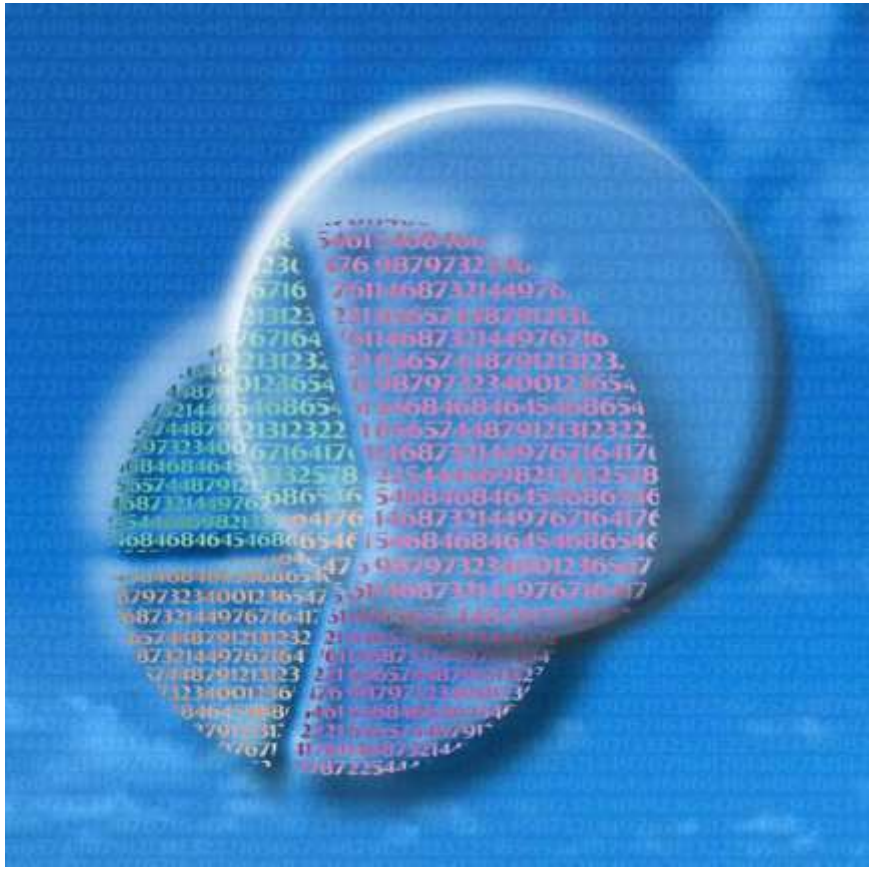


Sensitivity analysis: An introduction

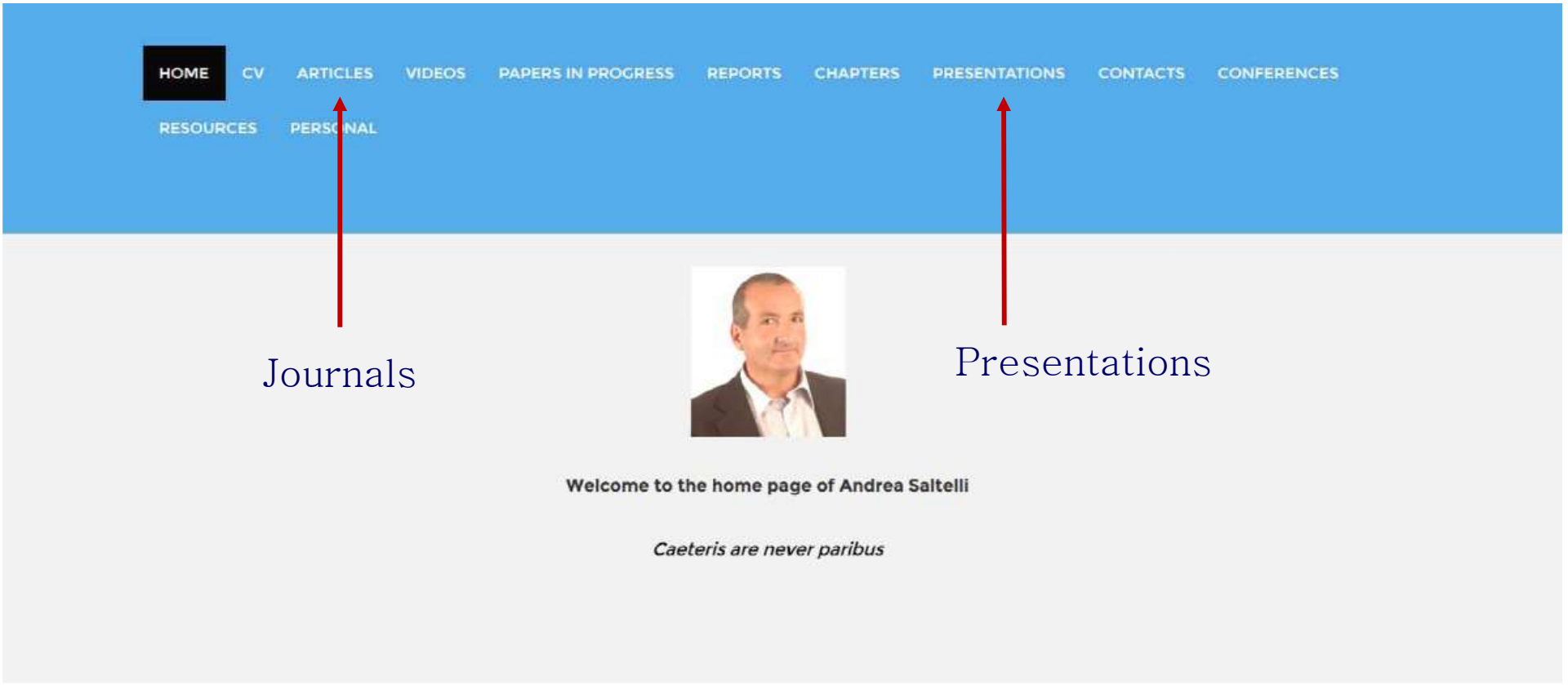
Andrea Saltelli
Centre for the Study of the Sciences and the Humanities (SVT) – University of Bergen (UIB)

Institut de Ciència i Tecnologia Ambientals (ICTA) – Universitat Autonoma de Barcelona (UAB)

Summer School on Sensitivity Analysis – SAMO 2016, Villa Orlandi, Anacapri – July 4-8, 2016

andrea.saltelli@uib.no
@andreasaltelli

www.andreasaltelli.eu



Where to find materials

andrea.saltelli@uib.no

sensitivity analysis,
sensitivity auditing,
science for policy,
impact assessment, ...

Welcome to the Ninth SAMO Summer school (previous schools in Venice, Florence and Ranco between 1999 and 2014)

The SAMO community

Many stories. OECD intercomparisons exercises between 1989 and 1993:

"Level E" on models and

"Level S" on Sensitivity analysis; some disagreement…

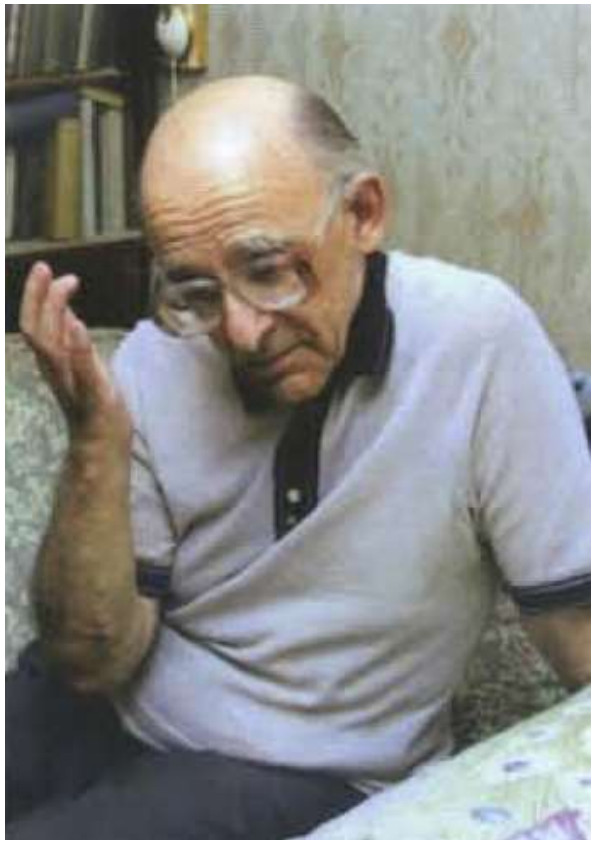
OECD (1989) OECD/NEA PSAC User group, *PSACOIN Level E intercomparison* (eds B. W. Goodwin, J. M. Laurens, J. E. Sinclair, D. A. Galson and E. Sartori). Nuclear Energy Agency, Organisation for Economic Co-operation and Development, Paris.

OECD (1993) OECD/NEA PSAG User group, *PSACOIN Level S intercomparison* (eds A. Alonso, P. Robinson, E. J. Bonano and D. A. Galson), Nuclear Energy Agency, Organisation for Economic Cooperation and Development, Paris.

… and friends

Enrico Sartori, John Helton, Tamas Turaniy, Toshimitsu Homma, Terry Andres, Roberto Pastres, Pedro Prado, Ilya M. Sobol', Sergei Kucherenko, Emanuele Borgonovo, Bertrand Iooss, Nathalie Saint Geours, Luc Pronzato, Clémentine Prieur, Bruno Sudret, Jeremy Oakley, Peter Young, Elmar Plischke, Thierry Mara, …

At the JRC: Stefano Tarantola, Francesca Campolongo, Paola Annoni, Beatrice d'Hombres, William Becker, Daniel Albrecht, Rossana Rosati, Federico Ferretti, …



Ilya Meyerovich Sobol'
See https://en.wikipedia.org/wiki/Ilya_M._Sobol

The SA community; a conference every three years

SAMO 1995 (Belgirate, Italy)
SAMO 1998 (Venice, Italy)
SAMO 2001 (Madrid, Spain)
SAMO 2004 (Santa Fe, USA)
SAMO 2007 (Budapest, Hungary)
SAMO 2010 (Milan, Italy)
SAMO 2013 (Nice, France)
SAMO 2016 (Reunion, France)

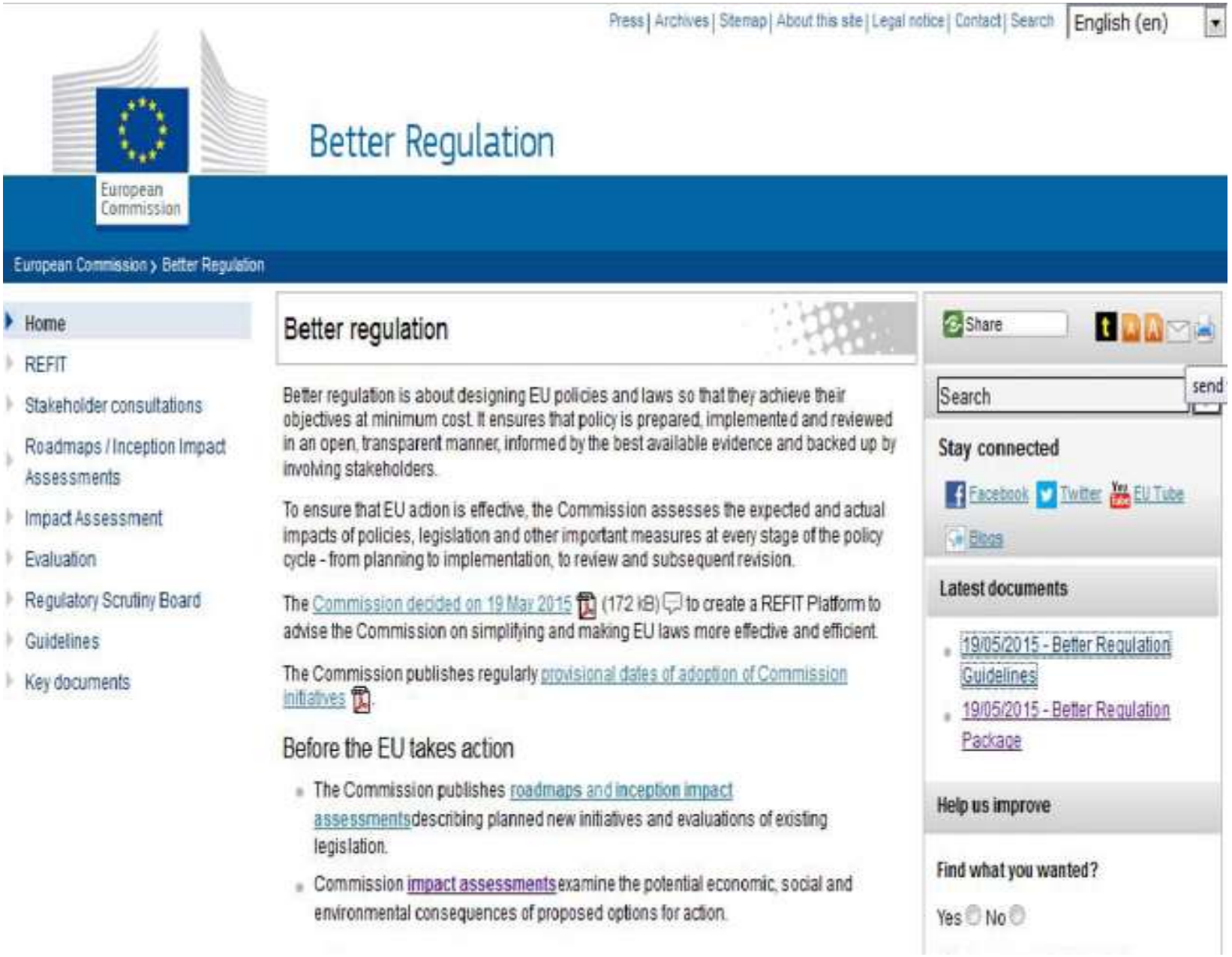
Next?

SAMO 2019: Barcelona, Spain [proposal]

Hospital de la Santa Creu i Sant Pau Barcelona, Catalonia, Spain, designed by the Catalan modernist architect Lluís Domènech i Montaner, built between 1901 and 1930 (Source Wikipedia).

Why sensitivity analysis

http://ec.europa.eu/smart-regulation/



http://ec.europa.eu/smart-regulation/guidelines/docs/br_toolbox_en.pdf

When testing the evidence behind inference some reasonable people suggest that ‘sensitivity analysis would help’

...

Edward E. Leamer, 1990, Let's Take the Con Out of Econometrics, American Economics Review, 73 (March 1983), 31-43.

<<I have proposed a form of organised sensitivity analysis that I call “global sensitivity analysis” in which a neighborhood of alternative assumptions is selected and the corresponding interval of inferences is identified.>>

Edward E. Leamer, 1990, Let's Take the Con Out of Econometrics, American Economics Review, 73 (March 1983), 31-43.

<<Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful.>>

Funtowicz & Ravetz's GIGO (Garbage In, Garbage Out) Science – or pseudo-science – "where uncertainties in inputs must be suppressed least outputs become indeterminate"

Leamer's 'Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful'.

