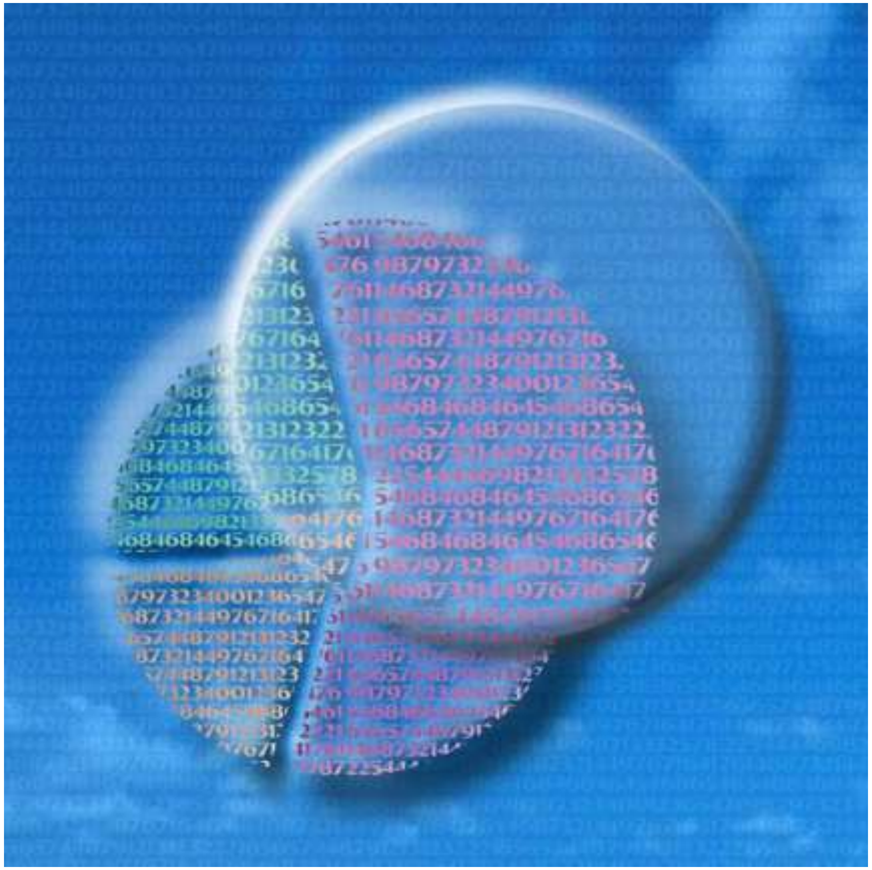

Sensitivity analysis: An introduction

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Where to find materials

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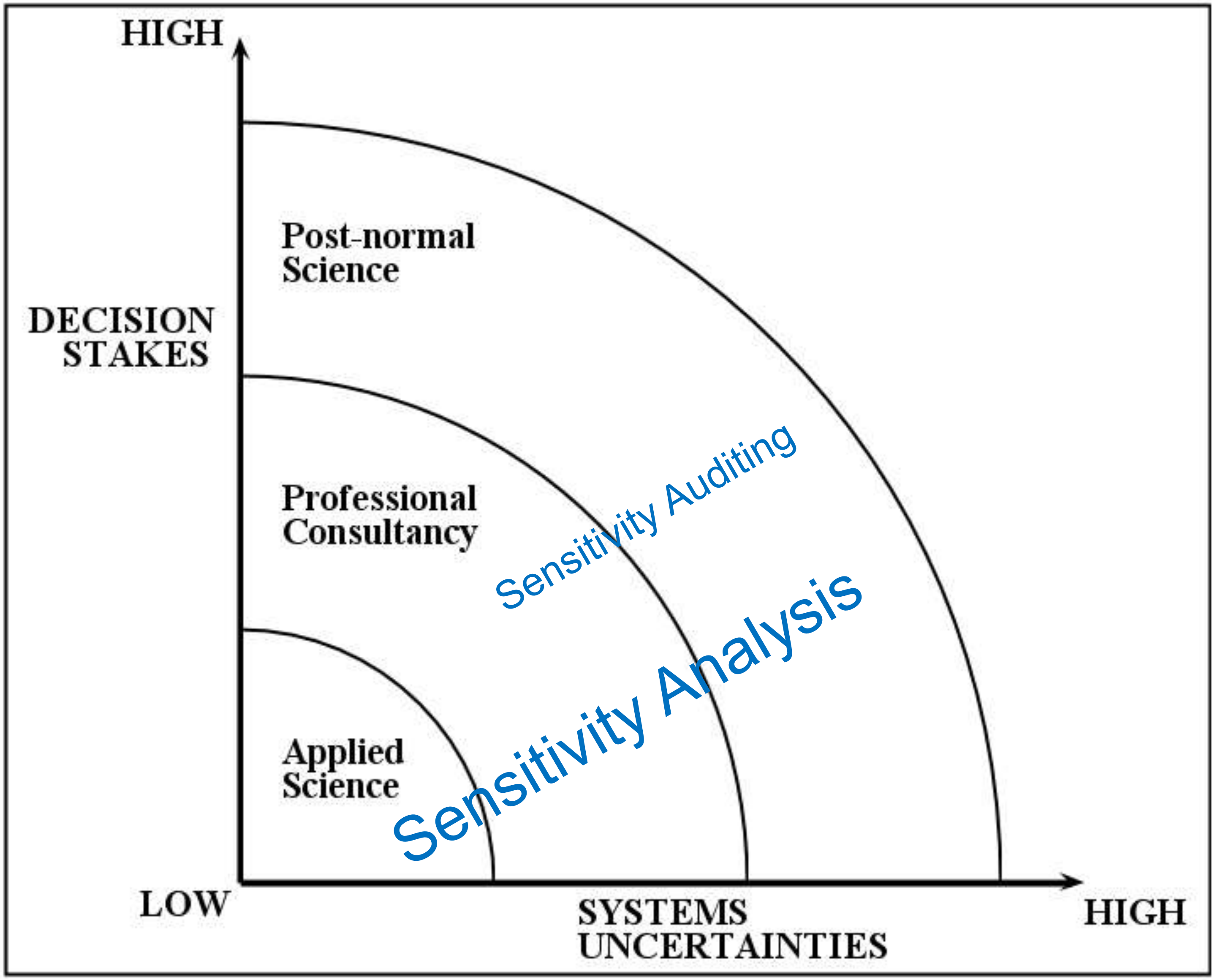
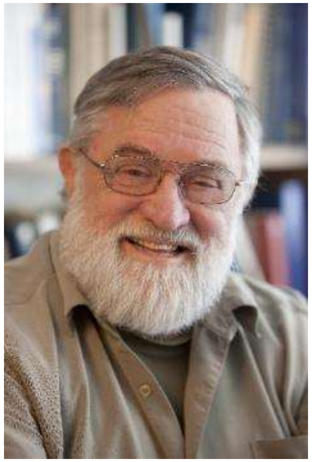
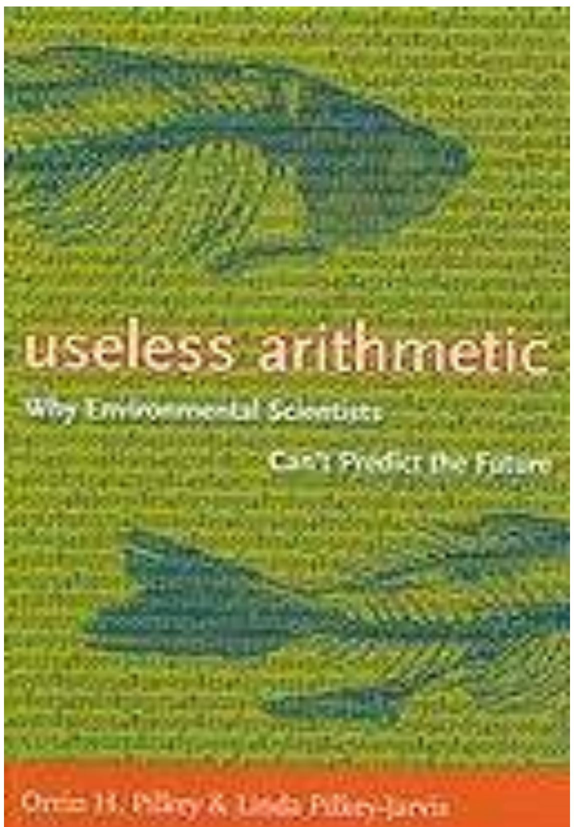
sensitivity analysis,
sensitivity auditing,
science for policy,
impact assessment, ...


HIGH
DECISION
STAKES
LOW
SYSTEMS
UNCERTAINTIES
HIGH
Post-normal
Science
Professional
Consultancy
Applied
Science
Sensitivity Auditing
Sensitivity Analysis

Limits of sensitivity analysis



Orrin H. Pilkey
Duke University,
NC



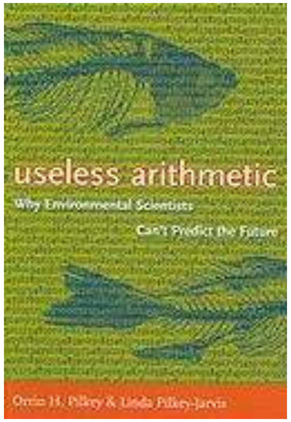
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

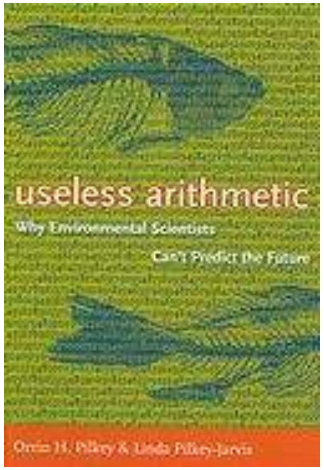
[…] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

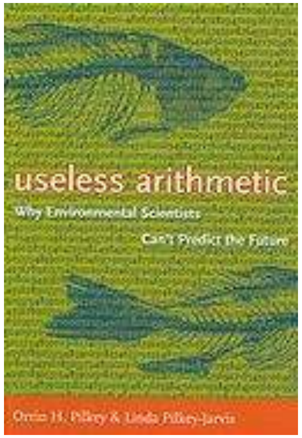
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

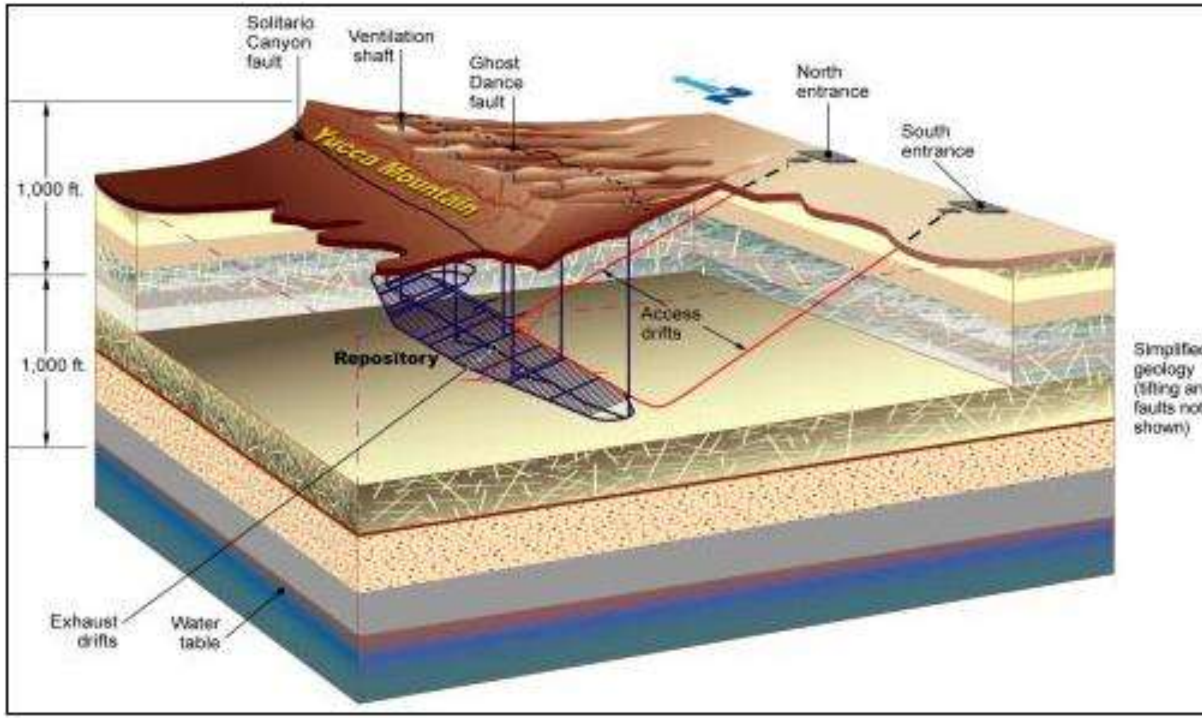
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.





