Máster Universitario en Administración y Dirección de Empresas
Full Time MBA

Quantitative methods for decision making

Professor Andrea Saltelli

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Elements of quantification for decision making with emphasis on operation research

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In this set of slides:

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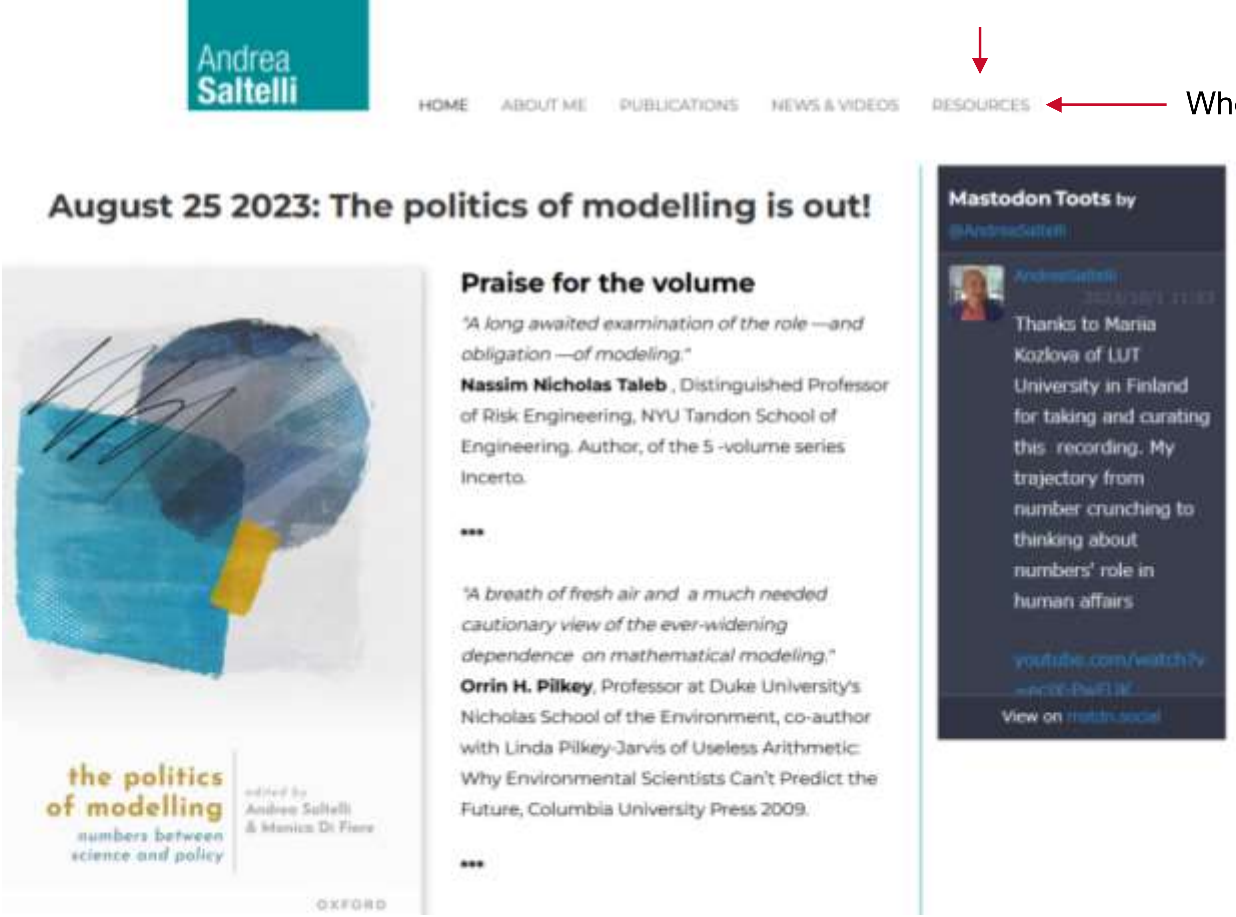
17.

Decision Analysis

Knight. Decision making with and without experimentation. Example: drilling or selling? Bayes in full. Decision trees. Multi Criteria Decision Analysis. Linearization. Borda count, Condorcet’s outranking matrix and Balinski-Laraki’s majority judgment. Hillier (2014) chapter 16 plus various authors.



Where to find this talk

The talk is also at

https://ecampus.bsm.upf.edu/,

where you find additional reading material

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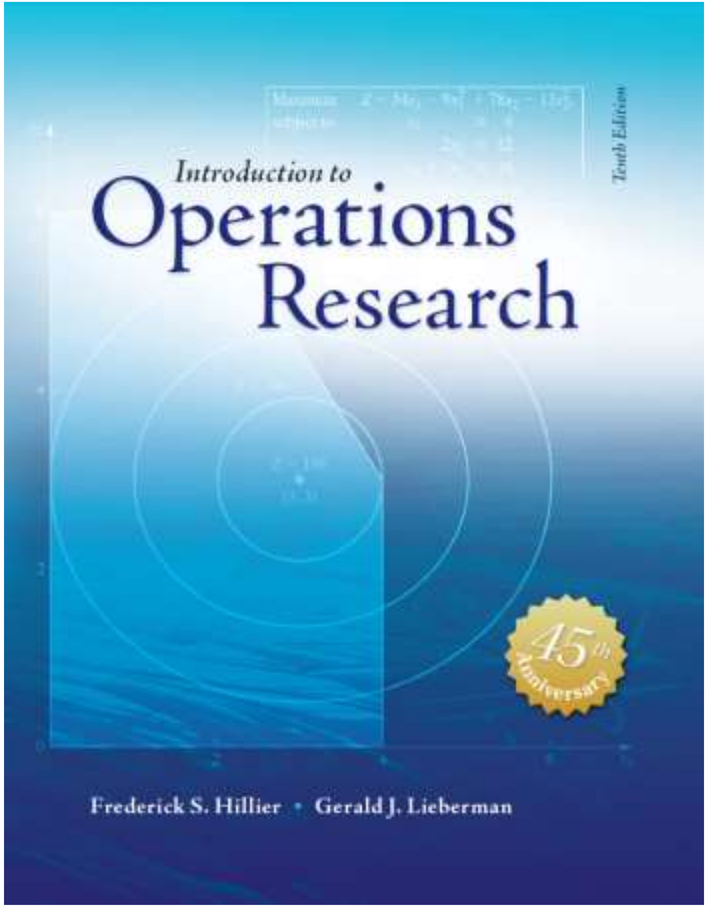
Where to find this book:

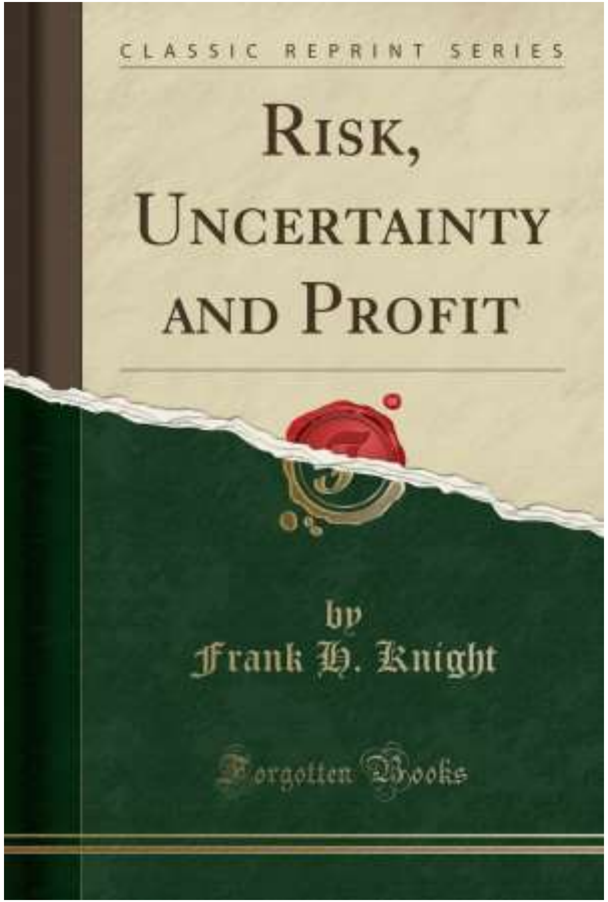
https://www.dropbox.com/sh/ddd48a8jguinbcf/AABF0s4eh1lPLVxdx0pes-0fa?dl=0&preview=Introduction+ to+ Operations+ Research+ -+ Frederick+ S.+ Hillier.pdf



Frank Knight (1921) distinguished risk from uncertainty

Risk = know outcomes & probabilities; roulette game

Uncertainty = unsure about the probabilities; starting a business



Frank H. Knight
1885-1972

Quote:

"We live in a world of contradiction and paradox, a fact of which perhaps the most fundamental illustration is this: that the existence of a problem of knowledge depends on the future being different from the past, while the possibility of the solution of the problem depends on the future being like the past."

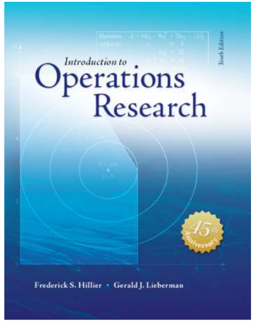
Frank H. Knight
1885–1972

A prototype example of decision under uncertainty; drilling or selling?

Source: https://ecsgeothermal.com/oil-drilling-on-land/

A company own land where there could be oil

Another company offers to purchase said land



Source: https://ecsgeothermal.com/oil-drilling-on-land/

TABLE 16.1 Prospective profits for the Goferbroke Company

Alternative \ Status of Land	Payoff: Oil	Payoff: Dry
Drill for oil	$700,000	−$100,000
Sell the land	$ 90,000	$ 90,000
Chance of status	1 in 4	3 in 4

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The table offers different payoffs associated to different decision (sell, drill) versus two possible states of nature (oil, no-oil)

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How to act on this table? Different alternatives are available.

