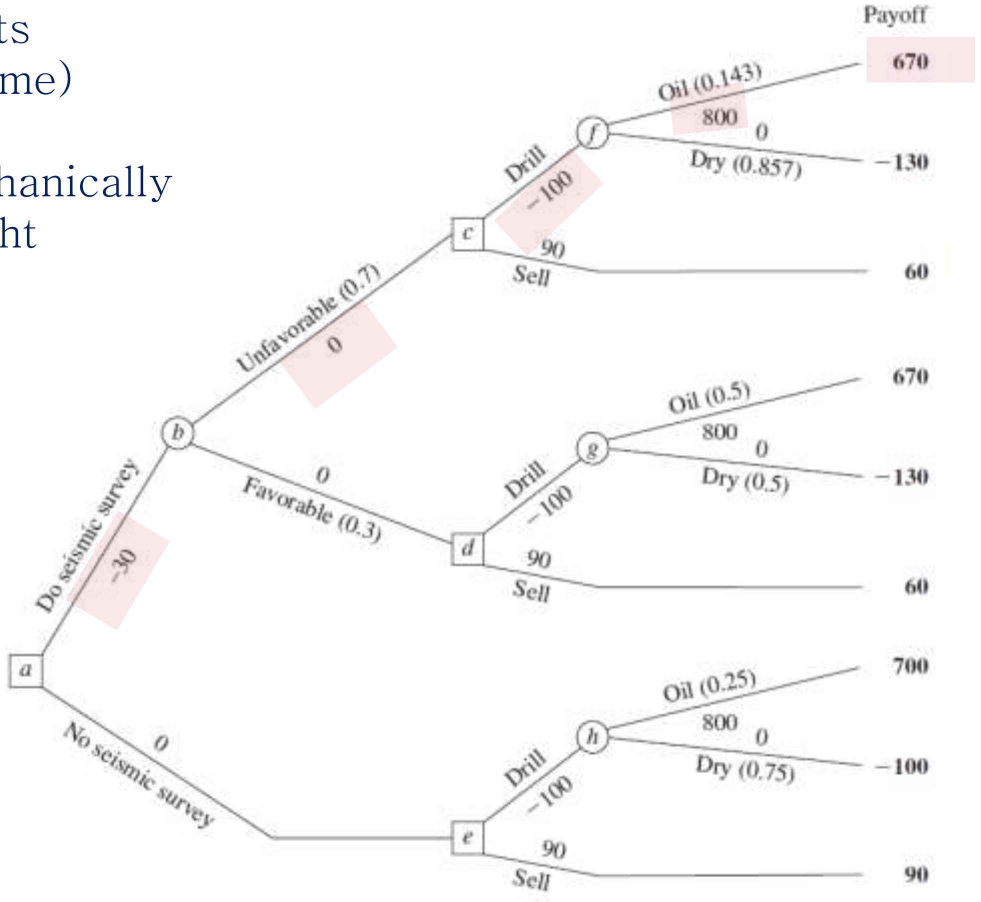
Decision tree with costs
(no probabilities this time)

Cost are compute mechanically moving from left to right



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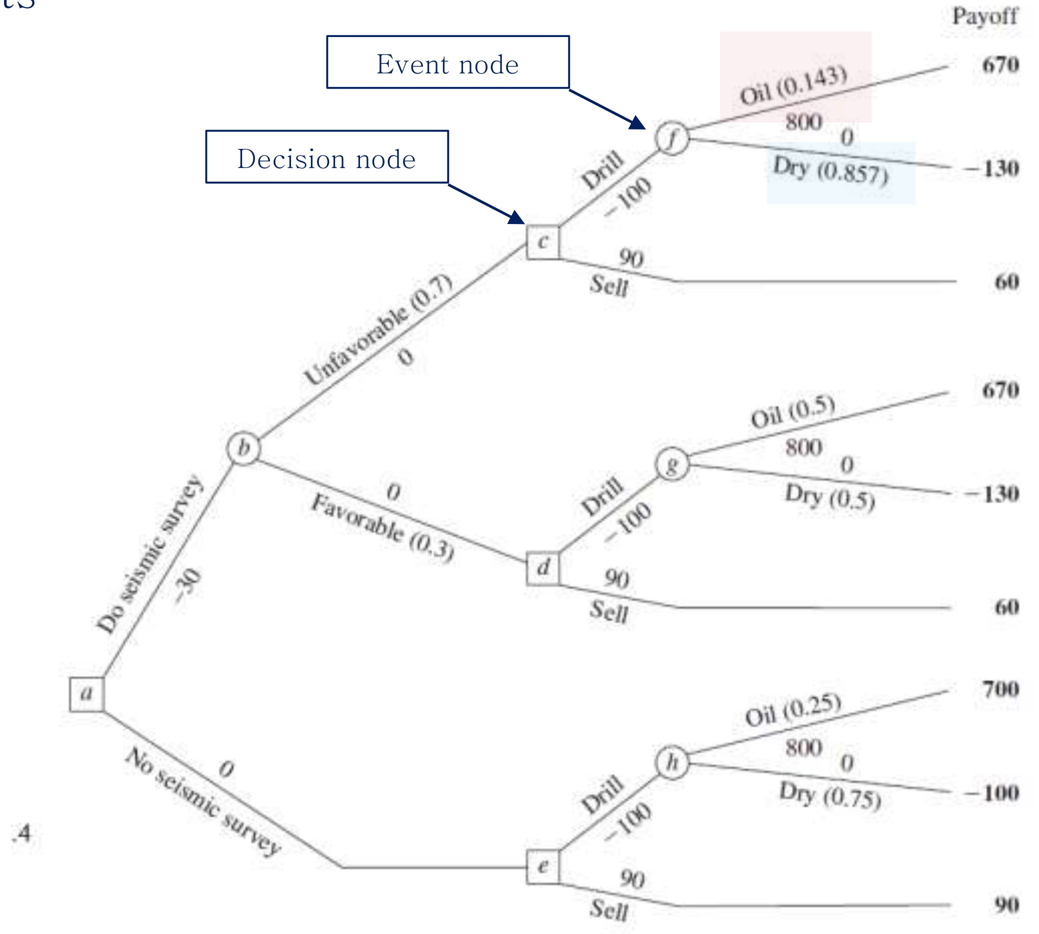
Decision tree with costs (adding probabilities)

$$p(Oil|FSS) = \frac{1}{2} = 0.5$$

$$p(Dry|FSS) = \frac{1}{2} = 0.5$$

$$p(Oil|USS) = \frac{1}{7} = 0.143$$

$$p(Dry|USS) = \frac{6}{7} = 0.857$$



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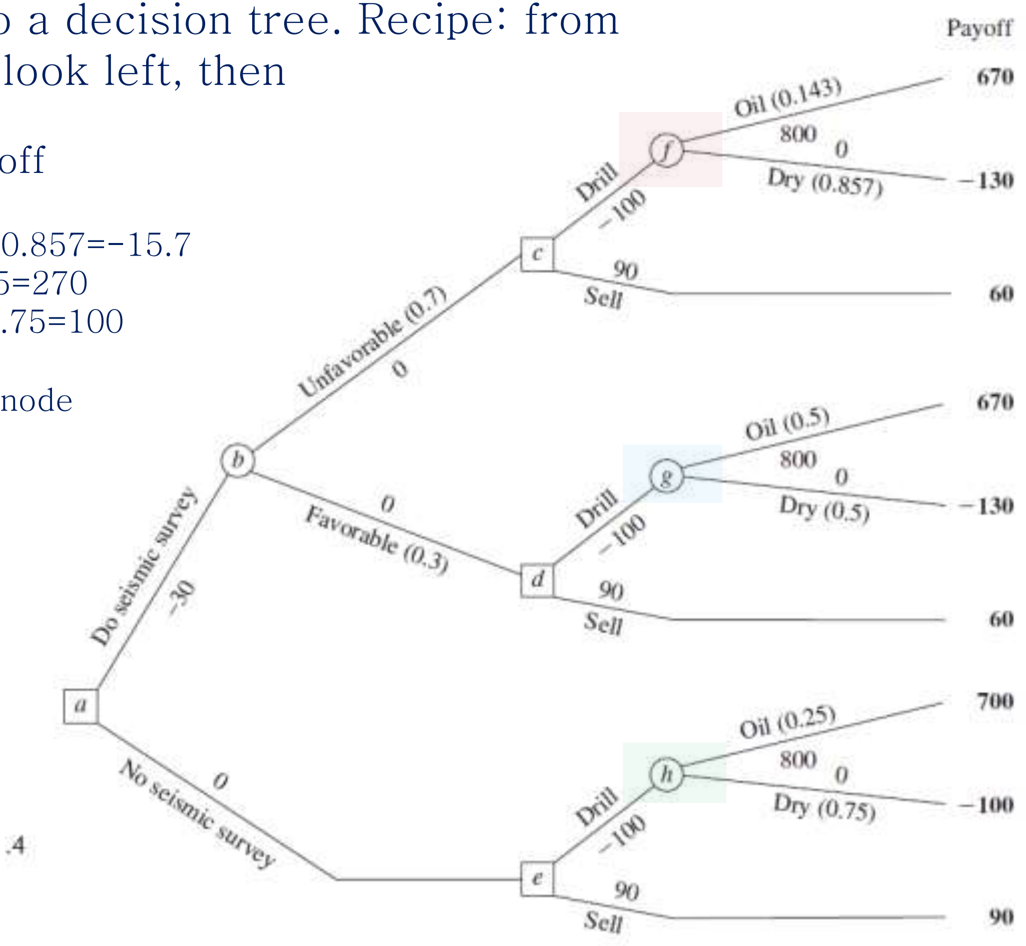
Transforming this into a decision tree. Recipe: from the rightmost column look left, then

If node=event compute payoff

$Payoff(f)$ =670*0.143+ (−130)*0.857=−15.7
$Payoff(g)$ =670*0.5+ (−130)*0.5=270
$Payoff(h)$ =700*0.25+ (−100)*0.75=100

Write these numbers above the node



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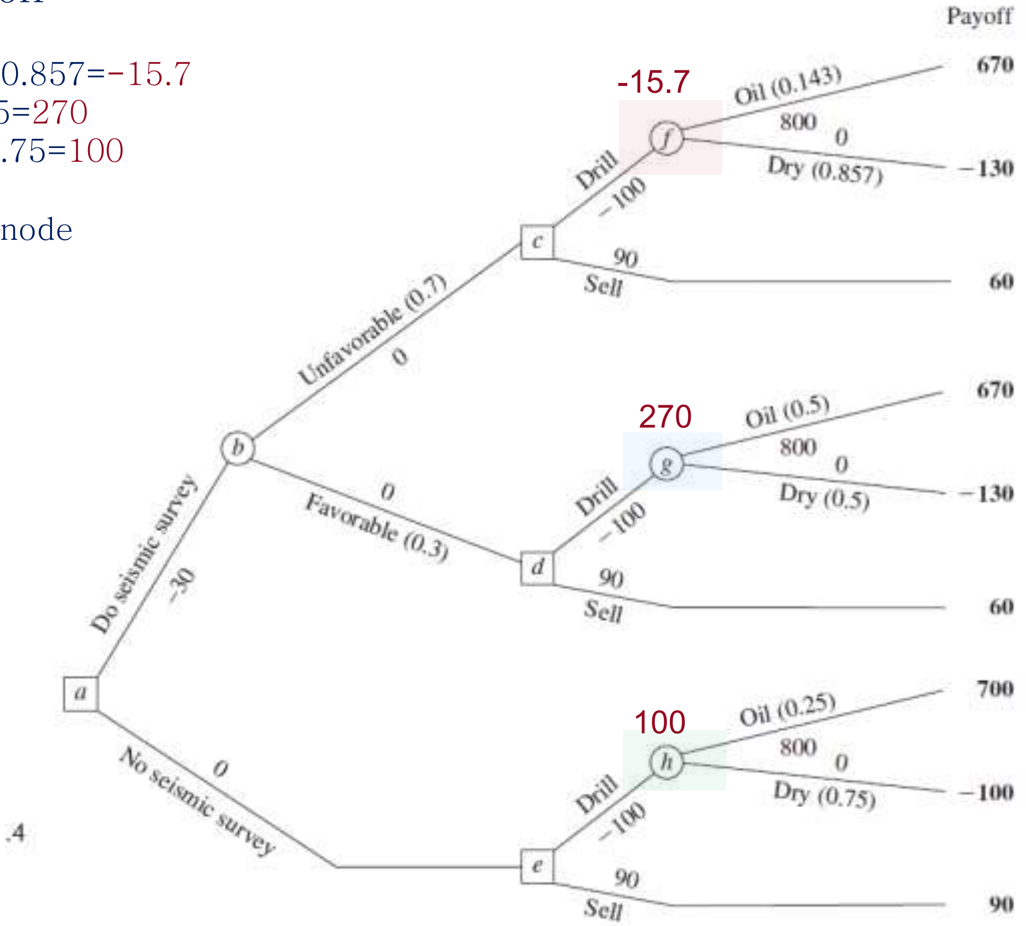

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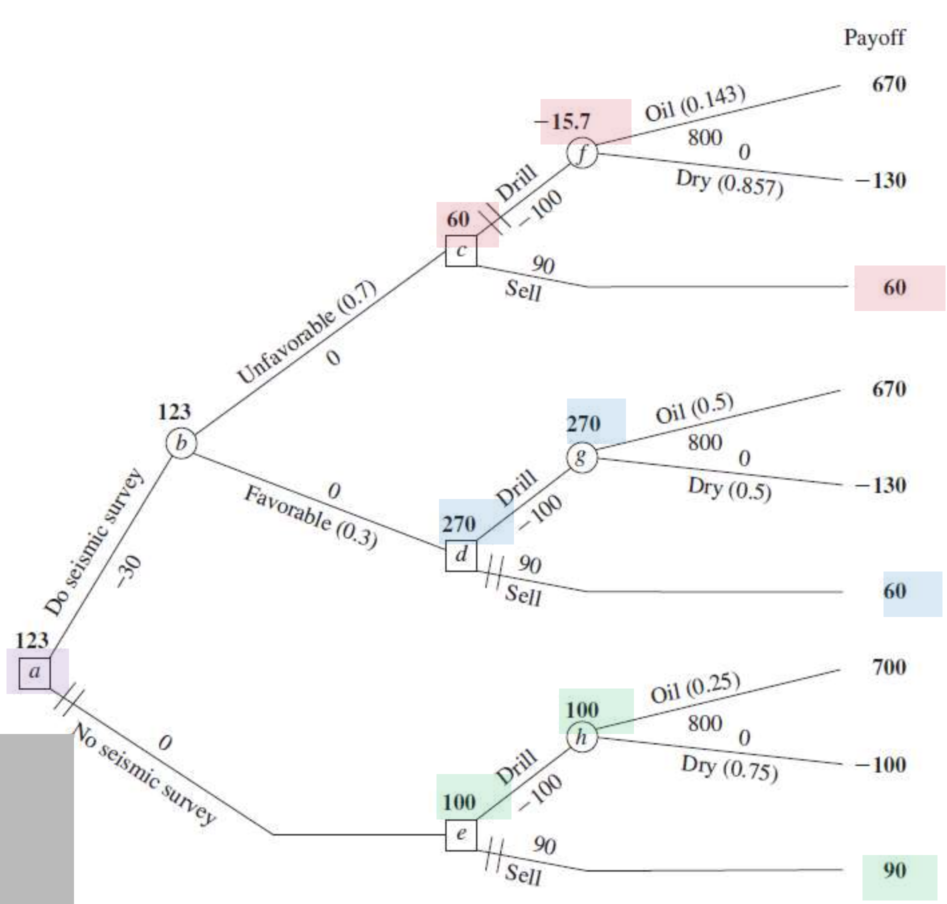
$Payoff(f)$ =670*0.143+ (−130)*0.857=−15.7
$Payoff(g)$ =670*0.5+ (−130)*0.5=270
$Payoff(h)$ =700*0.25+ (−100)*0.75=100

Write these numbers above the node

If node=decision then decide

$Decision(c) = Sell$
$Decision(d) = Drill$
$Decision(e) = Drill$

Report the payoff selected above the node
Move left
$Payoff(b)$ =60*0.7+ 270*0.3=123
$Decision(a) = Do\ survey$



Optimal policy:
Do the seismic survey.
If the result is unfavorable, sell the land.
If the result is favorable, drill for oil.
The expected payoff (including the cost of the seismic survey) is 123 ($123,000).