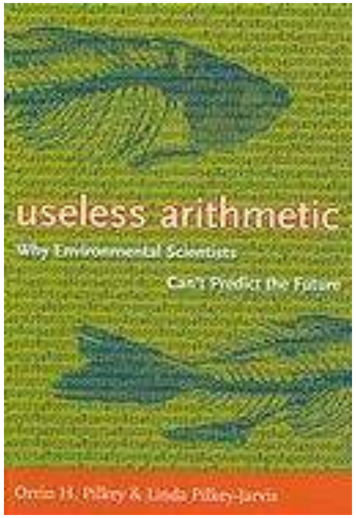
The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter

(the ^{36}Cl story)

Type III error in sensitivity: Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→… SA useless if it is instead ~ 3,000 millimetres per year.



“Scientific mathematical modelling should involve constant efforts to falsify the model”

Ref. ➔ Robert K. Merton’s ‘Organized skepticism ’

Communalism – the common ownership of scient40

ific discoveries, according to which scientists give up intellectual property rights in exchange for recognition and esteem (Merton actually used the term Communism, but had this notion of communalism in mind, not Marxism);

Universalism – according to which claims to truth are evaluated in terms of universal or impersonal criteria, and not on the basis of race, class, gender, religion, or nationality;

Disinterestedness – according to which scientists are rewarded for acting in ways that outwardly appear to be selfless;

Organized Skepticism – all ideas must be tested and are subject to rigorous, structured community scrutiny.

Statistical and mathematical modelling are at the hearth of

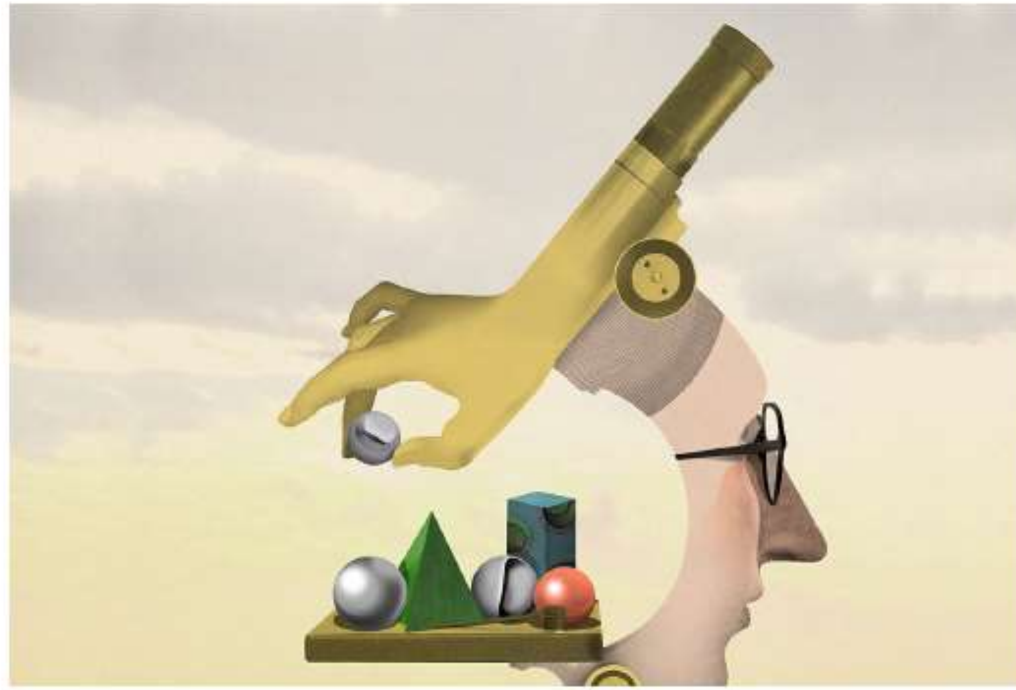
- science for policy
- storm about malpractices.

New Scientists talks of "statistical sausage factory"

FEATURE 13 April 2016

Why so much science research is flawed – and what to do about it

Dodgy results are fuelling flawed policy decisions and undermining medical advances. They could even make us lose faith in science. **New Scientist** investigates



An alarming amount of research is flawed
Brett Ryder

LEADER 13 April 2016

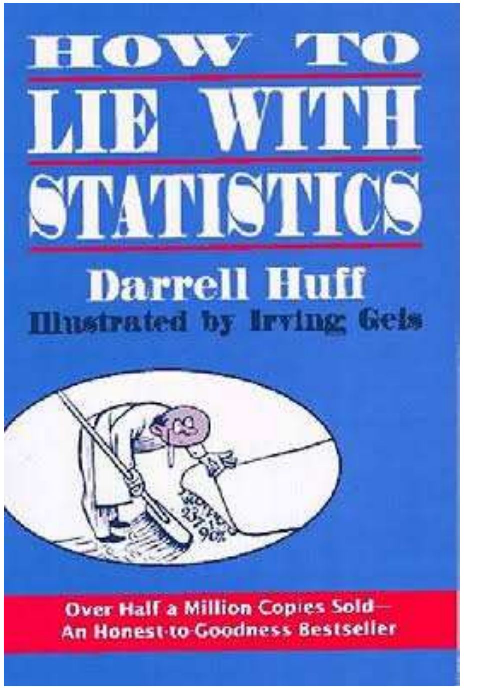
Science isn't as solid as it should be – but science can fix it

Unconscious biases and data-torturing are weakening our knowledge base – but unlike politicians and bankers, scientists aren't covering up their failings

New Scientist

Will any sensitivity analysis do the job?

Can I lie with sensitivity analysis as I can lie with statistics?



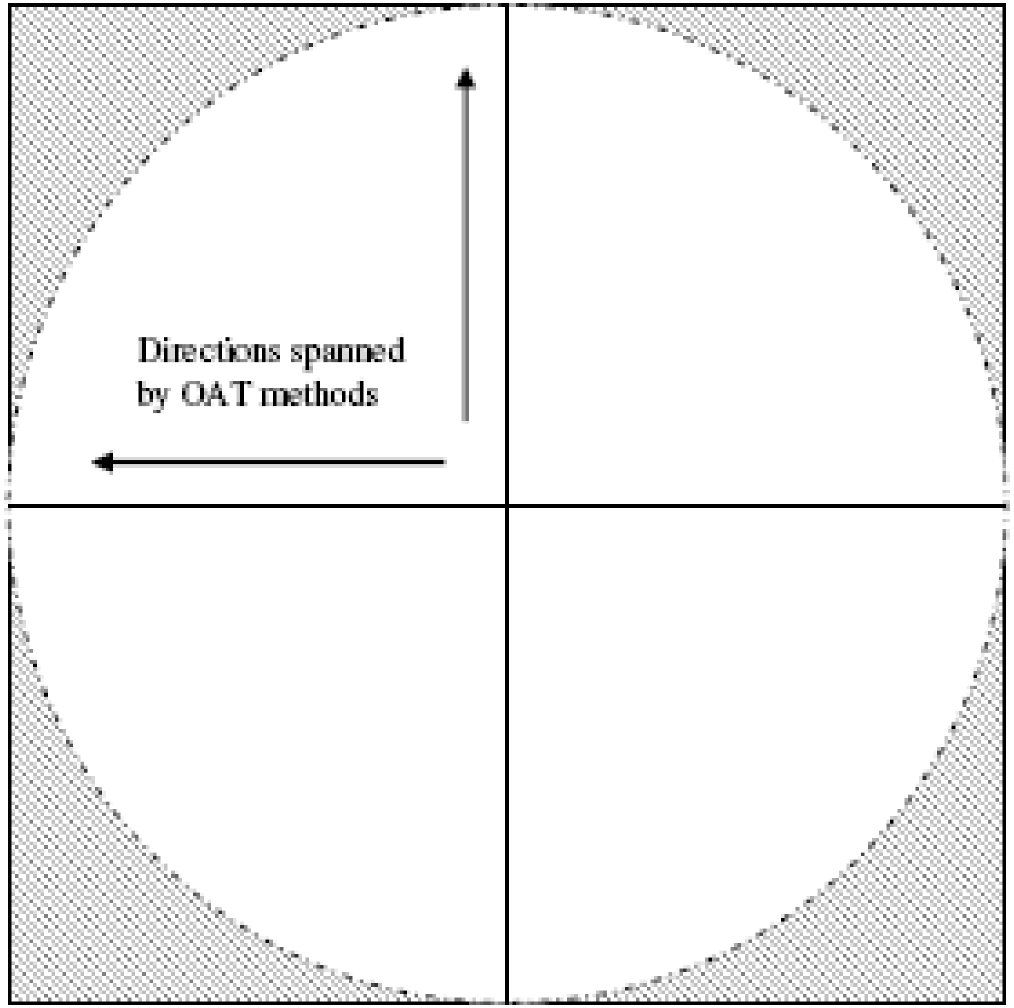
Saltelli, A., Annoni P., 2010, How to avoid a perfunctory sensitivity analysis, *Environmental Modeling and Software,* **25**, 1508-1517.

What do these have in common?

J. Campbell, *et al.*, *Science* **322**, 1085 (2008).
R. Bailis, M. Ezzati, D. Kammen, *Science* **308**, 98 (2005).
E. Stites, P. Trampont, Z. Ma, K. Ravichandran, *Science* **318**, 463 (2007).
J. Murphy, et al., *Nature* **430**, 768–772 (2004).
J. Coggan, *et al.*, *Science* **309**, 446 (2005).

OAT

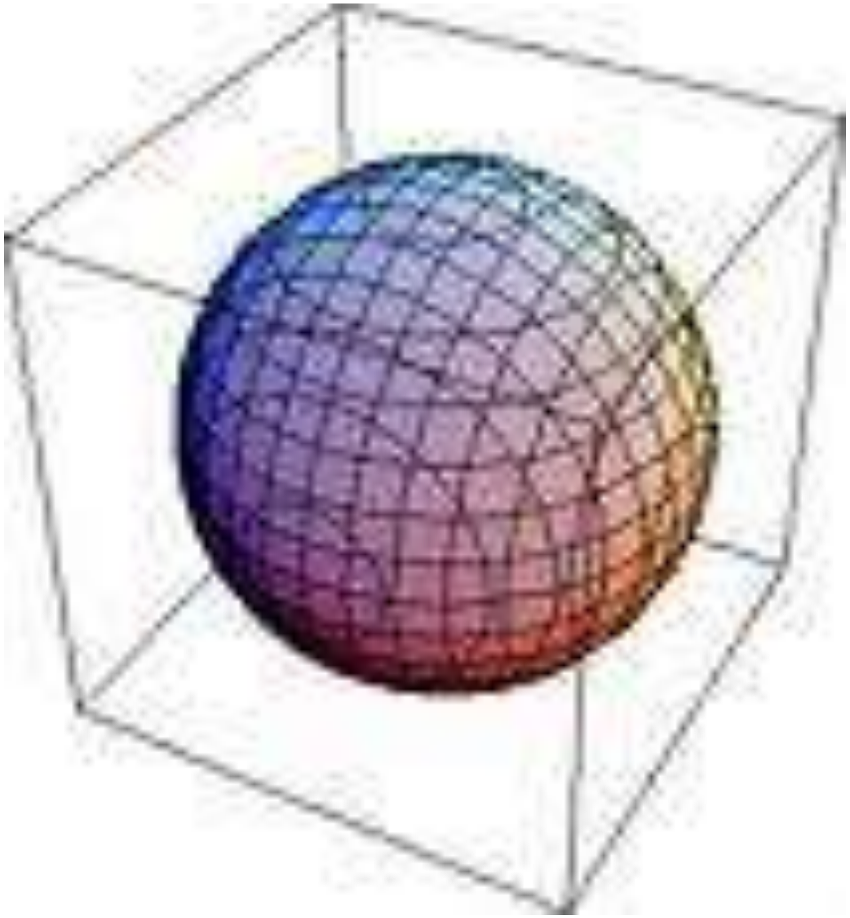
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions



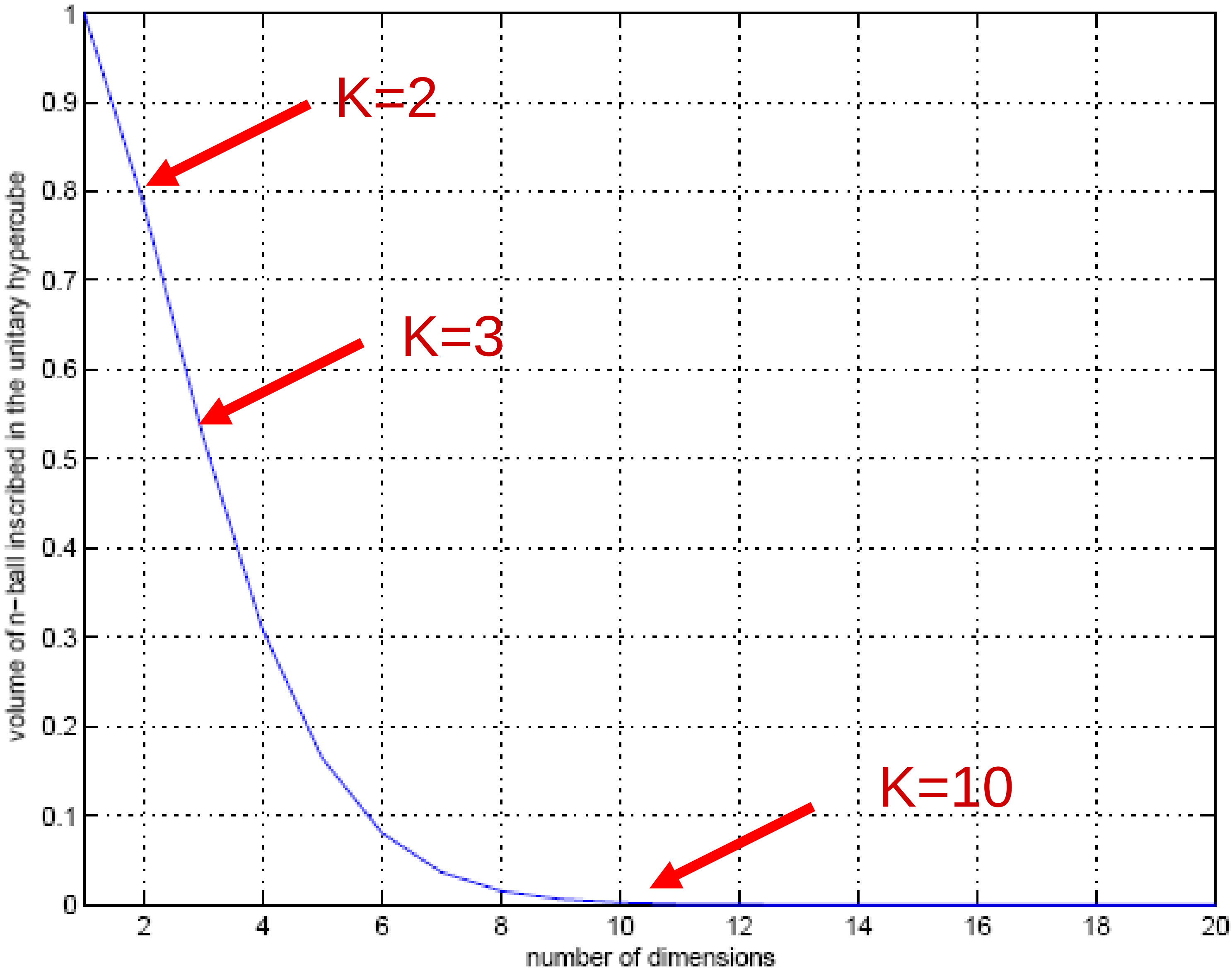
Volume sphere / volume cube =?

~ 1/2

OAT in 10 dimensions

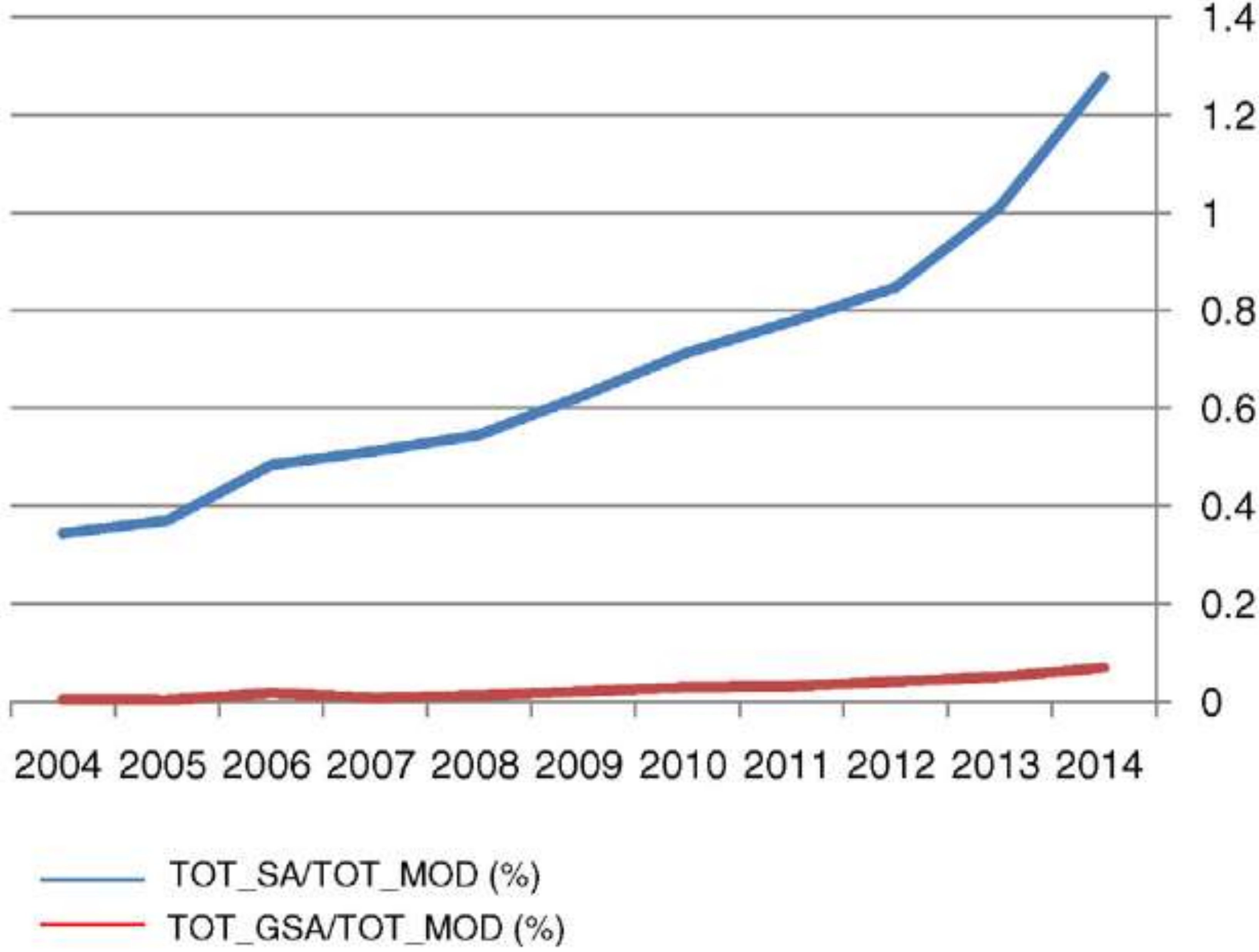
Volume hypersphere / volume ten dimensional hypercube ~ 0.0025

OAT in k dimensions



OAT is still the most largely used technique in SA. Out of every 100 papers with modelling & SA only 4 are ‘global’ in the sense discussed here.

Ferretti, F., Saltelli A., Tarantola, S., 2016, Trends in Sensitivity Analysis practice in the last decade, Science of the Total Environment, http://dx.doi.org/10.1016/j.scitotenv.2016.02.133


1.4
1.2
1
0.8
0.6
0.4
0.2
0
2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014
TOT_SA/TOT_MOD (%)
TOT_GSA/TOT_MOD (%)

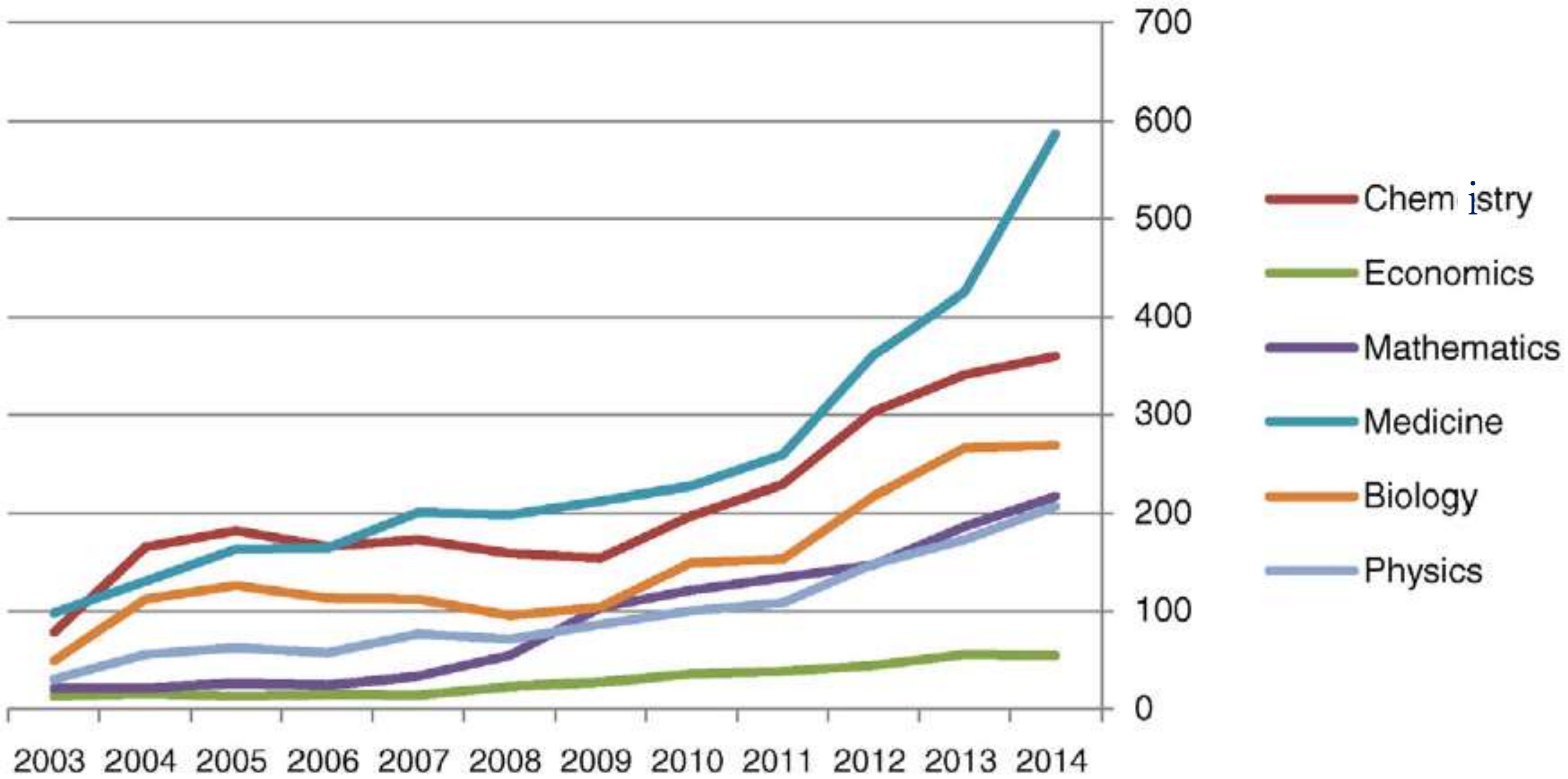


Fig. 4. GSA in the different scientific domains.

Secrets of sensitivity analysis

First secret: The most important question is the question.

Corollary 1: Sensitivity analysis is not "run" on a model but on a model once applied to a question.

First secret: The most important question is the question.

Corollary 2: The best setting for a sensitivity analysis is one when one wants to prove that a question cannot be answered given the model – e.g. it would be better to be in a setting of falsification than in one of confirmation (Oreskes et al., 1994).

[Normally the opposite is the case]

Second secret: Sensitivity analysis should not be used to hide assumptions

[it often is]

Third secret: If sensitivity analysis shows that a question cannot be answered by the model one should find another question/model which can be treated meaningfully.

[Often the love for the model prevails]

Badly kept secret:

There is always one more bug!

(Lubarsky's Law of Cybernetic Entomology)

Personal note: I never run a
SA without finding more bugs

Definition of uncertainty and sensitivity analysis.

Sensitivity analysis: The study of the relative importance of different input factors on the model output.