The Maximin Payoff Criterion

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For each decision look at the worst payoff over all possible states of nature … ← Dry for drill, indifferent for sell

…and choose the one with the best outcome ← Sell, as 90 is better than −100

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The Maximum likelihood approach

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Chance of status	1 in 4	3 in 4

Identify the most likely state of nature ← Dry, as $\frac{3}{4}$ is more than $\frac{1}{4}$ (prior probabilities)

…and choose the one with the best pay-off ← Sell, as 90 is better than −100

The Maximum likelihood approach

TABLE 16.4 Application of the maximum likelihood criterion to the first Goferbroke Co. problem

	State of Nature			
Alternative	**Oil**	**Dry**		
1. Drill for oil	700	−100	−100	
2. Sell the land	90	90	90	← Maximum in this column
Prior probability	0.25	0.75		
		↑ Maximum		

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Bayes' rule – the expected value approach

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Calculate the best expected payoff for each decision alternative

Payoff (Drill)=0.25∗700−0.75∗100=100

Payoff (Sell)=0.25∗90−0.75∗90=90

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Before we proceed with Reverend Bayes, remember the caveat of lesson one: expected value may lead to counter intuitive results

This is the story the St. Peterburg paradox (another game!)

Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli
(1700-1782)

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Bayes' rule – the expected value approach

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
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Prior probability	0.25	0.75

What do we do if we feel uneasy with these prior probabilities? What if instead of 0.25 the probability of oil is instead 0.15 or 0.35?

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What do we do if we feel uneasy with these prior probabilities? What if instead of 0.25 the probability of oil is instead 0.15 or 0.35?

The payoff for a generic value p of this prior is

$$p*700-(1-p)*100=800*p-100$$

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	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
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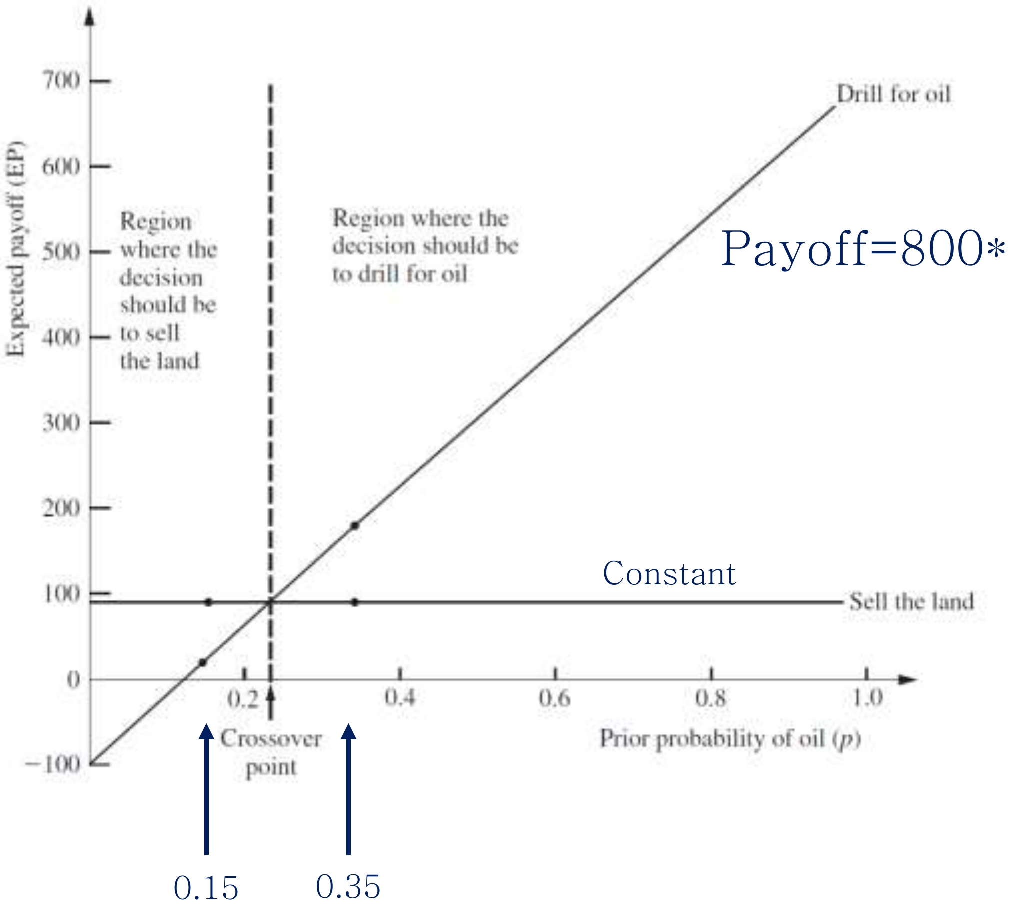
What if this is not 0.25, but for example 0.15 or 0.35?



Payoff=800*p−100

	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
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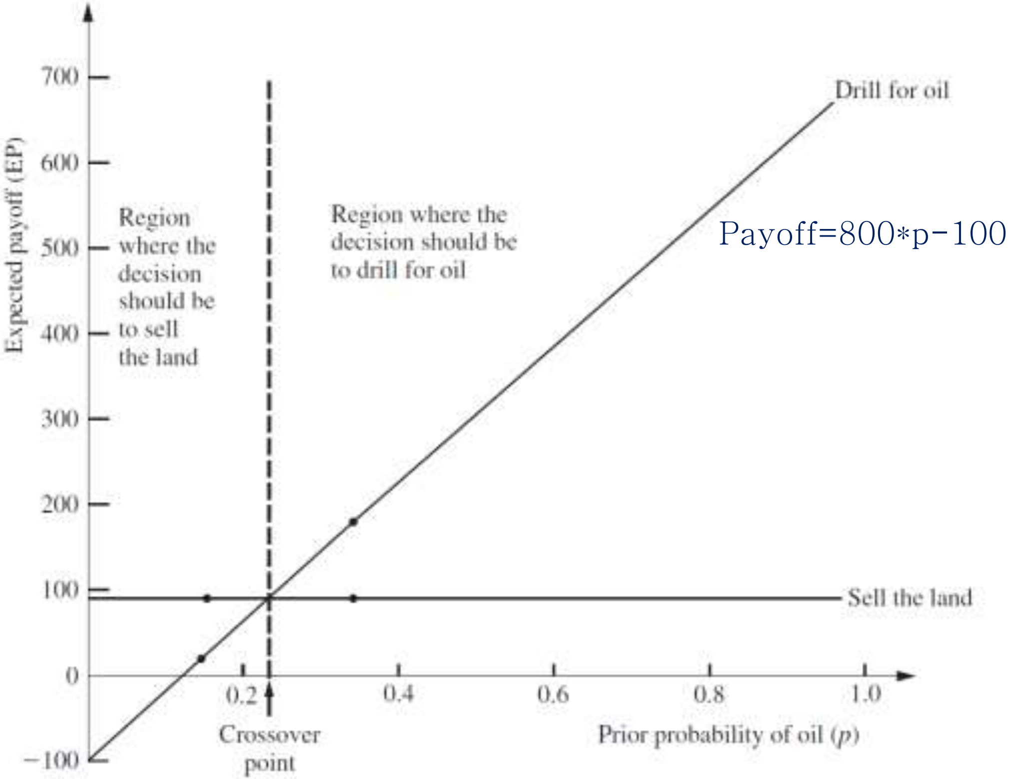
Prior probability	0.25	0.75

Exercise: compute cross over coordinates

Source: https://simpsons.fandom.com/wiki/Bart_Gets_Famous



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	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
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Prior probability	0.25	0.75