Pseudo-science: from this old book by STS scholars Silvio Funtowicz & Jerome R. Ravetz's (STS=studies of science and technology)

Funtowicz, S. O. and Ravetz, J. R., 1990. Uncertainty and quality in science for policy. Dordrecht: Kluwer.

Back to Leamer:
With the ashes of the mathematical models used to rate mortgage-backed securities still smoldering on Wall Street, now is an ideal time to revisit the sensitivity issues.

Tantalus on the Road to Asymptopia
Edward E. Leamer, 2010 *Journal of Economic Perspectives*, **24**, (2), 31–46.

"… my observation of economists at work who routinely pass their data through the filters of many models and then choose a few results for reporting purposes."

Ibidem

"One reason these methods are rarely used is their honesty seems destructive;"

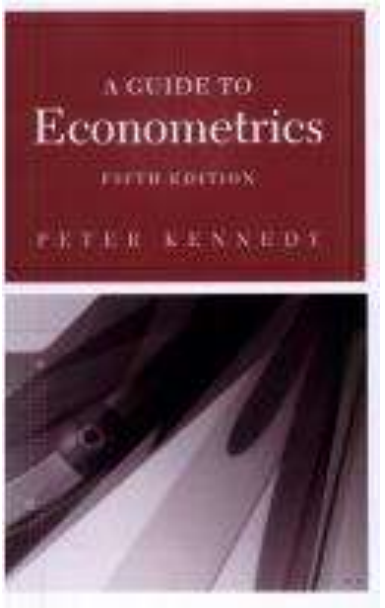
Ibidem

"or, to put it another way, a fanatical commitment to fanciful formal models is often needed to create the appearance of progress."

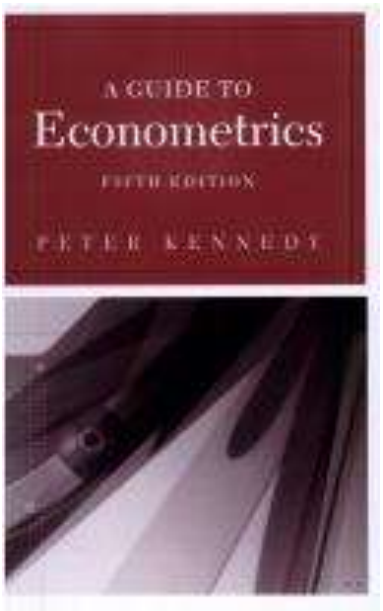
Ibidem

Peter Kennedy, A Guide to Econometrics.
Anticipating criticism by applying sensitivity analysis. This is one of the ten commandments of applied econometrics according to Peter Kennedy:

<<Thou shall confess in the presence of sensitivity.
Corollary: Thou shall anticipate criticism >>



<<When reporting a sensitivity analysis, researchers should explain fully their specification search so that the readers can judge for themselves how the results may have been affected. This is basically an `honesty is the best policy' approach, […]'.>>



Today: the ‘Mathiness’ discussion: blogs of Paul Romer, Judith Curry and Erik Reinert’s ‘scholasticism’ paper.

See https://paulromer.net/mathiness/

https://judithcurry.com/2015/08/12/the-adversarial-method-versus-feynman-integrity-2/

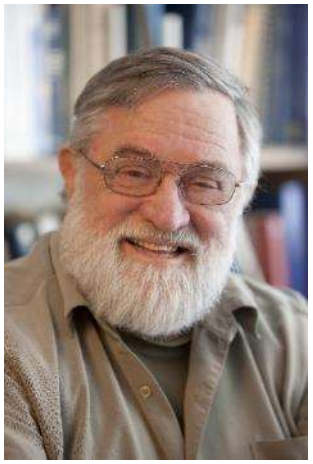
http://www.andreasaltelli.eu/file/repository/Full_Circle_scholasticism_2.pdf

Paul Romer

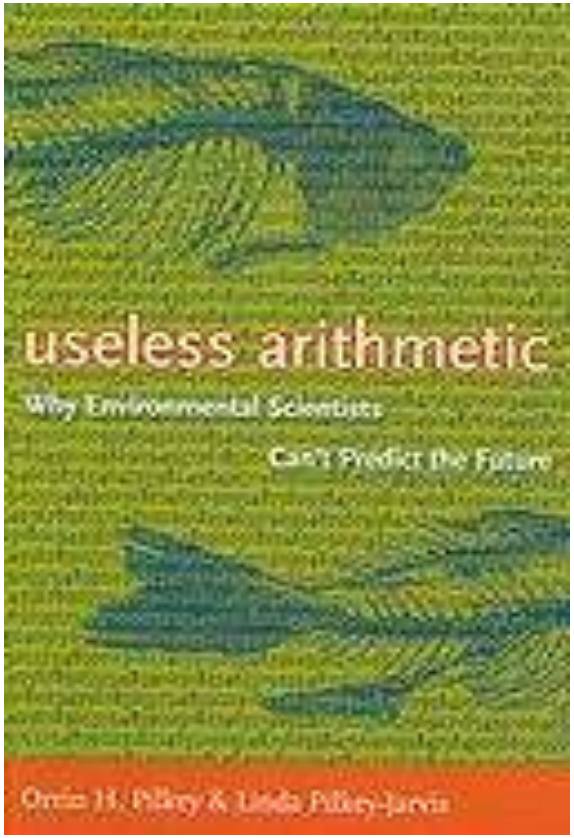
Judith Curry

Erik Reinert

Limits of sensitivity analysis



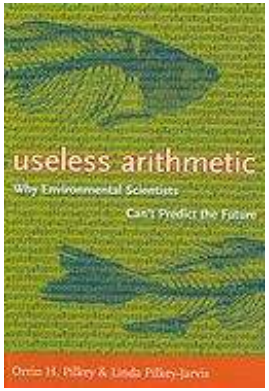
Orrin H. Pilkey
Duke University, NC



Useless Arithmetic: Why Environmental Scientists Can't Predict the Future

by Orrin H. Pilkey and Linda Pilkey-Jarvis

'Quantitative mathematical models used by policy makers and government administrators to form environmental policies are seriously flawed'

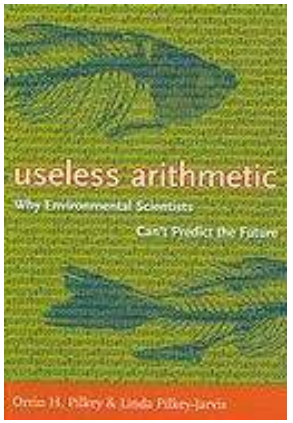


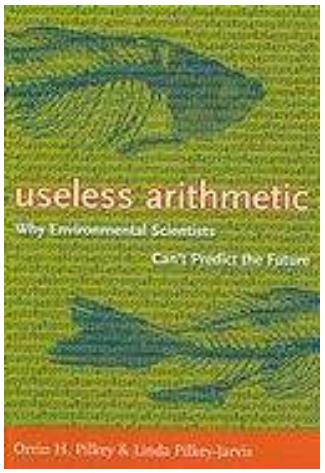
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

[…] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

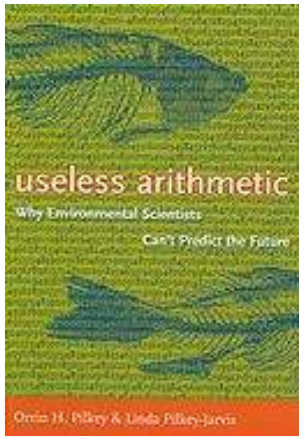
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

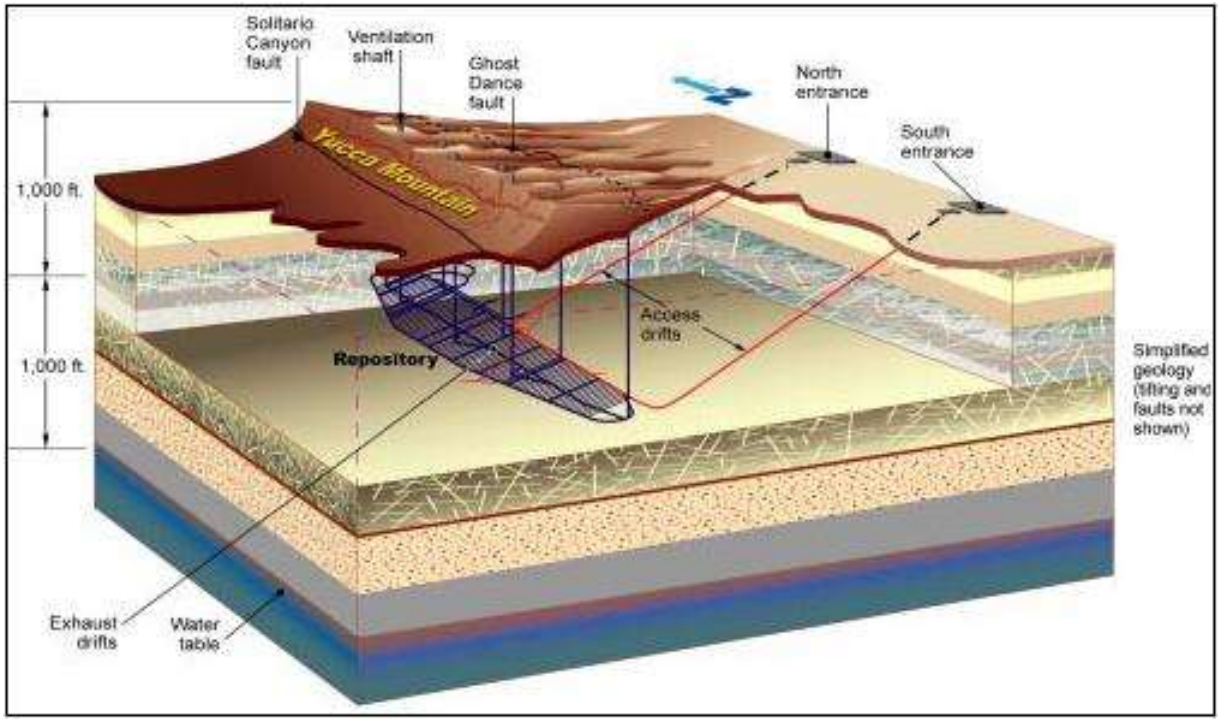
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.



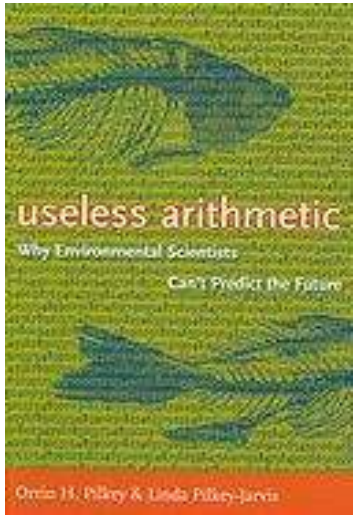


The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter (the ^{36}Cl story)

Type III error in sensitivity: Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→… SA useless if it is instead ~ 3,000 millimetres per year.



"Scientific mathematical modelling should involve constant efforts to falsify the model"

Ref. ➔ Robert K. Merton's 'Organized skepticism '

Communalism – the common ownership of scient40

ific discoveries, according to which scientists give up intellectual property rights in exchange for recognition and esteem (Merton actually used the term Communism, but had this notion of communalism in mind, not Marxism);

Universalism – according to which claims to truth are evaluated in terms of universal or impersonal criteria, and not on the basis of race, class, gender, religion, or nationality;

Disinterestedness – according to which scientists are rewarded for acting in ways that outwardly appear to be selfless;

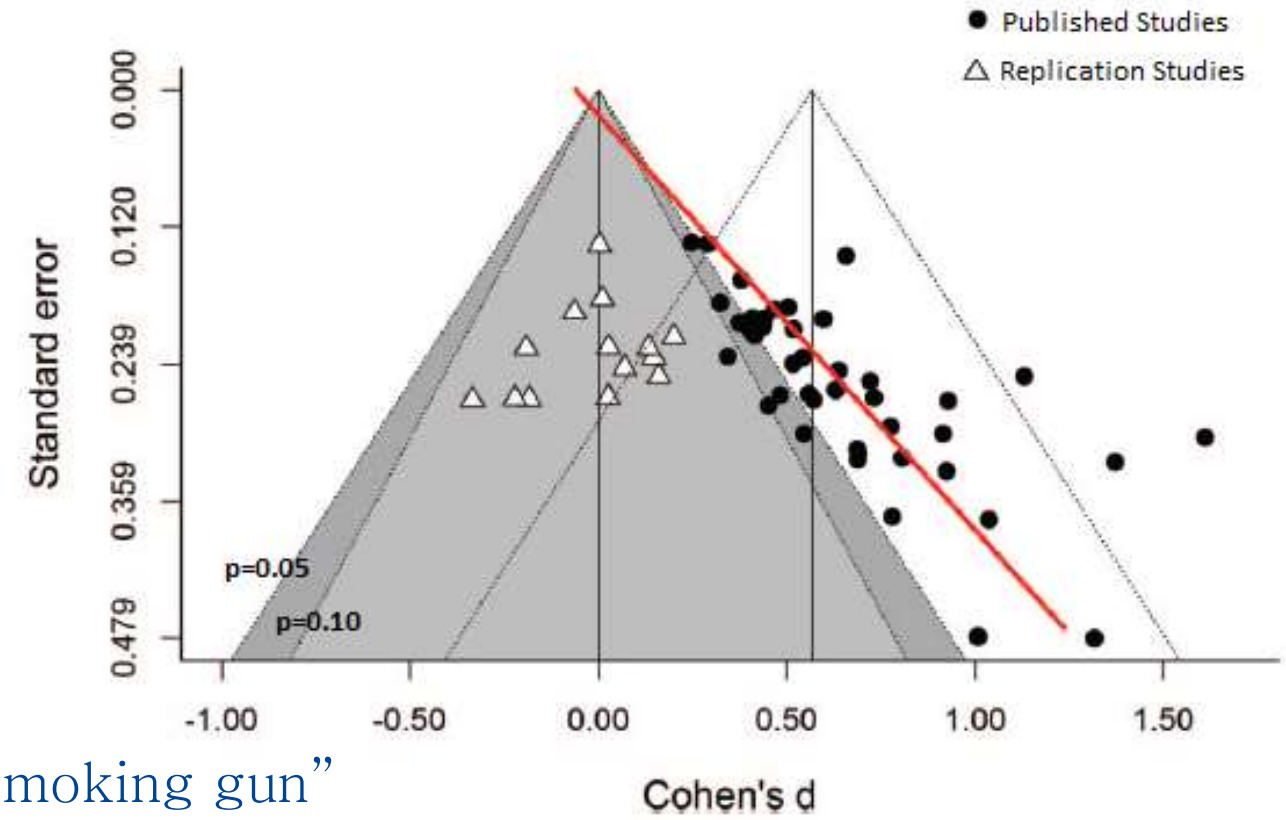
Organized Skepticism – all ideas must be tested and are subject to rigorous, structured community scrutiny.

Is this debate over? The reproducibility crisis in cancer research, organic chemistry, psychology, behavioural studies,… The p-values saga and its climax; the ASA statement and the 20 commentaries.