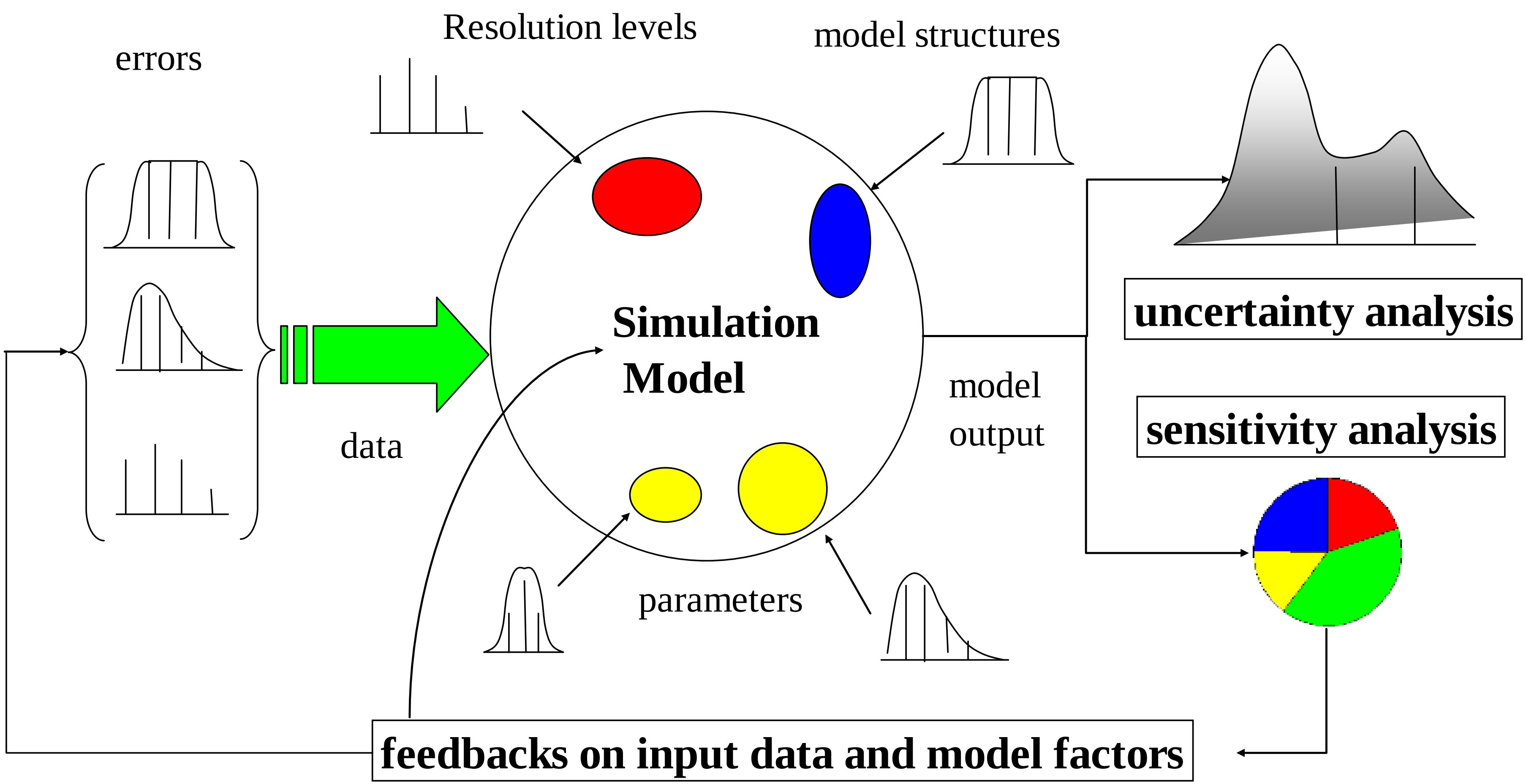
Uncertainty analysis: Focuses on just quantifying the uncertainty in model output.

[Global*] sensitivity analysis: "The study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input"

Saltelli A., 2002, Sensitivity Analysis for Importance Assessment, Risk Analysis, 22 (3), 1–12.

- Modelling in a Monte Carlo framework using quasi MC-points
- All uncertainties activated simultaneously; uncertainty and sensitivity together

An engineer's vision of UA, SA



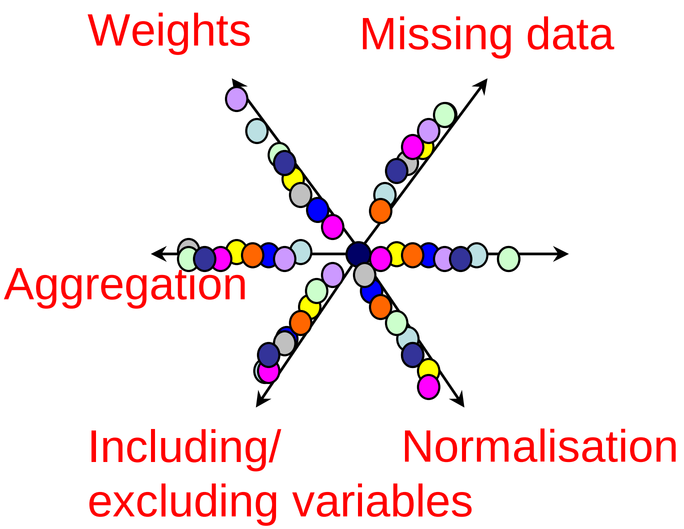
One can sample more than just factors ⋯

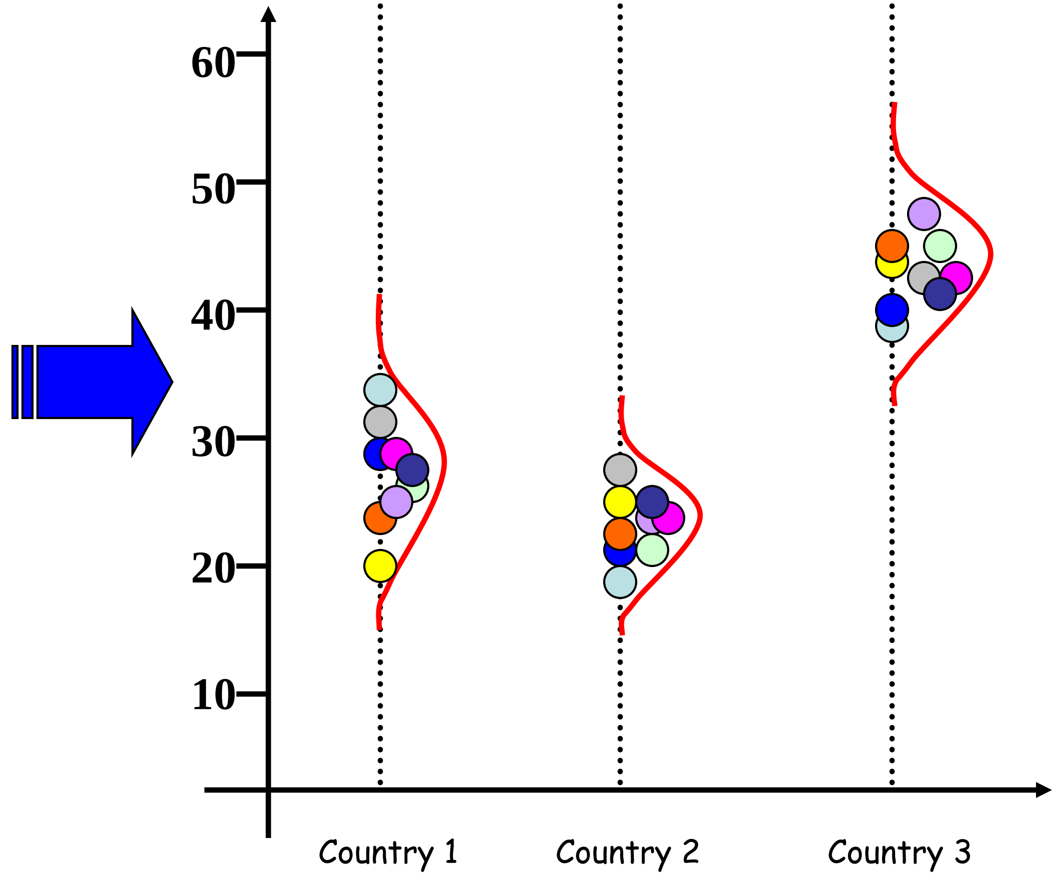
Using triggers one can sample modelling assumptions ⋯

Example: Y is a composite indicator

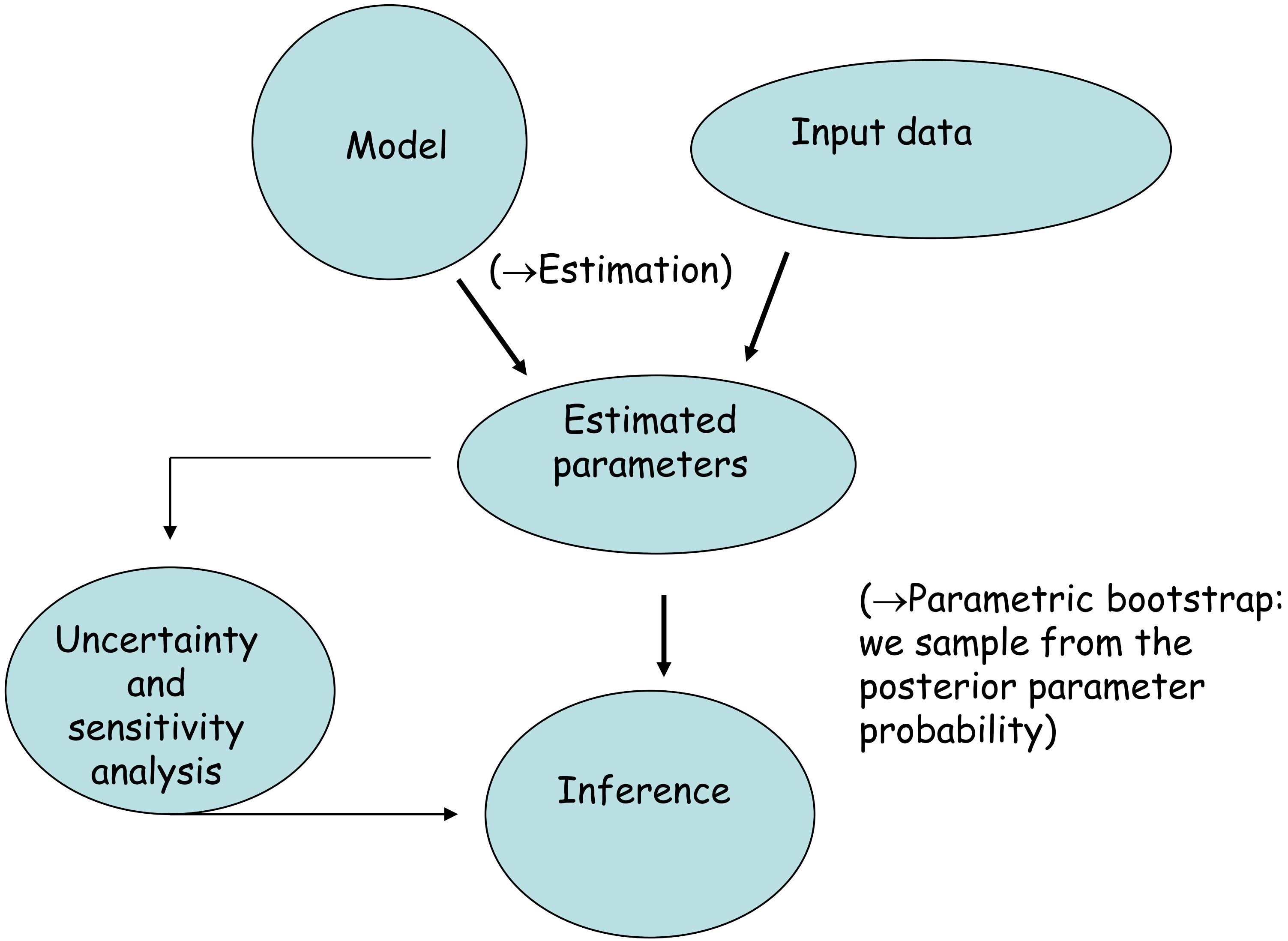
Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

Space of alternatives





Models maps assumptions onto inferences … (Parametric bootstrap version of UA/SA)



Sample matrix for parametric bootstrap.

$$\begin{array}{cccc} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{Nk} \end{array}$$

Each row is a sample trial for one model run. Each column is a sample of size N from the marginal distribution of the parameters as generated by the estimation procedure.

Model results:

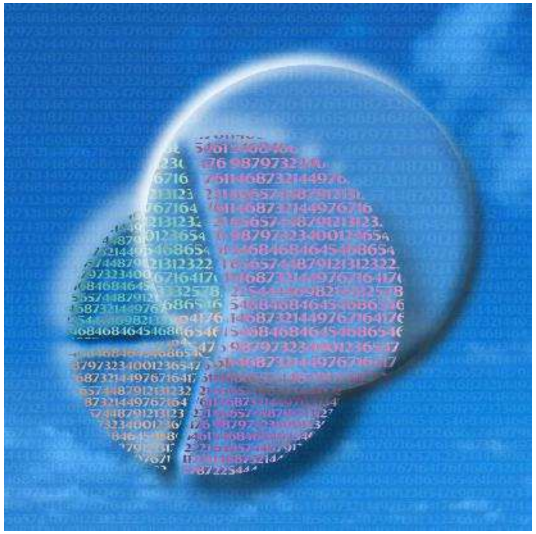
Each row is the error-free result of the model run.