How to deal with possible paradoxes when using expected value (our old slide again):

This is the story the St. Peterburg paradox (another game!)

Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli
(1700-1782)

➔Utility theory

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Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

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➔When using Utility Theory the rhetorical question above becomes the tool to elicit users preferences

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A common occurrence if that actors show a decreasing marginal utility for money (risk aversion)

To see if this is the case and to elicit the values for the utilities, the following alternatives are posed to the actor

Receiving $10,000 with certainty
Receiving 100,000 with probability p

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1) Receiving \$10,000 with certainty
2) Receiving \$100,000 with probability $\boldsymbol{p}$ (\$100,000 is the upper limit of the curve we intend to build)

The following question is posed: for what value of $\boldsymbol{p}$ would you consider options 1 and 2 equivalent. Imagine the answer is $\frac{1}{4}$ $(\boldsymbol{p = 0.25})$ ➔ the actor consider \$10,000 with certainty and \$100,000 with probability $\frac{1}{4}$ as equivalent

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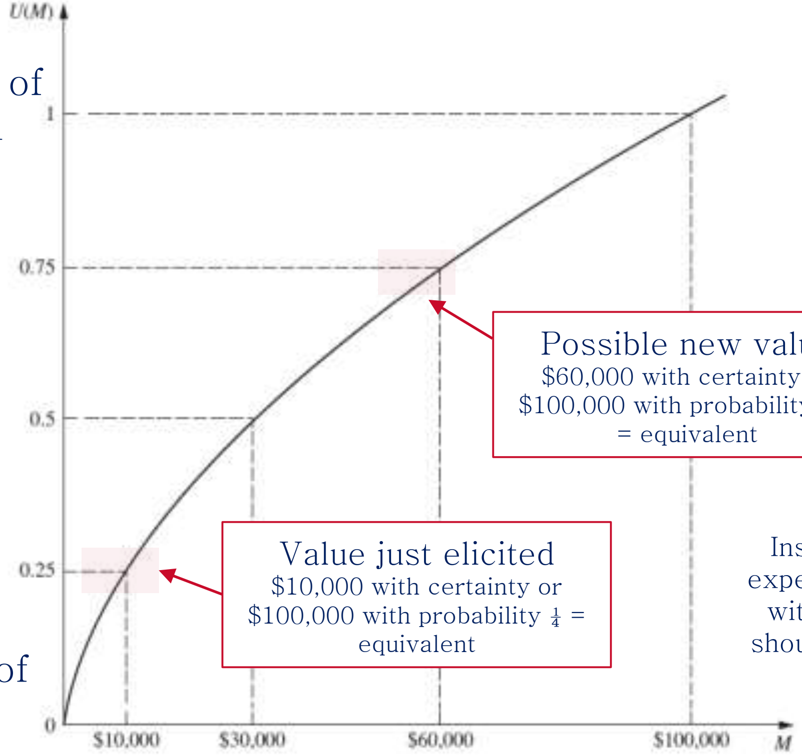
$10,000 with certainty or $100,000 with probability $\frac{1}{4}$ = equivalent

Maximum amount of money=Utility 1

Minimum amount of money=Utility 0



Possible new value
$60,000 with certainty or $100,000 with probability .75 = equivalent

Value just elicited
$10,000 with certainty or $100,000 with probability $\frac{1}{4}$ = equivalent

Instead, based on expected value 100K with **p=0.6** or 60K should be equivalent

Instead, based on expected value 100K with **p=0.1** or 10K should be equivalent

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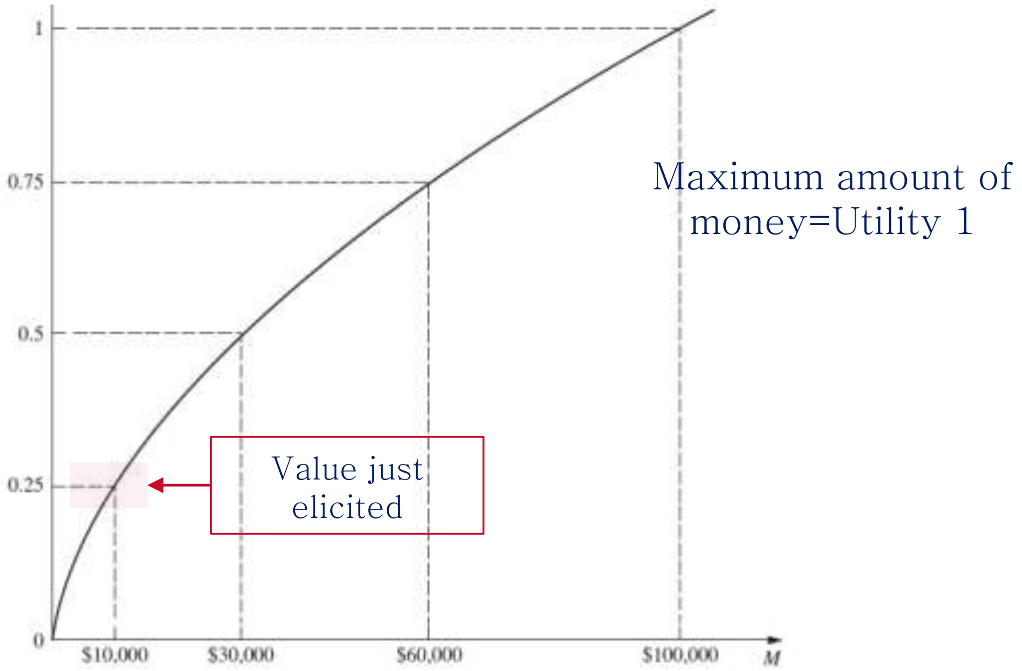
1) Receiving $10,000 with certainty
2) Receiving $100,000 with probability p ($100,000 is the upper limit of the curve we intend to build)

The following question is posed: for what value of p would you consider options 1 and 2 equivalent. Imagine the answer is $\frac{1}{4}$ ($p = 0.25$) ➔ the actor consider $10,000 with certainty of $100,000 with probability $\frac{1}{4}$ as equivalent

Repeating this for values different than $10,000 The utility curve can be built and used in decision analysis, simply replacing monetary payoff with utilities



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IN CLASS

For what p would you take 10,000 with probability p instead of the following with certainty

€	p
1,000	
3,000	
6,000	

For what p would you take 1,000 with probability p instead of the following with certainty

€	p
100	
300	
600	

For what p would you take 100,000 with probability p instead of the following with certainty

€	p
10,00	
30,00	
60,00	

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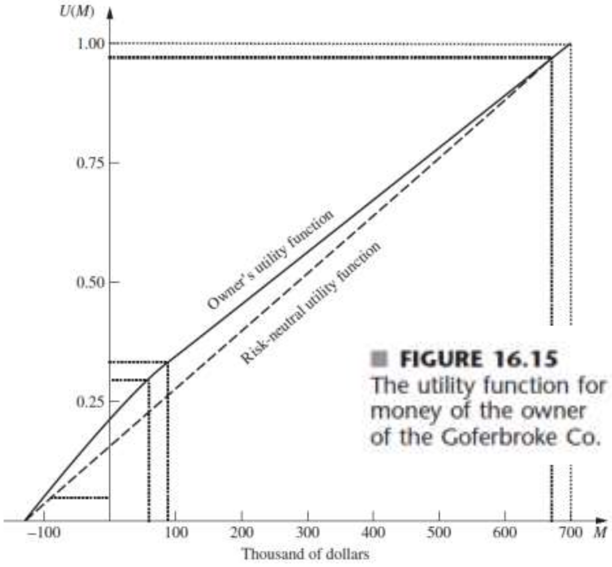
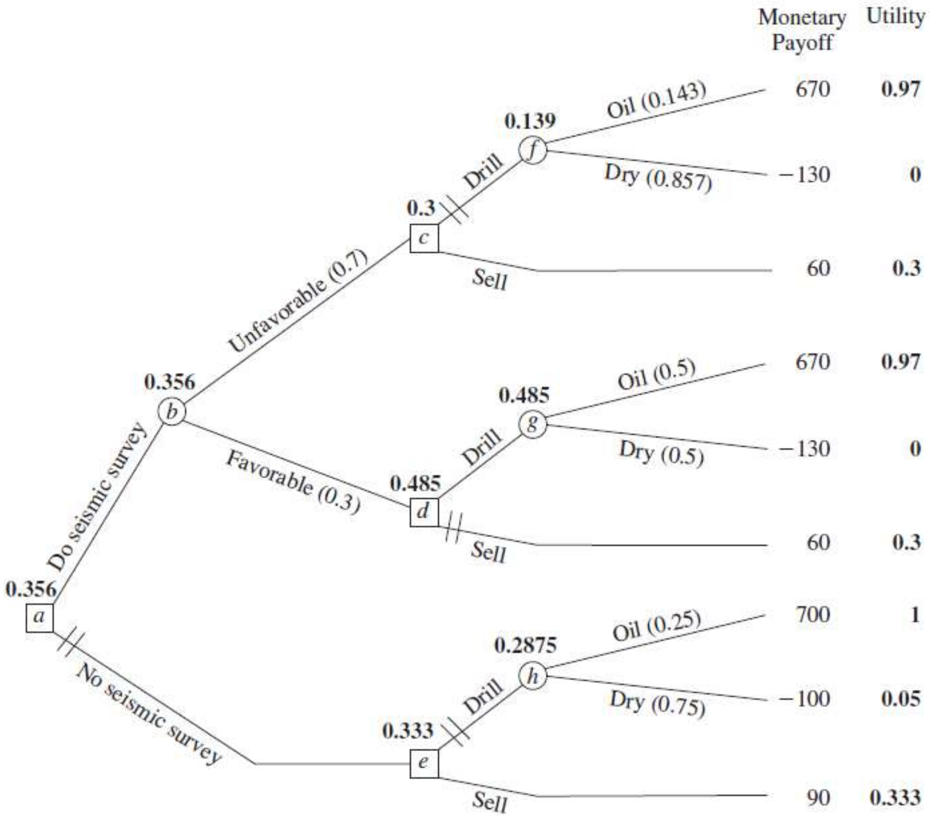


FIGURE 16.15 The utility function for money of the owner of the Goferbroke Co.



Eliciting from the owners of the problem their utility curve the monetary payoff is replaced with utilities scaled in 0-1

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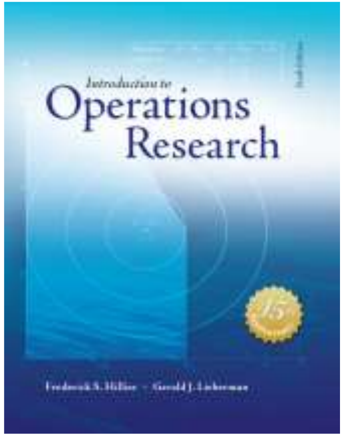
"However, many decision makers are not sufficiently comfortable with the relatively abstract notion of utilities, or with working with probabilities to construct a utility function, to be willing to use this approach. Consequently, utility theory is not yet used very widely in practice" (p. 715)



So what?

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This idiosyncrasy to reckon in terms of abstract utilities or probabilities needs to be kept in mind if decision are taken in teams, e.g. in *Decision Conferencing*

"With the assistance of a computerized group decision support system, the analyst builds and solves models on the spot, and then performs sensitivity analysis to respond to what-if questions from the group" (Hillier p. 716)

ChatGPT

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"With the assistance of a computerized group decision support system, the analyst builds and solves models on the spot, and then performs sensitivity analysis to respond to what-if questions from the group" (Hillier p. 716)

➔ Total quality management (W. Edwards Deming), quality circles

In science for policy settings: participatory methods, post-normal science's extended peer communities, Jasanoff's technologies of humility …

Wikipedia
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Extended peer community

Main menu hide

Article Talk

Main page

From Wikipedia, the free encyclopedia

Wikipedia
The Free Encyclopedia

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Quality circle

Wikipedia
The Free Encyclopedia

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Post-normal science

Essay | Published: 31 October 2007

Technologies of humility

nature

Sheila Jasanoff

Nature **450**, 33 (2007) | Cite this article

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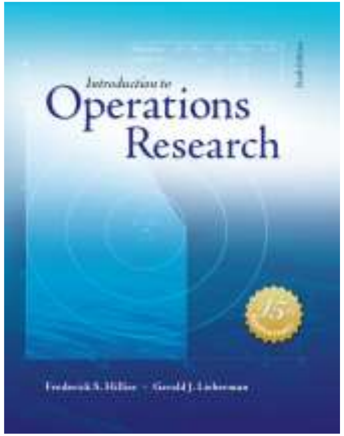
Multiple criteria decision analysis



Using simultaneously more than one criterion

E.g. a company wishing to meet simultaneously goals of

- Profit
- Employment
- Capital investments

A company wishing to meet simultaneously goals of

- Profit $\geq$ 125 (millions of dollars)
- Employment = 4 (hundreds of employees)
- Capital investments $\leq$55 investment goal

in the commercialization of three products (decision variables) x_1, x_2, x_3
Goals can be one sided upper (capital investment) or lower (profit) or two sided (employment).

The relation between decision variables and goals is defined as:

$$12x_1 + 9x_2 + 15x_3 \geq 125$$
$$5x_1 + 3x_2 + 4x_3 = 40$$
$$5x_1 + 7x_2 + 8x_3 \leq 55$$

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The relation between decision variables and goals is defined as:

$$12x_1 + 9x_2 + 15x_3 \geq 125$$
$$5x_1 + 3x_2 + 4x_3 = 40$$
$$5x_1 + 7x_2 + 8x_3 \leq 55$$

Note: MCDA section and this example are not available in the online version; this comes for the 11^{th} version

A penalty weight is attached to violating the goal, i.e.