Wasserstein, R.L. and Lazar, N.A., 2016. 'The ASA's statement on p-values: context, process, and purpose', The American Statistician, DOI:10.1080/00031305.2016.1154108.

Misuse of the P value — a common test for judging the strength of scientific evidence — is contributing to the number of research findings that cannot be reproduced, the American Statistical Association (ASA) warned on 8 March.

Baker, M., 2016, Statisticians issue warning on P values, Nature, 531, 151.



"P-hacking's smoking gun"

Shanks et al. (2015) JEP:General

J Exp Psychol Gen. 2015 Oct 26. "Romance, Risk, and Replication: Can Consumer Choices and Risk-Taking Be Primed by Mating Motives?", Shanks DR, Vadillo MA, Riedel B, Clymo A, Govind S, Hickin N, Tamman AJ, Puhlmann LM.: http://www.ncbi.nlm.nih.gov/pubmed/26501730

Statistical and mathematical modelling are at the hearth of

- science for policy
- storm about malpractices.

New Scientists talks of "statistical sausage factory"

FEATURE 13 April 2016

Why so much science research is flawed – and what to do about it

Dodgy results are fuelling flawed policy decisions and undermining medical advances. They could even make us lose faith in science. **New Scientist** investigates

An alarming amount of research is flawed
Brett Ryder

LEADER 13 April 2016

Science isn't as solid as it should be – but science can fix it

Unconscious biases and data-torturing are weakening our knowledge base – but unlike politicians and bankers, scientists aren't covering up their failings

New Scientist

New Scientist

Published 22 June 2016

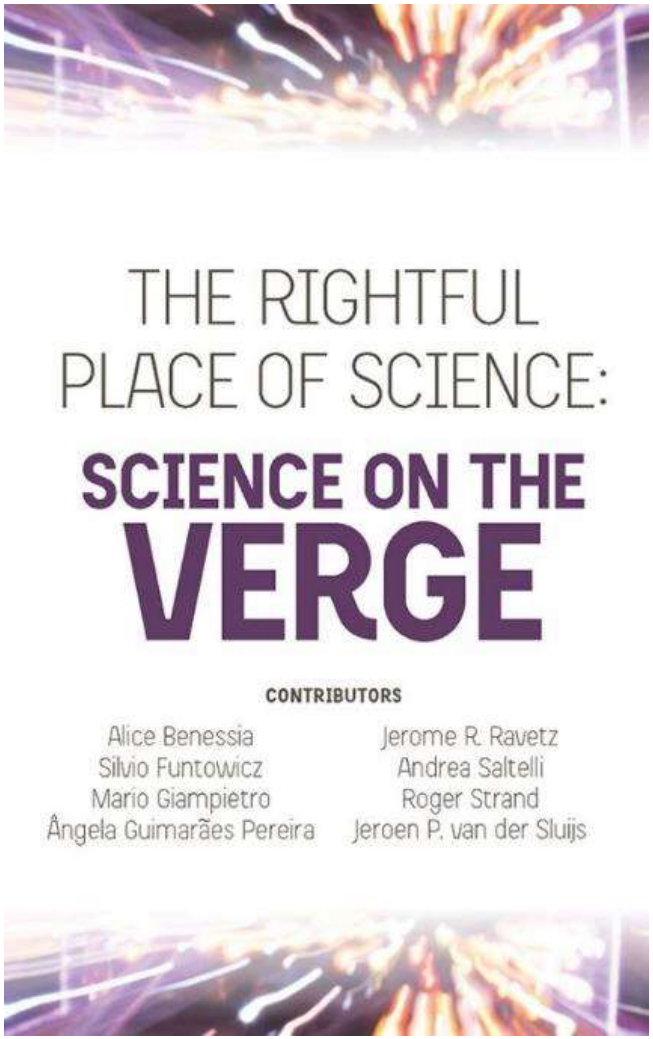
A new community for science

From Andrea Saltelli, Jerome R. Ravetz and Silvio Funtowicz

We would like to complement your analysis of a crisis in science relating to studies that can't be replicated (16 April, p 5 and p 38). One of us, Jerome Ravetz, predicted in 1971 in his book *Scientific* Knowledge and its Social Problems that the system of internal quality control of science would not easily withstand the evolution toward big science.

Quality in science depends on the existence of a community of scholars linked by norms and standards, and willing to stand by these. The historian Philip Mirowski in Science-Mart (2011), fills in the blanks of Ravetz's analysis with details of how science's internal quality control system stalled when "market" replaced "community" as a unifying principle, driven by firms funding research.

https://www.newscientist.com/letter/mg23030791-600-7-a-new-community-for-science/



The Rightful Place of Science: Science on the Verge

Paperback – 20 Feb 2016

by Andrea Saltelli (Author), Alice Benessia (Author), & 7 more

1 customer review

See all formats and editions

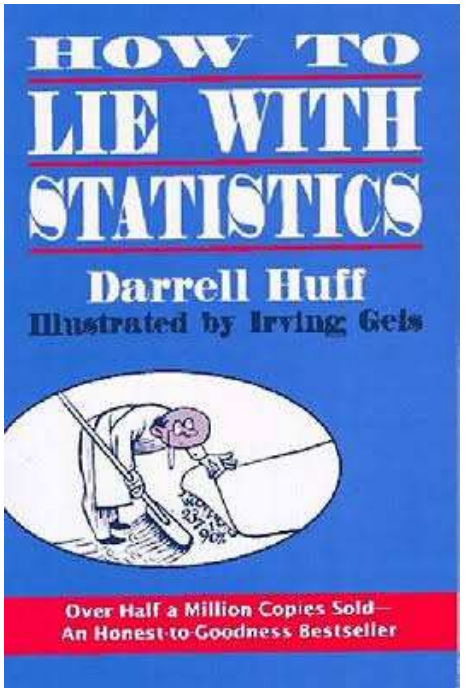
Kindle Edition	Paperback
£3.61	£6.99

http://www.amazon.com/Rightful-Place-Science-Verge/dp/0692596380/ref=sr_1_1?s=books&ie=UTF8&qid=1456255907&sr=1-1&keywords=saltelli

http://www.andreasaltelli.eu/science-on-the-verge

Will any sensitivity analysis do the job?

Can I lie with sensitivity analysis as I can lie with statistics?



Saltelli, A., Annoni P., 2010, How to avoid a perfunctory sensitivity analysis, *Environmental Modeling and Software,* 25, 1508-1517.

From: Saltelli, A., Annoni P., 2010
How to avoid a perfunctory sensitivity analysis,
Environmental Modeling and Software, **25**, 1508-1517.

What do these have in common?

J. Campbell, *et al.*, *Science* **322**, 1085 (2008).
R. Bailis, M. Ezzati, D. Kammen, *Science* **308**, 98 (2005).
E. Stites, P. Trampont, Z. Ma, K. Ravichandran, *Science* **318**, 463 (2007).
J. Murphy, et al., *Nature* **430**, 768–772 (2004).
J. Coggan, *et al.*, *Science* **309**, 446 (2005).

OAT

OAT methods – derivatives – local

$$S_1 = \frac{\partial Y}{\partial x_j}$$

Effect on Y of perturbing x_j around its nominal value x_j^0

$$S_2 = \frac{\partial Y}{\partial x_j}\frac{x_j^0}{Y^0}$$

Relative effect on Y of perturbing x_j by a fixed fraction of its nominal value x_j^0

$$S_3 = \frac{\partial Y}{\partial x_j}\frac{std(x_j)}{std(Y)}$$

Relative effect on Y of perturbing x_j by a fixed fraction of its standard deviation

Why not just changing one factor at a time (OAT)?

•OMB A4: Use a numerical sensitivity analysis to examine how the results of your analysis vary with plausible changes in assumptions, choices of input data, and alternative analytical approaches. Sensitivity analysis is especially valuable when the information is lacking to carry out a formal probabilistic simulation. Sensitivity analysis can be used to find 'switch points' -- critical parameter values at which estimated net benefits change sign or the low cost alternative switches. Sensitivity analysis usually proceeds by changing one variable or assumption at a time, but it can also be done by varying a combination of variables simultaneously to learn more about the robustness of your results to widespread changes. Again, however, major rules above the $1 billion annual threshold require a formal treatment.

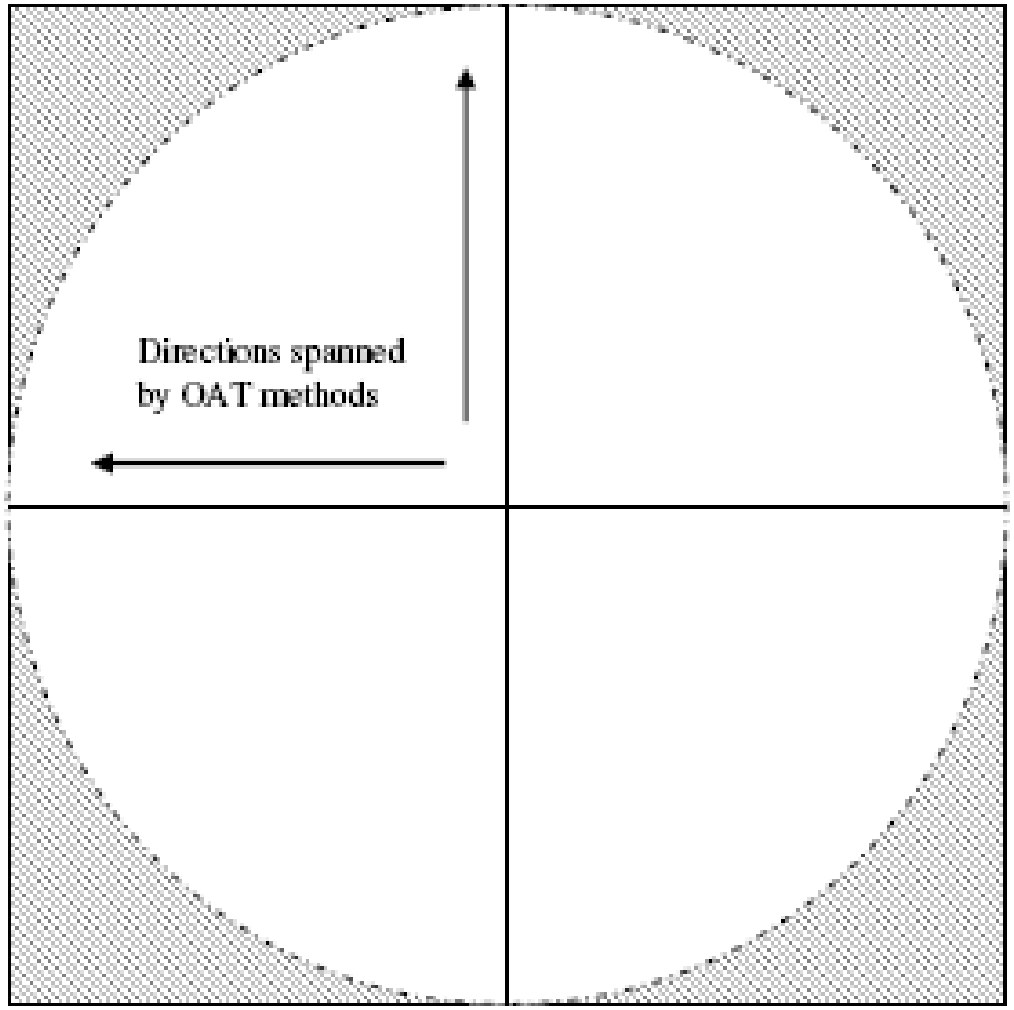
http://www.whitehouse.gov/omb/circulars_a004_a-4/ 2003

Why not just changing one factor at a time (OAT)?

•**OMB A4**: […]Sensitivity analysis usually proceeds by changing one variable or assumption at a time, but it can also be done by varying a combination of variables simultaneously to learn more about the robustness of your results to widespread changes.

http://www.whitehouse.gov/omb/circulars_a004_a-4/ 2003

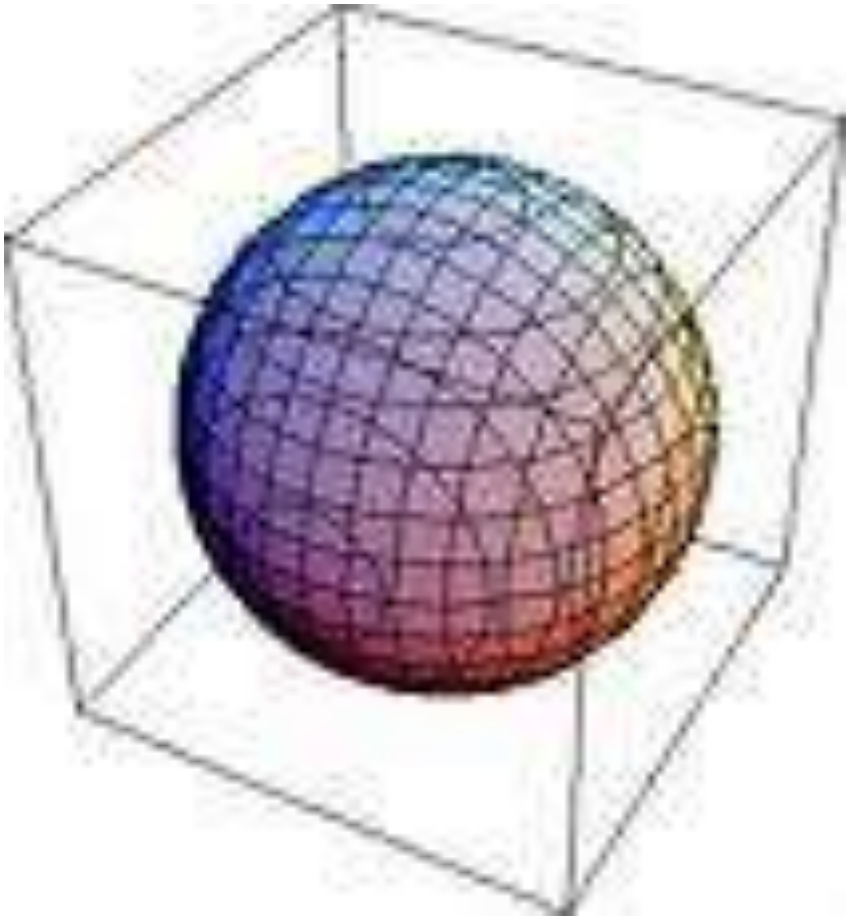
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions



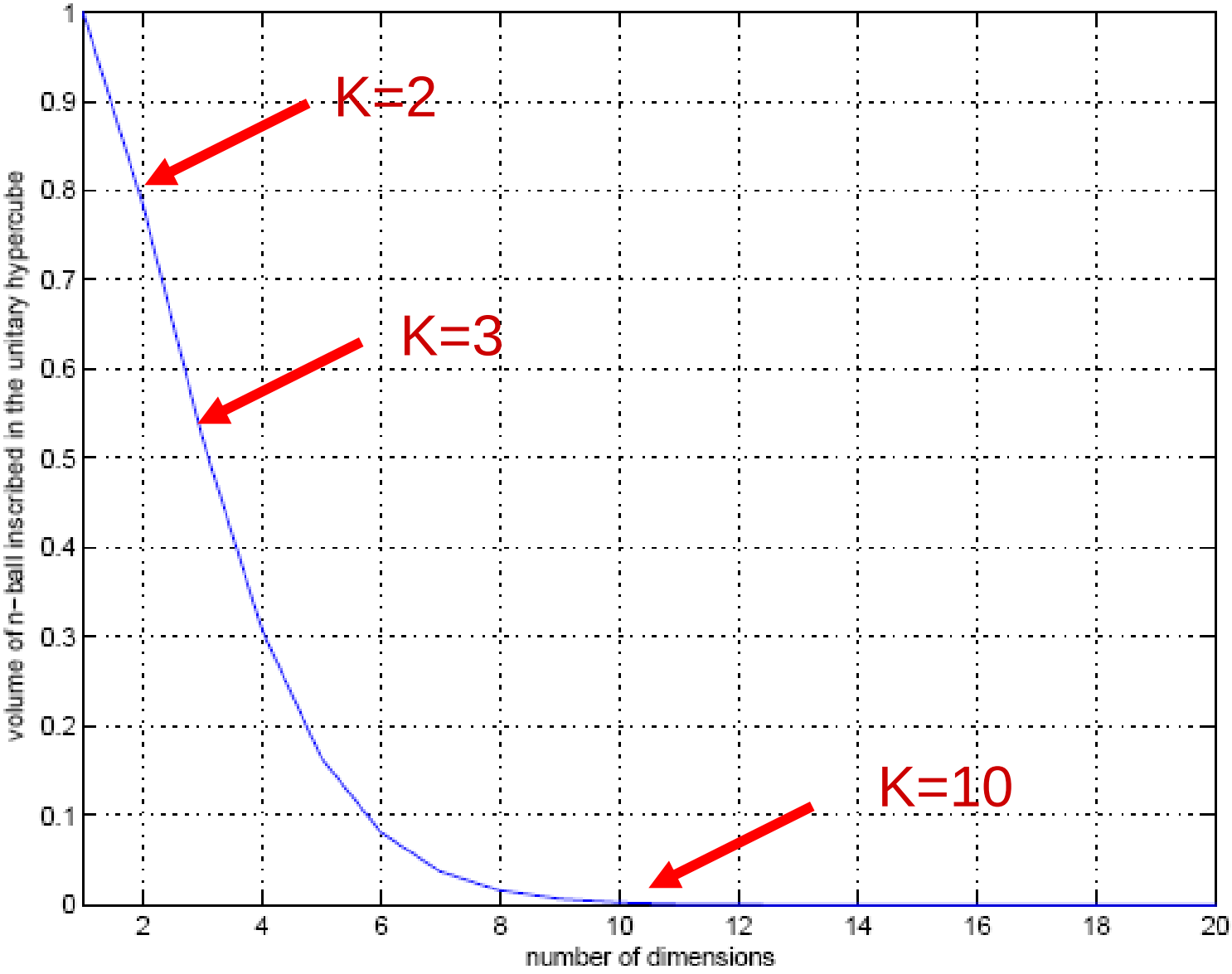
Volume sphere / volume cube =?

~ 1/2

OAT in 10 dimensions

Volume hypersphere / volume ten dimensional hypercube ~ 0.0025

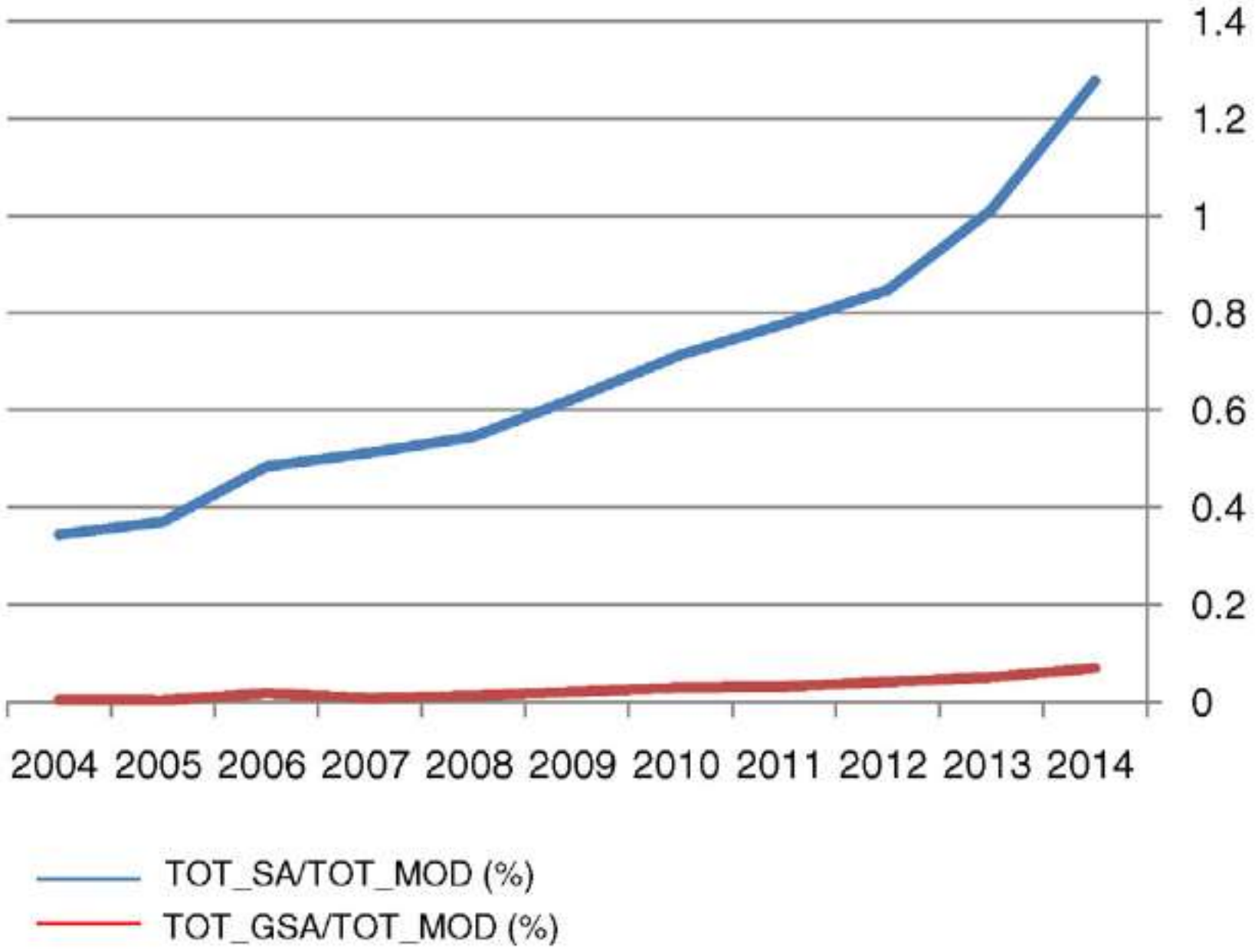
OAT in k dimensions



How are we doing in 2016?

…OAT is still the most largely used technique in SA, … clear increase in the use of GSA with preference for regression and variance-based techniques.

Ferretti, F., Saltelli A., Tarantola, S., 2016, Trends in Sensitivity Analysis practice in the last decade, Science of the Total Environment, http://dx.doi.org/10.1016/j.scitotenv.2016.02.133


1.4
1.2
1
0.8
0.6
0.4
0.2
0
2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014
TOT_SA/TOT_MOD (%)
TOT_GSA/TOT_MOD (%)

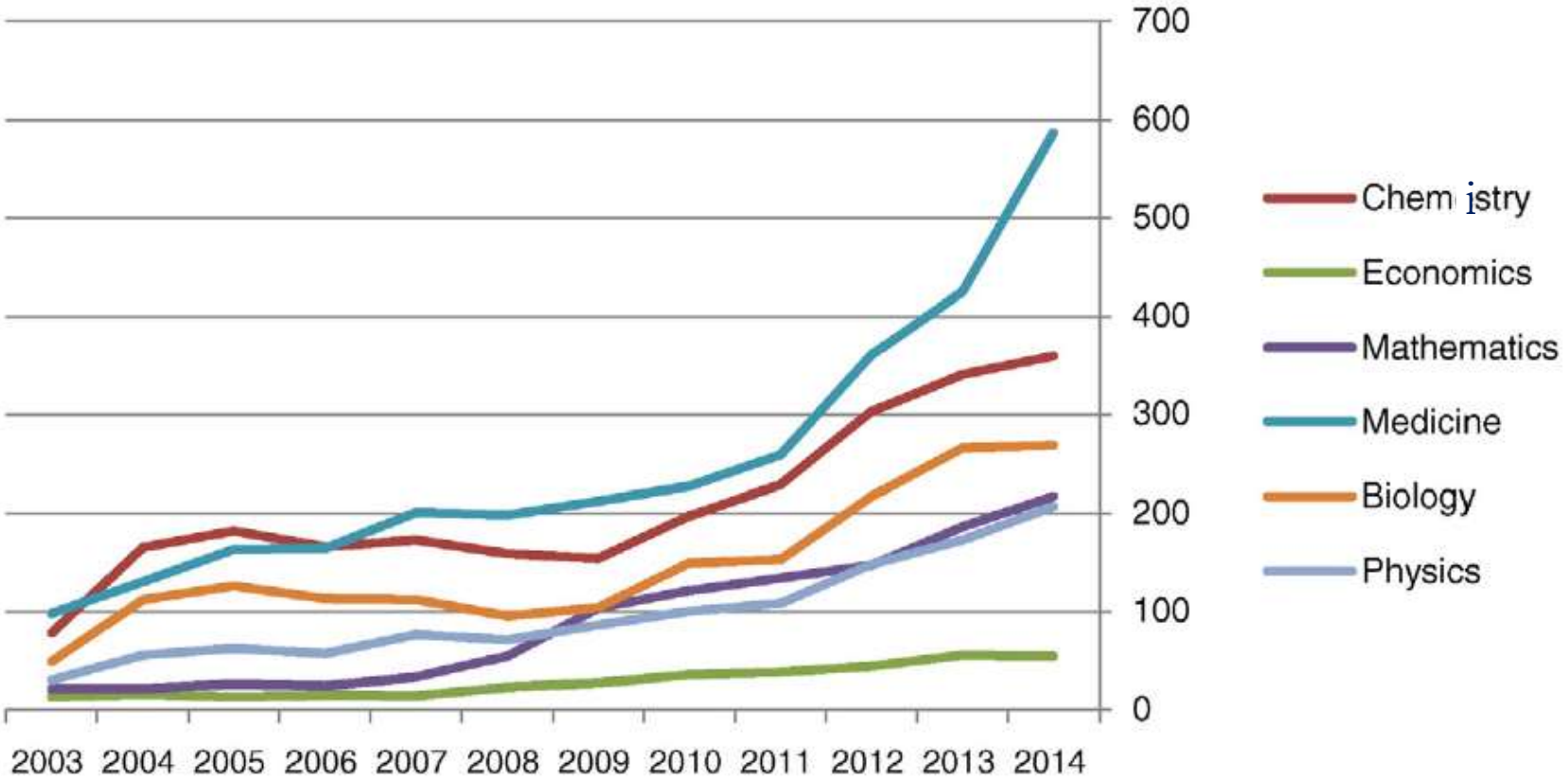


Fig. 4. GSA in the different scientific domains.

Definition of uncertainty and sensitivity analysis.

Sensitivity analysis: The study of the relative importance of different input factors on the model output.

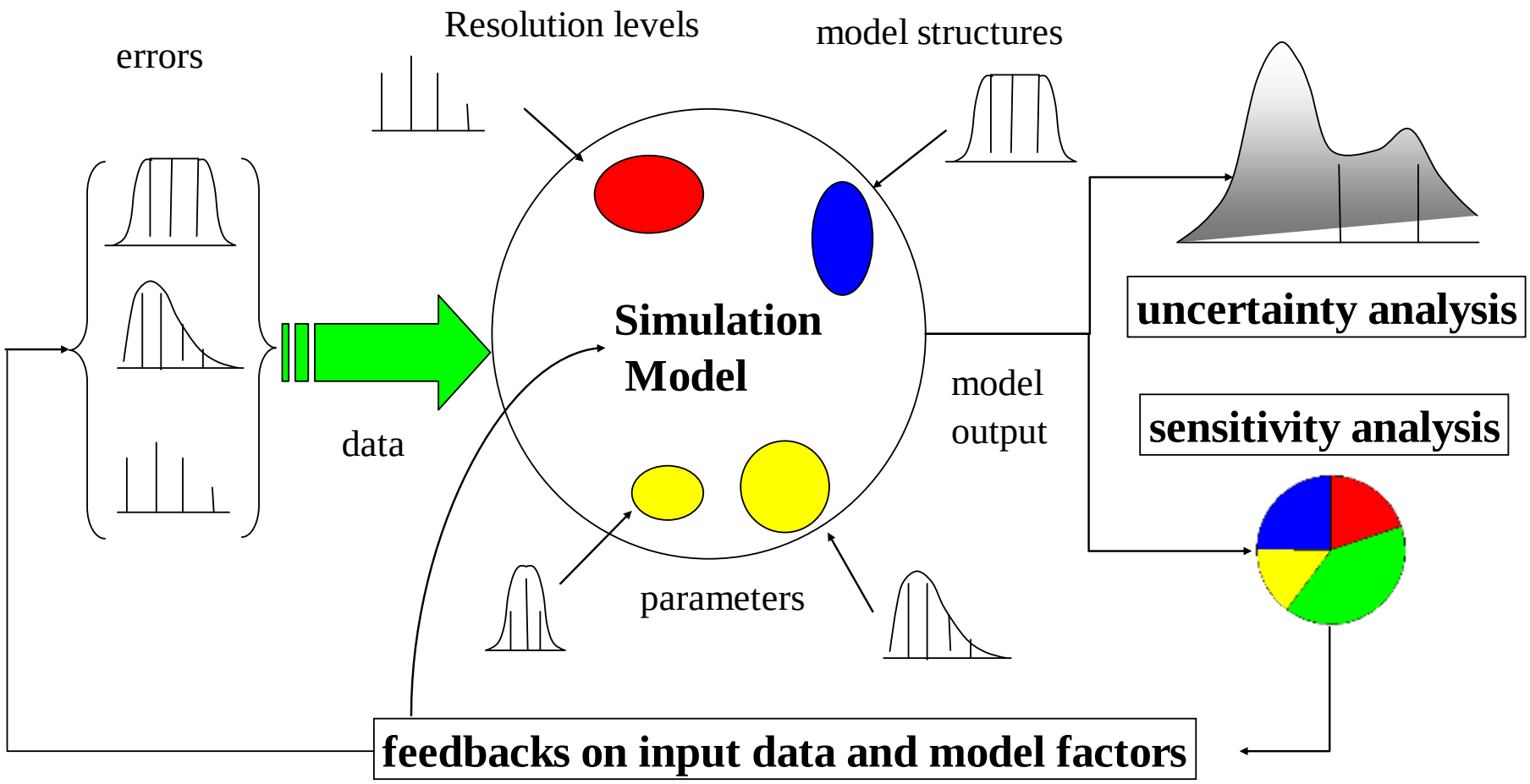
Uncertainty analysis: Focuses on just quantifying the uncertainty in model output.