$$y_1$$

$$y_2$$

$$\cdots$$

$$y_N$$

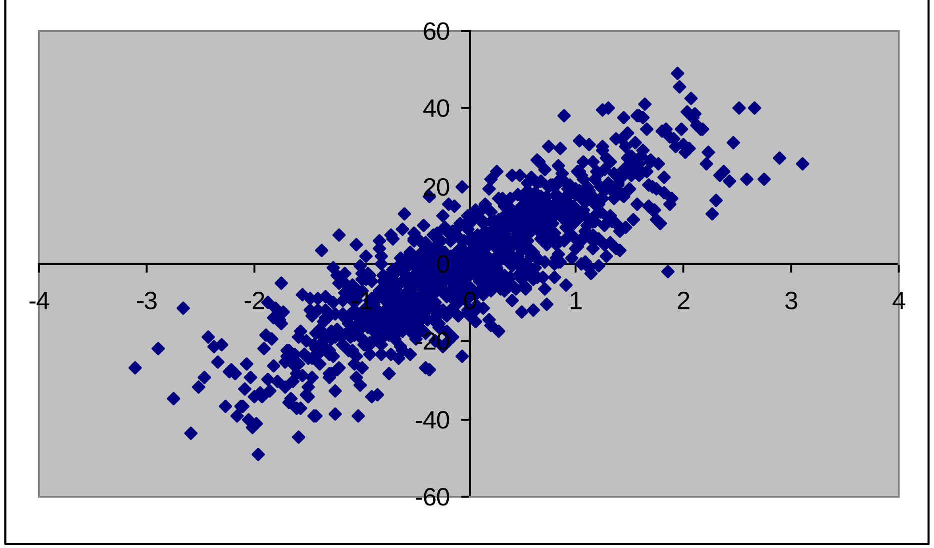


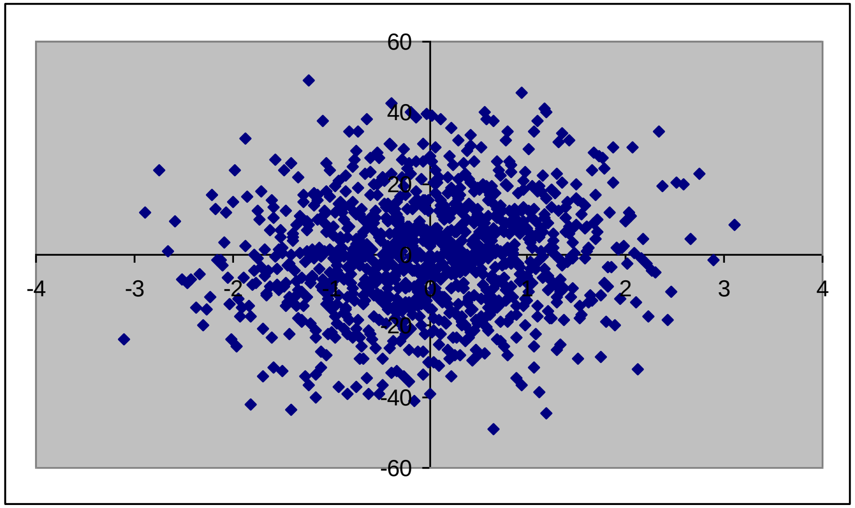
Our preferred methods for SA: variance based

An intuitive derivation of sensitivity indices

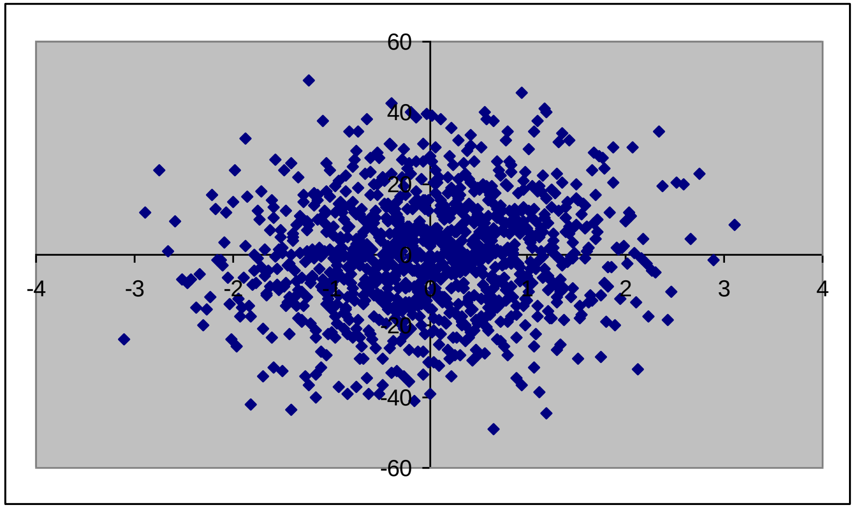
$$\begin{matrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{Nk} \end{matrix}$$

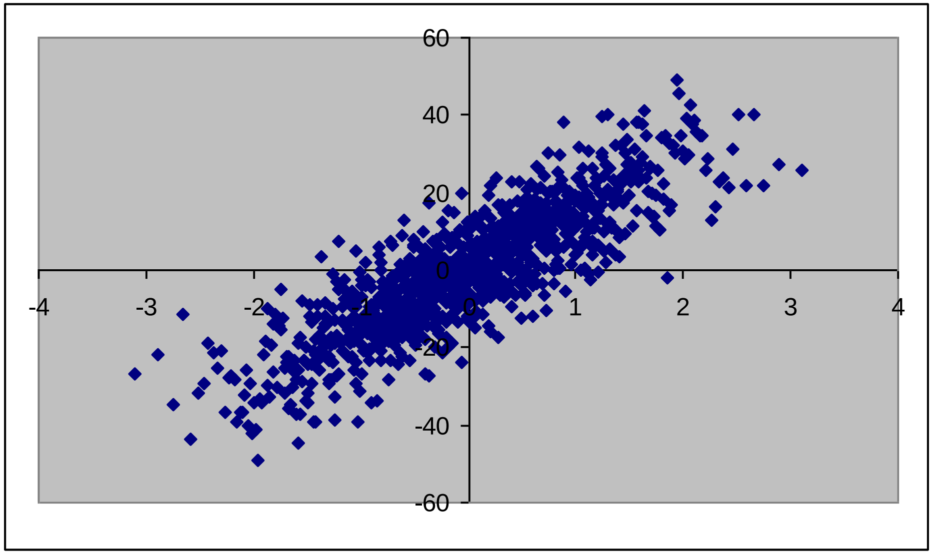
$$\begin{matrix} y_1 \\ y_2 \\ \cdots \\ y_N \end{matrix}$$





Scatterplots of y versus sorted factors

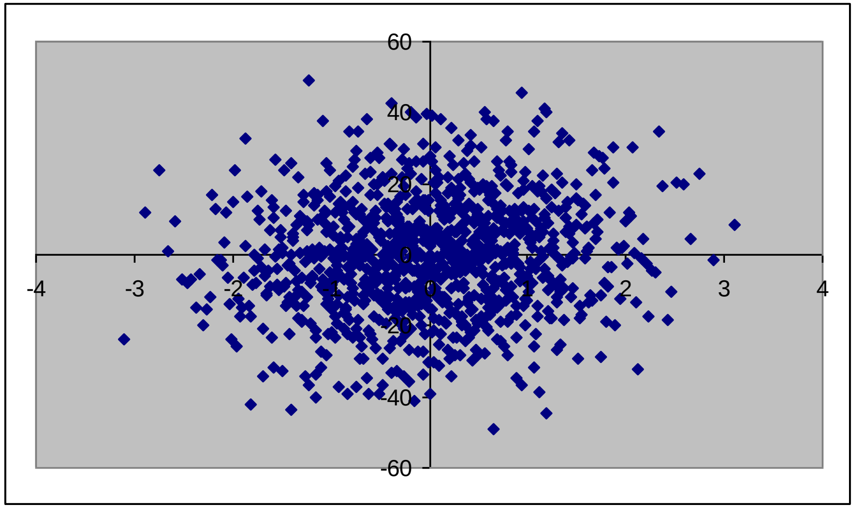


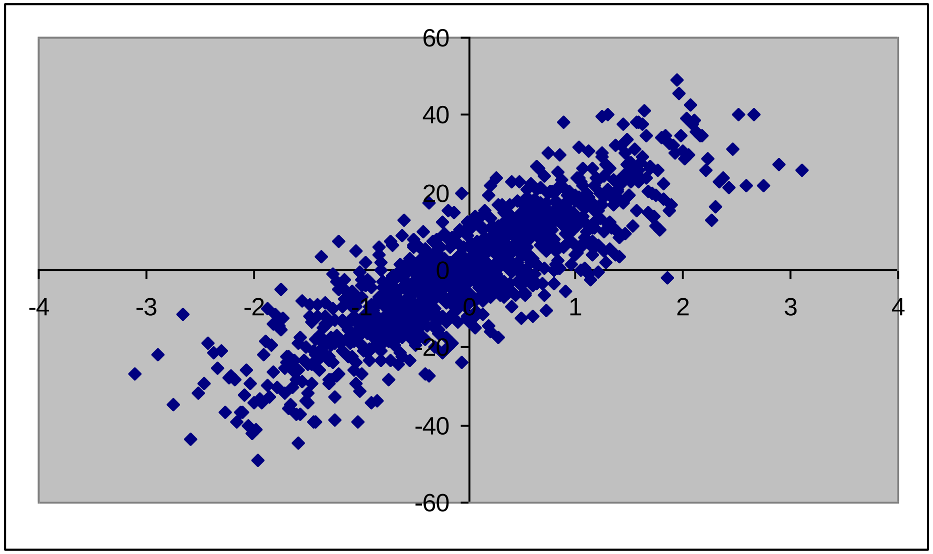


The ordinate axis is always Y

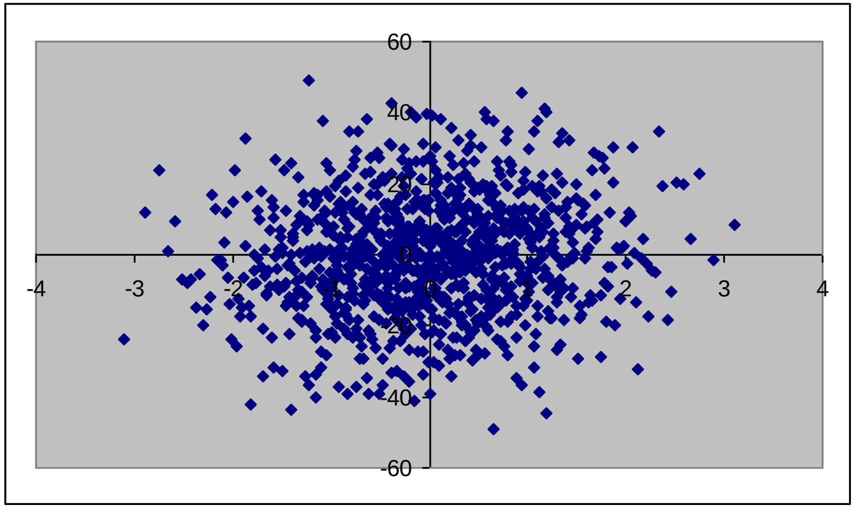
The abscissa are the various factors X_i in turn.

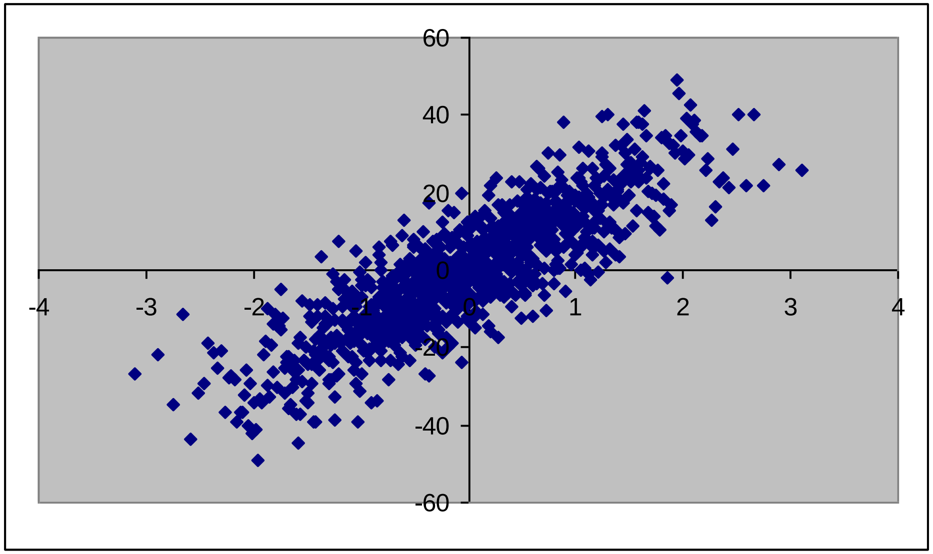
The points are always the same





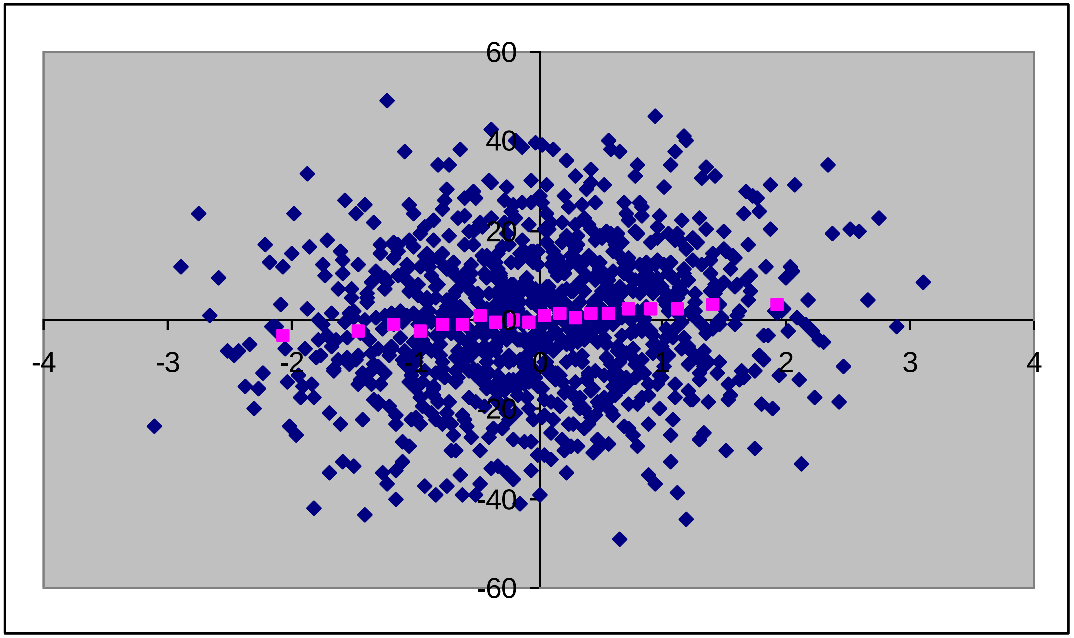
Which factor is more important?