Weight=5 per unit below profit goal
Weight=3 per unit over investment goal
Weight=4 per unit over employment goal
Weight=2 per unit below employment goal

So the problem is linearized as

Minimize $Z = 5(amount\ under\ profit\ goal) + 3(amount\ over\ investment\ goal) + 4(amount\ over\ employment\ goal) + 2(\ amount\ below\ employment\ goal)$

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So the problem is linearized as

$$\text{Minimize } Z = 5(\textit{amount under profit goal}) + 3(\textit{amount over investment goal}) + 4(\textit{amount over employment goal}) + 2(\textit{amount below employment goal})$$

The simplicity of this approach hides a hornet nest of problems … which is the reason why since the XIV century scholars have laboured to stay away from linear aggregations

So the problem is linearized as

Minimize $Z = 5(\textit{amount under profit goal}) + 3(\textit{amount over investment goal}) + 4(\textit{amount over employment goal}) + 2(\textit{amount below employment goal})$

E.g. the weights above 5,3,4,2 do not translate into importance when the variables have different ranges of uncertainties or are not independent

Suggestion: list different viable options and rank them using methods such as Borda, Condorcet, Balinski-Laraki …

Some of these methods have a long history
(including in Catalonia)

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Ramon Llull (Catalan, ca. 1232 – ca. 1315) proposed first what would then become known as the method of Condorcet. **Nicholas of Kues** (1401 – August 11, 1464), also referred to as Nicolaus Cusanus and Nicholas of Cusa developed what would later be known as the method of Borda. **Nicolas de Condorcet**, (17 September 1743 – 28 March 1794) developed the eponymous method. **Jean-Charles, chevalier de Borda** (May 4, 1733 – February 19, 1799) developed the Borda count

Images from Wikipedia Commons

An impact matrix

	Indic.	GDP ↑	Unemp. Rate ↓	Solid wastes ↓	Income dispar. ↓	Crime rate ↓
Country						
A		25,000	0.15	0.4	9.2	40
B		45,000	0.10	0.7	13.2	52
C		20,000	0.08	0.35	5.3	80
weights		.166	.166	0.333	.166	.166

We can say that
GDP 'votes' for B>A>C (countries / options)
UR 'votes' for C>B>A
SW 'votes' for C>B>A
ID 'votes' for C>A>B
CR 'votes' for A>B>C

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	Indic.	GDP	Unemp. Rate	Solid wastes	Income dispar.	Crime rate
Country						
A		25,000	0.15	0.4	9.2	40
B		45,000	0.10	0.7	13.2	52
C		20,000	0.08	0.35	5.3	80
weights		.166	.166	0.333	.166	.166

# of indicators	2	1	1	1
1st position	*c*	*b*	*c*	*a*
2nd position	*a*	*a*	*b*	*b*
3rd position	*b*	*c*	*a*	*c*

GDP: B>A>C
UR: C>B>A
SW: C>A>B
ID: C>A>B
CR: A>B>C

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# of indicators	2	1	1	1
1st position	*c*	*b*	*c*	*a*
2nd position	*a*	*a*	*b*	*b*
3rd position	*b*	*c*	*a*	*c*

→

Rank	*a*	*b*	*c*
1st	1	1	3
2nd	3	2	0
3rd	1	2	2

Different ways to organize the same information: building a frequency matrix

Three countries [options/candidates] and five indicators [criteria/voters]

# of indicators	2	1	1	1
1st position	*c*	*b*	*c*	*a*
2nd position	*a*	*a*	*b*	*b*
3rd position	*b*	*c*	*a*	*c*

Rank	*a*	*b*	*c*
1st	1	1	3
2nd	3	2	0
3rd	1	2	2

In this case Borda gives 3 minus 1 for each first rank , 2 minus 1 for each second rank and zero to the third

a gets $2*1+1*3=5$
b gets $2*1+1*2=4$
c gets $2*3+1*0=6$

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But lets try Borda on a more interesting case: (from Moulin, 21 criteria 4 options, cited in Munda 2008)

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21 criteria 4 alternatives

Note:3+ 5+ 7+ 6=21

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

Rank	*a*	*b*	*c*	*d*	Points
1st	8	7	6	0	3
2nd	0	9	5	7	2
3rd	0	5	10	6	1
4th	13	0	0	8	0

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Borda count – Frequency matrix (Moulin, 21 criteria 4 options)

Columns add up to the number of criteria / voters=21

3 points if first
2 if second
1 if third
0 if last

Rank	*a*	*b*	*c*	*d*	Points
1st	**8**	**7**	**6**	**0**	**3**
2nd	**0**	**9**	**5**	**7**	**2**
3rd	**0**	**5**	**10**	**6**	**1**
4th	**13**	**0**	**0**	**8**	**0**

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Borda score:

$$a = 8 \times 3 = 24$$

$$b = 5 + 9 \times 2 + 7 \times 3 = 44$$

$$c = 10 + 5 \times 2 + 6 \times 3 = 38$$

$$d = 6 + 7 \times 2 = 20$$

Borda solution:
b→c→a→d

Rank	*a*	*b*	*c*	*d*	Points
1st	**8**	**7**	**6**	**0**	**3**
2nd	**0**	**9**	**5**	**7**	**2**
3rd	**0**	**5**	**10**	**6**	**1**
4th	**13**	**0**	**0**	**8**	**0**

Frequency matrix (21 criteria 4 alternatives)

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The Borda count was developed independently several times, (e.g. by Nicolaus Cusanus beginning XV century) but is named for Jean-Charles de Borda, who devised the system in 1770.

Jean-Charles, chevalier de Borda

It is currently used for the election of two ethnic minority members of the National Assembly of Slovenia (https://www.electoral-reform.org.uk/how-do-elections-work-in-slovenia/), in the Pacific Island of Nauru and …

University of Michigan and the University of Cambridge.

Electronics Engineers (IEEE)

The World Chess Federation (FIDE)

The Mathematical Association of America

The Eurovision Song Contest

UNESCO

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Borda was a mariner and a scientist. Worked on chronometers. Between 1777 and 1778, he participated in the American Revolutionary War.

The French Academy of Sciences used Borda's method to elect its members for about two decades [till Napoleon Bonaparte became president…]

Condorcet's outscoring matrix (21 criteria 4 alternatives)

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

Frequency matrix

b better than *a* 7+6=13 times

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{array}$$

Outscoring matrix

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How to move from frequency to outscoring ?

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

Frequency matrix

$$\begin{bmatrix} & a & b & c & d \\ a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{bmatrix}$$

Outscoring matrix

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Condorcet's outscoring matrix
(21 criteria 4 alternatives)

For each pair of countries a concordance index is computed by counting how many indicators/voters are in favour of each country (e.g. 13 voters prefer b to a).

Note the "constant sum property" in the outranking matrix (13+ 8=21 number of indicators/voters)

$$
\begin{bmatrix}
 & a & b & c & d \\
a & 0 & 8 & 8 & 8 \\
b & 13 & 0 & 10 & 21 \\
c & 13 & 11 & 0 & 14 \\
d & 13 & 0 & 7 & 0
\end{bmatrix}
$$

Outranking matrix

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How to use Condorcet's outscoring matrix (21 criteria 4 alternatives)

Pairs with concordance index > 50% of the indicators/voters are considered: majority threshold = 11 (i.e. a number of voters > 50% of voters=21)

Thus *aP* …*none, bPa= 13, bPd=21(=always), cPa=13, cPb=11, cPd=14, dPa=13*.

c is better than a,b,d so it is the winner
b is better than the remaining a,d, it is the second best
d is better than a.

$$
\begin{array}{c|cccc}
 & a & b & c & d \\
\hline
a & 0 & 8 & 8 & 8 \\
b & 13 & 0 & 10 & 21 \\
c & 13 & 11 & 0 & 14 \\
d & 13 & 0 & 7 & 0
\end{array}
$$

➔Condorcet solution: c → b→ d→ a

Count row-wise discarding entries < 11 as there are 21voters/criteria

Borda solution: b → c → a → d
Condorcet solution: c → b → d → a

Can we choose between Borda and Condorcet on some theoretical and/or practical grounds?

… for another day

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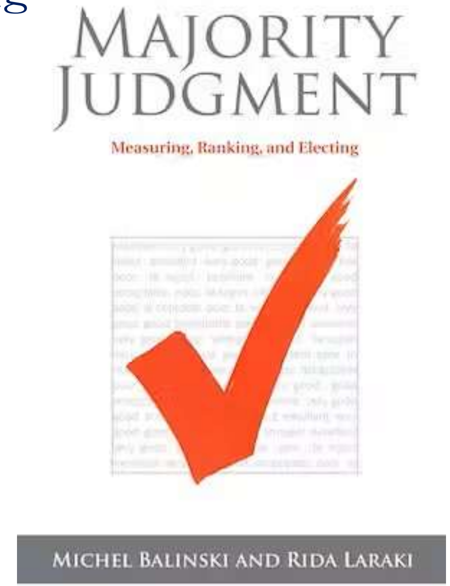
Who should have won the 2007 French Election according to Balinski and Laraki?

Nicolas, Sarkozy, Ségolène Royal, François Bayrou



The winner is a classical 'majority of voters' election is strongly favoured by most but also strongly disfavoured by many … a better method is needed (same argument used in all MCDA methods)

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Each voter provides a ranking of all candidates or options. The method seeks a "central" ranking that best represents the collective preference

Steps:
Collect rankings from all voters (Like in Borda)
Determine the medium rank for each candidate
Aggregate rankings based on medium rank

Voter	Candidate A	Candidate B	Candidate C
1	1	3	2
2	2	1	3
3	2	3	1
4	3	1	2
5	1	2	3
Borda Rank	4	5	4
Medium Rank	9/5=1.8	2	11/5=2.2

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18.

Programming and Planning. PERT and CPM

…. Hillier chapter 22.

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PERT (program evaluation and review technique)

CPM (critical path method)

<ins>Perhaps the most widely used OR techniques</ins> (PERT may be required beside GANTT(∗) charts for project submission e.g. at EU level)

(∗) Developed by Henry L. Gantt in the 1910

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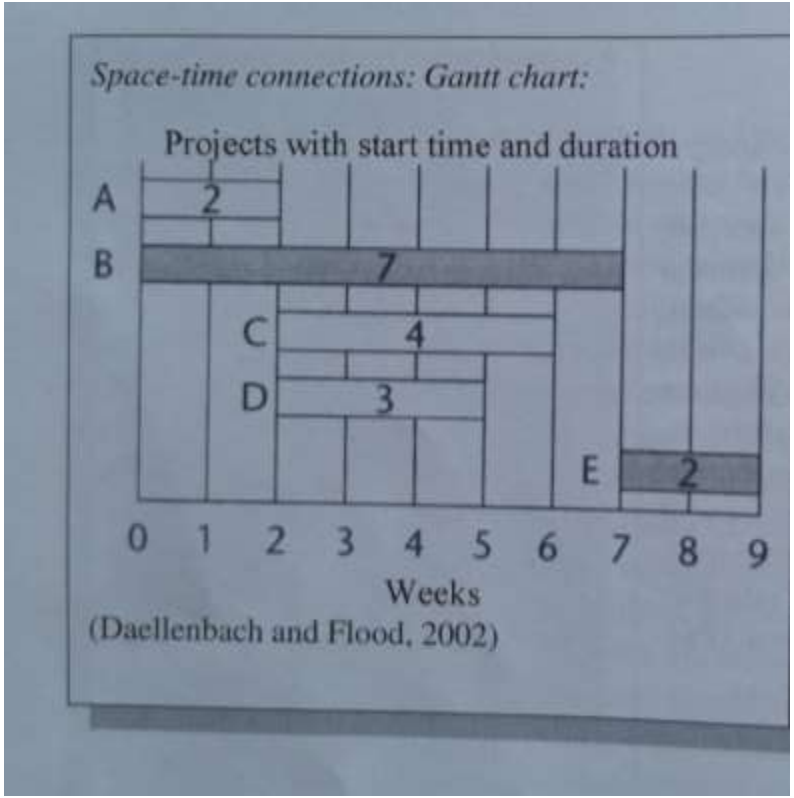
Worked with Frederick Taylor and possibly contributes as well with PERT and CPM developments (Gass and Assad, p. 116))

Henry L. Gantt (1861-1919)
Source: Wikipedia Commons



Source: p. 24, Gass, Saul I., and Arjang A. Assad. 2006. An Annotated Timeline Of Operations Research: An Informal History. 1st Corrected ed. 2005. Corr. 2nd printing 2006 edition. New York: Springer-Verlag New York Inc.

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Why PERT?

1. Project Planning: breaking down the project into tasks, estimating the time required for each task, and identifying dependencies between tasks.

2. Resource Allocation

3. Risk Assessment – helps in identifying critical paths

4. Communication – presenting a well-structured plan

5. Monitoring and Control: track deviations from the schedule, and make necessary adjustments

Project Management with PERT – CPM (critical path method), Hiller chapter 22

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A prototype example: Construction work with penalty for delays and premium for speedy completion (Hillier section 22.1)

TABLE 22.1 Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
C	Put up the rough wall	*B*	10 weeks
D	Put up the roof	*C*	6 weeks
E	Install the exterior plumbing	*C*	4 weeks
F	Install the interior plumbing	*E*	5 weeks
G	Put up the exterior siding	*D*	7 weeks
H	Do the exterior painting	*E, G*	9 weeks
I	Do the electrical work	*C*	7 weeks
J	Put up the wallboard	*F, I*	8 weeks
K	Install the flooring	*J*	4 weeks
L	Do the interior painting	*J*	5 weeks
M	Install the exterior fixtures	*H*	2 weeks
N	Install the interior fixtures	*K, L*	6 weeks

Example: Cannot put up a roof without walls

The contract foresees

A penalty of $300,000 if construction not completed by 47 weeks
A bonus of $150,000 if it is completed within 40 weeks

TABLE 22.1 Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
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J	Put up the wallboard	*F, I*	8 weeks
K	Install the flooring	*J*	4 weeks
L	Do the interior painting	*J*	5 weeks
M	Install the exterior fixtures	*H*	2 weeks
N	Install the interior fixtures	*K, L*	6 weeks

Questions for the analyst (1):

1. How can the project be **displayed graphically** to better visualize the flow of the activities?
2. What is the **total time** required to complete the project if no delays occur?
3. When do the individual activities need to start and finish (**at the latest**) to meet this project completion time?
4. When can the individual activities start and finish (**at the earliest**) if no delays occur?
5. Which are the **critical bottleneck** activities where any delays must be avoided to prevent delaying project completion?

Questions for the analyst (2):

6. For the other (=non bottleneck) activities, **how much delay can be tolerated** without delaying project completion?
7. Given the uncertainties in accurately estimating activity durations, what is the **probability of completing the project by the deadline**?
8. If **extra money** is spent to **expedite** the project, what is the least expensive way of attempting to meet the target completion time (40 weeks)?
9. How should ongoing **costs** be **monitored** to try to keep the project within budget?

1. How can the project be **displayed graphically** to better visualize the flow of the activities? Let's try it in class

TABLE 22.1 Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
C	Put up the rough wall	*B*	10 weeks
D	Put up the roof	*C*	6 weeks
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K	Install the flooring	*J*	4 weeks
L	Do the interior painting	*J*	5 weeks
M	Install the exterior fixtures	*H*	2 weeks
N	Install the interior fixtures	*K, L*	6 weeks

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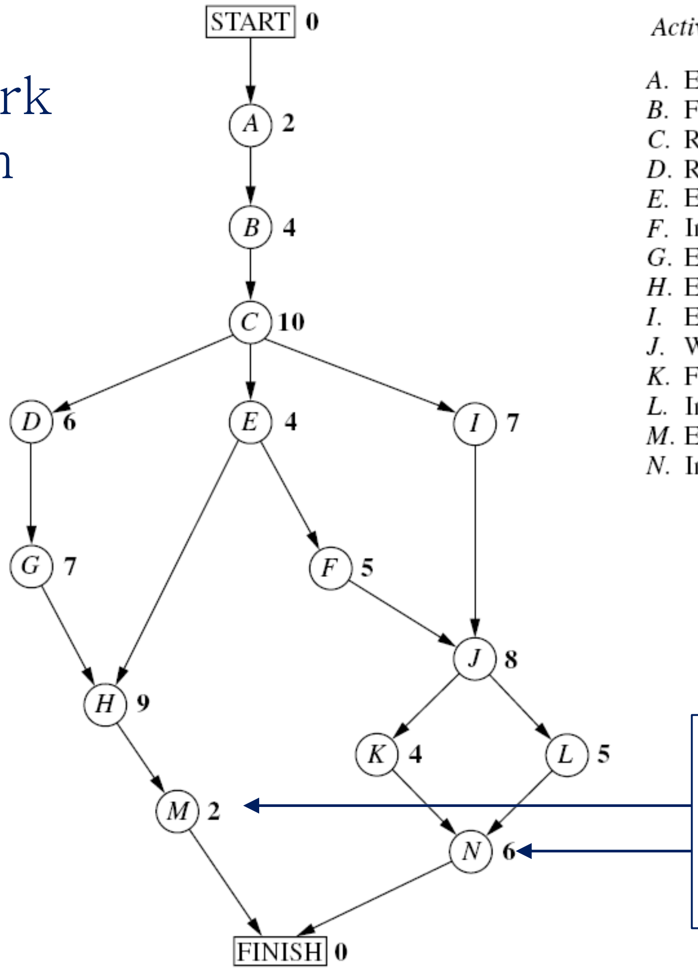
Project network representation

Is this a directed or undirected network?