Exercise: compute cross over coordinates

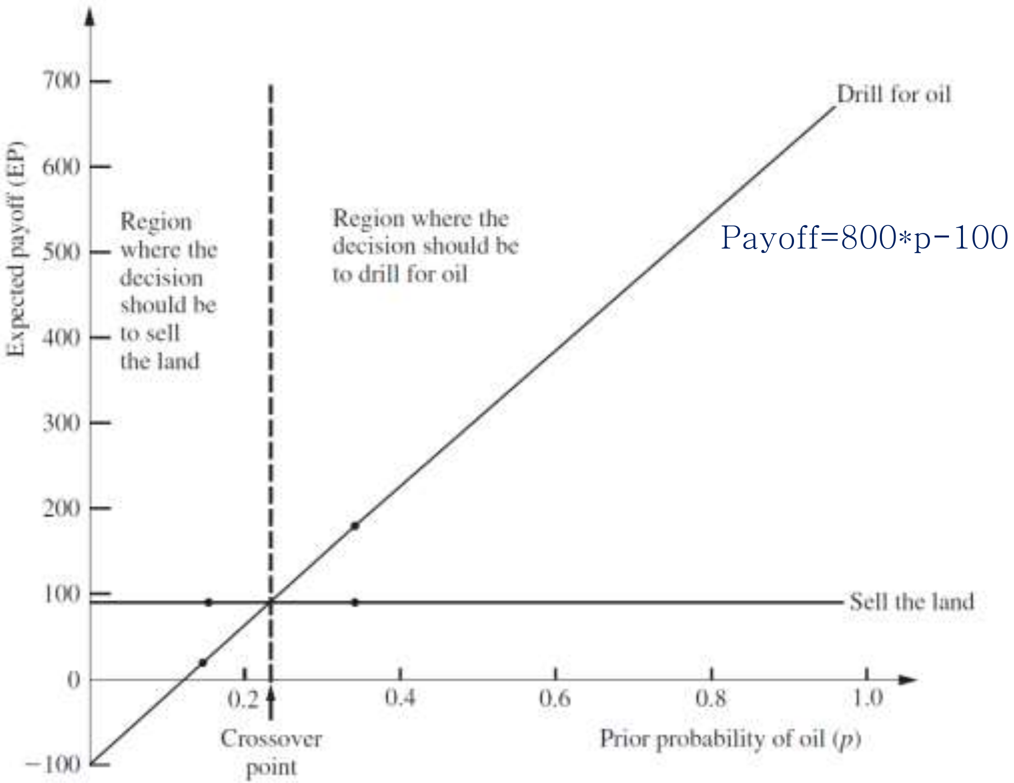
Intersection of

$$y = 800x - 100$$

and

$$y = 90$$

$$x = 190/800 = .2375$$



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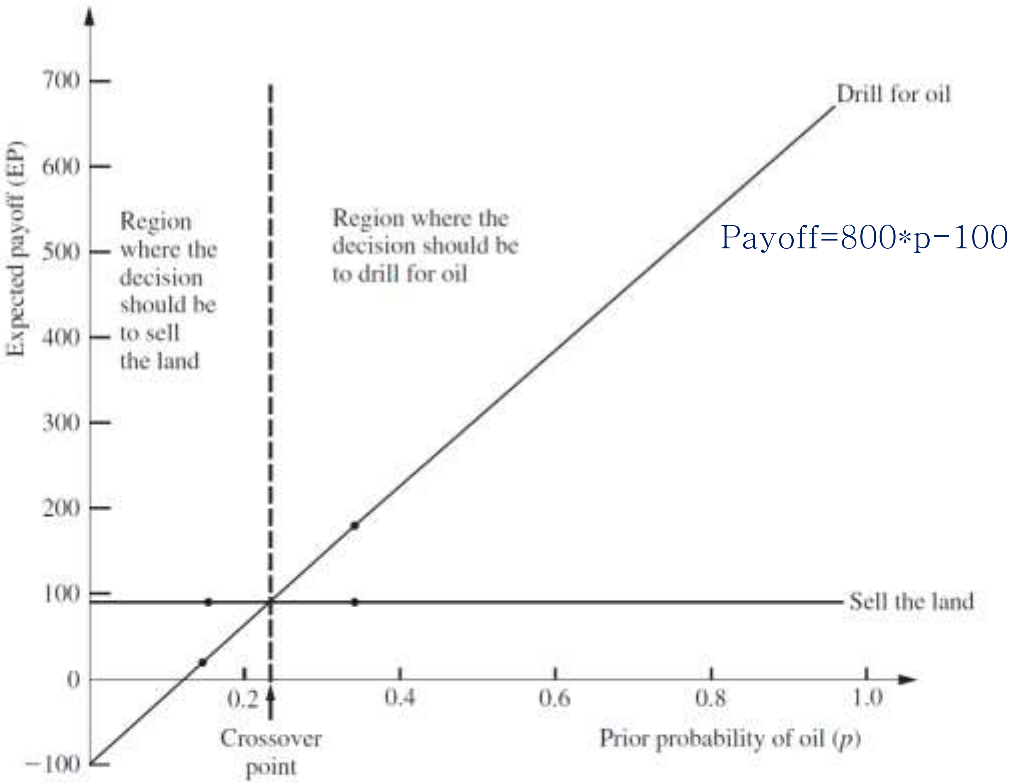
	State of Nature	
Alternative	**Oil**	**Dry**
1. Drill for oil	700	−100
2. Sell the land	90	90
Prior probability	0.25	0.75

Conclusion:

if $p < .2375$ then sell

if $p > .2375$ then drill



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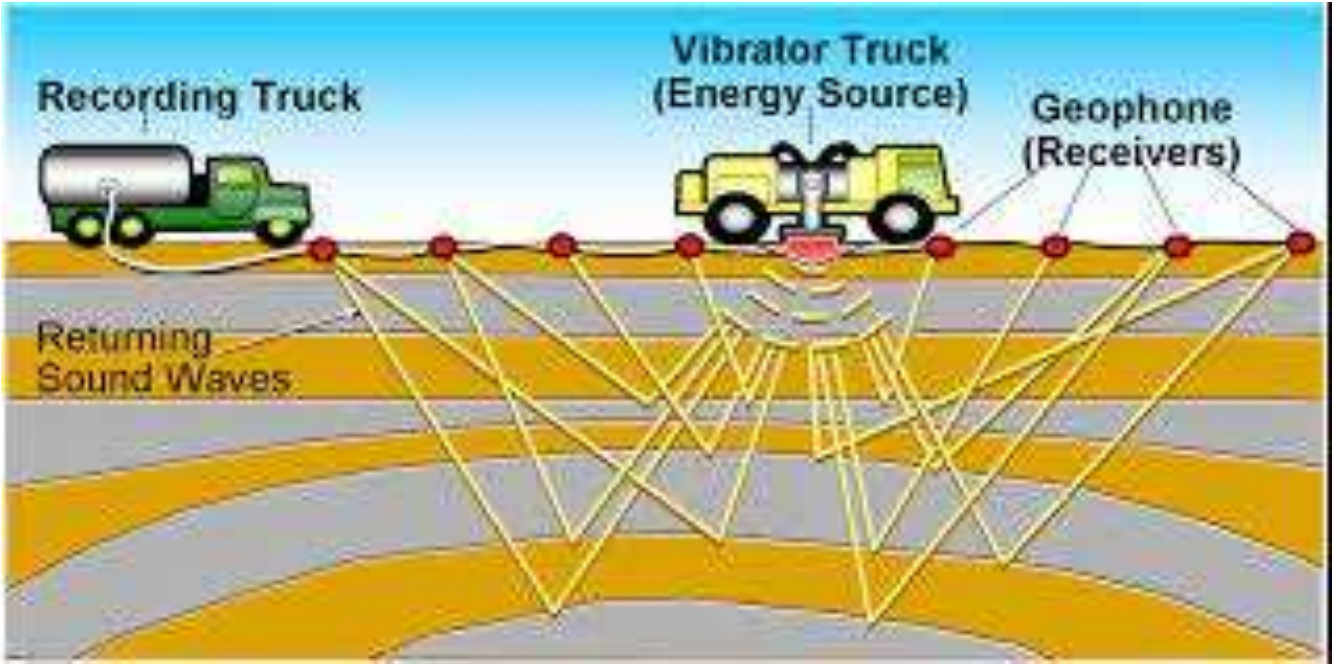
Decision making with experimentation



Source: https://gov.nu.ca/sites/default/files/2017_seismic_eng.pdf

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Perhaps before deciding whether to sell or drill some prospection study should be done, such as seismic surveying

This would come to a cost, so even in this case, before the survey, it would be wise to crunch some numbers

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The cost of the seismic survey is \$30,000.

Experience says that:
USS: **U**nfavorable **S**eismic **S**oundings ➔ oil is fairly unlikely.
FSS: **F**avorable **S**eismic **S**oundings ➔ oil is fairly likely.

Again experience translates this into

$p(USS|Oil) = 0.4$ and $p(FSS|Oil) = 1 - 0.4 = 0.6$

$p(USS|Dry) = 0.8$ and $p(FSS|Dry) = 1 - 0.8 = 0.2$

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As it is written, the famous theorem 'looks' symmetric in A and B $\cdots$

$$p(A|B)p(B) = p(B|A)p(A) = p(A,B)$$

Conditional Prior Joint

In fact the way it is used in practice is rather asymmetric, and aims to update A based on B being true, B being for example an experiment and A a theory

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

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When B is the outcome of an experiment and A is a state of nature $p(A|B)$ becomes the probability that given the outcome B (for example a favourable outcome FSS) then we indeed have A – the oil in this case;

we do not know $p(A|B)$ but we do know $p(B|A)$, in this case the probability that if there is oil the test will be favourable

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

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we do not know $p(A|B)$ but we do know $p(B|A)$, in this case the probability that if there is oil the test will be favourable

We also know $P(Oil)$, as this is the old prior, the probability of oil being there before the survey

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

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We also know $P(Oil)$, as this is the old priot, the probability of oil being there before the survey

$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

We only lack $P(FSS)$. This is a delicate point. The unconditional probability of favourable drilling is the total probability of this outcome in all cases, e.g. both oil and no-oil

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$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

We only lack $P(FSS)$. This is a delicate point. The unconditional probability of favourable drilling is the total probability of this outcome in all cases, e.g. both oil and no-oil

How about:

$$P(FSS) = p(FSS|Oil)P(Oil) + p(FSS|No-oil)P(No-oil)$$

This is indeed the total, and hence unconditional, probability of FSS – that is to say all possible ways in which FSS can come about

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Putting these two together:

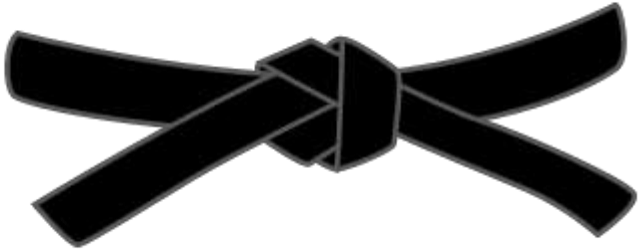
$$p(Oil|FSS)P(FSS) = p(FSS|Oil)P(Oil)$$

$$P(FSS) = p(FSS|Oil)P(Oil) + p(FSS|No - oil)P(No - oil)$$

we get

$$p(Oil|FSS) = \frac{p(FSS|Oil)P(Oil)}{p(FSS|Oil)P(Oil) + p(FSS|No - oil)P(No - oil)}$$