[Global*] sensitivity analysis: “The study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input”

Saltelli A., 2002, Sensitivity Analysis for Importance Assessment, Risk Analysis, 22 (3), 1-12.

- Modelling in a Monte Carlo framework using quasi MC-points
- All uncertainties activated simultaneously; uncertainty and sensitivity together

An engineer's vision of UA, SA



One can sample more than just factors …

Using triggers one can sample modelling assumptions …

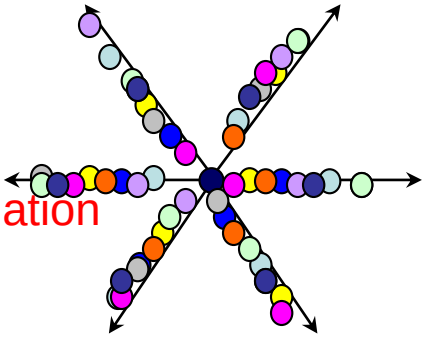
Example: Y is a composite indicator

Assumption	**Alternatives**
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

Space of alternatives

Weights

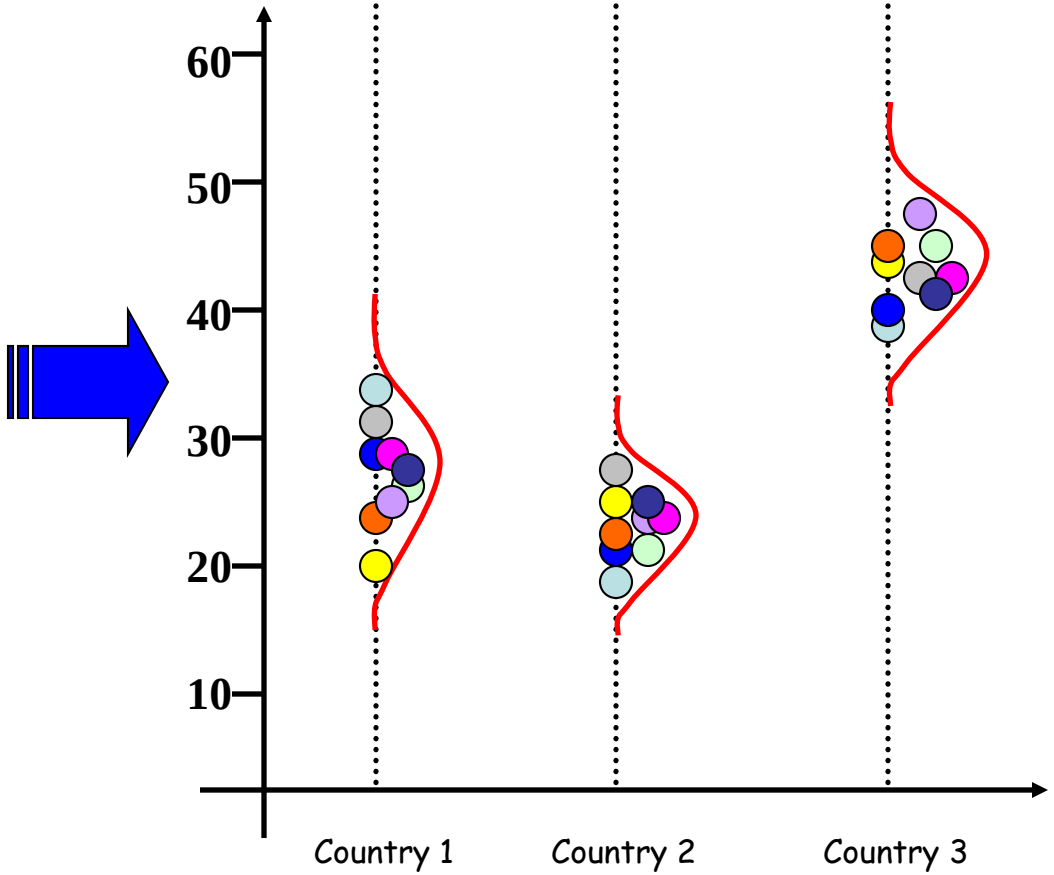
Missing data



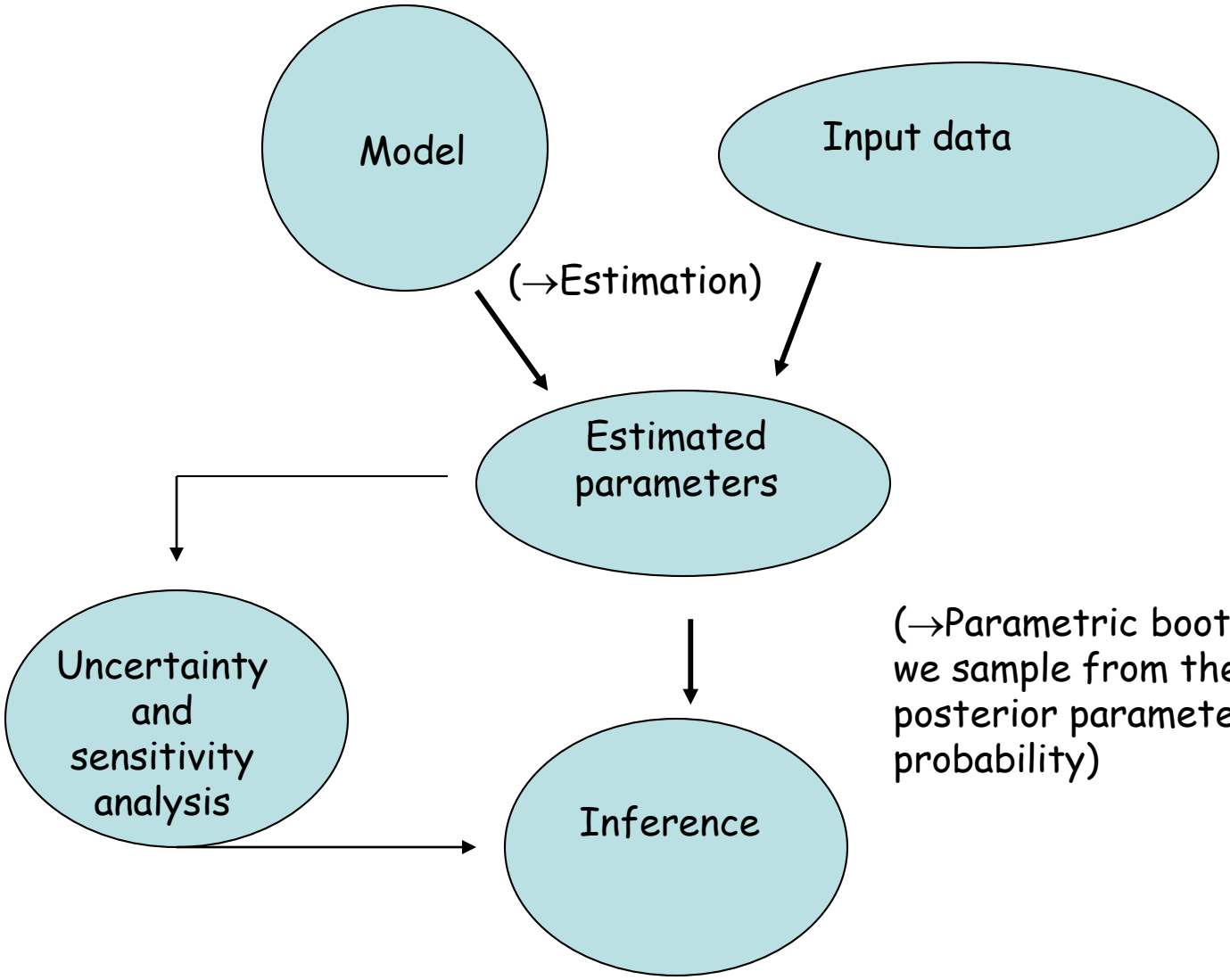
Aggregation

Including/ excluding variables

Normalisation



Models maps assumptions onto inferences … (Parametric bootstrap version of UA/SA)



Sample matrix for parametric bootstrap.

$$\begin{matrix} x_{11} & x_{12} \cdots & x_{1k} \\ x_{21} & x_{21} \cdots & x_{21} \\ \cdots & \cdots & \cdots \\ x_{N1} & x_{N1} & x_{N1} \end{matrix}$$

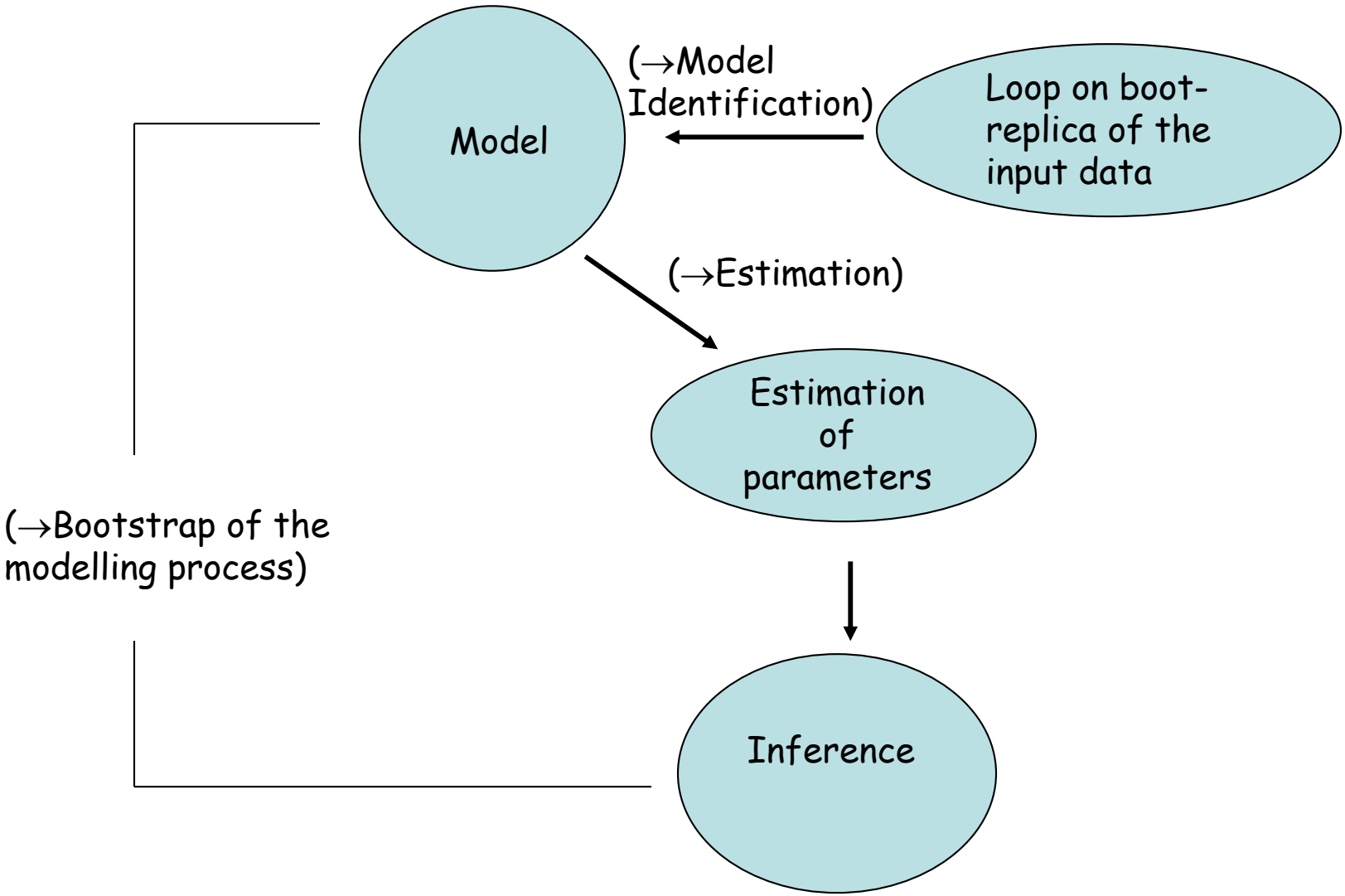
Each row is a sample trial for one model run. Each column is a sample of size N from the marginal distribution of the parameters as generated by the estimation procedure.

Model results:

Each row is the error-free result of the model run.

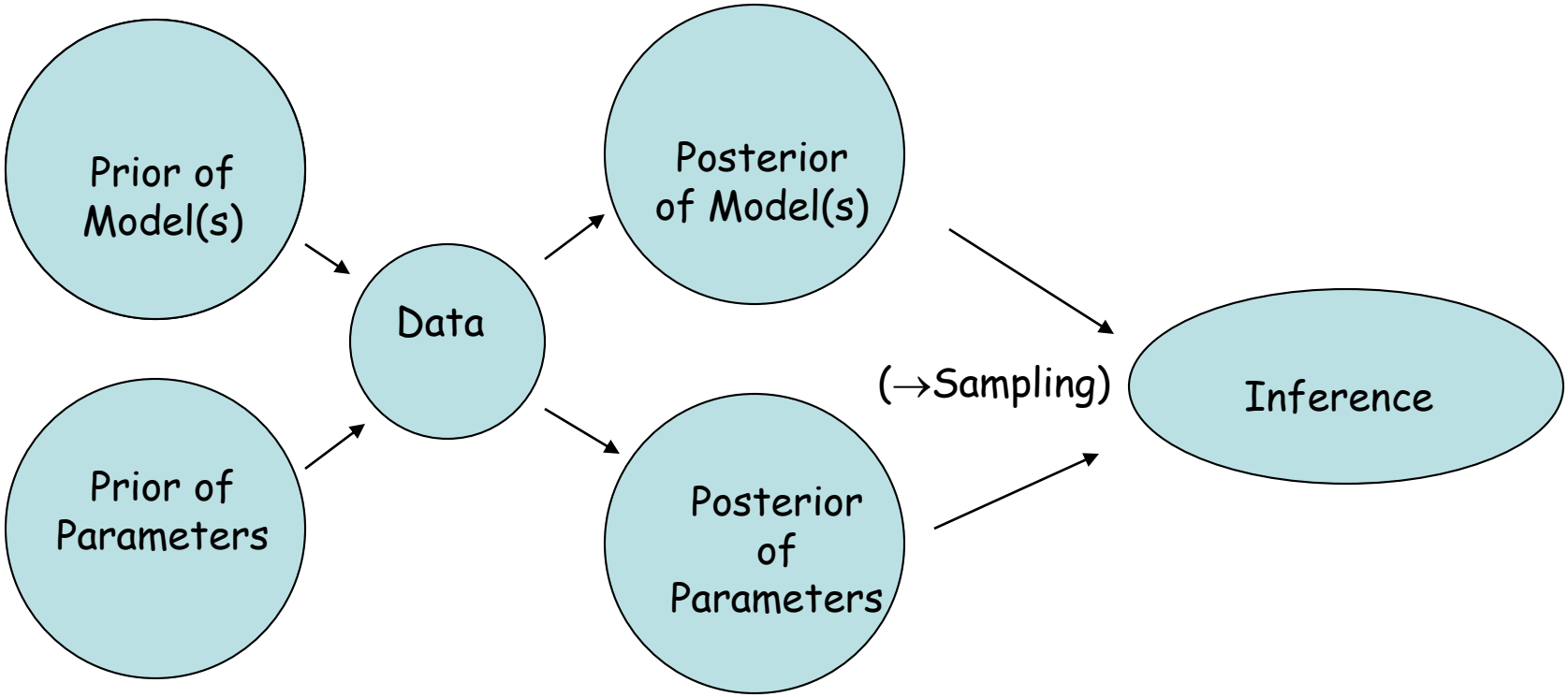
$$y_1$$
$$y_2$$
$$\dots$$
$$y_N$$

Bootstrapping-of-the-modelling-process

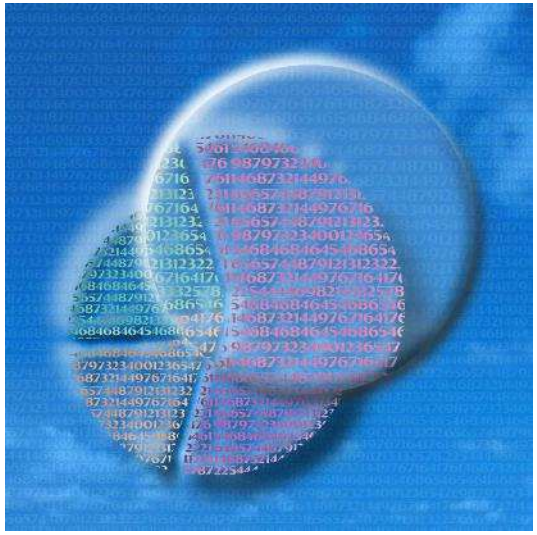


Chatfield, C., 1995, Model Uncertainty, Data Mining and Statistical Inference, Journal of the Royal Statistical Society. Series A (Statistics in Society), 158, No. 3, 419-466.