Activity Code

A. Excavate
B. Foundation
C. Rough wall
D. Roof
E. Exterior plumbing
F. Interior plumbing
G. Exterior siding
H. Exterior painting
I. Electrical work
J. Wallboard
K. Flooring
L. Interior painting
M. Exterior fixtures
N. Interior fixtures

Note: the project finishes here, zero weeks to Finish from either M or N

FIGURE 22.1
The project network for the Reliable Construction Co. project.

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Question 1

1. How can the project be displaye graphically to better visualize the
the activities?

■ **TABLE 22.1** Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
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E	Install the exterior plumbing	*C*	4 weeks
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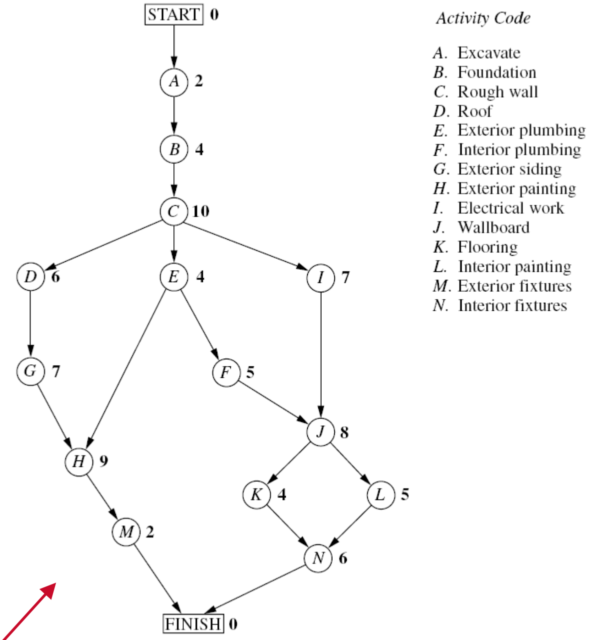

■ **FIGURE 22.1** The project network for the Reliable Construction Co. project.

This is way more informative than this

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Question 2 and 5 now:

Question 2: What is the total time required to complete the project if no delays occur?

Question 5: Which are the critical bottleneck activities where any delays must be avoided to prevent delaying project completion?

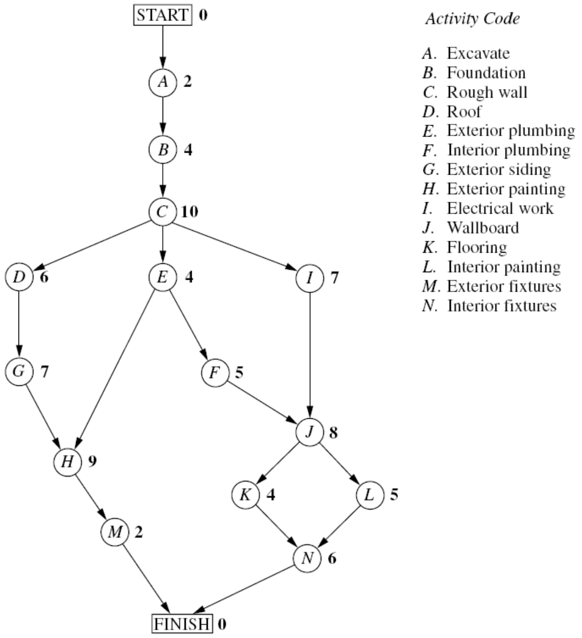

FIGURE 22.1
The project network for the Reliable Construction Co. project.

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TABLE 22.1 Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
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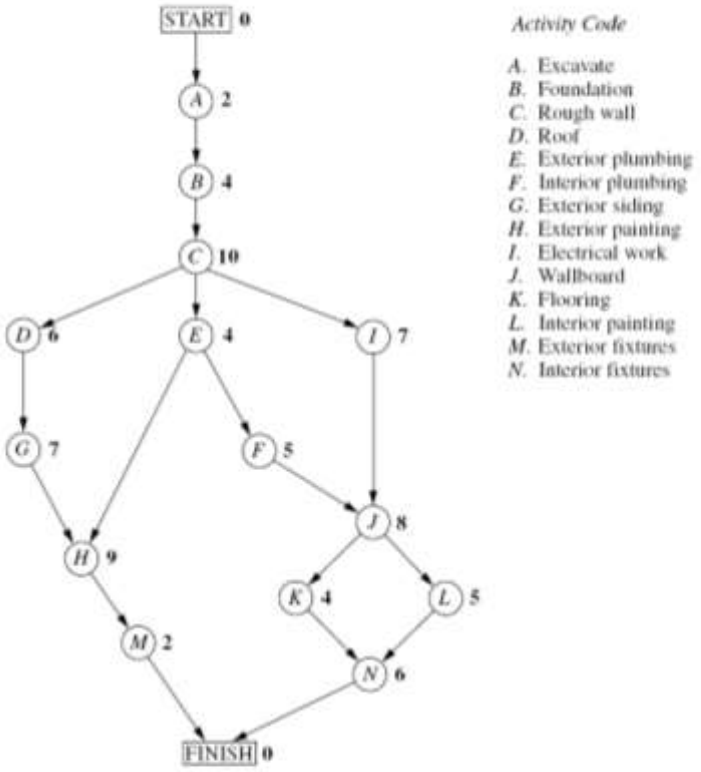


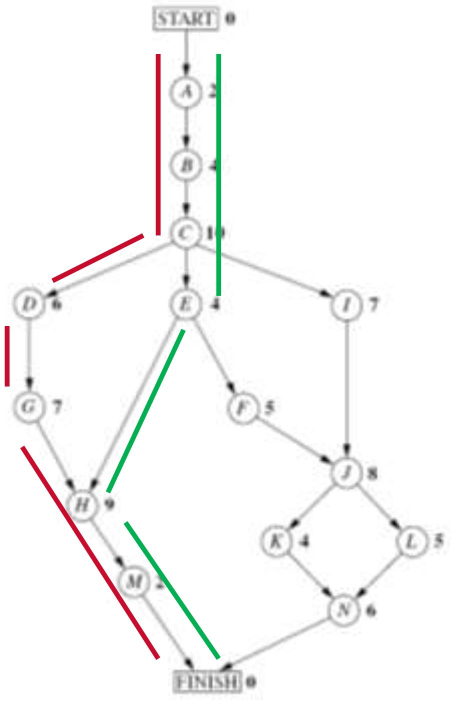
Question 2: What is the total time required to complete the project if no delays occur?

Adding the durations in the table gives 79 weeks but the network tells us that some activities can be run in parallel

Adding the durations in the table gives 79 weeks but the network tells us that some activities can be run in parallel

Lets trace all possible paths from Start to Finish

The first two paths are shown in red and green



■ **TABLE 22.2** **The paths and path lengths through Reliable's project network**

Path	Length
START →A→B→C→D→G→H→M→ FINISH	2 + 4 + 10 + 6 + 7 + 9 + 2 = 40 weeks
START →A→B→C→E→H→M→ FINISH	2 + 4 + 10 + 4 + 9 + 2 = 31 weeks
START →A→B→C→E→F→J→K→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 4 + 6 = 43 weeks
START →A→B→C→E→F→J→L→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 5 + 6 = 44 weeks
START →A→B→C→I→J→K→N→ FINISH	2 + 4 + 10 + 7 + 8 + 4 + 6 = 41 weeks
START →A→B→C→I→J→L→N→ FINISH	2 + 4 + 10 + 7 + 8 + 5 + 6 = 42 weeks

Note that the path are not inclusive of all steps, and that for example activity H in the green path needs both G and E (that is on another path) to be completed; yet …

… the estimated project duration equals the length of the **longest path** through the project Network (=critical path, in blue)

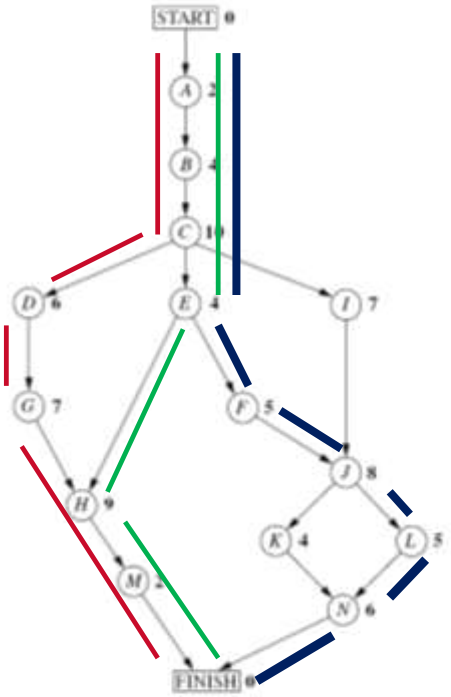


TABLE 22.2 **The paths and path lengths through Reliable's project network**

Path	Length
START →A→B→C→D→G→H→M→ FINISH	2 + 4 + 10 + 6 + 7 + 9 + 2 = 40 weeks
START →A→B→C→E→H→M→ FINISH	2 + 4 + 10 + 4 + 9 + 2 = 31 weeks
START →A→B→C→E→F→J→K→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 4 + 6 = 43 weeks
START →A→B→C→E→F→J→L→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 5 + 6 = 44 weeks
START →A→B→C→I→J→K→N→ FINISH	2 + 4 + 10 + 7 + 8 + 4 + 6 = 41 weeks
START →A→B→C→I→J→L→N→ FINISH	2 + 4 + 10 + 7 + 8 + 5 + 6 = 42 weeks

… the estimated project duration equals the length of the longest path through the project Network (=critical path)

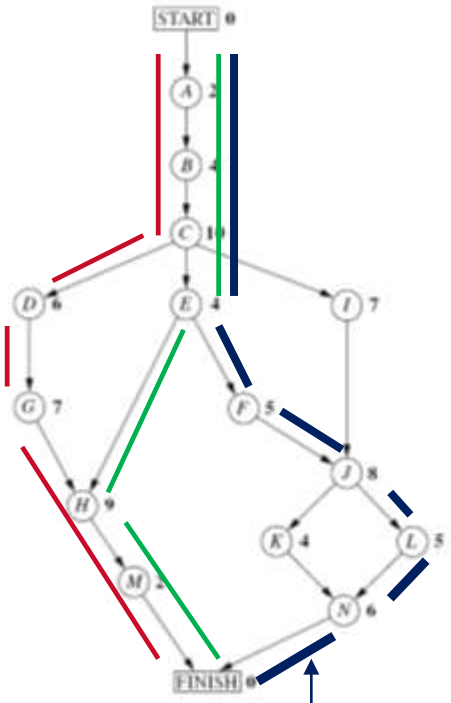
All the shorter paths will reach the FINISH node no later than this critical path

all the shorter paths will reach the FINISH node no later than this

➔ Project duration = 44 weeks and the critical bottlenecks are the nodes of this path

TABLE 22.2 **The paths and path lengths through Reliable's project network**

Path	Length
START →A→B→C→D→G→H→M→ FINISH	2 + 4 + 10 + 6 + 7 + 9 + 2 = 40 weeks
START →A→B→C→E→H→M→ FINISH	2 + 4 + 10 + 4 + 9 + 2 = 31 weeks
START →A→B→C→E→F→J→K→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 4 + 6 = 43 weeks
START →A→B→C→E→F→J→L→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 5 + 6 = 44 weeks
START →A→B→C→I→J→K→N→ FINISH	2 + 4 + 10 + 7 + 8 + 4 + 6 = 41 weeks
START →A→B→C→I→J→L→N→ FINISH	2 + 4 + 10 + 7 + 8 + 5 + 6 = 42 weeks



The activities on this path can be performed sequentially without interruption, otherwise, this would not be the longest path

Question 2 and 5

Question 2: What is the total time required to complete the project if no delays occur?

Question 5: Which are the critical bottleneck activities where any delays must be avoided to prevent delaying project completion?

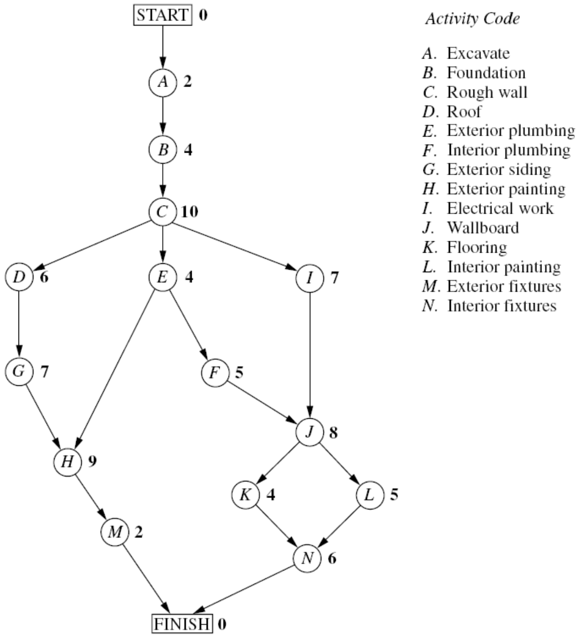


FIGURE 22.1
The project network for the Reliable Construction Co. project.

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Scheduling Individual Activities with PERT/CPM

Question 4

4. When can the individual activities start and finish (at the earliest) if no delays occur?

Logging earliest start time (**ES**) and the earliest finish time (**EF**) for each activity, starting with those with a single predecessor

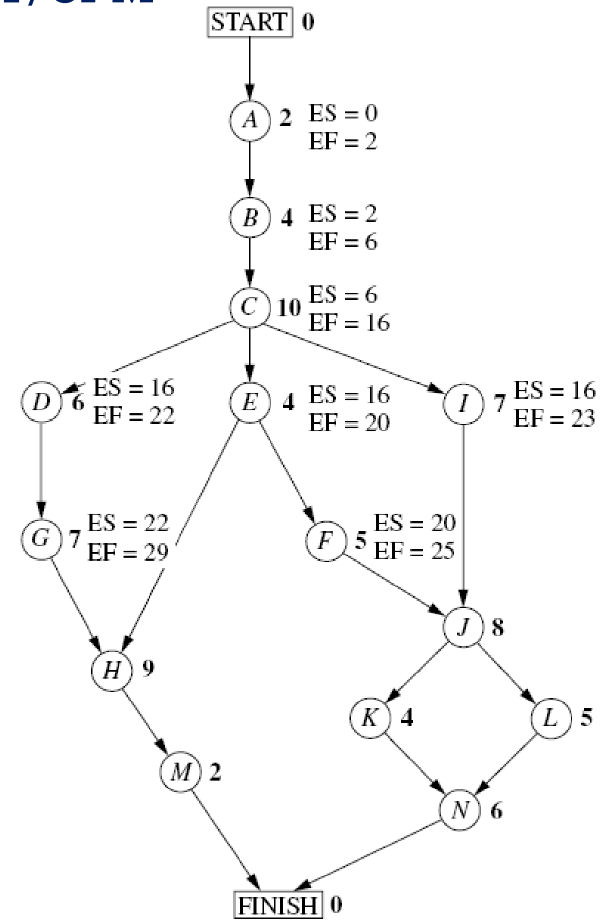

FIGURE 22.2
Earliest start time (ES) and earliest finish time (EF) values for the initial activities in Fig. 22.1 that have only a single immediate predecessor.