You have just done your first Bayesian updating



Plugging the numbers

0.6 0.25

$$p(Oil|FSS) = \frac{p(FSS|Oil)P(Oil)}{p(FSS|Oil)P(Oil) + p(FSS|No-oil)P(No-oil)}$$

0.6 0.25 0.20 0.75

And this gives $p(Oil|FSS) = \frac{1}{2}$

Hence since

$$p(Oil|FSS) = \frac{1}{2}$$

then

$$p(Dry|FSS) = \frac{1}{2}$$

And following a similar path for the negative survey outcome USS

$$p(Oil|USS) = \frac{1}{7} = 1.4$$

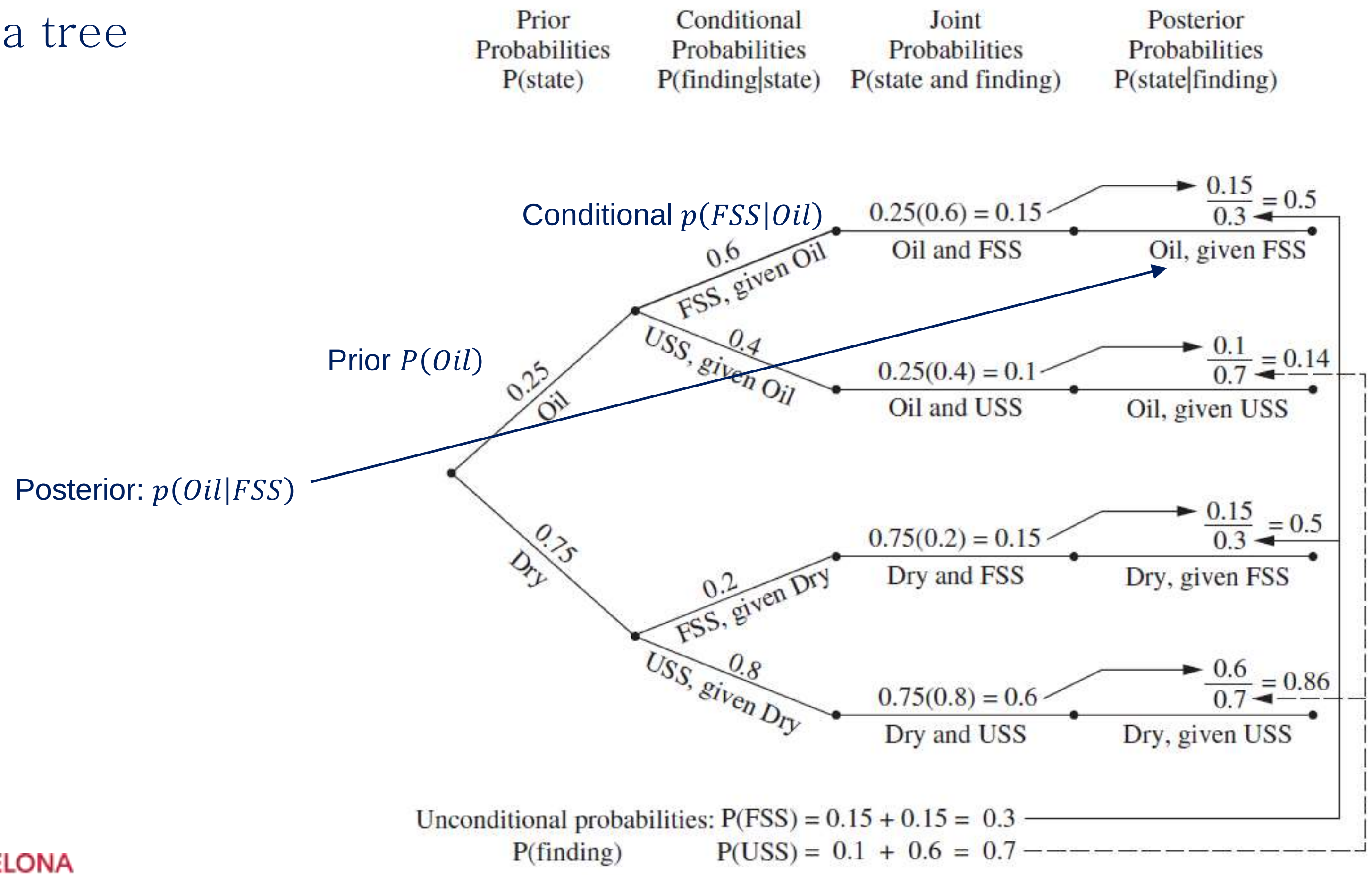
$$p(Dry|USS) = \frac{6}{7} = .86$$

Indeed the survey is a game changer when compared to the prior probabilities $P(Oil)$=0.25 and $P(Dry) = 0.75$

➔ Probably nobody would be a taker for drill if USS is true

➔ One half is much better than one in four if FSS is true

All in a tree


Prior Probabilities P(state)
Conditional Probabilities P(finding|state)
Joint Probabilities P(state and finding)
Posterior Probabilities P(state|finding)
Conditional $p(FSS|Oil)$
Prior $P(Oil)$
Posterior: $p(Oil|FSS)$
0.25 Oil
0.75 Dry
0.6 FSS, given Oil
0.4 USS, given Oil
0.2 FSS, given Dry
0.8 USS, given Dry
0.25(0.6) = 0.15
Oil and FSS
0.25(0.4) = 0.1
Oil and USS
0.75(0.2) = 0.15
Dry and FSS
0.75(0.8) = 0.6
Dry and USS
0.15/0.3 = 0.5
Oil, given FSS
0.1/0.7 = 0.14
Oil, given USS
0.15/0.3 = 0.5
Dry, given FSS
0.6/0.7 = 0.86
Dry, given USS
Unconditional probabilities: P(FSS) = 0.15 + 0.15 = 0.3
P(finding) P(USS) = 0.1 + 0.6 = 0.7

We now need to use all these

$$p(Oil|FSS) = \frac{1}{2}$$

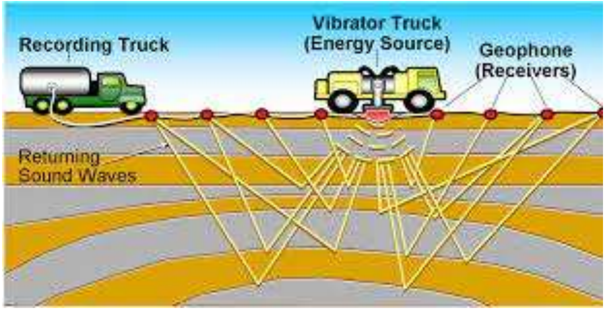
$$p(Dry|FSS) = \frac{1}{2}$$

$$p(Oil|USS) = \frac{1}{7}$$

$$p(Dry|USS) = \frac{6}{7}$$

to take a decision, about drill, sell, and survey

Source: https://ecsgeothermal.com/oil-drilling-on-land/



Source: https://gov.nu.ca/sites/default/files/2017_seismic_eng.pdf

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This is now straightforward:

Payoffs if unfavourable survey (USS):

$$E(Payoff - Drill|USS) = \frac{1}{7}(600) + \frac{6}{7}(-100) - 30 = -15.7$$

$$E(Payoff - Sell|USS) = \frac{1}{7}(90) + \frac{6}{7}(90) - 30 = 60$$

Payoffs if favourable survey (FSS):

$$E(Payoff - Drill|FSS) = \frac{1}{2}(600) + \frac{1}{2}(-100) - 30 = 270$$

$$E(Payoff - Sell|FSS) = \frac{1}{2}(90) + \frac{1}{2}(90) - 30 = 60$$

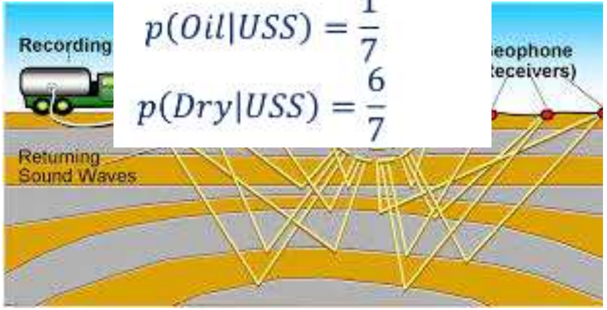
Sou … and/

$$p(Oil|FSS) = \frac{1}{2}$$

$$p(Dry|FSS) = \frac{1}{2}$$

$$p(Oil|USS) = \frac{1}{7}$$

$$p(Dry|USS) = \frac{6}{7}$$

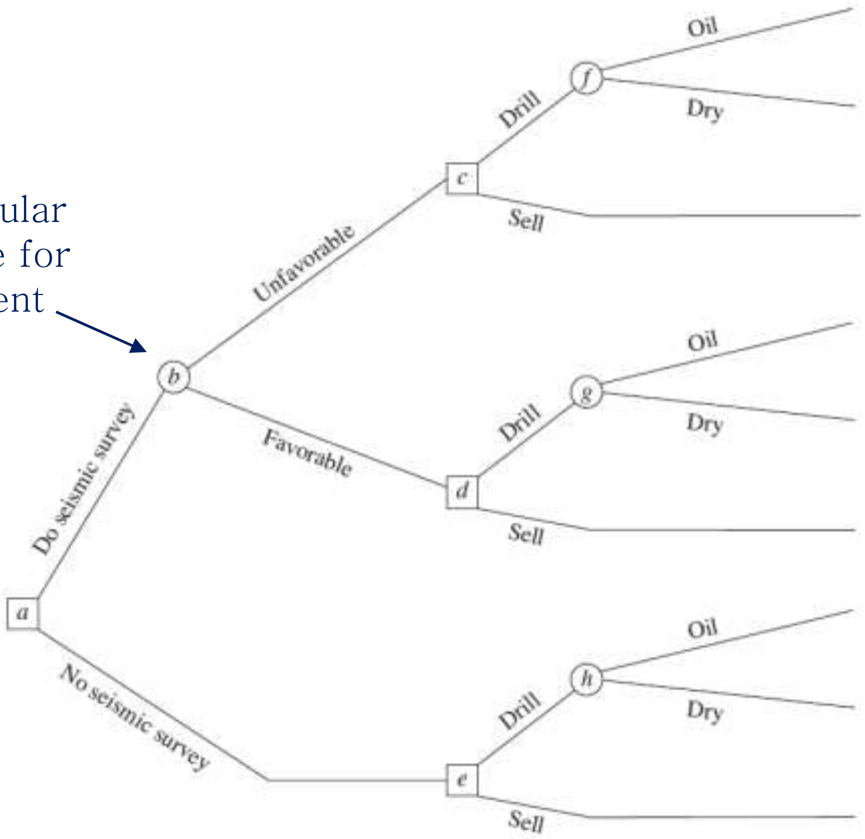


Source: https://gov.nu.ca/sites/default/files/2017_seismic_eng.pdf

Decision tree for the same problem (you have seen this already):