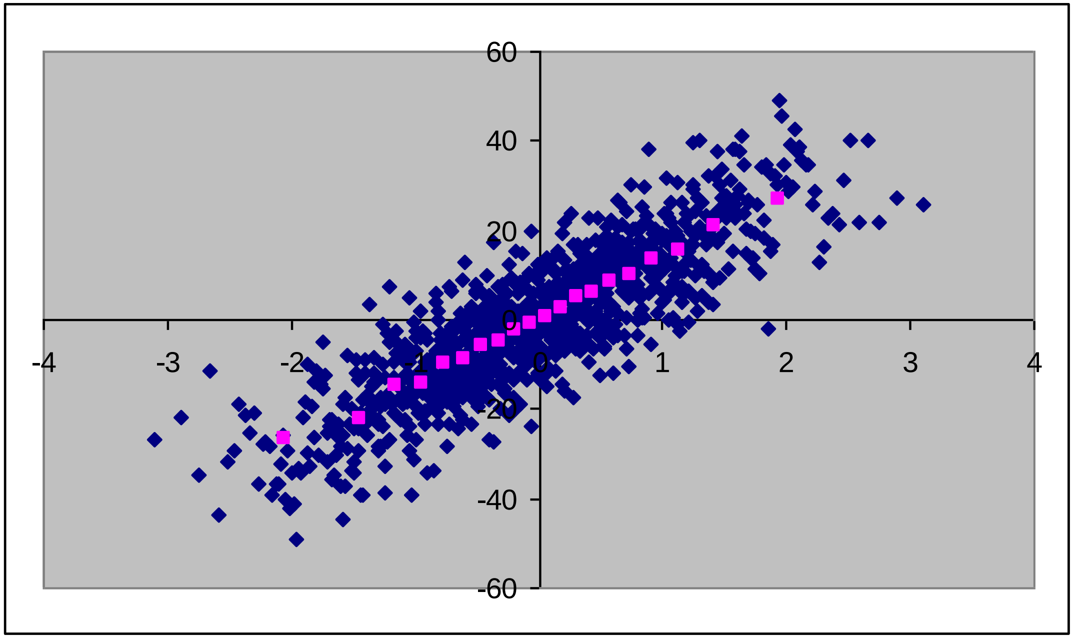




These are ~1,000 points

Divide them in 20 bins of ~ 50 points

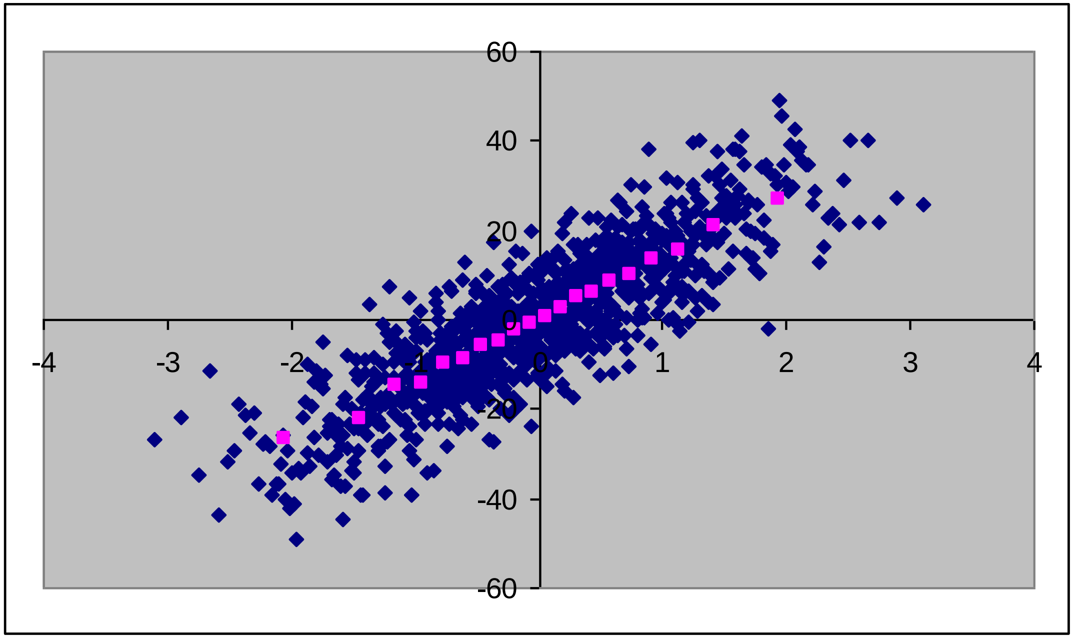




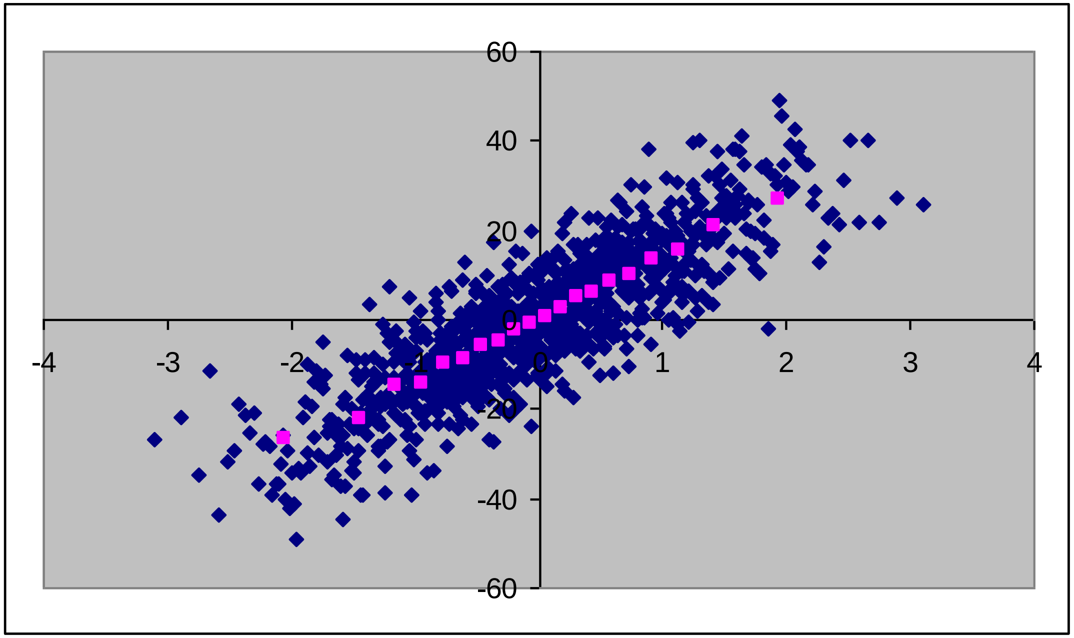
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



Each pink point is ~ $E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)$



Take the variance
of the pinkies

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)$$

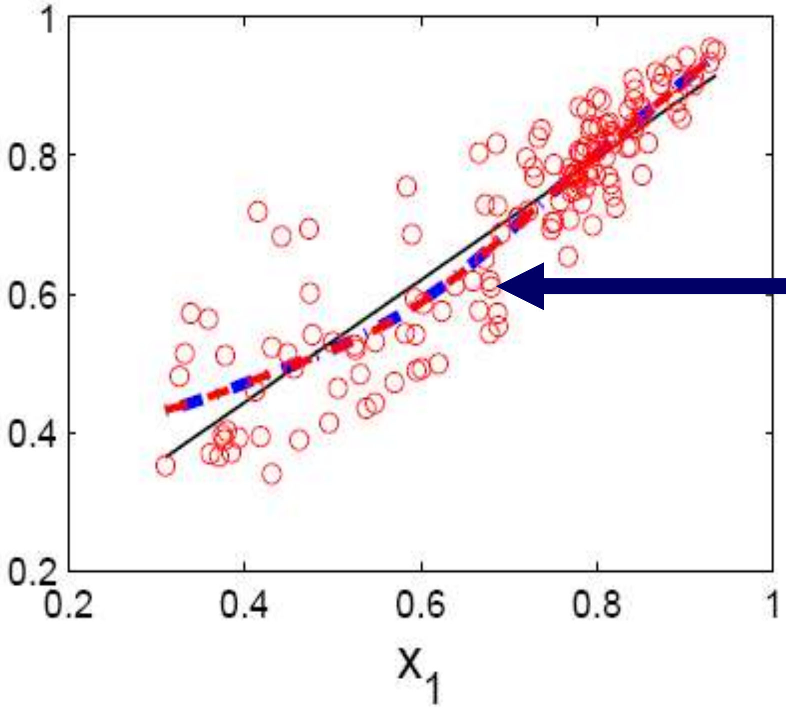
Pearson's correlation ratio

Smoothed curve

$$S_i \equiv \eta_i^2 := \frac{V_{x_i}\left(E_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{V\left(y\right)}$$

First order sensitivity index

Unconditional variance



Smoothed curve

$$E_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)$$

First order sensitivity index:

$$\frac{V_{x_i}\left(E_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{V\left(y\right)}$$

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)$$

First order effect, or top marginal variance=

= the expected reduction in variance that would be achieved if factor Xi could be fixed.

Why?

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)+$$
$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)=V(Y)$$

Easy to prove using $V(Y)=E(Y^2)-E^2(Y)$

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)+$$

$$+\boxed{E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)}=V(Y)$$

This is what variance would be left (on average) if Xi could be fixed…

… then this …

$$\boxed{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)} + E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) = V(Y)$$

… must be the expected reduction in variance that would be achieved if factor Xi could be fixed

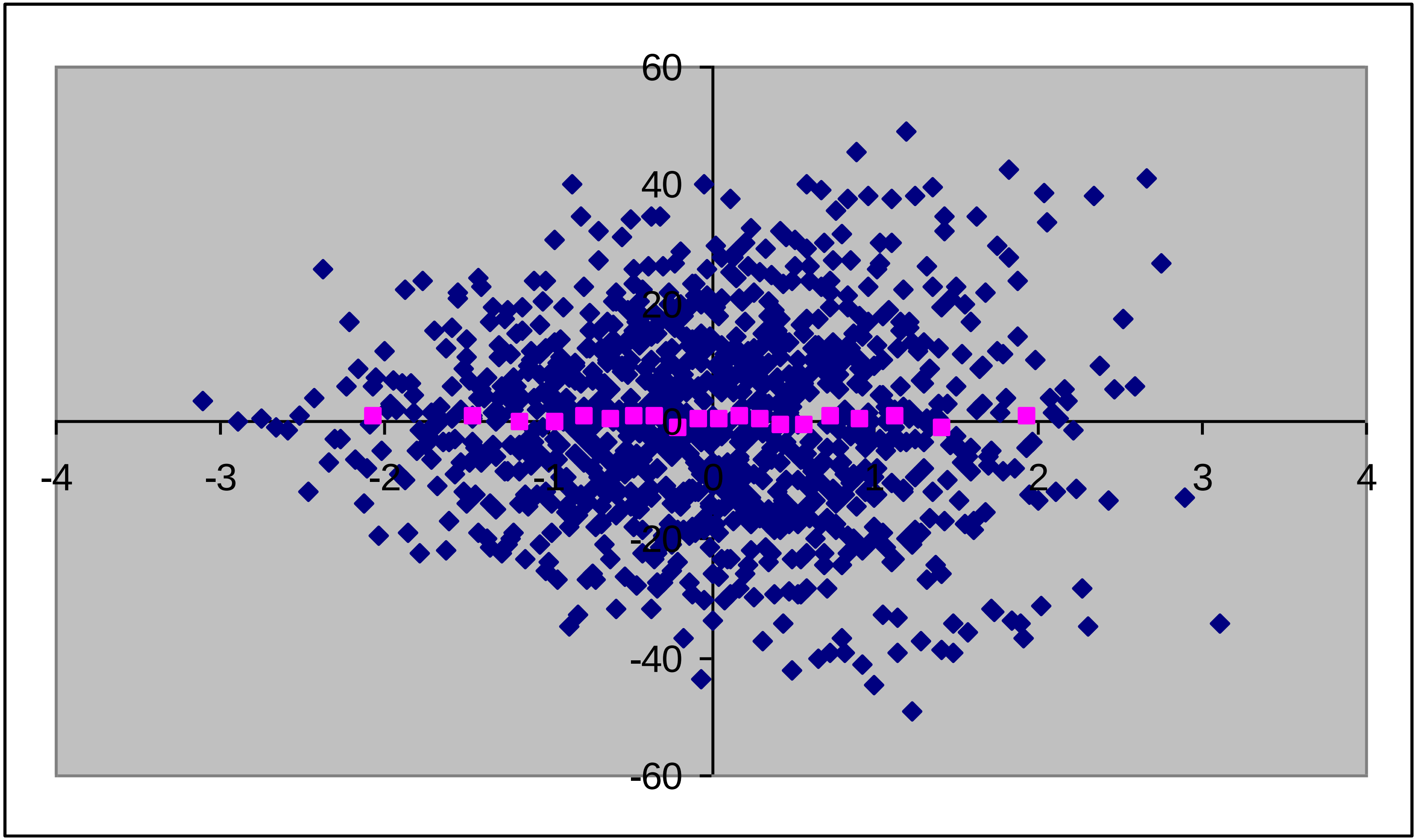
For <u>additive</u> models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) \approx V(Y)$$

… which is also how additive models are defined

How about non additive models?