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What when there are two predecessors (Example E and G for node H)?
Take the maximum EF (Example 29 of G > 20 of E)

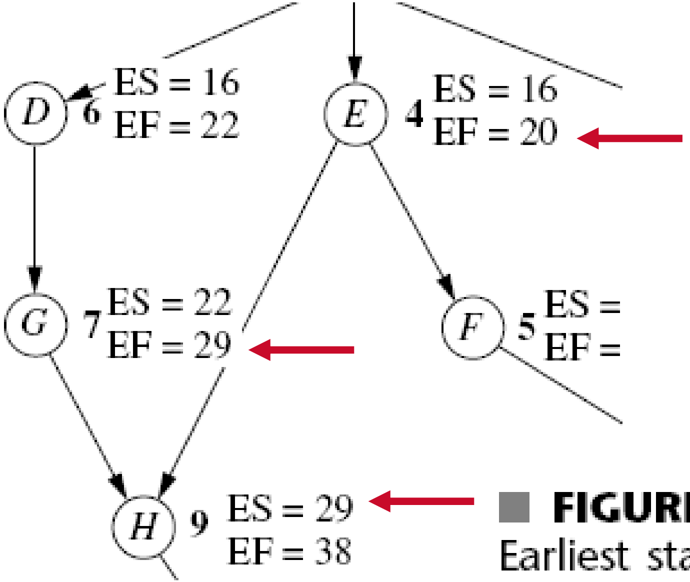
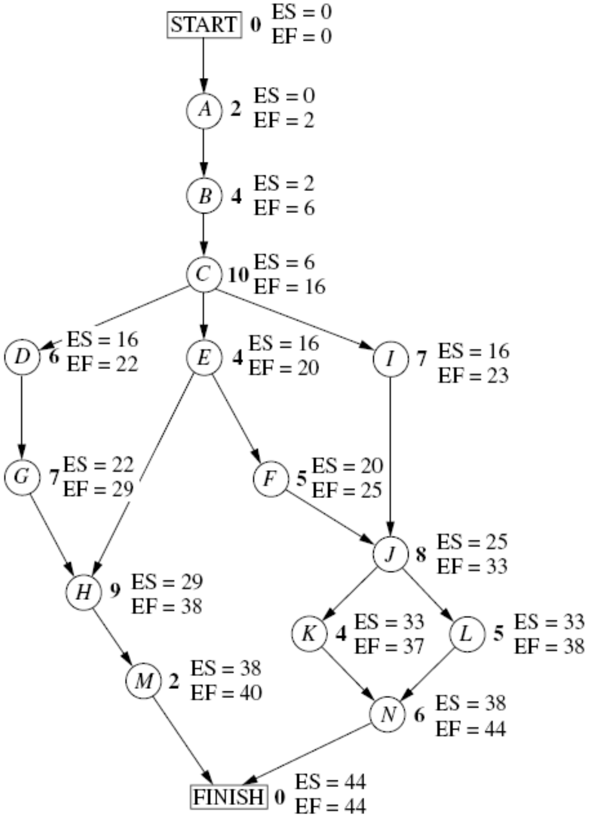


FIGURE 22.3
Earliest start time (ES) and earliest finish time (EF) values for all the activities (plus the START and FINISH nodes) of the Reliable Construction Co. project.



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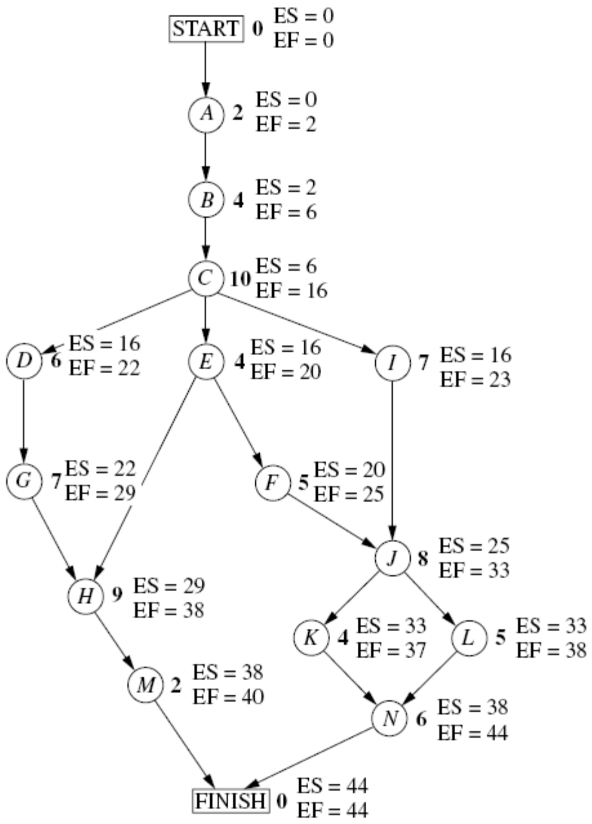
Question 4

4. When can the individual activities start and finish (at the earliest) if no delays occur?

This is a forward pass through the network



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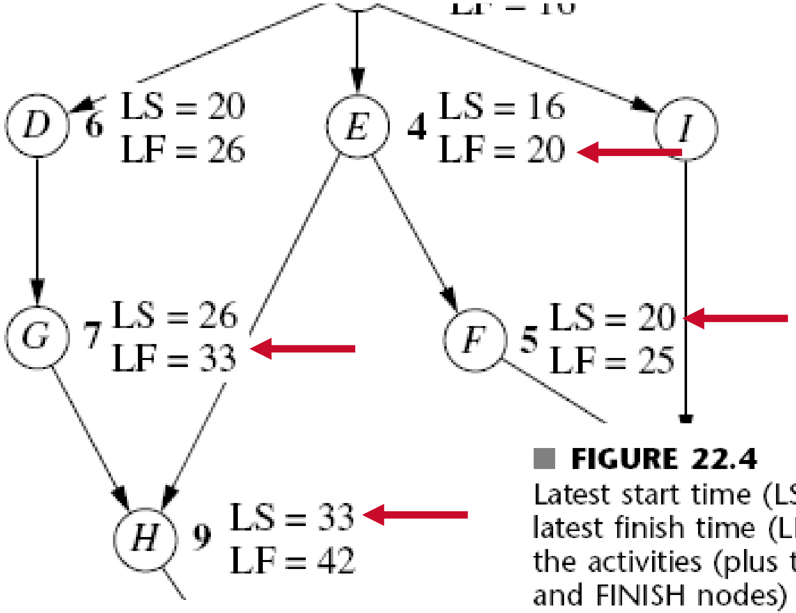
What if there are delays? It is convenient to have for each activity also the latest start (**LS**) time and latest finish (**LF**) time, computed as not to engender delay of the subsequent nodes. We proceed backward



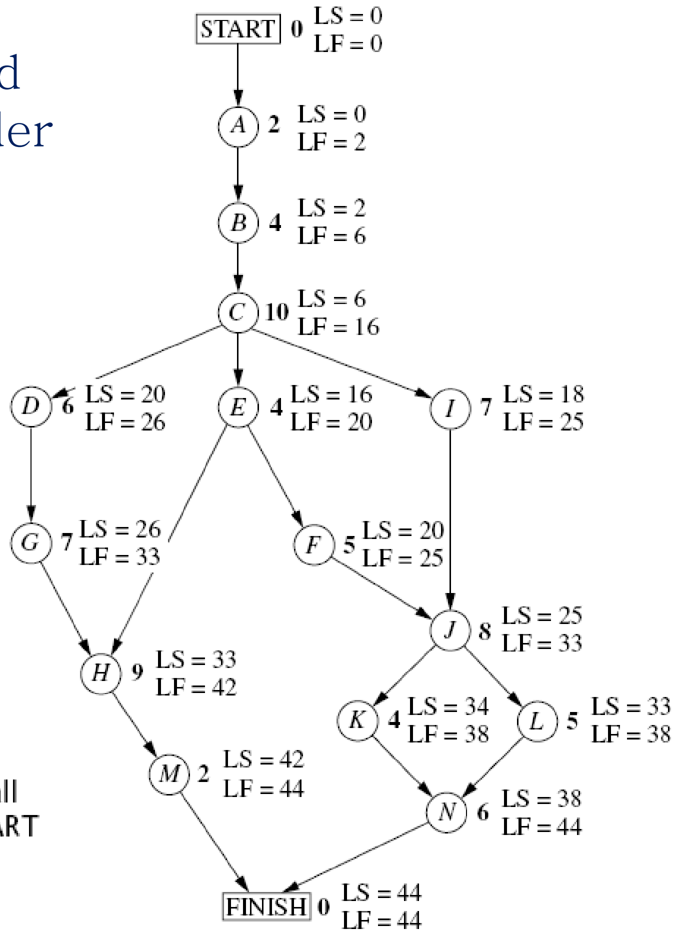


FIGURE 22.4
Latest start time (LS) and latest finish time (LF) for all the activities (plus the START and FINISH nodes) of the Reliable Construction Co. project.

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What if there are delays? It is convenient to l
for each activity also the latest start (LS) tim
and latest finish (LF) time, computed as not
engender delay of the subsequent nodes. We
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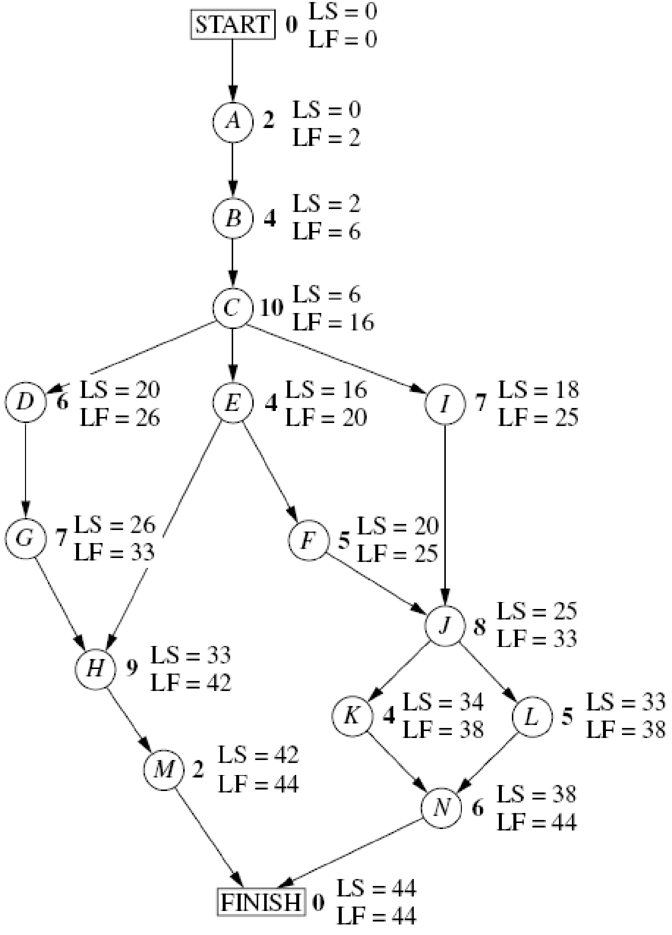
This is a backward pass through the network



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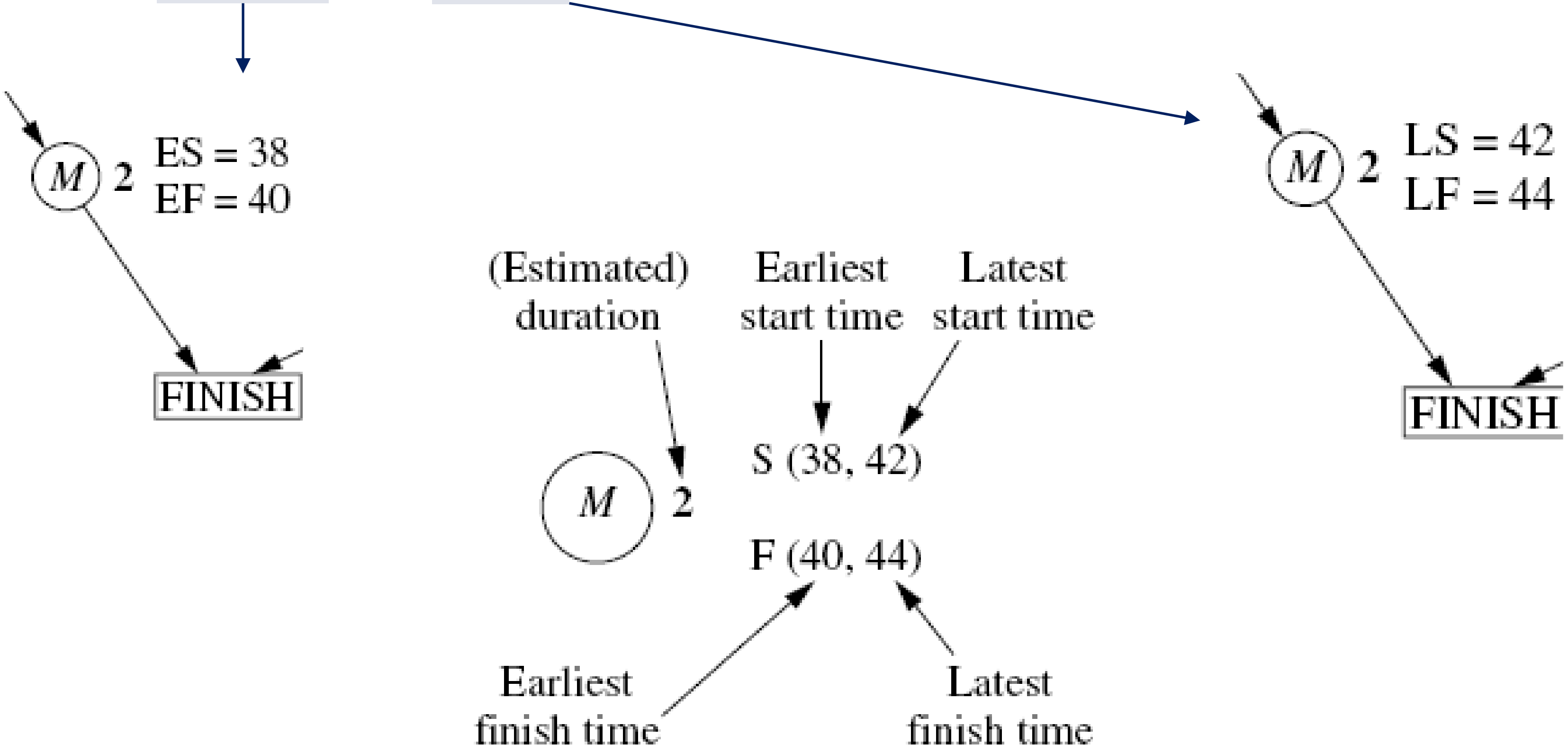
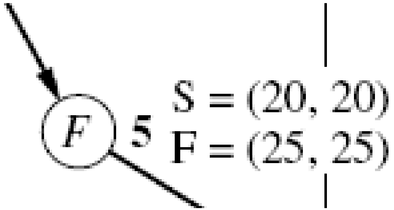
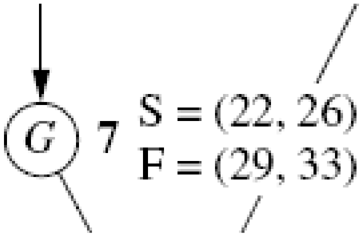
Combining earliest and latest time information as to identify and quantify **slack**