Circular node for event

Square node for decision



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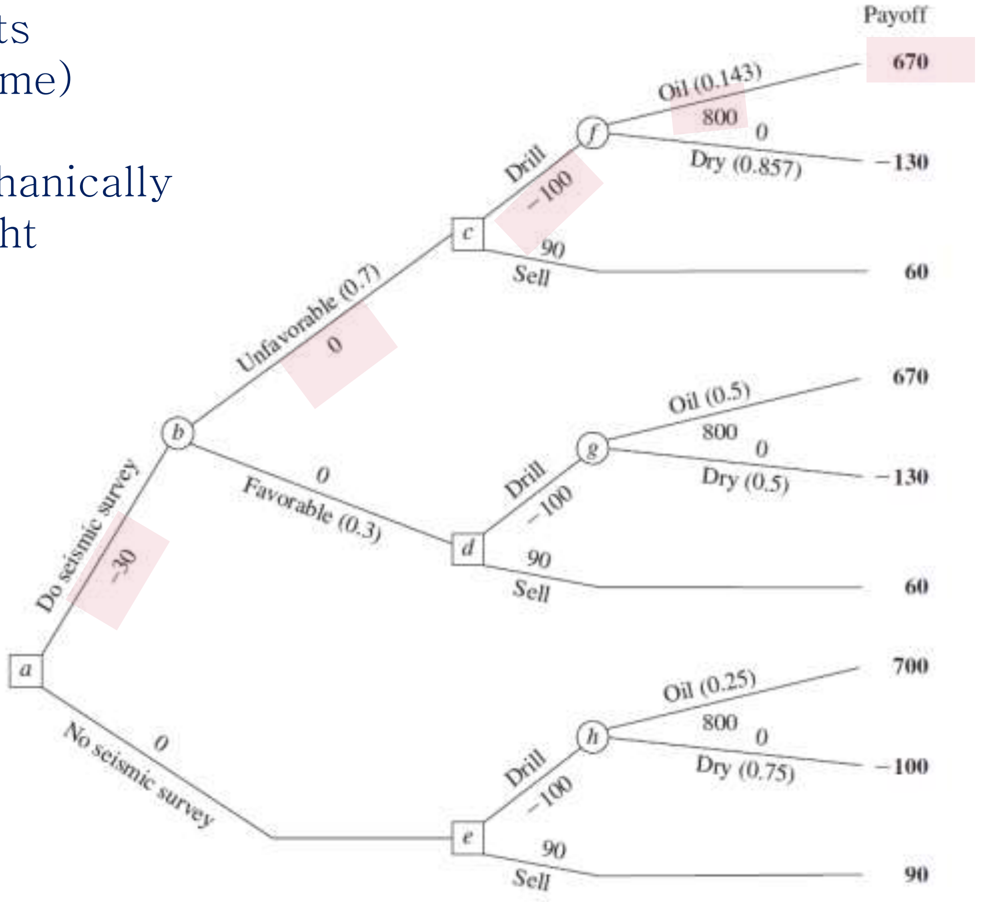
Decision tree with costs
(no probabilities this time)

Cost are compute mechanically moving from left to right



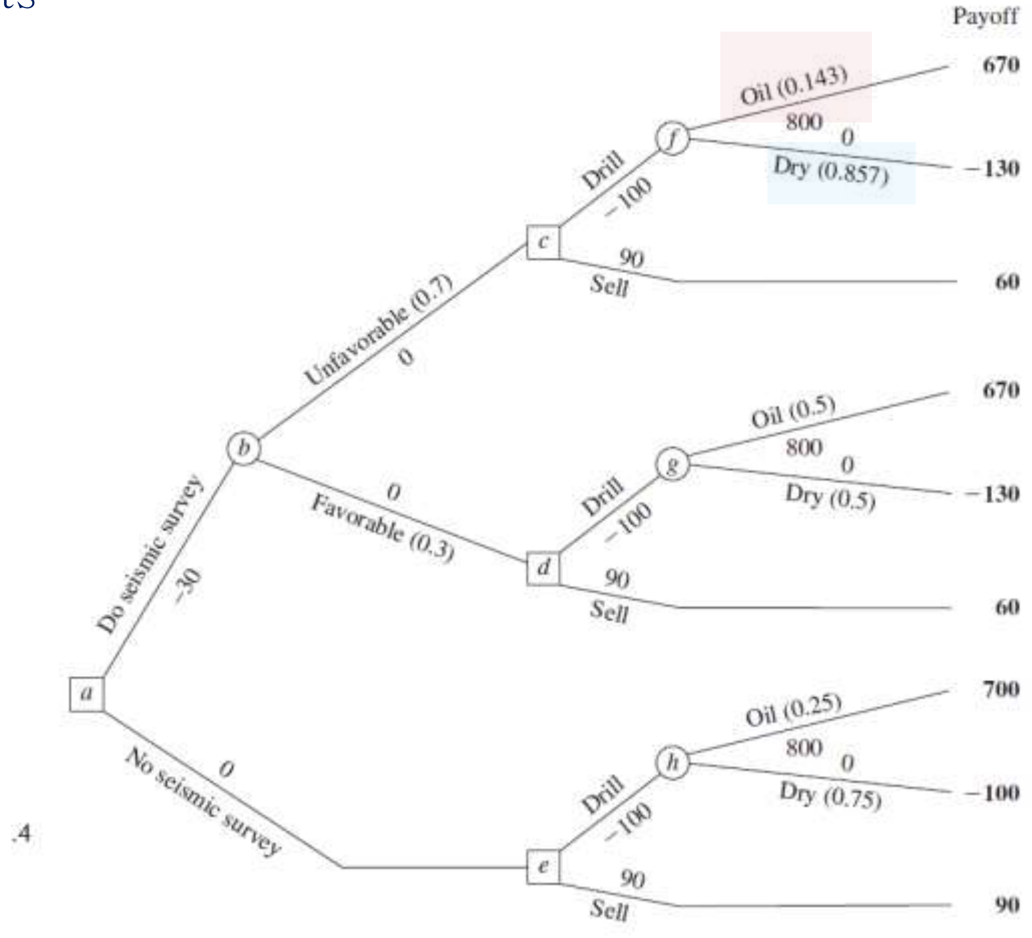
Decision tree with costs (adding probabilities)

$$p(Oil|FSS) = \frac{1}{2} = 0.5$$

$$p(Dry|FSS) = \frac{1}{2} = 0.5$$

$$p(Oil|USS) = \frac{1}{7} = 0.143$$

$$p(Dry|USS) = \frac{6}{7} = 0.857$$



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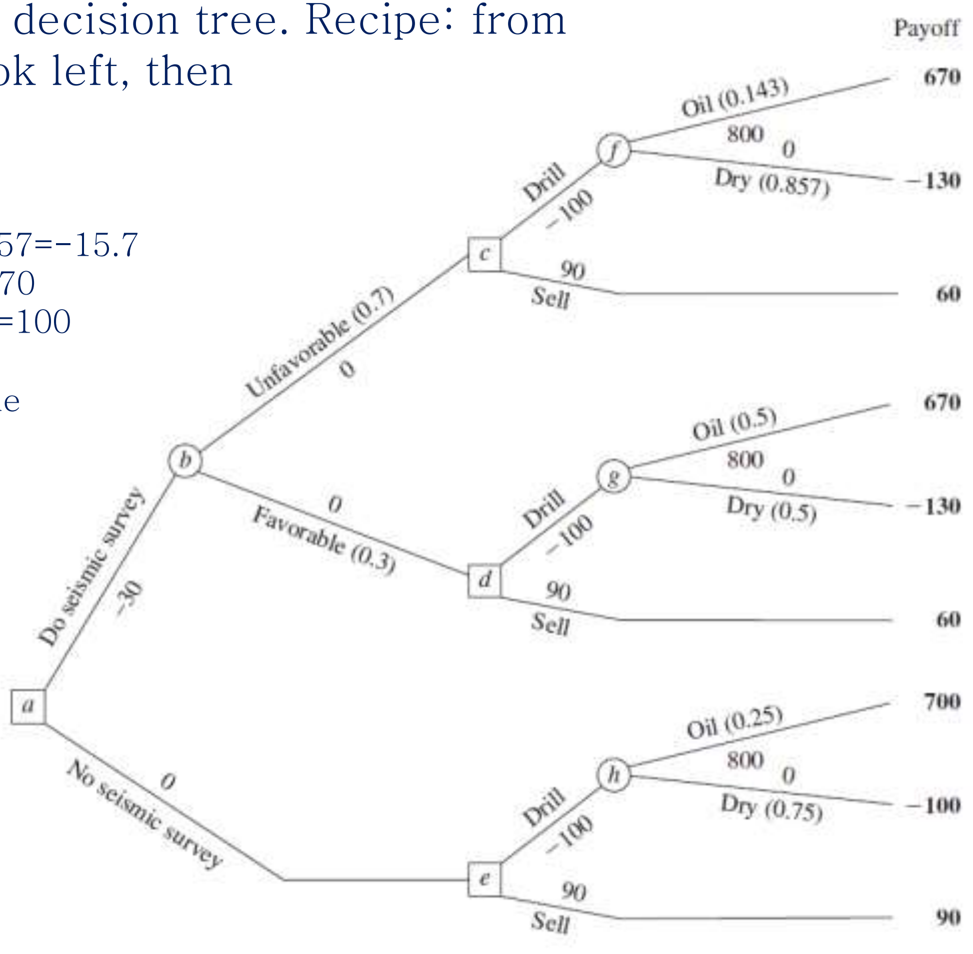
Transforming this into a decision tree. Recipe: from the rightmost column look left, then