- Is $S_i = 0$?
- Is this factor non-important?



There are terms which capture two-way, three way, ⋯ interactions among variables.

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_{i} V_i + \sum_{i,j>i} V_{ij} + \ldots + V_{123\ldots k}$$

Variance decomposition (ANOVA)

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right) = V_i$$

$$V_{X_i X_j}\left(E_{\mathbf{X}_{\sim ij}}\left(Y \middle| X_i, X_j\right)\right) =$$

$$= V_i + V_j + V_{ij}$$

…

Variance decomposition (ANOVA)

When the factors are independent the total variance can be decomposed into main effects and interaction effects up to the order k, the dimensionality of the problem.

Variance decomposition (ANOVA)

When the factors are <u>not</u> independent the decomposition loses its unicity (and hence its appeal)

If fact interactions terms are awkward to handle: second order terms are as many as $k(k-1)/2 \cdots$

Wouldn’t it be handy to have just a single ‘importance’ terms for all effects, inclusive of first order and interactions?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a model $Y=f(X_1,X_2,X_3)$

Instead of

$$V=V_1+V_2+V_3+$$
$$+V_{12}+V_{13}+V_{23}+$$
$$+V_{123}$$

and

$$1=S_1+S_2+S_3+$$
$$+S_{12}+S_{13}+S_{23}+$$
$$+S_{123}$$

We have:

$S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$

(and analogue formulae for S_{T2}, S_{T3}) which can be computed without knowing S_1, S_{12}, S_{13}, S_{123}

S_{T1} is called a total effect sensitivity index

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y \middle| \mathbf{X}_{\sim i}\right)\right)$$

Total effect, or bottom marginal variance=

= the expected variance that would be left if all factors but Xi could be fixed.

$$\frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)\right)}{V(Y)} = S_i$$

$$\frac{E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y \middle| \mathbf{X}_{\sim i}\right)\right)}{V(Y)} = S_{Ti}$$

Rescaled to [0,1], under the name of first order and total order sensitivity coefficient

Variance based measures are:
-well scaled,
-concise,
-easy to communicate.

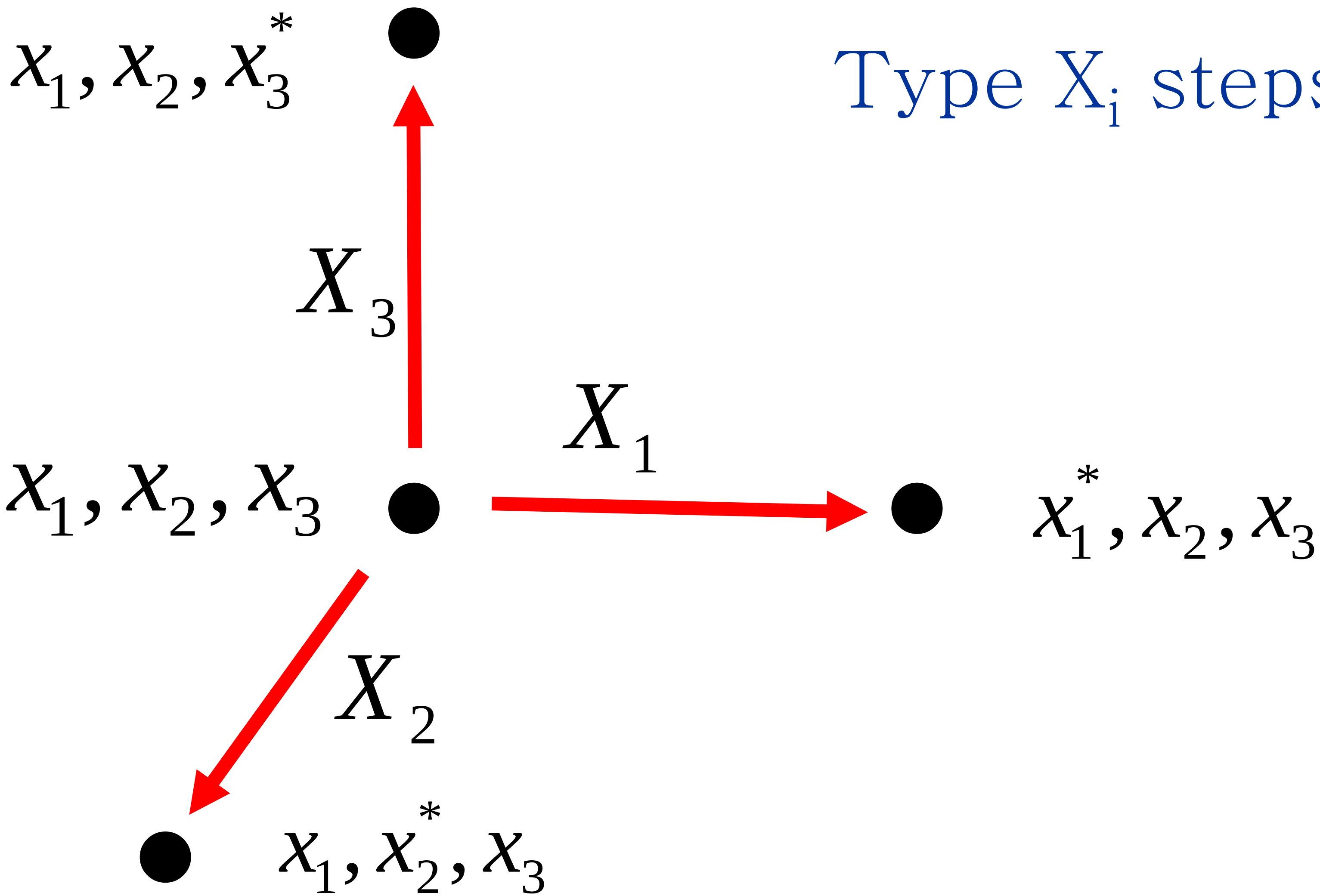
Further
- S_i reduces to squared standard regression coefficients for linear model.
- S_{Ti} detect and describe interactions and
- Becomes a screening test at low sample size (See Campolongo F, Saltelli A, Cariboni, J, 2011, From screening to quantitative sensitivity analysis. A unified approach, *Computer Physics Communication*, 182 (4), pp. 978-988.)

Both indices can be computed via Monte Carlo

We use quasi random sequences developed by I.M. Sobol’

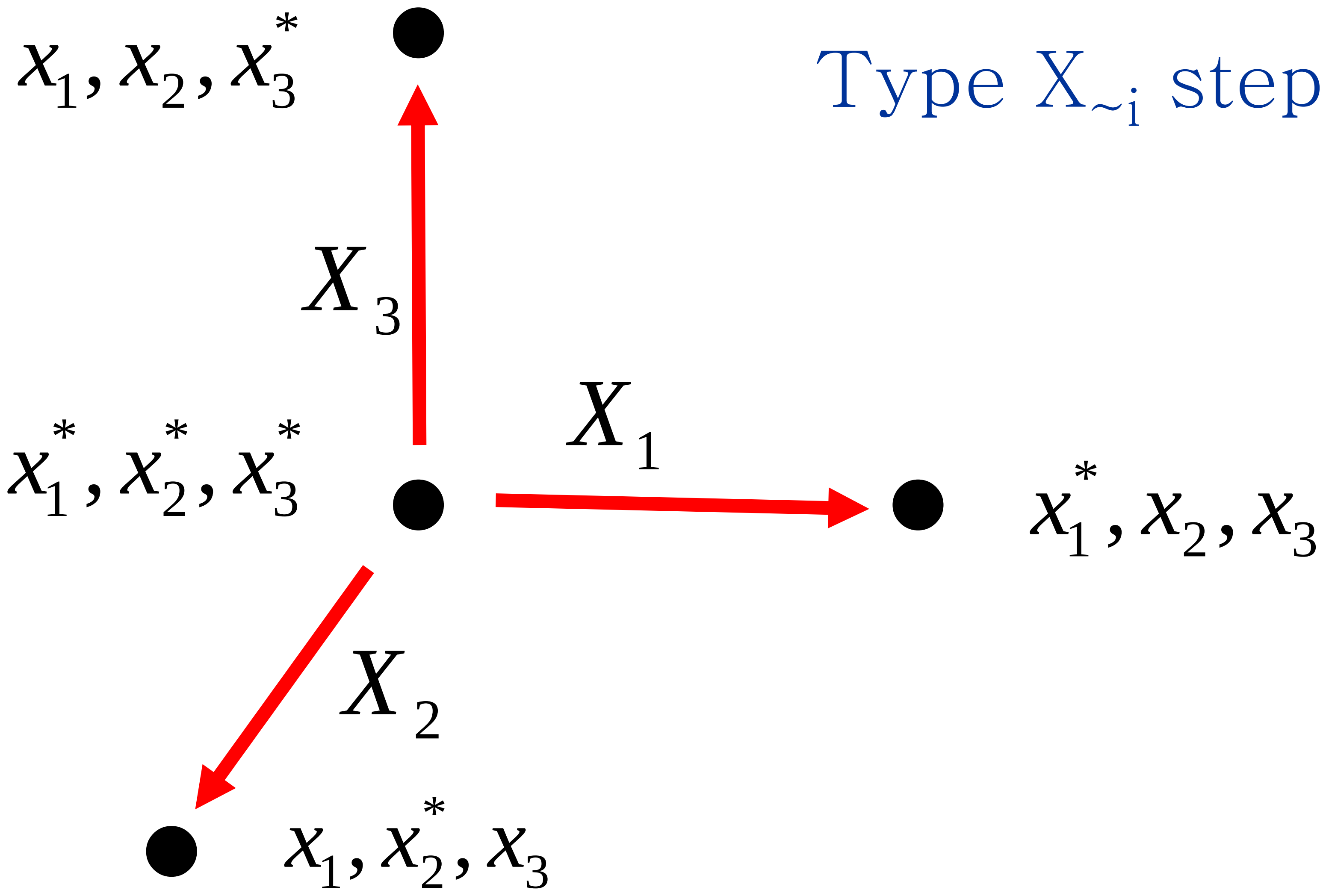
Computing S_{Ti}

Type X_i steps



Computing S_i

Type $X_{\sim i}$ steps



Why these measures?

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y \mid X_i\right)\right)$$

Factors prioritization

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y \mid \mathbf{X}_{\sim i}\right)\right)$$

Fixing (dropping) non important factors

Saltelli A. Tarantola S., 2002, On the relative importance of input factors in mathematical models: safety assessment for nuclear waste disposal, *Journal of American Statistical Association*, **97** (459), 02-709.

More about the settings:

•Factor prioritisation → $$S_i \equiv \frac{V\left(E\left(Y \middle| X_i\right)\right)}{V_Y}$$

If the cost of 'discovering' factors were the same for all factors which factor should I try to discover first?

•<u>Factor fixing:</u> Can I fix a factor [or a subset of input factors] at any given value over their range of uncertainty without reducing significantly the output?

$$S_{Ti} \equiv \frac{E\left(V\left(Y \middle| \mathbf{X}_{\sim i}\right)\right)}{V_Y}$$

Factor fixing is useful to achieve model simplification and 'relevance'.