Bayesian Model Averaging



Hoeting, J.A., Madigan, D., Raftery, A.E. and Volinsky, C.T., 1999, Bayesian Model Averaging: A Tutorial Statistical Science, 1999, Vol. 14, No. 4, 382–417



Our preferred methods for SA: variance based

Variance based methods' best formalization is based on the work of Ilya M. Sobol' (1990), who extended the work of R.I. Cukier (1973).

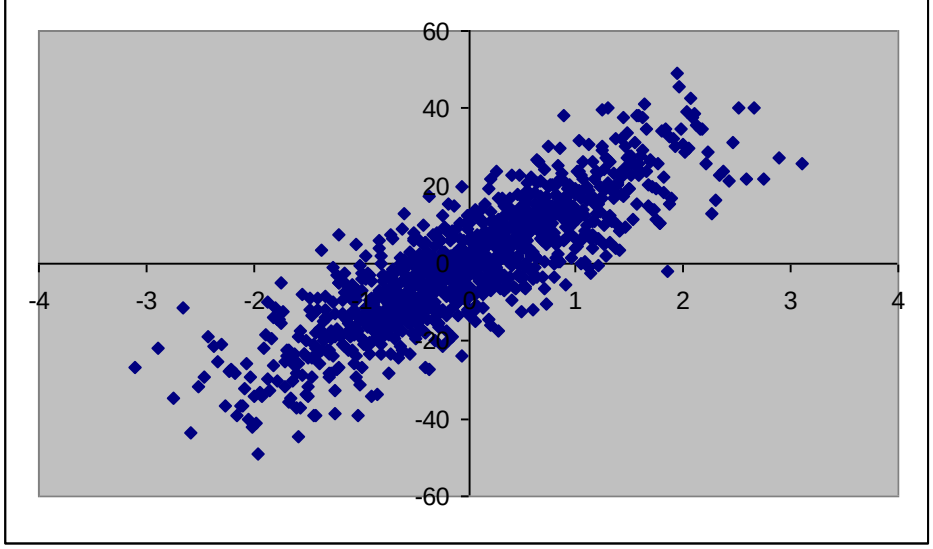
Total sensitivity indices by T. Homma and myself (1996).

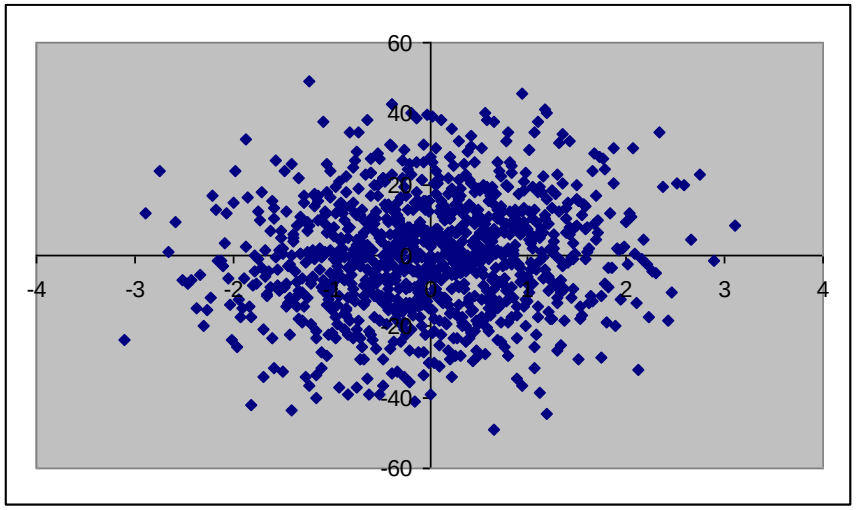
Today a rich literature and many investigators on the topic.

An intuitive derivation of sensitivity indices

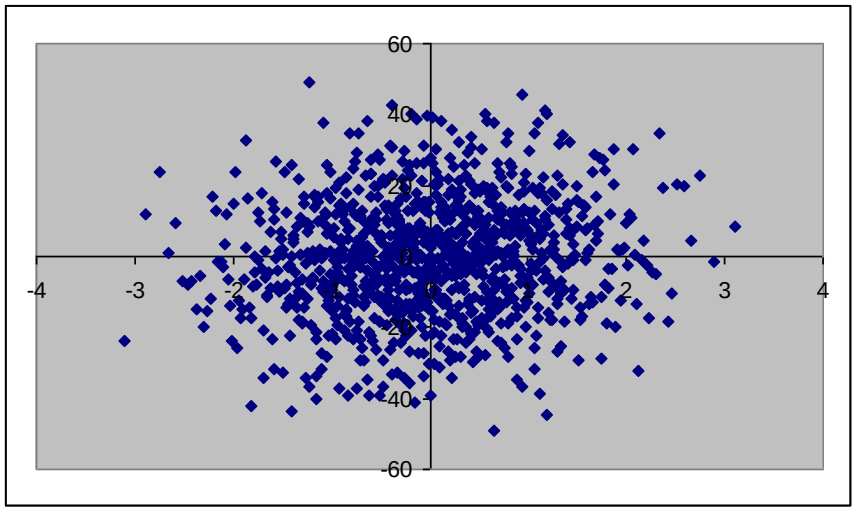
$$\begin{matrix} x_{11} & x_{12} \cdots & x_{1k} \\ x_{21} & x_{21} \cdots & x_{21} \\ \cdots & \cdots & \cdots \\ x_{N1} & x_{N1} & x_{N1} \end{matrix}$$

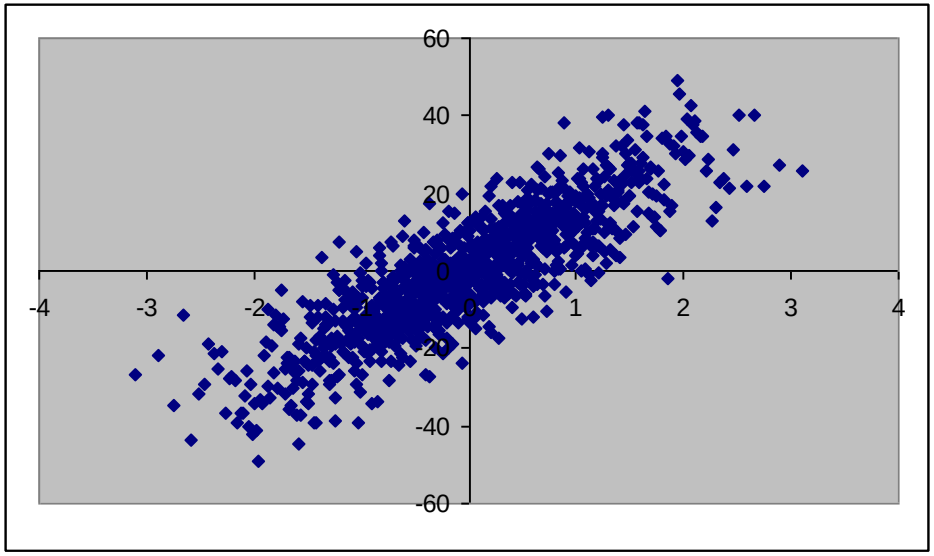
$$\begin{matrix} y_1 \\ y_2 \\ \cdots \\ y_N \end{matrix}$$





Scatterplots of y versus sorted factors

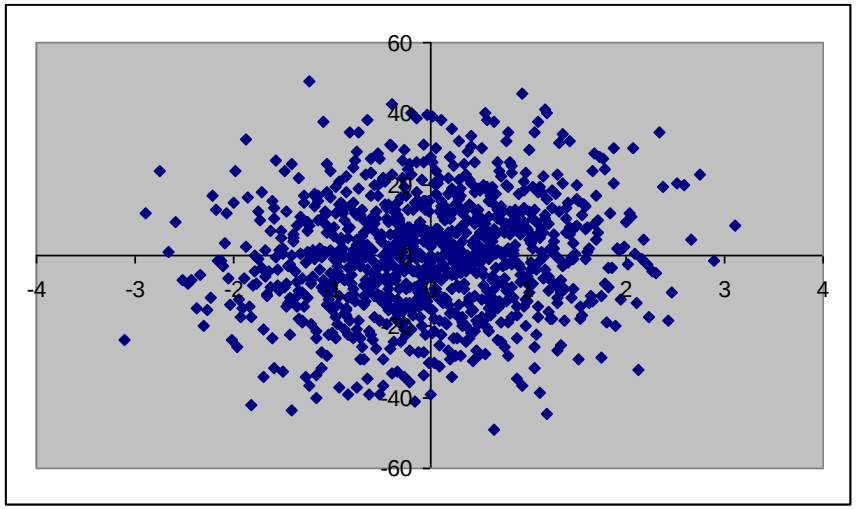


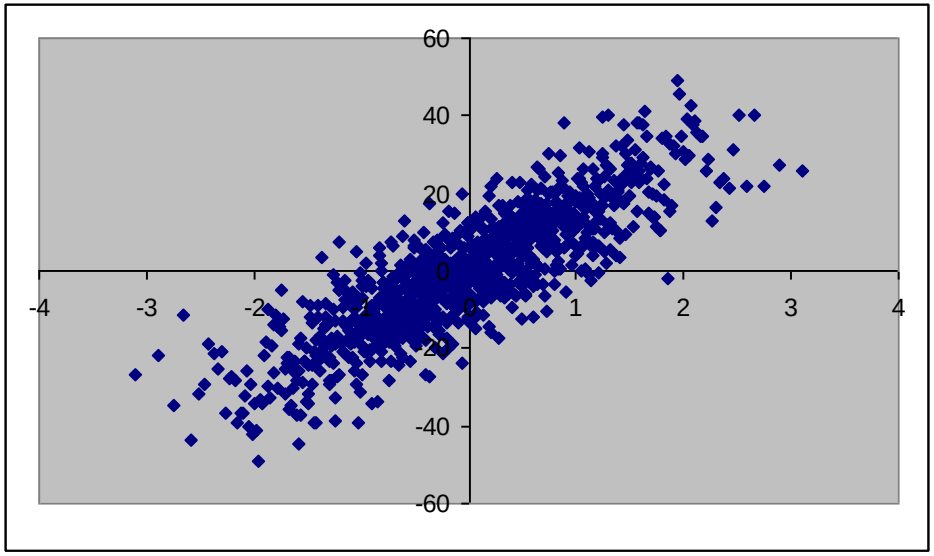


The ordinate axis is always Y

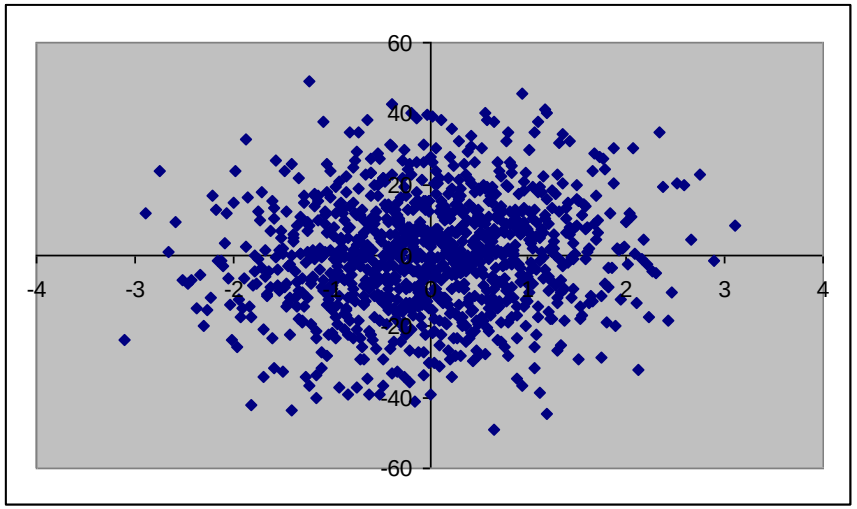
The abscissa are the various factors X_i in turn.