If node=event compute payoff

$Payoff(f)$ =670*0.143+ (−130)*0.857=−15.7
$Payoff(g)$ =670*0.5+ (−130)*0.5=270
$Payoff(h)$ =700*0.25+ (−100)*0.75=100

Write these numbers above the node



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If node=event compute payoff

$Payoff(f)$ =670∗0.143+ (−130)∗0.857=−15.7
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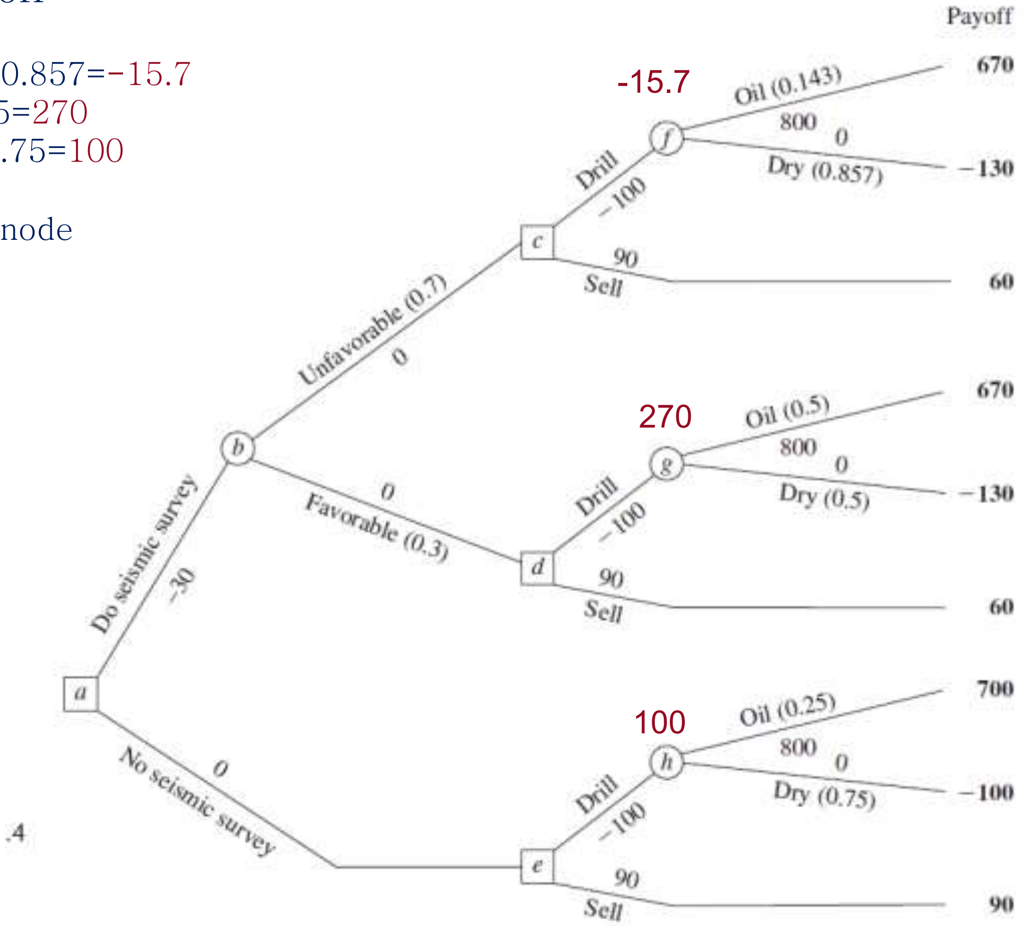

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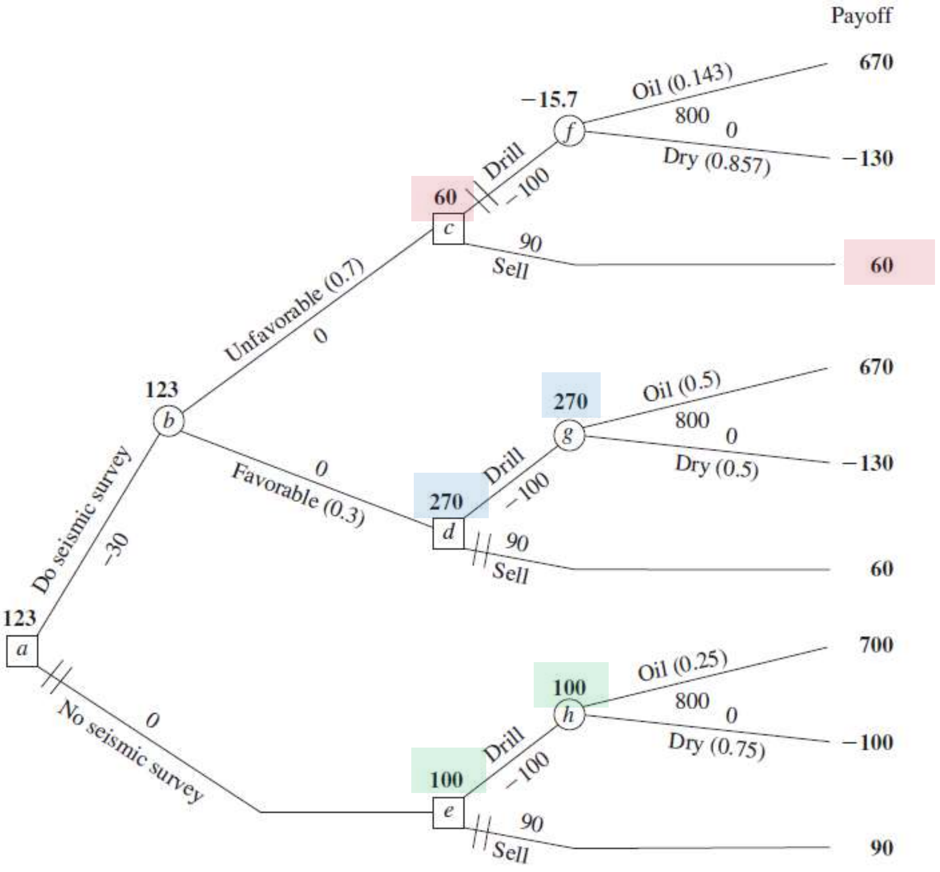
Write these numbers above the node

If node=decision then decide
$Decision(c) = Sell$
$Decision(d) = Drill$
$Decision(e) = Drill$

Report the payoff selected above the node
Move left
$Payoff(b)$ =60*0.7+ 270*0.3=123
$Decision(a) = Do\ survey$



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How to deal with possible paradoxes when using expected value (our old slide again):

This is the story the St. Peterburg paradox (another game!)

Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli
(1700-1782)

➔Utility theory

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This is the story the St. Peterburg paradox (another game!)

Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli (1700-1782)

➔When using Utility Theory the rhetorical question above becomes the tool to elicit users preferences

→When using Utility Theory the rhetorical question above becomes the tool to elicit users preferences

A common occurrence if that actors show a decreasing marginal utility for money (risk aversion)

To see if this is the case and to elicit the values for the utilities, the following alternatives are posed to the actor

Receiving $10,000 with certainty
Receiving 100,000 with probability p

To see if this is the case and to elicit the values for the utilities, the following alternatives are posed to the actor

1) Receiving $10,000 with certainty
2) Receiving $100,000 with probability p ($100,000 is the upper limit of the curve we intend to build)

The following question is posed: for what value of p would you consider options 1 and 2 equivalent. Imagine the answer is $\frac{1}{4}$ $(p = 0.25)$ ➔ the actor consider $10,000 with certainty and $100,000 with probability $\frac{1}{4}$ as equivalent

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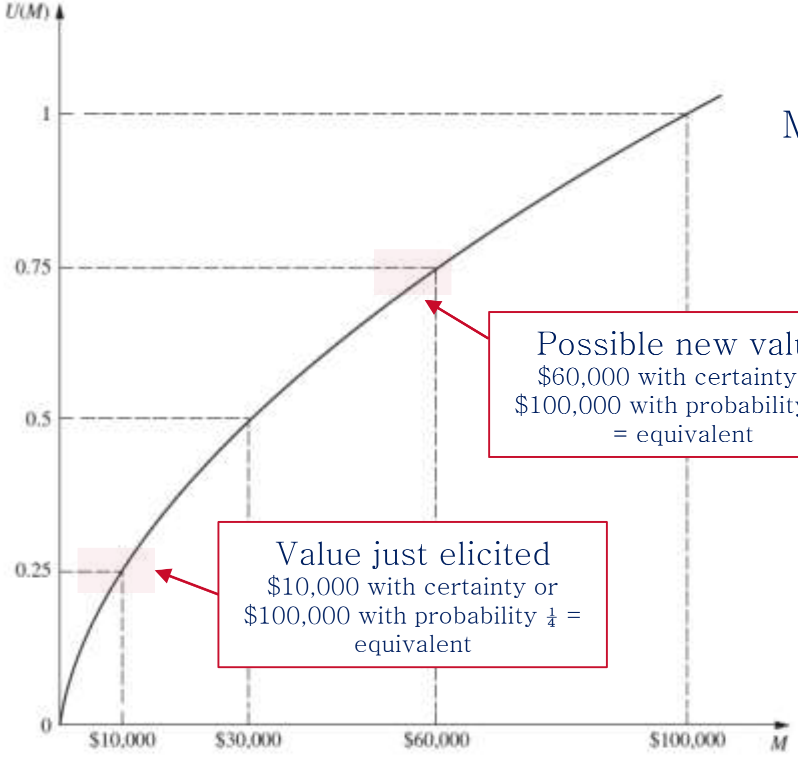
$10,000 with certainty or $100,000 with probability ¼ = equivalent