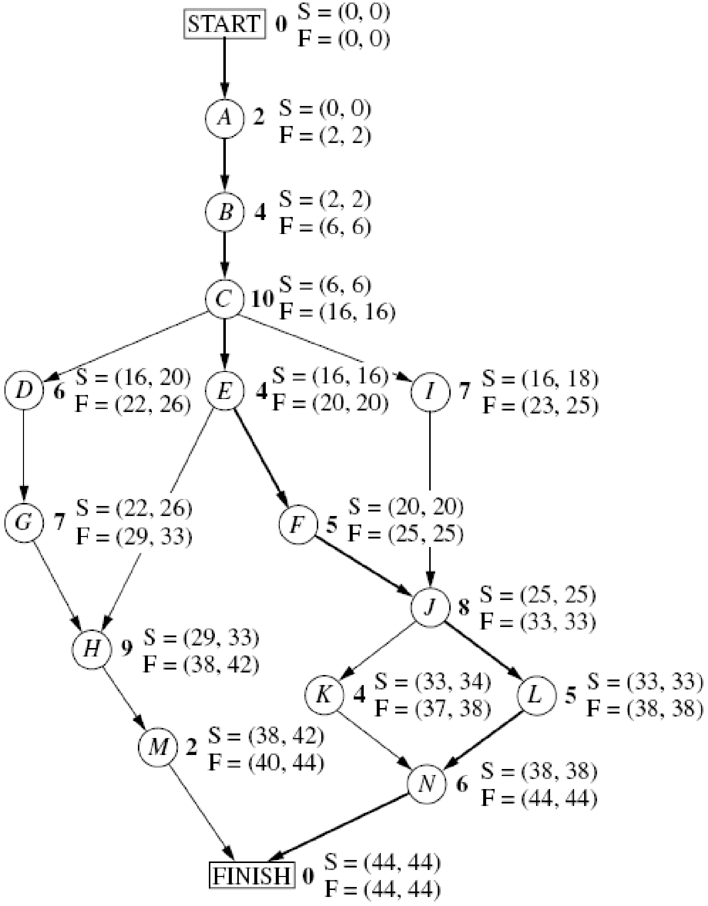
The full picture



No slack; critical path



Slack; not critical path



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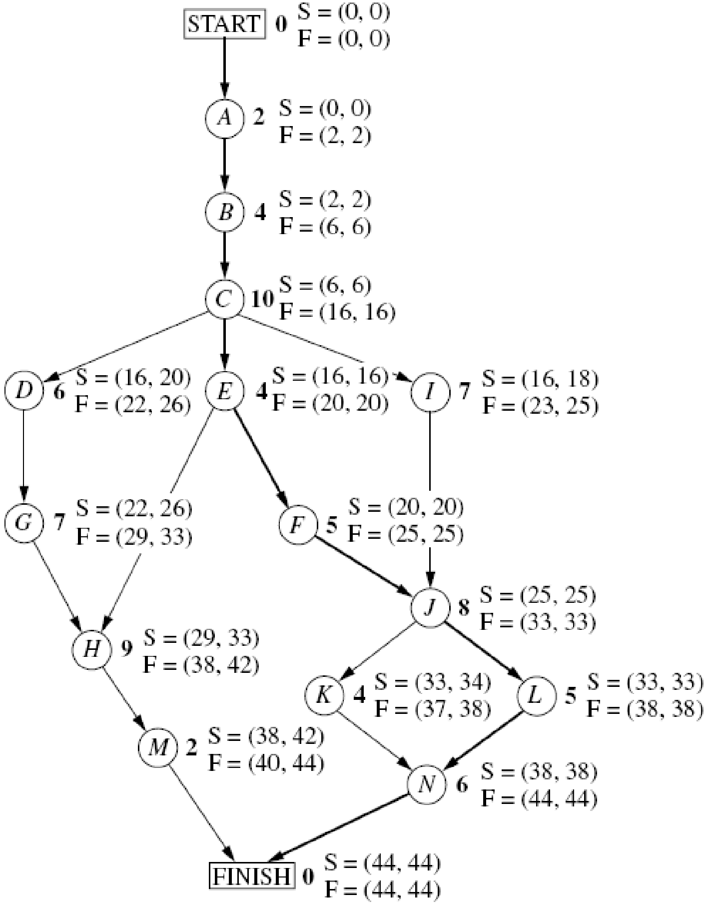
The full picture

■ **TABLE 22.3** Slack for Reliable's activities

Activity	Slack (LF − EF)	On Critical Path?
A	0	Yes
B	0	Yes
C	0	Yes
D	4	No
E	0	Yes
F	0	Yes
G	4	No
H	4	No
I	2	No
J	0	Yes
K	1	No
L	0	Yes
M	4	No
N	0	Yes



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Solved

1. Project display
2. Time to completion
3. Latest start and finish time per activity
4. Earliest start and finish time per activity
5. Bottlenecks
6. Slacks

The full picture

■ **TABLE 22.3** Slack for Reliable's activities

Activity	Slack (LF − EF)	On Critical Path?
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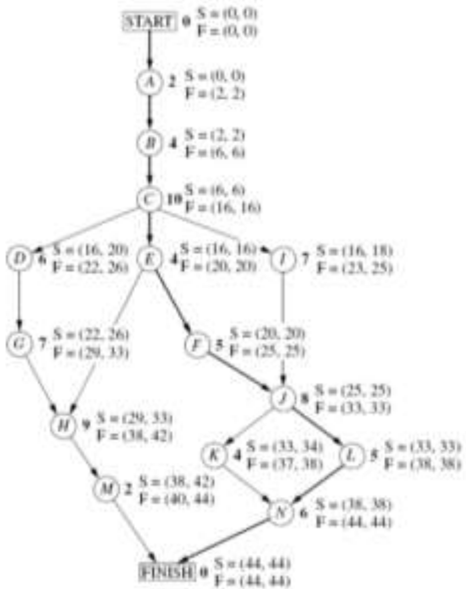
H	4	No
I	2	No
J	0	Yes
K	1	No
L	0	Yes
M	4	No
N	0	Yes



Task	Task Description	Tasks that Must Precede	Time
A	Buy the mozzarella cheese*		30 minutes
B	Slice the mozzarella	*A*	5 minutes
C	Beat 2 eggs		2 minutes
D	Mix eggs and ricotta cheese	*C*	3 minutes
E	Cut up onions and mushrooms		7 minutes
F	Cook the tomato sauce	*E*	25 minutes
G	Boil large quantity of water		15 minutes
H	Boil the lasagna noodles	*G*	10 minutes
I	Drain the lasagna noodles	*H*	2 minutes
J	Assemble all the ingredients	*I, F, D, B*	10 minutes
K	Preheat the oven		15 minutes
L	Bake the lasagna	*J, K*	30 minutes

*There is none in the refrigerator.

(a) Construct the project network for preparing this dinner.

(b) Find all the paths and path lengths through this project network. Which of these paths is a critical path?

Exercise in class

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Task	Task Description	Tasks that Must Precede	Time
A	Buy the mozzarella cheese*		30 minutes
B	Slice the mozzarella	A	5 minutes
C	Beat 2 eggs		2 minutes
D	Mix eggs and ricotta cheese	C	3 minutes
E	Cut up onions and mushrooms		7 minutes
F	Cook the tomato sauce	E	25 minutes
G	Boil large quantity of water		15 minutes
H	Boil the lasagna noodles	G	10 minutes
I	Drain the lasagna noodles	H	2 minutes
J	Assemble all the ingredients	I, F, D, B	10 minutes
K	Preheat the oven		15 minutes
L	Bake the lasagna	J, K	30 minutes

*There is none in the refrigerator.

(a) Construct the project network for preparing this dinner.

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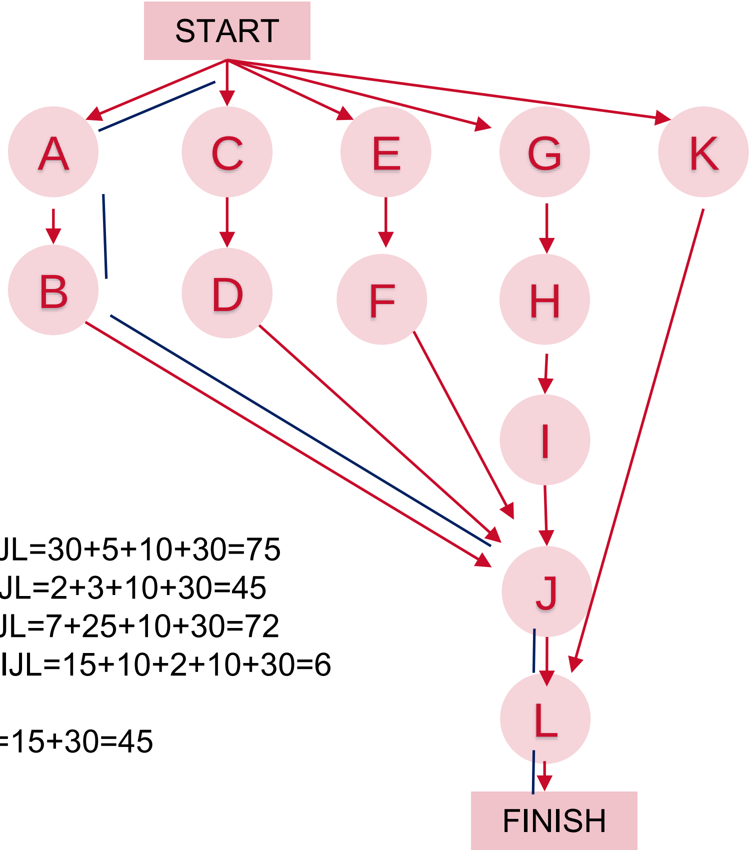

ABJL=30+5+10+30=75
CDJL=2+3+10+30=45
EFJL=7+25+10+30=72
GHIJL=15+10+2+10+30=6
7
KL=15+30=45

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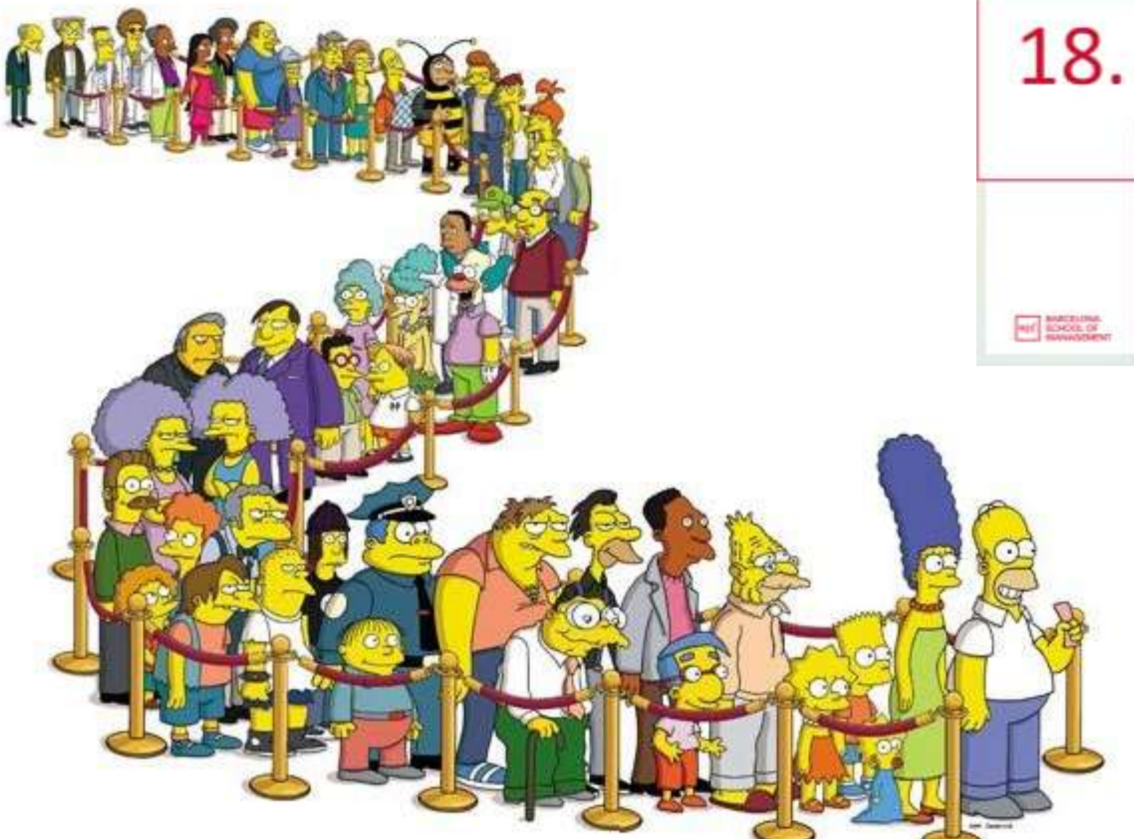
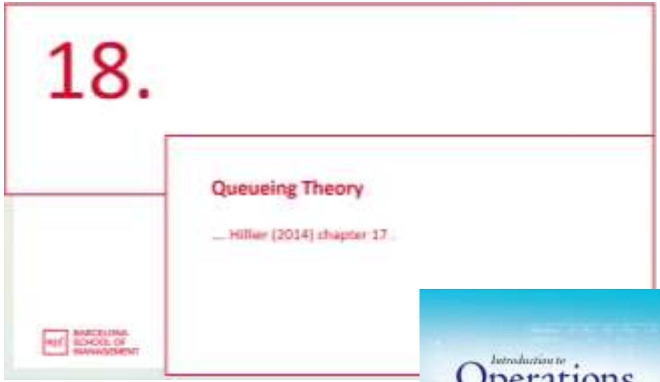
19.

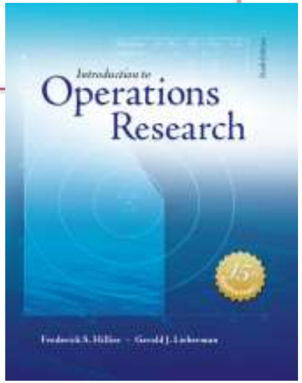
Queueing Theory

…. Hillier (2014) chapter 17 .



The Simpsons, Twentieth Century Fox Film Corporation





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MANAGEMENT

FIGURE 17.1
The basic queueing process.

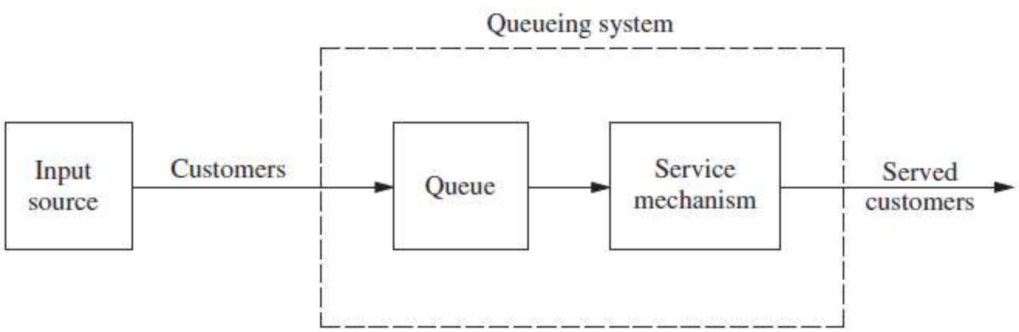
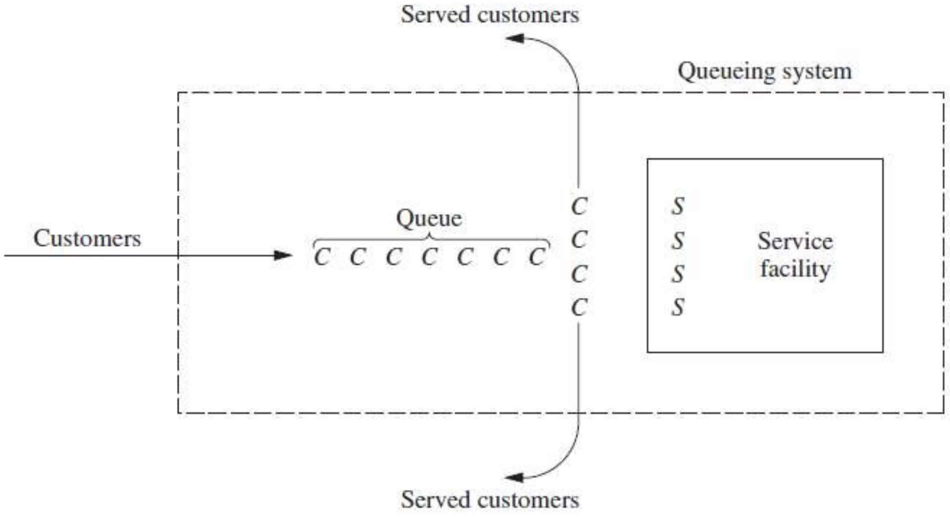


FIGURE 17.2
An elementary queueing system (each customer is indicated by a *C* and each server by an *S*).