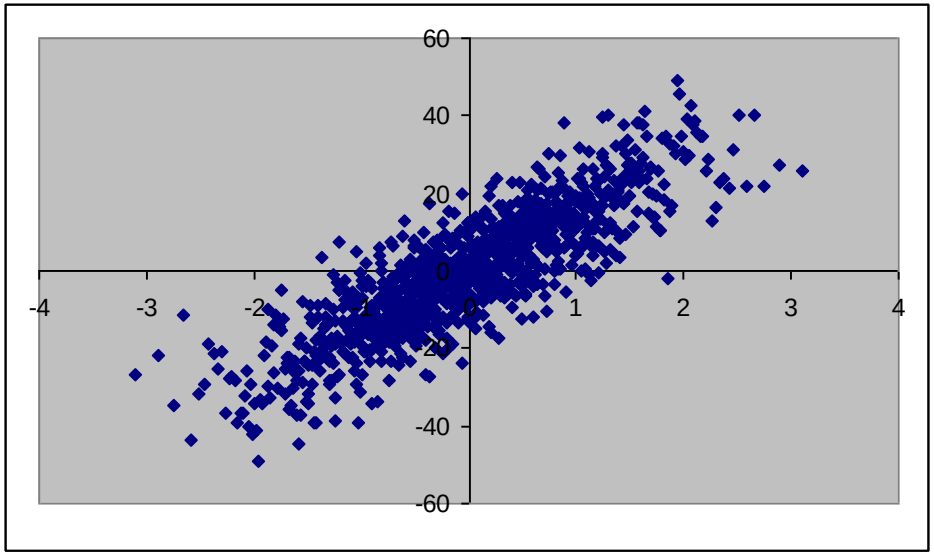
The points are always the same





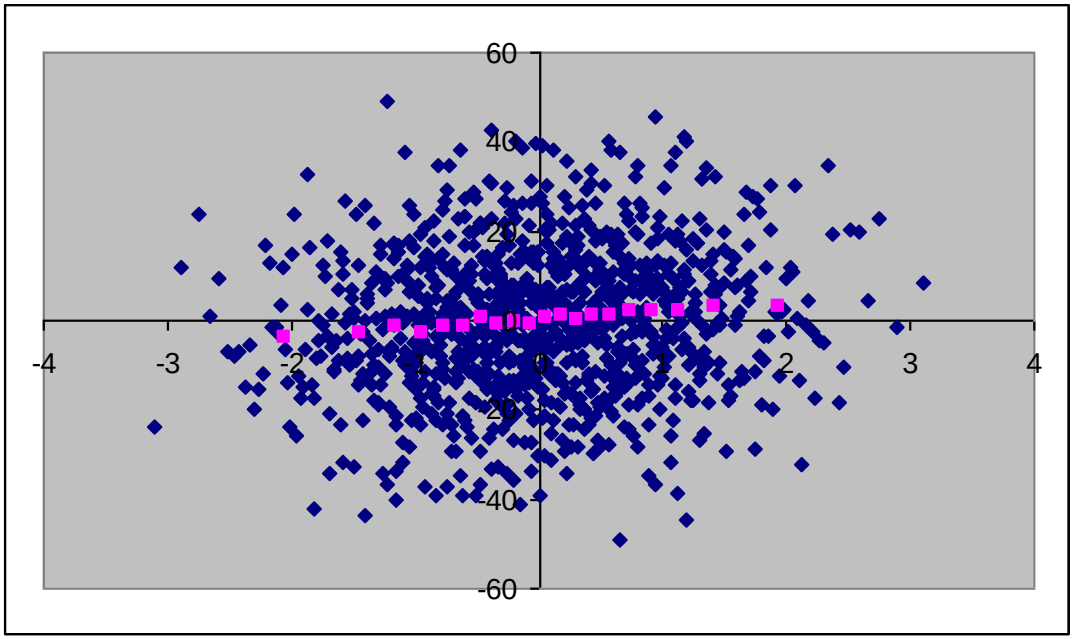
Which factor is more important?

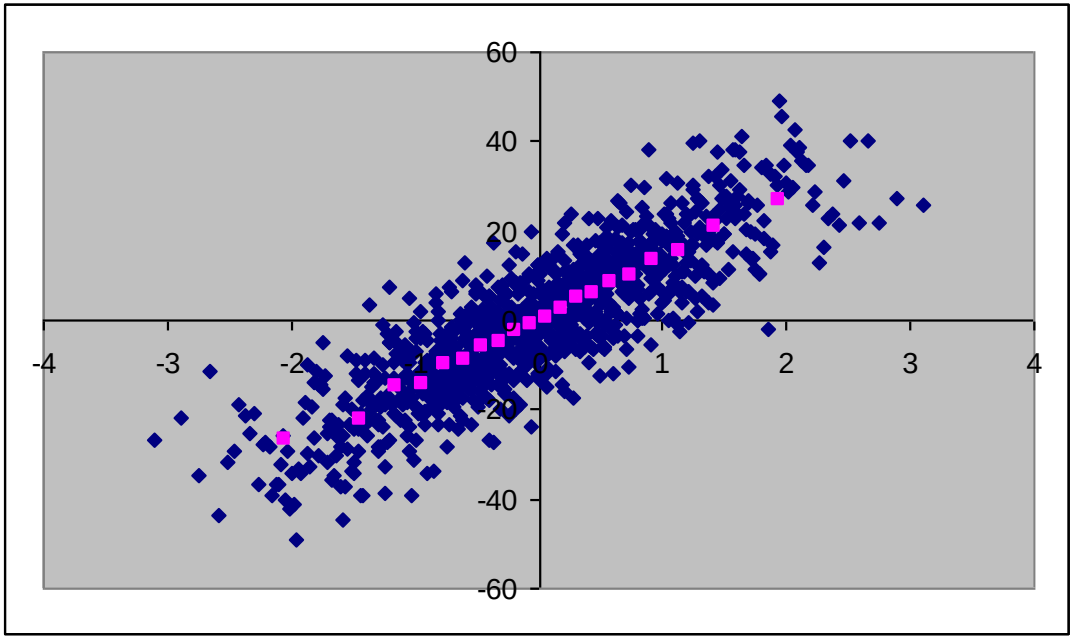




These are ~1,000 points

Divide them in 20 bins of ~ 50 points

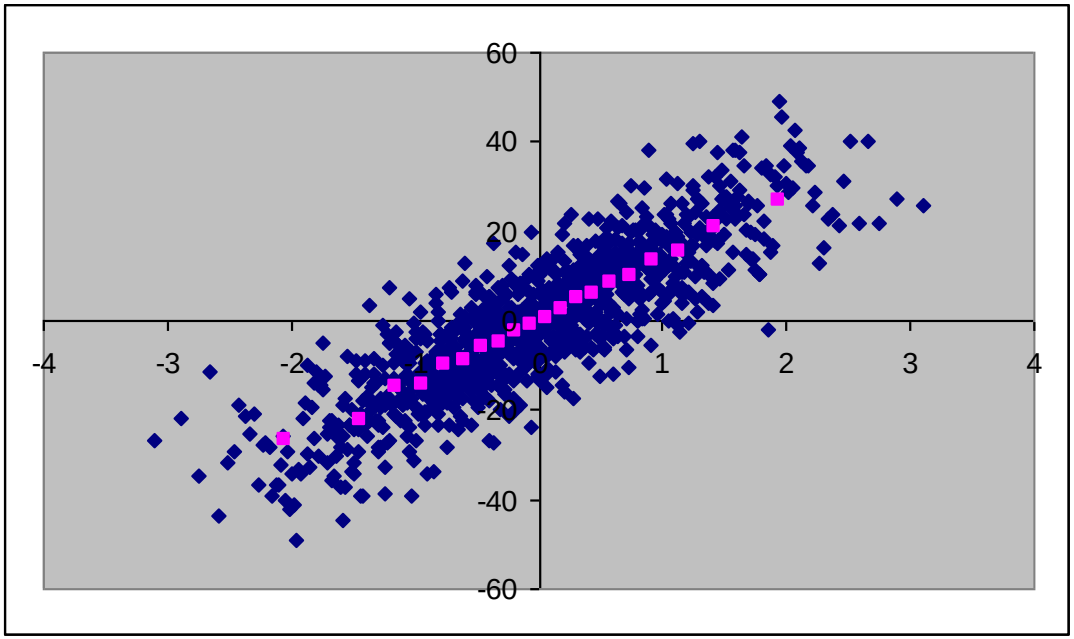




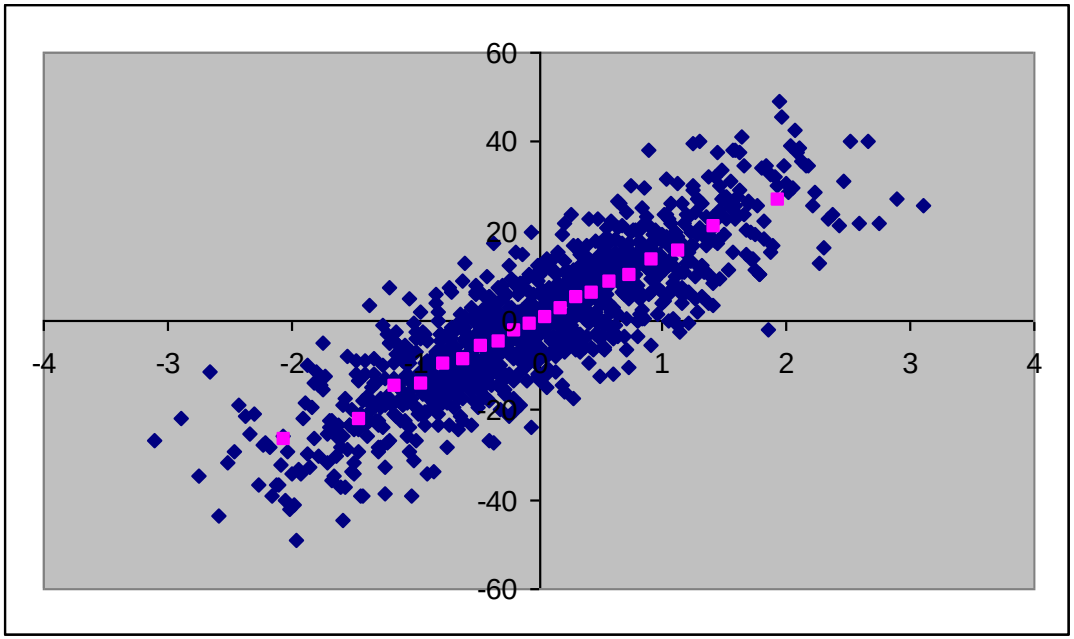
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



Each pink point is ~ $E_{\mathbf{x}_{\sim i}}\left(Y \middle| X_i\right)$



Take the variance of the pinkies

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y\middle|X_i\right)\right)$$

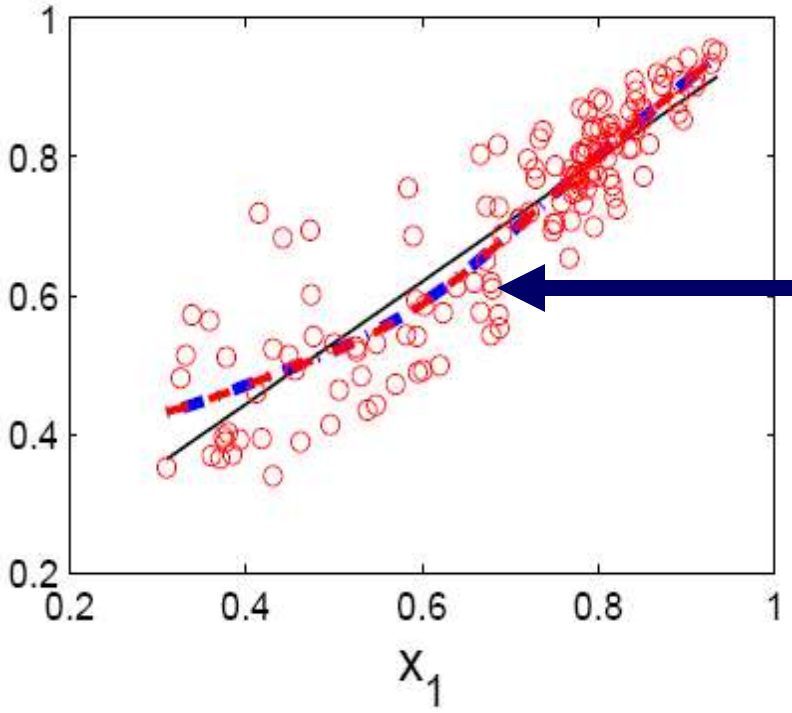
Pearson's correlation ratio

Smoothed curve

$$S_i \equiv \eta_i^2 := \frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

First order sensitivity index

Unconditional variance



Smoothed curve

$$\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)$$

First order sensitivity index:

$$\frac{\mathrm{V}_{x_i}\left(\mathrm{E}_{\mathbf{x}_{\sim i}}\left(y \mid x_i\right)\right)}{\mathrm{V}\left(y\right)}$$

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

First order effect, or top marginal variance=

= the expected reduction in variance than would be achieved if factor Xi could be fixed.

Why?

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+$$
$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)=V(Y)$$

Easy to prove using $V(Y)=E(Y^2)-E^2(Y)$

Because:

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+$$

$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)=V(Y)$$

This is what variance would be left (on average) if Xi could be fixed…

… then this …

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+$$

$$+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)=V(Y)$$

… must be the expected reduction in variance than would be achieved if factor Xi could be fixed

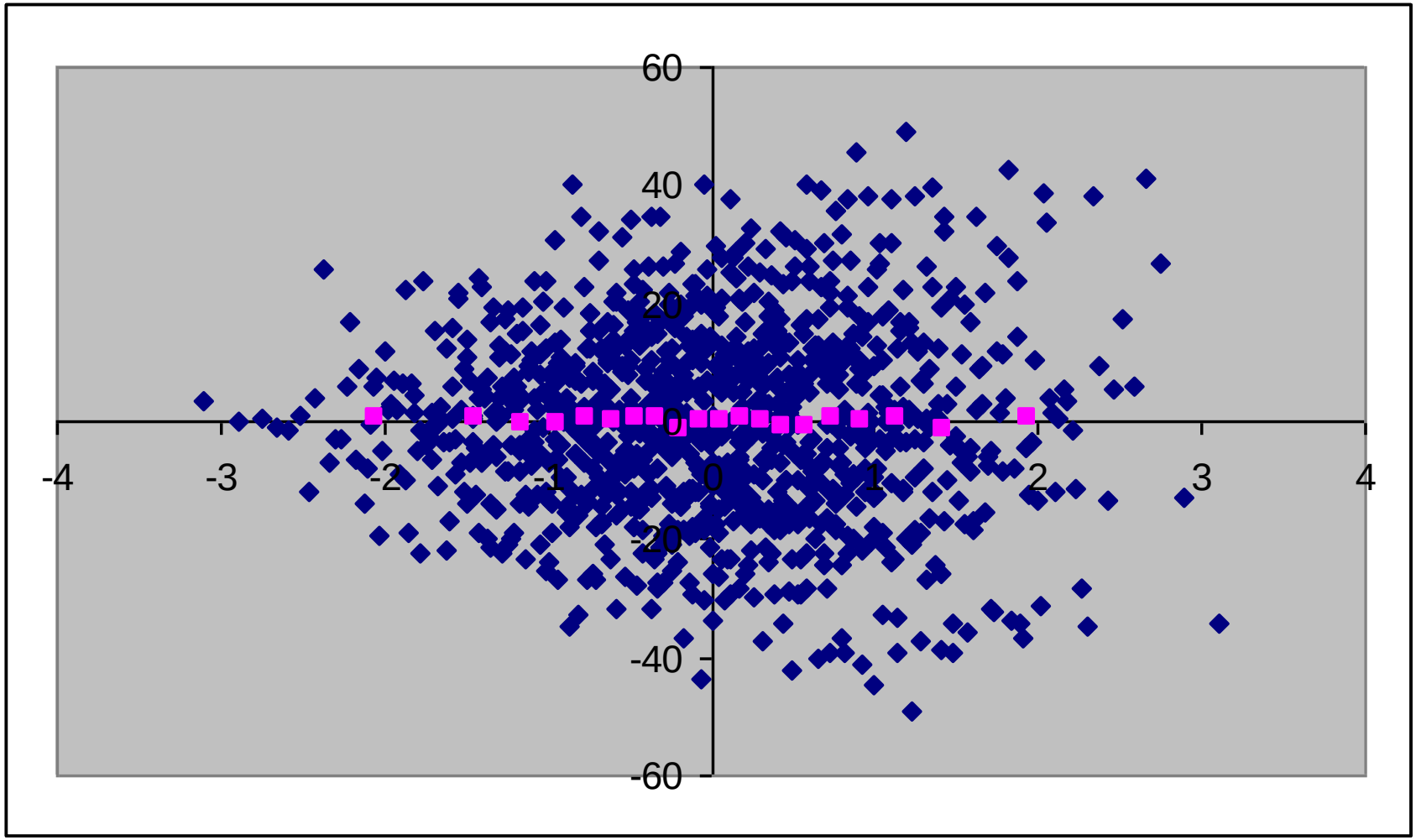
For additive models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)\approx V(Y)$$

… which is also how additive models are defined

How about non additive models?