Maximum amount of money=Utility 1

Possible new value
$60,000 with certainty or $100,000 with probability .75 = equivalent

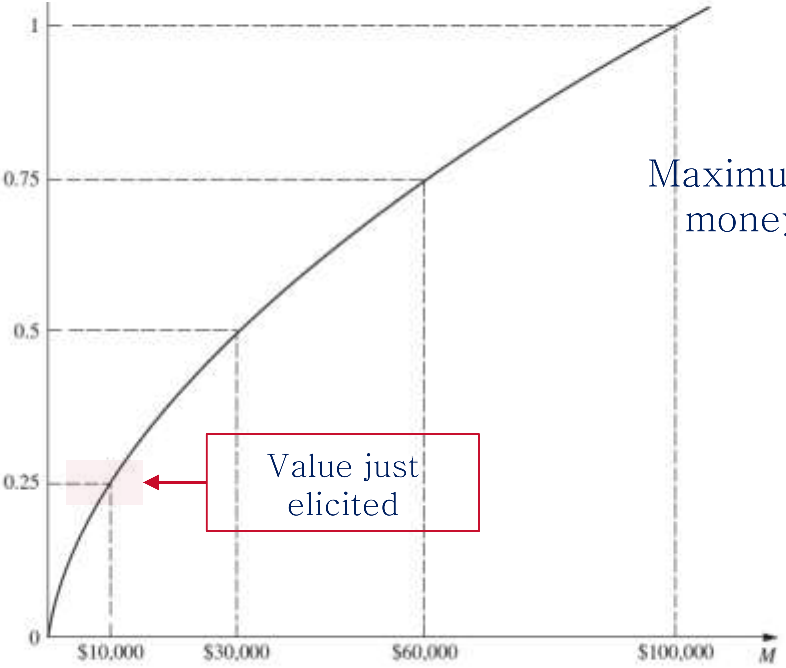
Value just elicited
$10,000 with certainty or $100,000 with probability ¼ = equivalent

Minimum amount of money=Utility 0

1) Receiving $10,000 with certainty
2) Receiving $100,000 with probability p ($100,000 is the upper limit of the curve we intend to build)

The following question is posed: for what value of p would you consider options 1 and 2 equivalent. Imagine the answer is ¼ ($p = 0.25$) → the actor consider $10,000 with certainty of $100,000 with probability ¼ as equivalent

Repeating this for values different than $10,000
The utility curve can be built and used in decision analysis, simply replacing monetary payoff with utilities



Value just elicited

Maximum amount of money=Utility 1

Minimum amount of money=Utility 0

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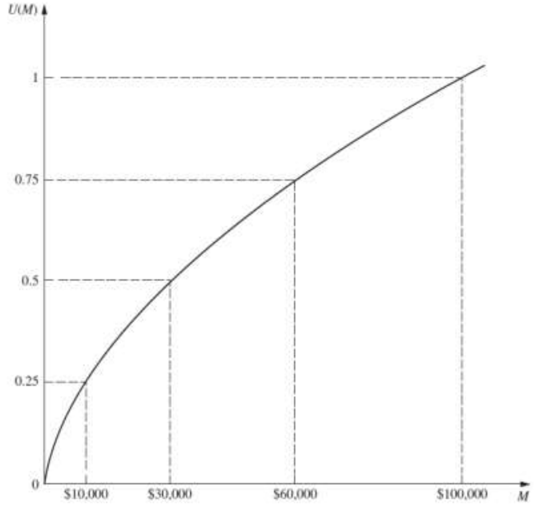

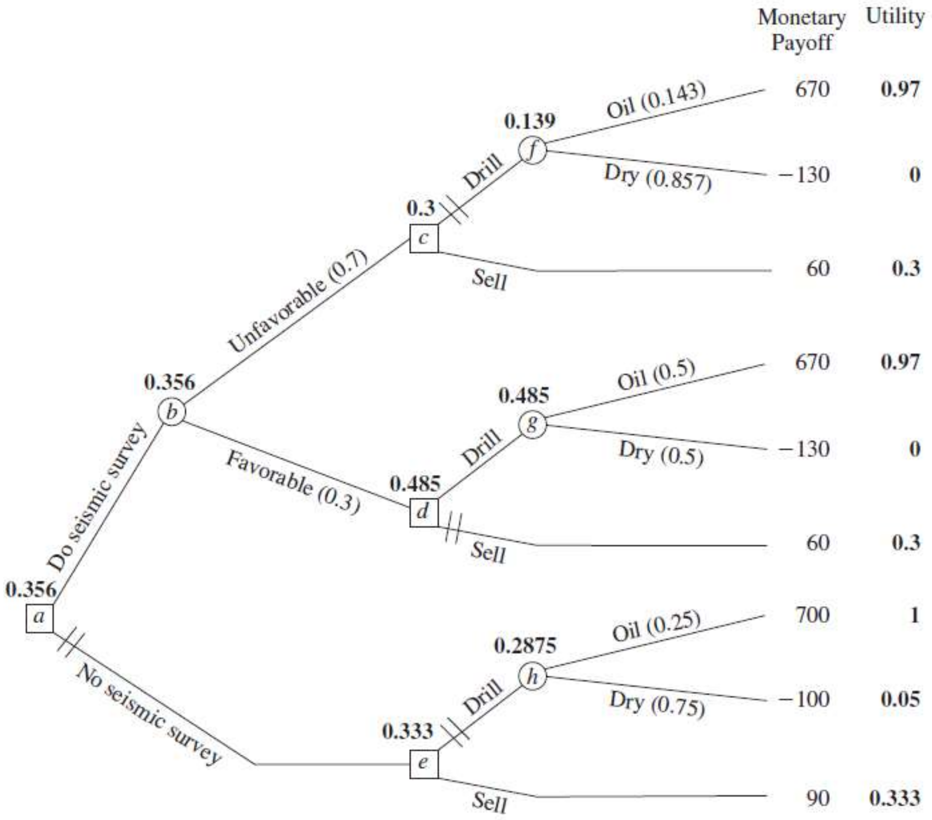

Using the utility curve the monetary payoff is replaced with utilities

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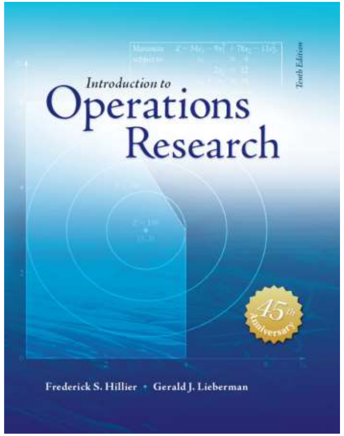
"However, many decision makers are not sufficiently comfortable with the relatively abstract notion of utilities, or with working with probabilities to construct a utility function, to be willing to use this approach. Consequently, utility theory is not yet used very widely in practice" (p. 715)



Source: https://www.alamy.com/

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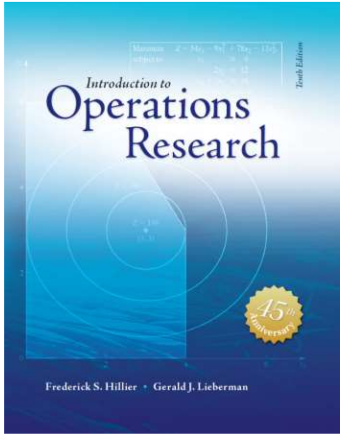
Source: https://www.alamy.com/

This idiosyncrasy to reckon in terms of abstract utilities or probabilities needs to be kept in mind if decision are taken in teams, e.g. in *Decision Conferencing*

Multiple criteria decision analysis



Using simultaneously more than one criterion

E.g. a company wishing to meet simultaneously goals of

- Profit
- Employment
- Capital investments

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A company wishing to meet simultaneously goals of

- Profit ≥ 125 (millions of dollars)
- Employment = 4 (hundreds of employees)
- Capital investments ≤55 investment goal

in the commercialization of three products (decision variables) x_1, x_2, x_3
Goals can be one sided upper (capital investment) or lower (profit) or two sided (employment).

The relation between decision variables and goals is defined as:

$$12x_1 + 9x_2 + 15x_3 \geq 125$$
$$5x_1 + 3x_2 + 4x_3 = 40$$
$$5x_1 + 7x_2 + 8x_3 \leq 55$$

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Note: MCDA section and this example are not available in the online version; this comes for the 11th version

A penalty weight is attached to violating the goal, i.e.