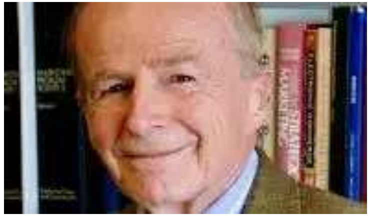
Key parameters of these systems are customers' "interarrival time" and "service time"

Fundamental relations in queueing: Frequency λ of arrivals (units per time) multiplied by the average waiting time in the queue W gives the numbers of units waiting in the queue L

$$L = \lambda W$$

Fairly intuitive: the slower the service in the queue, the higher the frequency of arrival, the longer the queue

Little's formula, from MIT's John D. C. Little



Feb 1 1928 - Sept 27 2024

FIGURE 17.1
The basic queueing process.

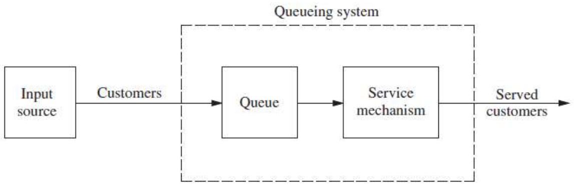


FIGURE 17.2
An elementary queueing system (each customer is indicated by a C and each server by an S).

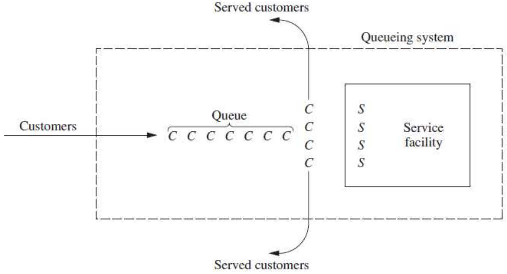


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FIGURE 17.1
The basic queueing process.

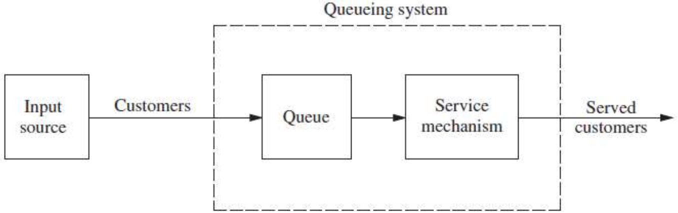

FIGURE 17.2
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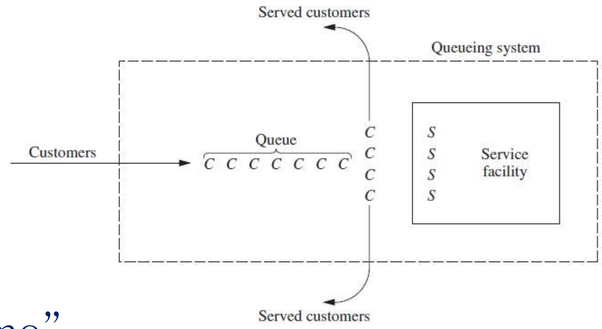


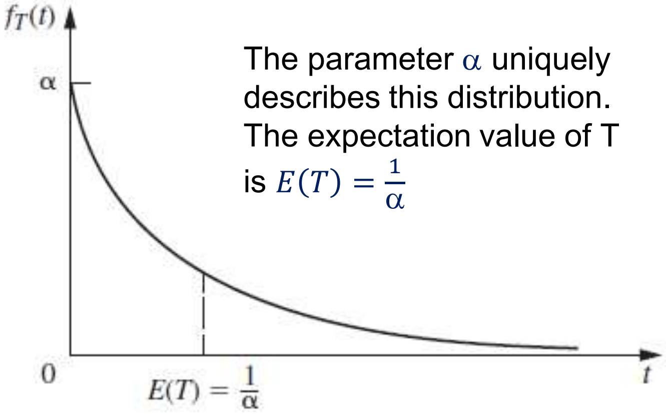
Ley T be either an "inter-arrival time" or a "service time". This time T can be often assumed to follow an exponential distribution

$$T \sim f_T(t) = \alpha e^{-\alpha t}$$

Math notations to say that T follows distribution f_T

FIGURE 17.3
Probability density function for the exponential distribution.

Mean value of T is $E(T) = \frac{1}{\alpha}$



The parameter α uniquely describes this distribution. The expectation value of T is $E(T) = \frac{1}{\alpha}$

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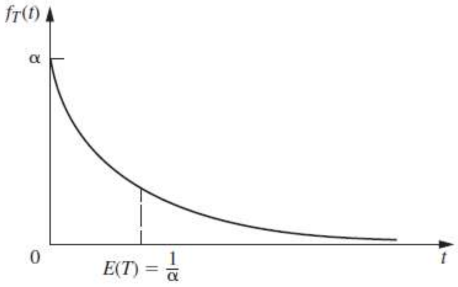
Ley T be either an “inter-arrival time” or a “service time”.
T can be often assumed to follow an exponential distribution

$$T \sim f_T(t) = \alpha e^{-\alpha t}$$

■ **FIGURE 17.3**
Probability density function for the exponential distribution.



Properties of this distribution

The parameter α has the dimension of 1/time; small values of α correspond to longer waits

Small T are more likely then large T, where the scale of what is big or small is relative to α

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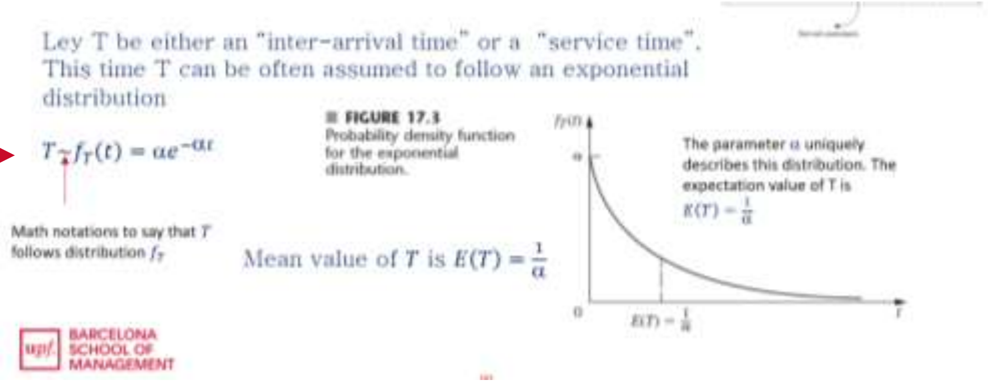
Assume this holds;
then if *X(t)* is the *number* of times an event (arrivals or completions) occurs over a specified time *t* then *X*(*t*) follows a *Poisson distribution* with parameter αt



$$P\{X(t) = n\} = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$$

Using these formulae different queueing problems can be formulated and solved

Siméon Denis Poisson
1781 – 1840)

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Before leaving the Poisson distribution $P\{X(t) = n\} = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$ an example:

It is normally stated as $P\{X = n\} = \frac{(\lambda)^n e^{-\lambda}}{n!}$ where P refers to events per unit time, λ is the average number of times an event occur per unit of time; so if λ is the average number of soldiers killed by a horse-kick in a given year, the probability of n soldier dying in a given year is given by $P\{X = n\} = \frac{(\lambda)^n e^{-\lambda}}{n!}$

Source: Wikipedia Commons

Ladislaus Bortkiewicz used this formula in the late 1800s, when investigating accidental deaths by horse kick of soldiers in the Prussian army. Also known for his work on Marxian theory

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An average of 0.61 soldiers died by horse kicks per year in each Prussian army corps. You want to calculate the probability that exactly two soldiers died in the VII Army Corps in 1898, assuming that the number of horse kick deaths per year follows a Poisson distribution $P\{X = n\} = \frac{(\lambda)^n e^{-\lambda}}{n!}$

$$P\{X = 2\} = \frac{(0.61)^2 e^{-0.61}}{2!} \sim 0.101$$

Source: Wikipedia Commons

Since Bortkiewicz's time, Poisson distributions have been used to describe many other things.

For example, a Poisson distribution could be used to explain or predict:

- Text messages per hour
- Male grizzly bears per hectare
- Machine malfunctions per year
- Website visitors per month
- Influenza cases per year

Source: https://www.scribbr.com/statistics/poisson-distribution/

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20.

Ethical considerations for OR

From Condorcet to Bentham. Sociology of quantification. Scheduling software and its consequences. Responsible modelling. Partly based on O'Neil, C. (2016) Weapons of math destruction. Random House Publishing Group.

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'Mathématique sociale': We still use today terms such as 'Condorcet method', 'Condorcet winner', 'Condorcet-ranking procedure'

From
Lesson #1

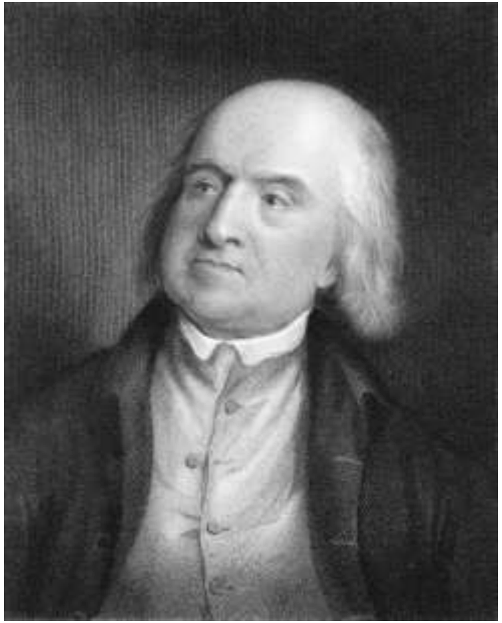
Nicolas de Caritat,
marquis de Condorcet
(1743- 1794)
'

Feldman, J., 2005, Condorcet et la mathematique sociale: enthousiasmes et bemols, Mathematics and Social Sciences, 172(4), 7-41, http://www.ehess.fr/revue-msh/pdf/N172R955.pdf

Munda G. (2007) - Social multi-criteria evaluation, Springer-Verlag, Heidelberg, New York, Economics Series

Condorcet's Mathématique sociale had its continuation in Jeremy Bentham's utilitarianism

Marquis de Condorcet (1743- 1794)



Felicific calculus: 'The greatest good for the greatest number'
(utility or hedonistic calculus)

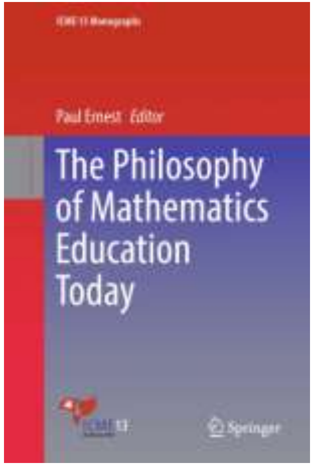
Jeremy Bentham
(1748-1832)

From Lesson #1

The Ethics of Mathematics: Is Mathematics Harmful?

Paul Ernest

© Springer International Publishing AG, part of Springer Nature 2018
P. Ernest (ed.), *The Philosophy of Mathematics Education Today*,
ICME-13 Monographs, https://doi.org/10.1007/978-3-319-77760-3_12



Mathematics – styles of thinking that can be damaging when applied beyond mathematics to social and human issues

Condorcet’s dream

Applications in society can be deleterious to our humanity unless very carefully monitored and checked

Algorithms

Impact of learning mathematics on learners’ thinking and life chances

Regimenting, stigmatizing

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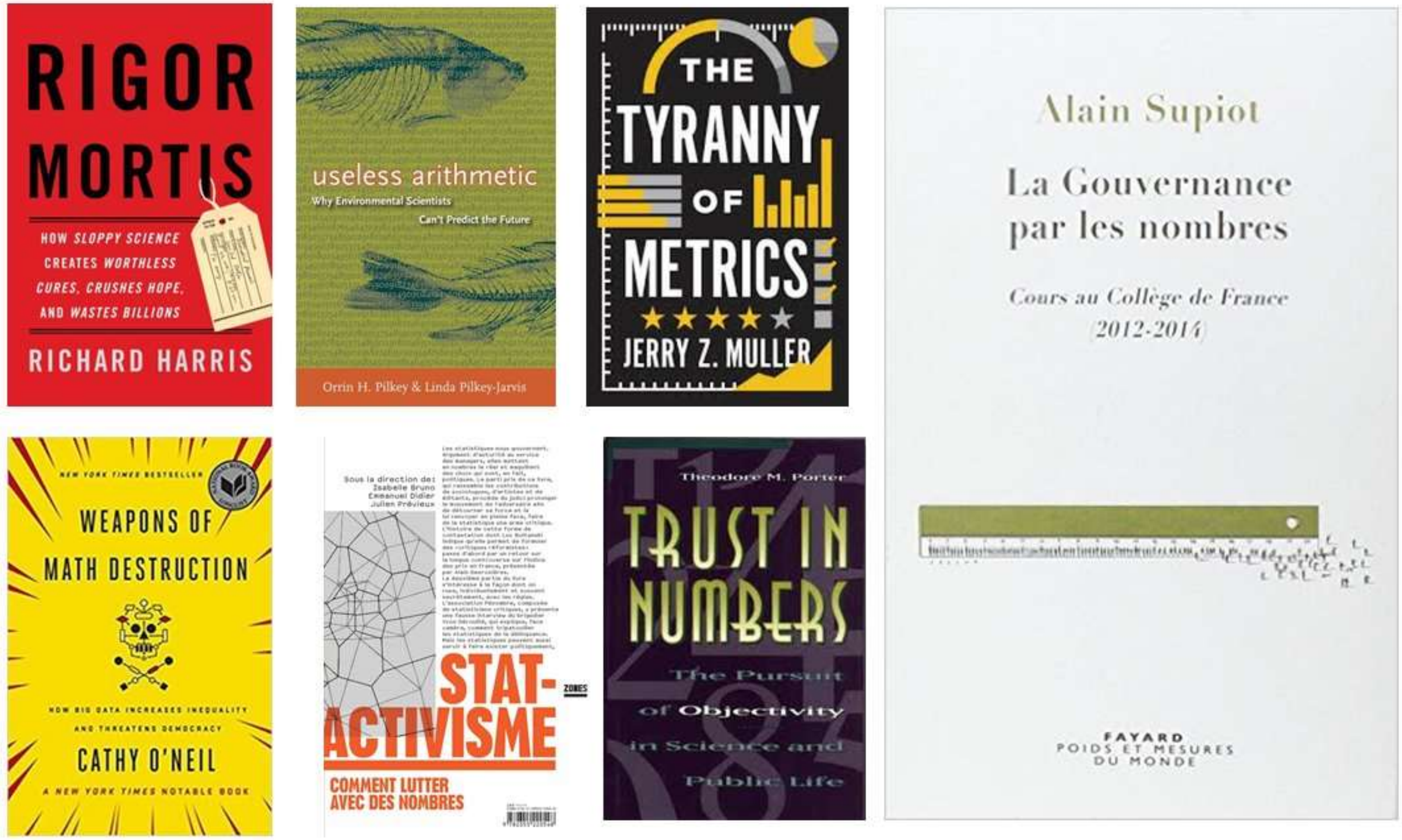
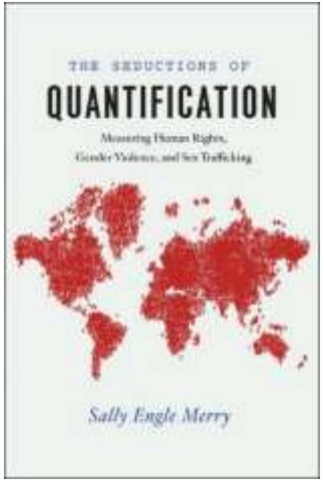
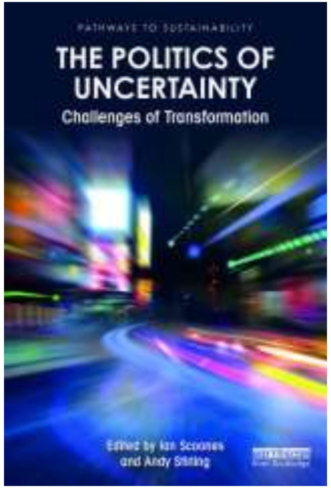
Sociology of quantification, for numbers, visible and invisible…