- Is S_i =0?
- Is this factor non-important?



There are terms which capture two-way, three way, … interactions among variables.

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_{i} V_i + \sum_{i,j>i} V_{ij} + \ldots + V_{123..k}$$

Variance decomposition (ANOVA)

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y\middle|X_i\right)\right)=V_i$$

$$V_{X_iX_j}\left(E_{\mathbf{X}_{\sim ij}}\left(Y\middle|X_iX_j\right)\right)=$$

$$=V_i+V_i+V_{ij}$$

...

Variance decomposition (ANOVA)

When the factors are independent the total variance can be decomposed into main effects and interaction effects up to the order k, the dimensionality of the problem.

Variance decomposition (ANOVA)

When the factors are not independent the decomposition loses its unicity (and hence its appeal)

If fact interactions terms are awkward to handle: second order terms are as many as $k(k-1)/2$ …

Wouldn't it be handy to have just a single 'importance' terms for all effects, inclusive of first order and interactions?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a model $Y=f(X_1,X_2,X_3)$

Instead of

$$V=V_1+V_2+V_3+$$
$$+V_{12}+V_{13}+V_{23}+$$
$$+V_{123}$$

and

$$1=S_1+S_2+S_3+$$
$$+S_{12}+S_{13}+S_{23}+$$
$$+S_{123}$$

We have:

$S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$

(and analogue formulae for S_{T2}, S_{T3}) which can be computed without knowing S_1, S_{12}, S_{13}, S_{123}

S_{T1} is called a total effect sensitivity index

How to get from first order to total order

From

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

Main effect of factor X_i

replacing X_i with $\boldsymbol{X}_{\sim i}$

To main effect of non-X_i

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

BUT:

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y\middle|\mathbf{X}_{\sim i}\right)\right)+$$
$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y\middle|\mathbf{X}_{\sim i}\right)\right)=V\left(Y\right)$$

If this is the main effect on non-X_i…

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

… all remaining variance must be due to X_i and its interactions

Main effects

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

Residuals

$$E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

Main (or first order) effect of X_i

Main effects

Residuals

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)+E_{X_i}\left(V_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right) = \mathbf{V(Y)}$$

$$V_{\mathbf{X}_{\sim i}}\left(E_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)+E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right) = \mathbf{V(Y)}$$

Total (or total order) effect of X_i

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

Total effect, or bottom marginal variance=

= the expected variance than would be left if all factors but Xi could be fixed.

$$\frac{V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)}{V(Y)} = S_i$$

$$\frac{E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)}{V(Y)} = S_{Ti}$$

Rescaled to [0,1], under the name of first order and total order sensitivity coefficient

Variance based measures are:
-well scaled,
-concise,
-easy to communicate.

Further
- S_i reduces to squared standard regression coefficients for linear model.
- S_{Ti} detect and describe interactions and
- Becomes a screening test at low sample size (See Campolongo F, Saltelli A, Cariboni, J, 2011, From screening to quantitative sensitivity analysis. A unified approach, *Computer Physics Communication*, 182 (4), pp. 978-988.)

➔ Lesson of Sergei Kucherenko

Both indices can be computed via Monte Carlo

We use quasi random sequences developed by I.M. Sobol’

Estimation procedures:

- No brute force. A double loop is not needed, though the measures are expresses as V(E(•)) and E(V(•)).
- For S_i quick estimation procedures are available which are k-independent.
- For S_{Ti} estimation procedures are mostly k-dependent (unless … active area of research…).

➔Lessons of William Becker

➔ Lessons of Elmar Plischke

➔Lessons of Sergei Kucherenco …

Why these measures?

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$

Factors prioritization

$$E_{\mathbf{X}_{\sim i}}\left(V_{X_i}\left(Y|\mathbf{X}_{\sim i}\right)\right)$$

Fixing (dropping) non important factors

Saltelli A. Tarantola S., 2002, On the relative importance of input factors in mathematical models: safety assessment for nuclear waste disposal, *Journal of American Statistical Association*, 97 (459), 02-709.

More about the settings:

- Factor prioritisation → $S_i \equiv \frac{V\left(E\left(Y|X_i\right)\right)}{V_Y}$

If the cost of 'discovering' factors were the same for all factors which factor should I try to discover first?

•Factor fixing: Can I fix a factor [or a subset of input factors] at any given value over their range of uncertainty without reducing significantly the output?

$$S_{Ti} \equiv \frac{E\left(V\left(Y|\mathbf{X}_{\sim i}\right)\right)}{V_Y}$$

Factor fixing is useful to achieve model simplification and 'relevance'.

We cannot use S_i to fix a factor; $S_i = 0$ is a necessary condition for X_i to be non-influential but not a sufficient one

X_i could be influent at the second order.

Factor fixing

Imagine that X_i is non-influential and we compute:

$$V_{X_i}\left(Y \mid \mathbf{X}_{\sim i} = \mathbf{x}^*_{\sim i}\right)$$

Non-X_i fixed to a point in the non-X_i space

But this must be zero because if X_i is non-influential than all depends from $X_{\sim i}$ and fixing it freezes the variance. Then:

$$E\left(V\left(Y \mid \mathbf{X}_{\sim i}\right)\right) = 0$$

and S_{Ti} is zero

Factor fixing

We have just proven that if X_i is non-influential then S_{Ti} is zero (necessary condition). Conversely if S_{Ti} is zero then

$$E\left(V\left(Y \mid \mathbf{X}_{\sim i}\right)\right) = 0$$

By definition. But a variance can only be a positive number and if an average of variances is zero than all variances must be zero, which proves that:

$$V\left(Y \mid \mathbf{X}_{\sim i} = \mathbf{x}^*_{\sim i}\right)$$

is also zero for any value of the fixed point $\mathbf{x}^*_{\sim i}$. This proves that nowhere in the space of $X_{\sim i}$ the factor Xi has any influence (sufficient condition).

Remarks on factor fixing: 1

Model simplification supported by factor fixing is useful.

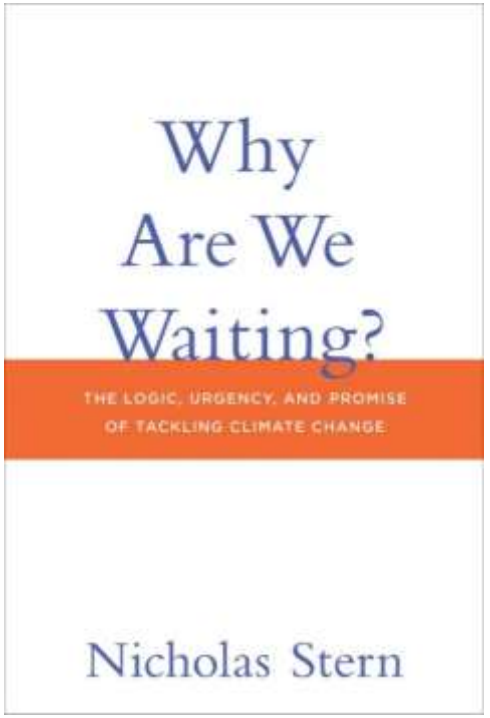
"As the complexity of a system increases … precision and significance (or relevance) become almost mutually exclusive characteristics"

Zadeh's incompatibility principle (1965).

Lofti Zadeh

Remarks on factor fixing: 1

Discussion on 'costing' climate against Economists' claim and desire to compute the cost in dollar of damage from (and offsetting of) climate change.



Saltelli, A., Stark, P.B., Becker, W., and Stano, P., 2015, Climate Models As Economic Guides Scientific Challenge or Quixotic Quest?, Issues in Science and Technology, Volume XXXI, Issue 3, spring 2015.

Saltelli, A., Funtowicz, S., Giampietro, M., Sarewitz, D., Stark, P.B., van der Sluijs, J.P., 2016, Climate costing is politics not science, Nature, 14 April, 532, 177.

Remarks on factor fixing: 2

The model 'relevance' problem

$$R = \frac{\text{number of factors that truly induce variations in the output of interest}}{\text{total number of factors in the model}}$$

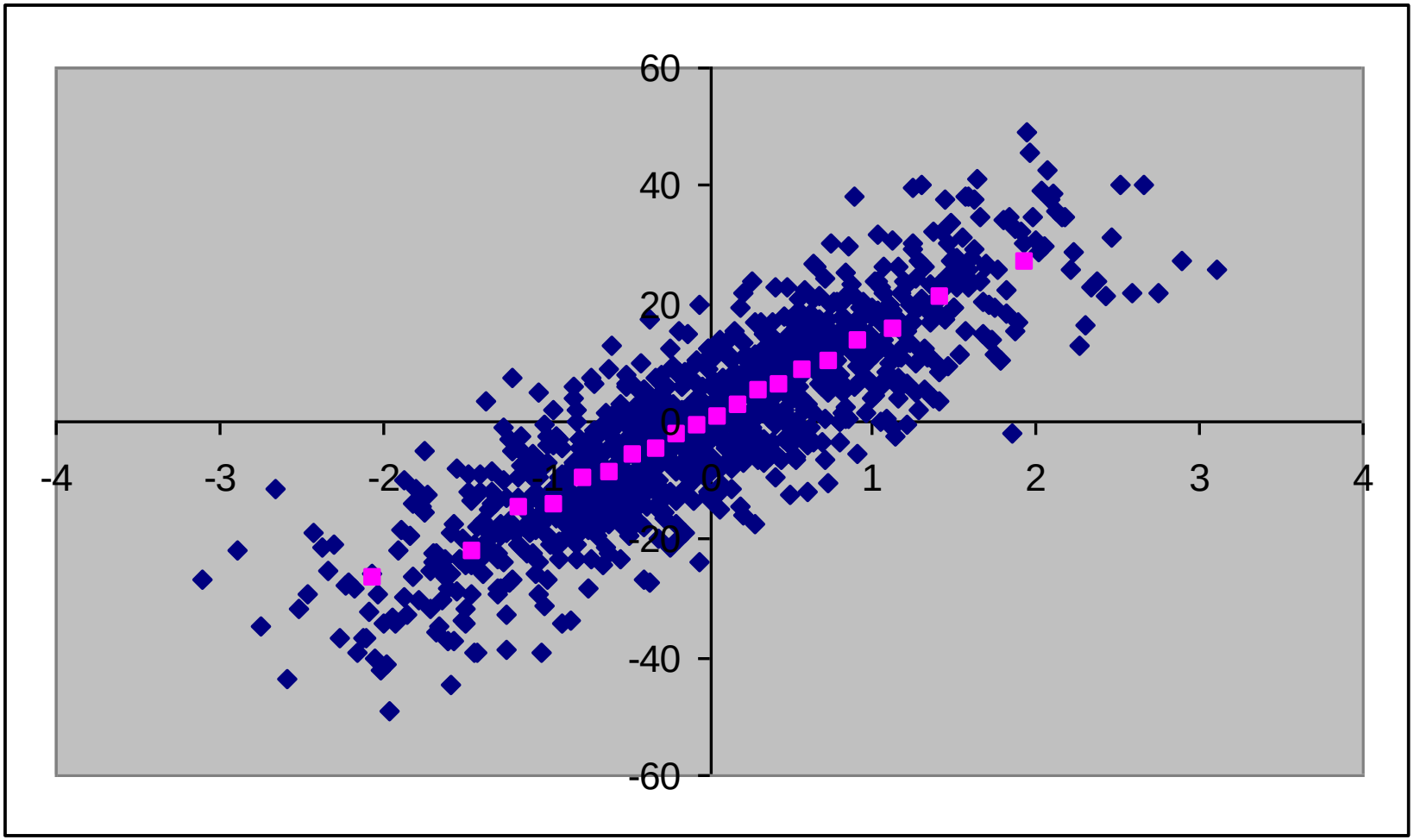
Low R could flag a model meant to intimidate.

Bruce Beck

Summary for variance based measures:

1. Easy-to-code, Monte Carlo – better on quasi-random points. Estimate of the error available.

2. The main effect can be made cheap; its computational cost does not depend upon k.



Easy to smooth and interpolate!

Summary for variance based measures:

3. The total effect is more expensive; its computational cost is $(k+1)N$ where N is one of the order of one thousand (unless e.g. using emulators …).

Afterthoughts

Sensitivity analysis cannot is not “run” on a model but on a model once applied to a case.

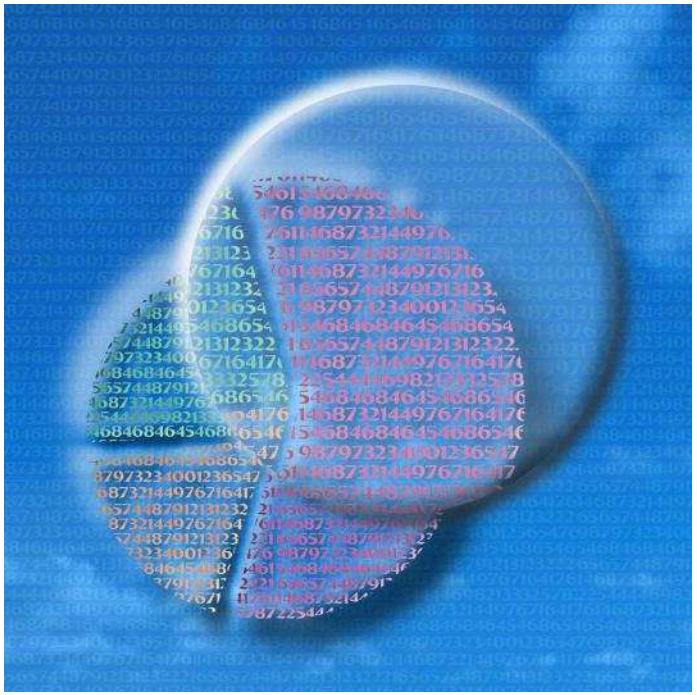
➔It is meaningful in relation of a statement which the model is called to support.

Sensitivity analysis should not be used to strengthen a reductionist compression of reality.

It can never proof that a model is ‘true’. Its best used is to falsify a model (Oreskes).

Discussion points

- Why doing a sensitivity analysis if it can undermine an laborious quantification exercise?
- What do I do if this happens to be the case?



END