Weight=5 per unit below profit goal
Weight=3 per unit over investment goal
Weight=4 per unit over employment goal
Weight=2 per unit below employment goal

So the problem is linearized as

Minimize $Z = 5(\textit{amount under profit goal}) + 3(\textit{amount over investment goal}) + 4(\textit{amount over employment goal}) + 2(\textit{amount below employment goal})$

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Extreme caution should be used in this kind of linearization, as the use of penalty weights in a linear model may lead to paradoxes – we just 'scratch' the problem here and suggest to use different approaches

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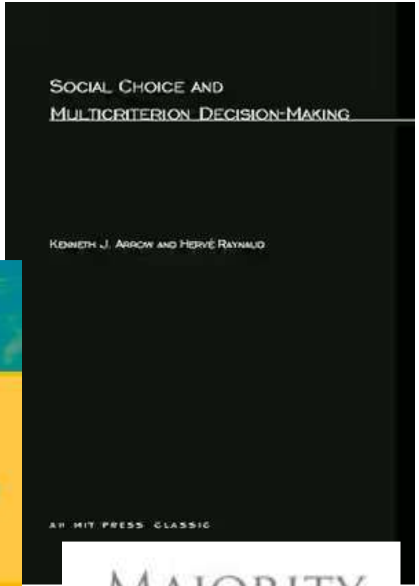
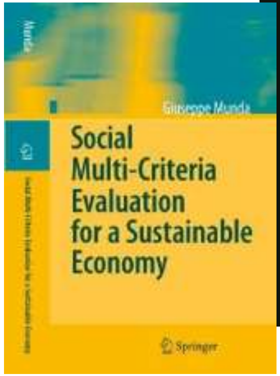
So the problem is linearized as

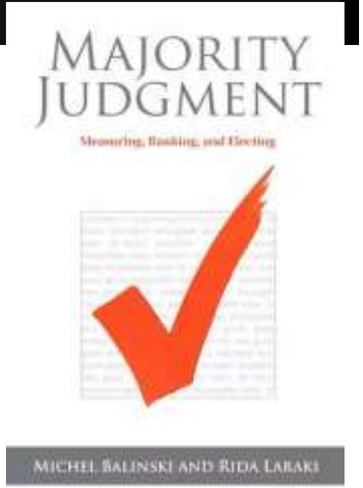
$$\text{Minimize } Z = 5(\textit{amount under profit goal}) + 3(\textit{amount over investment goal}) + 4(\textit{amount over employment goal}) + 2(\textit{amount below employment goal})$$

Extreme caution should be used in this kind of linearization, as the use of penalty weights in a linear form may lead to paradoxes – e.g. when the items above have appreciable covariance

Suggestion: list different viable options and rank them using methods such as Borda, Condorcet, Balinski-Laraki …







Some of these methods have a long history
(including in Catalonia)

Ramon Llull (Catalan, ca. 1232 – ca. 1315) proposed first what would then become known as the method of Condorcet. **Nicholas of Kues** (1401 – August 11, 1464), also referred to as Nicolaus Cusanus and Nicholas of Cusa developed what would later be known as the method of Borda. **Nicolas de Condorcet**, (17 September 1743 – 28 March 1794) developed the eponymous method. **Jean-Charles, chevalier de Borda** (May 4, 1733 – February 19, 1799) developed the Borda count

Images from Wikipedia Commons

An impact matrix

	Indic.	GDP ↑	Unemp. Rate ↓	Solid wastes ↓	Income dispar. ↓	Crime rate ↓
Country						
A		25,000	0.15	0.4	9.2	40
B		45,000	0.10	0.7	13.2	52
C		20,000	0.08	0.35	5.3	80
weights		.166	.166	0.333	.166	.166

We can say that
GDP 'votes' for B>A>C (countries / options)
UR 'votes' for C>B>A
SW 'votes' for C>B>A
ID 'votes' for C>A>B
CR 'votes' for A>B>C

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	Indic.	GDP	Unemp. Rate	Solid wastes	Income dispar.	Crime rate
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weights		.166	.166	0.333	.166	.166

GDP: B>A>C
UR: C>B>A
SW: C>A>B
ID: C>A>B
CR: A>B>C

# of indicators	2	1	1	1
1st position	c	b	c	a
2nd position	a	a	b	b
3rd position	b	c	a	c

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# of indicators	2	1	1	1
1st position	*c*	*b*	*c*	*a*
2nd position	*a*	*a*	*b*	*b*
3rd position	*b*	*c*	*a*	*c*

→

Rank	*a*	*b*	*c*
1st	1	1	3
2nd	3	2	0
3rd	1	2	2

Different ways to organize the same information: building a frequency matrix

Three countries [options/candidates] and five indicators [criteria/voters]

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# of indicators	2	1	1	1
1st position	*c*	*b*	*c*	*a*
2nd position	*a*	*a*	*b*	*b*
3rd position	*b*	*c*	*a*	*c*

→

Rank	*a*	*b*	*c*
1st	1	1	3
2nd	3	2	0
3rd	1	2	2

In this case Borda gives 3 minus 1 for each first rank , 2 minus 1 for each second rank and zero to the third

a gets 2*1+ 1*3=5
b gets 2*1+ 1*2=4
c gets 2*3+ 1*0=6

But lets try Borda on a more interesting case: (from Moulin, 21 criteria 4 options, cited in Munda 2008)

21 criteria 4 alternatives

Note:3+ 5+ 7+ 6=21

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

→

Rank	*a*	*b*	*c*	*d*	Points
1st	8	7	6	0	3
2nd	0	9	5	7	2
3rd	0	5	10	6	1
4th	13	0	0	8	0

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Borda count – Frequency matrix (Moulin, 21 criteria 4 options)

Columns add up to the number of criteria / voters=21

3 points if first
2 if second
1 if third
0 if last

Rank	*a*	*b*	*c*	*d*	Points
1st	**8**	**7**	**6**	**0**	**3**
2nd	**0**	**9**	**5**	**7**	**2**
3rd	**0**	**5**	**10**	**6**	**1**
4th	**13**	**0**	**0**	**8**	**0**

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Borda score:

$$a = 8 \times 3 = 24$$

$$b = 5 + 9 \times 2 + 7 \times 3 = 44$$

$$c = 10 + 5 \times 2 + 6 \times 3 = 38$$

$$d = 6 + 7 \times 2 = 20$$

Borda solution:
b→c→a→d

Rank	*a*	*b*	*c*	*d*	Points
1st	**8**	**7**	**6**	**0**	**3**
2nd	**0**	**9**	**5**	**7**	**2**
3rd	**0**	**5**	**10**	**6**	**1**
4th	**13**	**0**	**0**	**8**	**0**

Frequency matrix (21 criteria 4 alternatives)

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The Borda count was developed independently several times, (e.g. by Nicolaus Cusanus beginning XV century) but is named for Jean-Charles de Borda, who devised the system in 1770.

It is currently used for the election of two ethnic minority members of the National Assembly of Slovenia (https://www.electoral-reform.org.uk/how-do-elections-work-in-slovenia/)

It is used throughout the world by various organisations and competitions [e.g. in academia]

Jean-Charles, chevalier de Borda

Borda was a mariner and a scientist. Worked on chronometers. Between 1777 and 1778, he participated in the American Revolutionary War.

The French Academy of Sciences used Borda's method to elect its members for about two decades [till Napoleon Bonaparte became president…]

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Condorcet's outscoring matrix (21 criteria 4 alternatives)

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

Frequency matrix

$$\begin{bmatrix} & a & b & c & d \\ a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{bmatrix}$$

Outscoring matrix

B better than a 7+6=13 times

How to move from frequency to outscoring ?

# of indicators	3	5	7	6
1st position	*a*	*a*	*b*	*c*
2nd position	*b*	*c*	*d*	*b*
3rd position	*c*	*b*	*c*	*d*
4th position	*d*	*d*	*a*	*a*

Frequency matrix

$$\begin{bmatrix} & a & b & c & d \\ a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{bmatrix}$$

Outscoring matrix

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Condorcet's outscoring matrix
(21 criteria 4 alternatives)

For each pair of countries a concordance index is computed by counting how many indicators/voters are in favour of each country (e.g. 13 voters prefer b to a).

Note the "constant sum property" in the outranking matrix (13+ 8=21 number of indicators/voters)

$$\begin{bmatrix} & a & b & c & d \\ a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{bmatrix}$$

Outranking matrix

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How to use Condorcet's outscoring matrix (21 criteria 4 alternatives)

Pairs with concordance index > 50% of the indicators/voters are considered: majority threshold = 11 (i.e. a number of voters > 50% of voters=21)

Thus *aP none, bPa= 13, bPd=21(=always), cPa=13, cPb=11, cPd=14, dPa=13.*

c is better than a,b,d so it is the winner
b is better than the remaining a,d, it is the second best
d is better than a.

➔Condorcet solution: c → b→ d→ a

$$\begin{bmatrix} & a & b & c & d \\ a & 0 & 8 & 8 & 8 \\ b & 13 & 0 & 10 & 21 \\ c & 13 & 11 & 0 & 14 \\ d & 13 & 0 & 7 & 0 \end{bmatrix}$$

Count row-wise discarding entries < 11 as there are 21voters/criteria

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Borda solution: b → c → a → d

Condorcet solution: c → b → d → a

Can we choose between Borda and Condorcet on some theoretical and/or practical grounds?

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Homework

1. Both a dice and a coin are launched simultaneously in an experiment. We count a coin falling head as one and falling tail as a zero. If we call success the outcome seven (dice=six, coin=H), which is the chance of success in one experiment? Which is the chance of two successes in 4 experiments?

2. Read Hiller's chapter 17 Queueing Theory (pages 731-739)and write down ten practical problems that can be framed as queueing problems

3. Knowing that the average sales from a salesman are 0.9 items per day use the Poisson distribution to compute the probability that she sells (0,1,2,3,4,5,6) items in a given day.

Thank you

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https://www.youtube.com/channel/UCz26ZK04xchekUy4GevA3DA

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