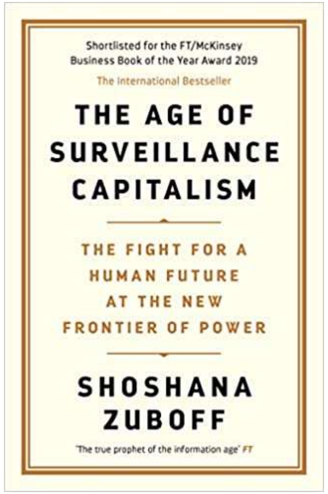
Algorithms, models, metrics, statistics…

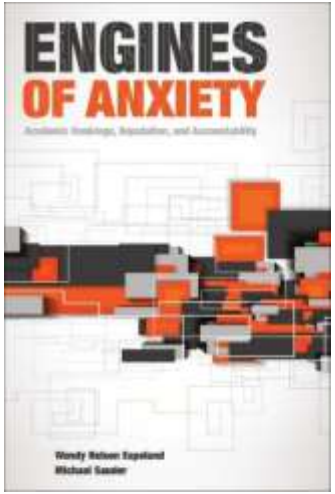


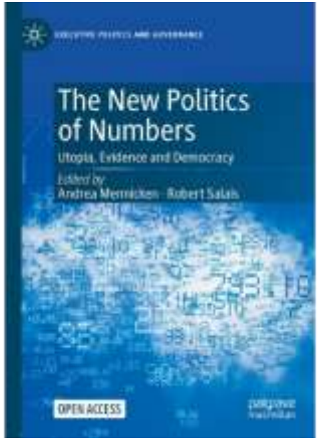
Algorithms, models, metrics, statistics…

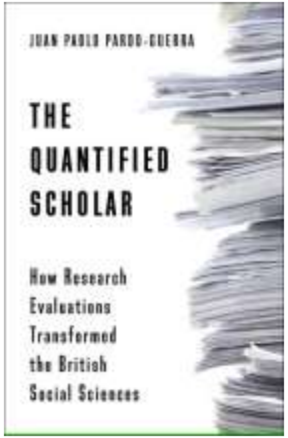


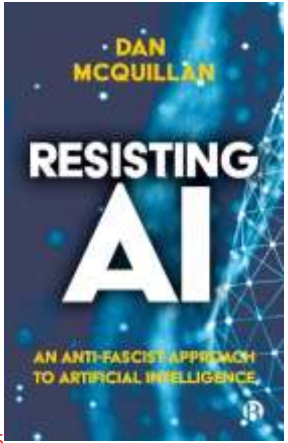












… our world is structured by numbers, visible and invisible, where truth is conveyed and reality constructed

Numbers are seductive, performative, confer to their masters' epistemic power and legitimacy

Governing the modern state, or even contesting it, without numbers is impossible

Numbers are the prevalent means to express value in our societies … Access & production of numbers reflect and reinforce power imbalances

Source: Saltelli, A., Andreoni, A., Drechsler W., Ghosh, J., Kattel, R., Kvangraven, I. H., Rafols, I., Reinert, E. S., Stirling, A. and Xu, T. (2021). Why ethics of quantification is needed now. UCL Institute for Innovation and Public Purpose, Working Paper Series (IIPP WP 2021/05) https://www.ucl.ac.uk/bartlett/public-purpose/publications/2021/jan/why-ethics-quantification-needed-now.

IIPP UCL Institute for Innovation and Public Purpose

WORKING PAPER
WP 2021/05

Numbers capture our attention; they illuminate the part of reality which is being numerified, and fatally push those parts into the background which come without the clothing of …

… numbers are so deeply entrenched in our existence that we barely reflect on them critically them anymore — too close to us, they have become part of the very lens through which we attend to and comprehend the world.

Source: Saltelli, A., Andreoni, A., Drechsler W., Ghosh, J., Kattel, R., Kvangraven, I. H., Rafols, I., Reinert, E. S., Stirling, A. and Xu, T. (2021). Why ethics of quantification is needed now. UCL Institute for Innovation and Public Purpose, Working Paper Series (IIPP WP 2021/05) https://www.ucl.ac.uk/bartlett/public-purpose/publications/2021/jan/why-ethics-quantification-needed-now.

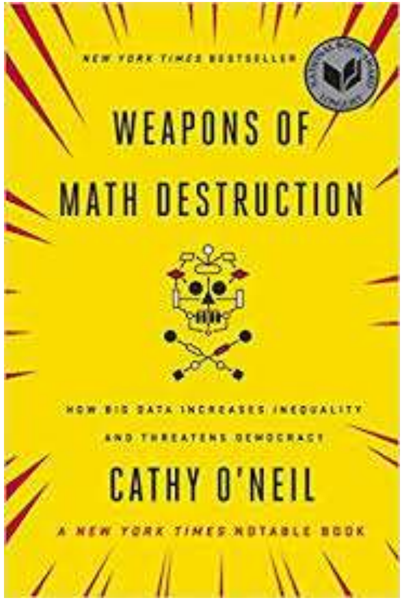
Numbers and their ‘reactivity’
(Espeland and Sauder, 2016)

Incumbent numbers affect what society will measure in the future (Merry 2016)

Numbers “create the environment that justifies their assumptions”
(O’Neil, 2016)

Alarm for Weapons of Math Destruction

Cathy O'Neil



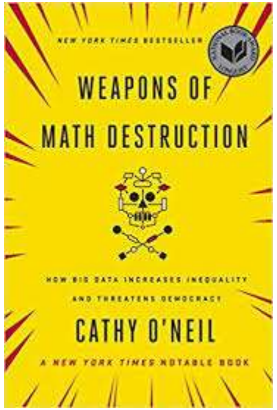
O'Neil, C. (2016). Weapons of math destruction : how big data increases inequality and threatens democracy. Random House Publishing Group.

OR and “clopening” (portmanteau of opening and closing)

Software scheduling programs – continuous adjusting of working assignment (day by day) may be unfavourable to workers making hard for them to plan for study, work, children

Clopening implies the same worker closes a shop and the open it the next morning at companies like Starbucks, McDonald’s and Walmart

O’Neil, C. (2016). Weapons of math destruction : how big data increases inequality and threatens democracy. Random House Publishing Group.



NYT running the story of one worker working difficult hours

The New York Times

Working Anything but 9 to 5

Scheduling Technology Leaves Low-Income Parents With Hours of Chaos

By **JODI KANTOR**, Photographs by **SAM HODGSON**

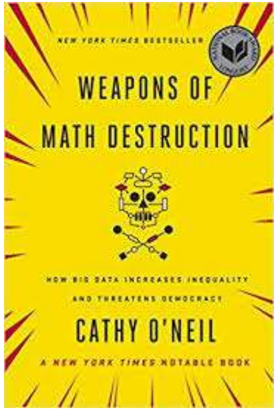
AUGUST 13, 2014

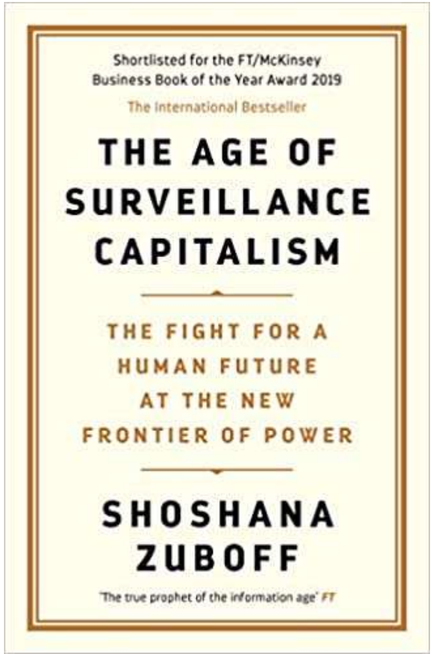
Available on eCampus

https://www.nytimes.com/interactive/2014/08/13/us/starbucks-workers-scheduling-hours.html “managers’ pay is contingent upon the efficiency of their staff measured by revenue for employee hour” p. 126

“I consider scheduling software one of the more appalling WMD [Weapon of Math Destruction]” p. 128

O’Neil, C. (2016). Weapons of math destruction : how big data increases inequality and threatens democracy. Random House Publishing Group.

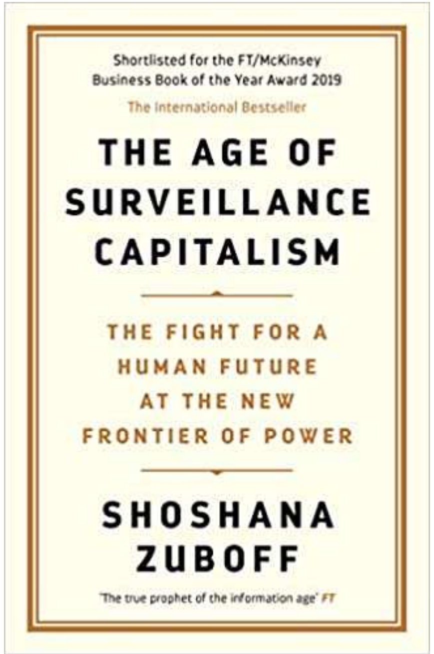




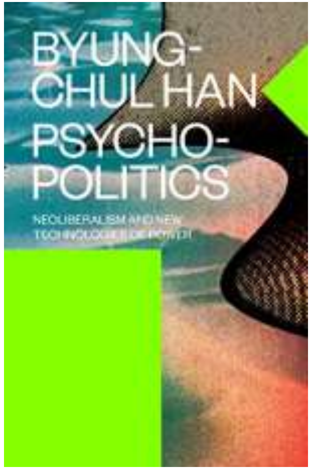
A project of domination of consumers and voters is made possible by artificial intelligence, big data & cognitive psychology

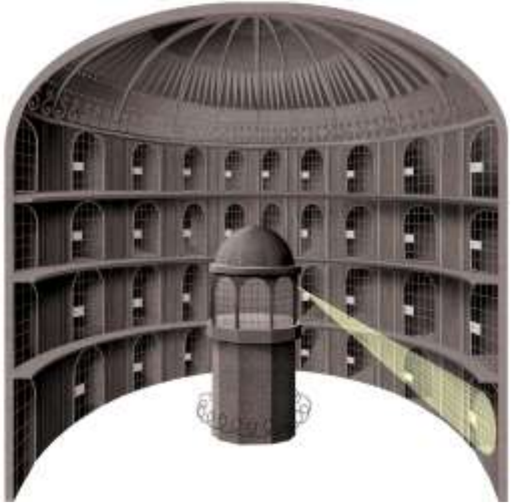
Inequality, power asymmetries and the world of surveillance capitalism

… and the surveillance is voluntarily accepted



Byung Chul Han 'virtual panopticon'





The New York Review of Books

The Boss Will See You Now

Zephyr Teachout

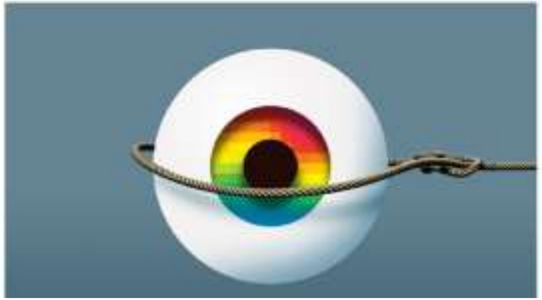
We are experiencing a major turning point in the surveillance of workers, driven by wearable tech, artificial intelligence, and Covid.

The New York Review of Books

The Coming Tech Autocracy

Sue Halpern
November 7, 2024 issue

A functional government, committed to safeguarding its citizens, might be keen to create a regulatory agency for AI or pass comprehensive legislation, but we in the United States do not have such a government.



The New York Times

Working Anything but 9 to 5

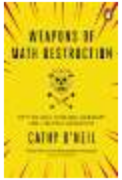
Scheduling Technology Leaves Low-Income Parents With Hours of Chaos

All available on eCampus

Some reading

The New York Review of Books

Some watching


WEAPONS OF MATH DESTRUCTION
CATHY O'NEIL

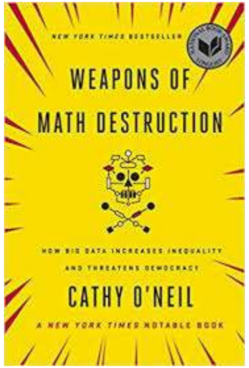

NETFLIX
UNLIMITED TV SHOWS & MOVIES
JOIN NOW
CODED BIAS
Coded Bias
2020 | 12+ | 1h 25m | Science & Nature Docs
This documentary investigates the bias in algorithms after M.I.T. Media Lab researcher Joy Buolamwini uncovered flaws in facial recognition technology.

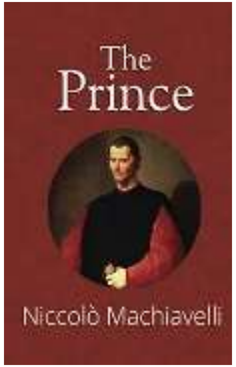
Algorithmic Justice League

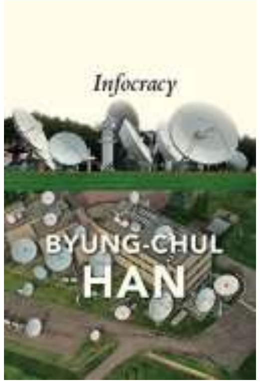
https://www.ajl.org/

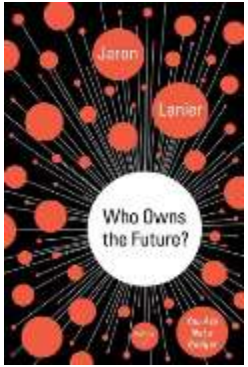
A useful illustration of strategies of capture, starring O'Neil, Zuboff, Lanier, and GAFA technologists…

… such as Tristan Harris, former design ethicist at Google, explaining from inside how social media pursue addiction to maximize profit and manipulates people's behaviour









Worth reading? These are small!

Thank you

www.andreasaltelli.eu
https://orcid.org/0000-0003-4222-6975
@AndreaSaltelli@mstdn.social
https://www.youtube.com/channel/UCz26ZK04xchekUy4Gev
A3DA

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