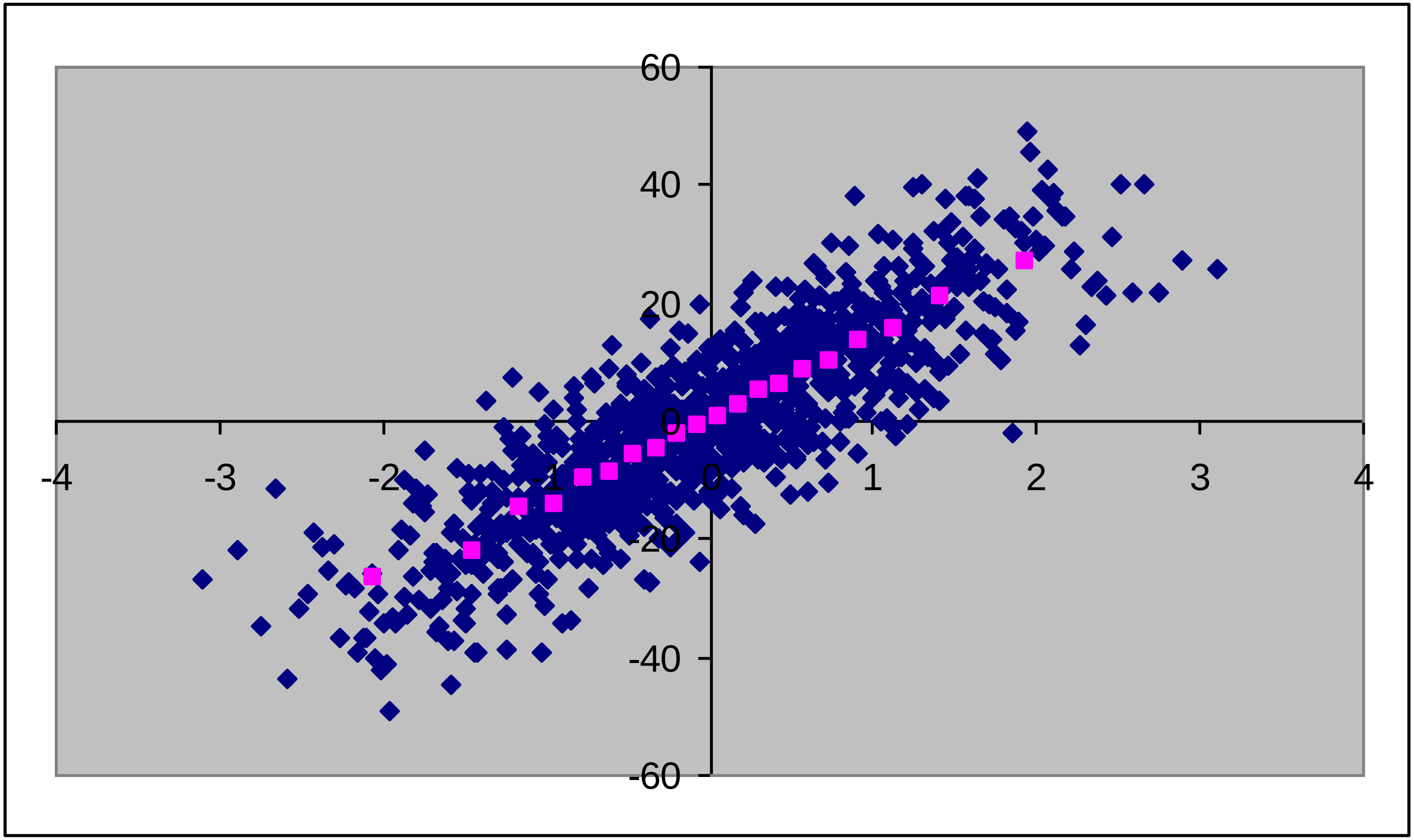
We cannot use S_i to fix a factor; $S_i = 0$ is a necessary condition for X_i to be non-influential but not a sufficient one

X_i could be influent at the second order.

Summary for variance based measures:

1. Easy-to-code, Monte Carlo – better on quasi-random points. Estimate of the error available.

2. <u>The main effect</u> can be made cheap; its computational cost does not depend upon k.



Easy to smooth and interpolate!

Summary for variance based measures:

3. <u>The total effect</u> is more expensive; its computational cost is $(k+1)N$ where N is one of the order of one thousand (unless e.g. using emulators $\cdots$).

How about MuSIASEM?

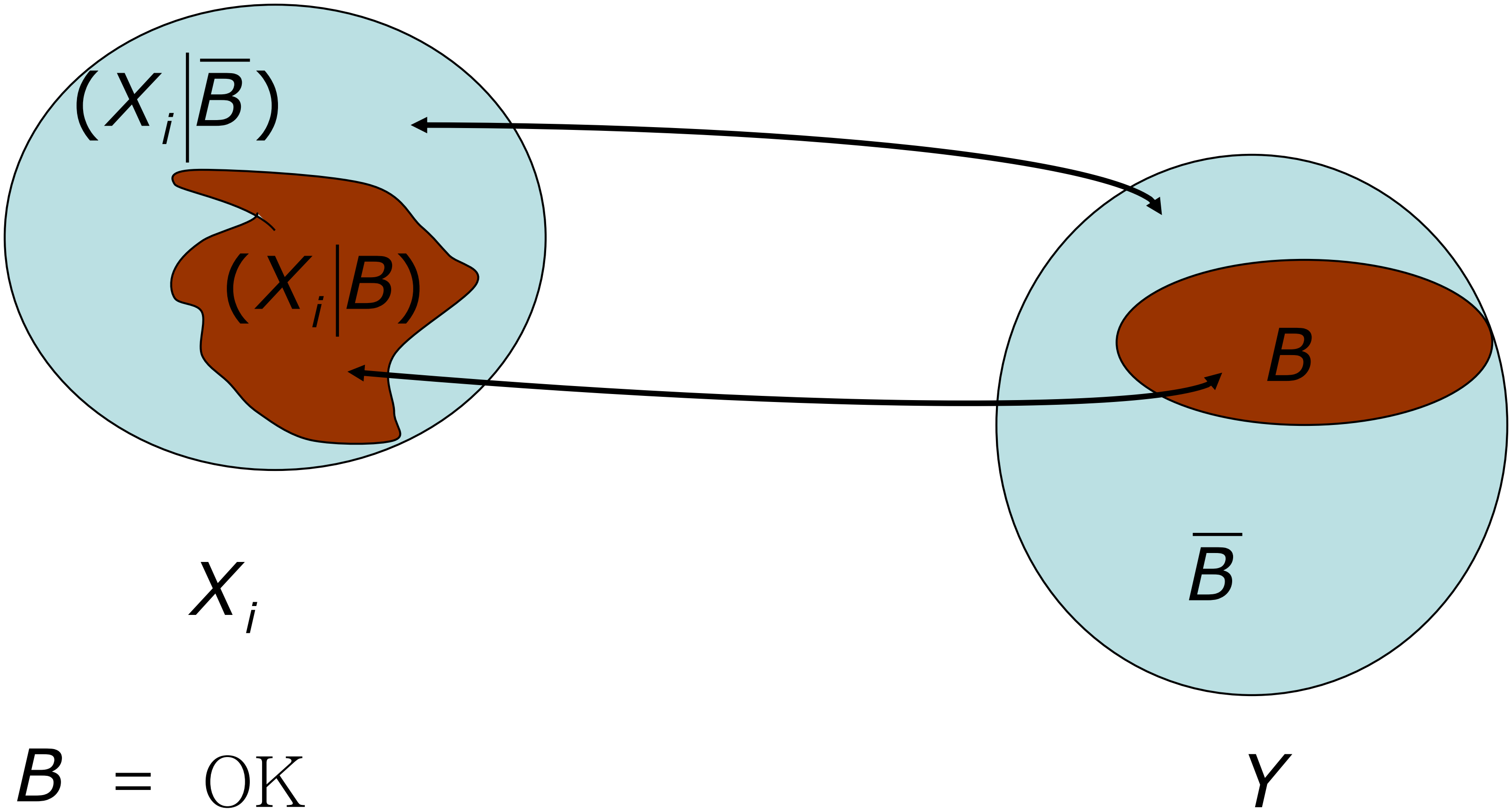
Either apply variance based measures to ‘design’ variables/factors.

Example: to make sure that missing data imputation does not affect the inference given the uncertainty in everything else.

How about MuSIASEM?

Or apply a different technique based on Monte Carlo filtering

Monte Carlo filtering



B = OK

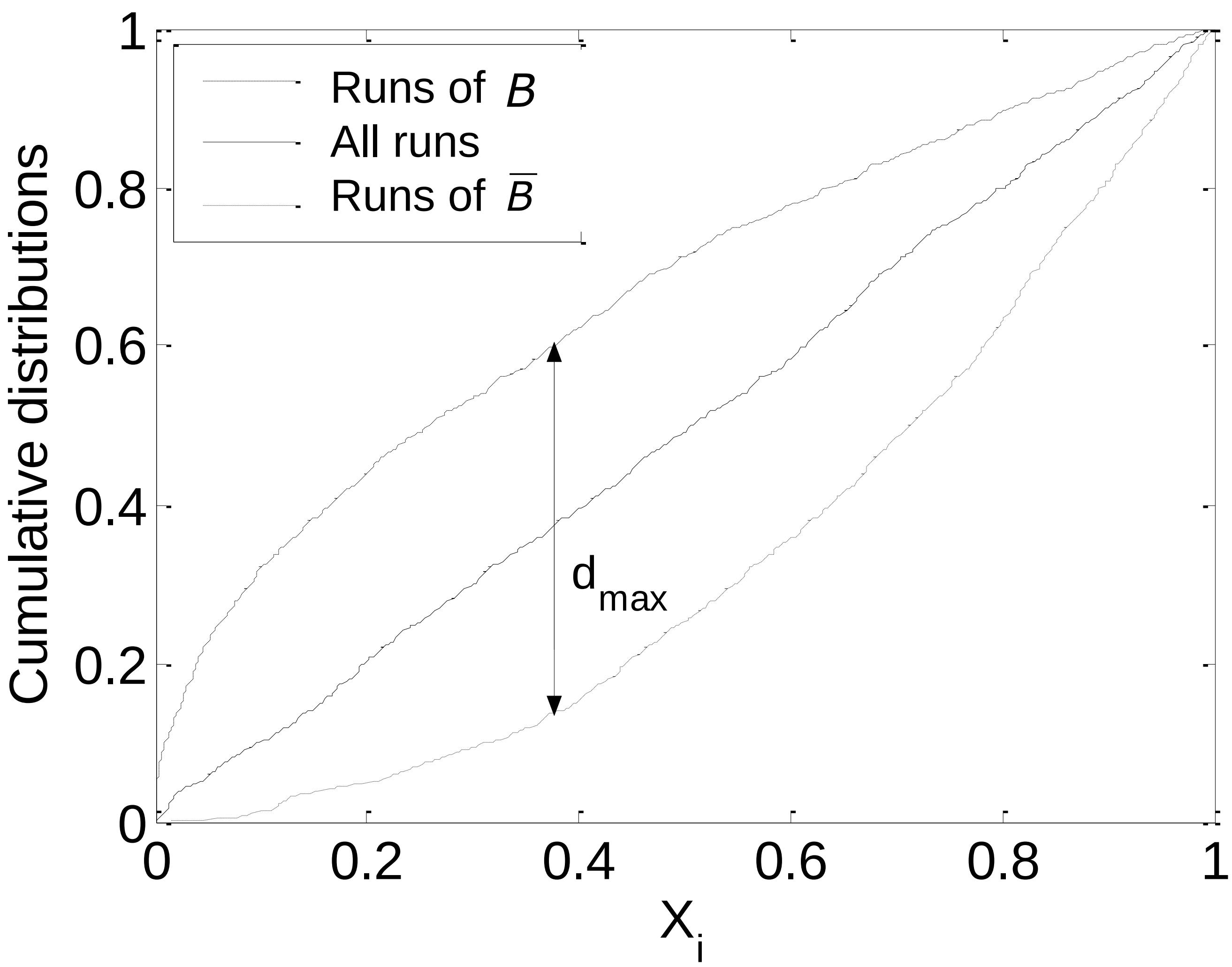
$\overline{B}$ = not OK

Monte Carlo filtering

Step by step:

- Classifying simulations as either B or $\bar{B}$. This allows distinguishing two sub-sets for each Xi: $(X_i | B)$ and $(X_i | \bar{B})$
- The Smirnov two-sample test (two-sided version) is performed for each factor independently, analyzing the maximum distance between the cumulative distributions of the B and $\bar{B}$ sets.

Monte Carlo filtering



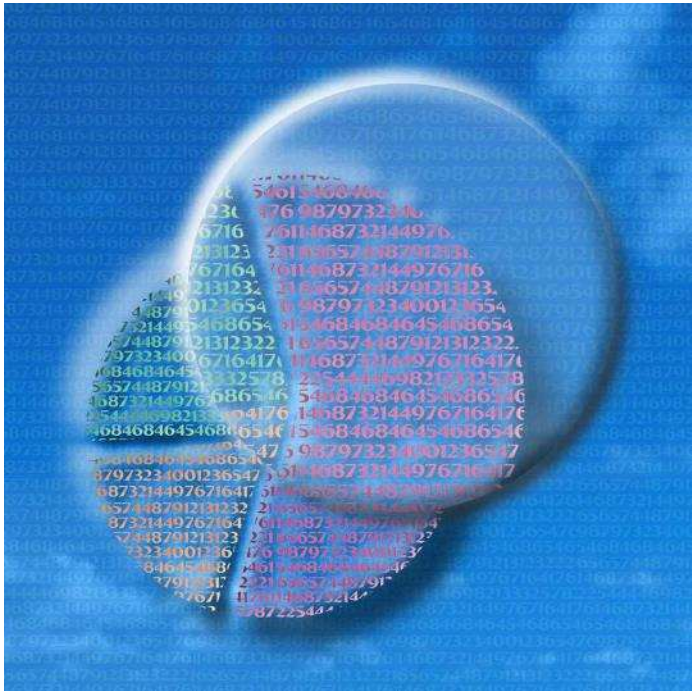
Next SAMO Conference: SAMO 2016 (Reunion, France)
November 30th December 3rd

Next?

SAMO 2019: Barcelona, Spain [proposal]

Sergei Kucherenko

Hospital de la Santa Creu i Sant Pau Barcelona, Catalonia, Spain, designed by the Catalan modernist architect Lluís Domènech i Montaner, built between 1901 and 1930 (Source Wikipedia).



END