Máster Universitario en Administración y Dirección de Empresas
Full Time MBA

Quantitative methods for decision making

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upf. BARCELONA SCHOOL OF MANAGEMENT

Elements of quantification for decision making with emphasis on operation research

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Andrea Saltelli

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Where to find this talk

August 25 2023: The politics of modelling is out!

Praise for the volume

"A long awaited examination of the role —and obligation —of modeling."
Nassim Nicholas Taleb , Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series Incerto.

***

"A breath of fresh air and a much needed cautionary view of the ever-widening dependence on mathematical modeling."
Orrin H. Pilkey, Professor at Duke University's Nicholas School of the Environment, co-author with Linda Pilkey-Jarvis of Useless Arithmetic: Why Environmental Scientists Can't Predict the Future, Columbia University Press 2009.

***

Mastodon Toots by

Thanks to Mariia Kozlova of LUT University in Finland for taking and curating this recording. My trajectory from number crunching to thinking about numbers' role in human affairs

View on

In this set of slides:

01 A playful introduction

02 A brief recap of probability

03 A mini-history of quantification and operation research

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A playful introduction

Methods were games at birth. Pre-analytic assumptions. The Seven Bridges of Königsberg. The problem of Luca Pacioli.

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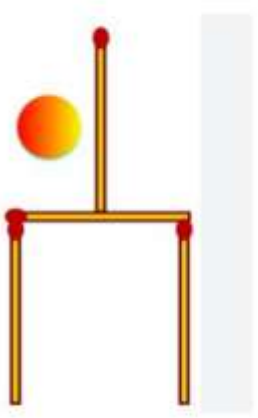
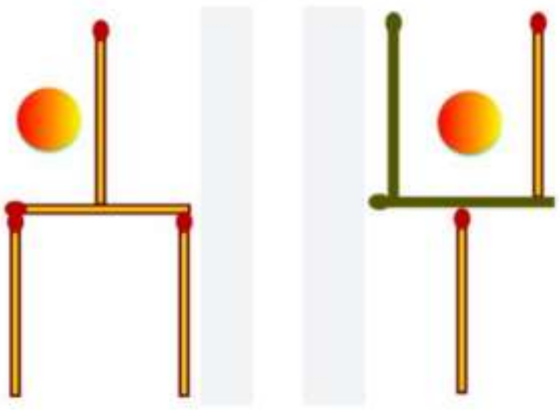


Put the ball in the collector
moving only two stitches



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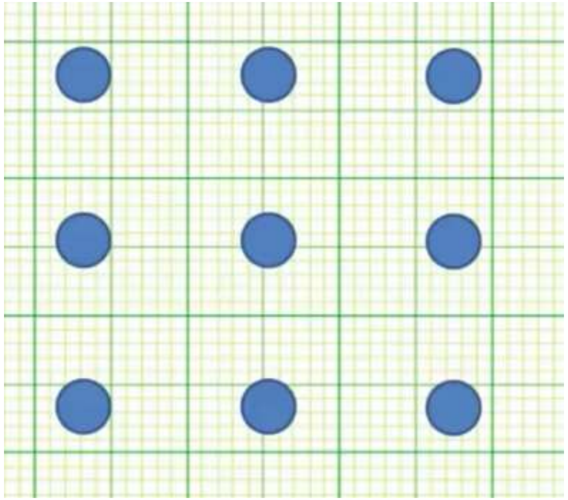

Without lifting your pencil, connect the nine points with four consecutive strokes

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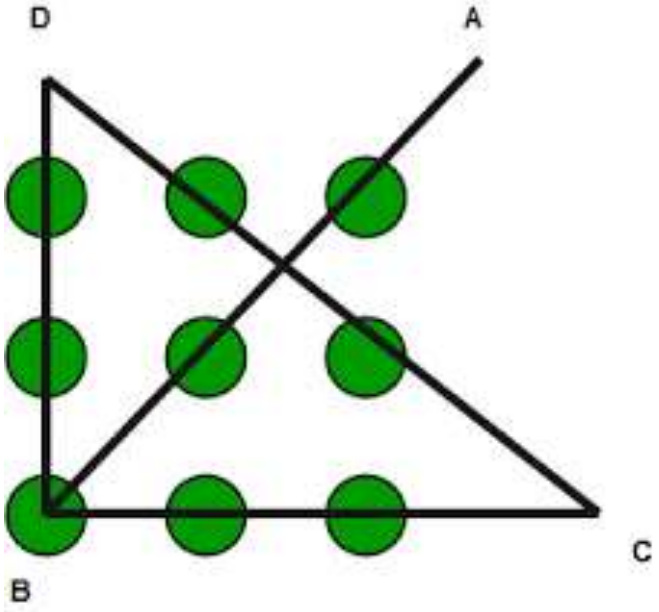
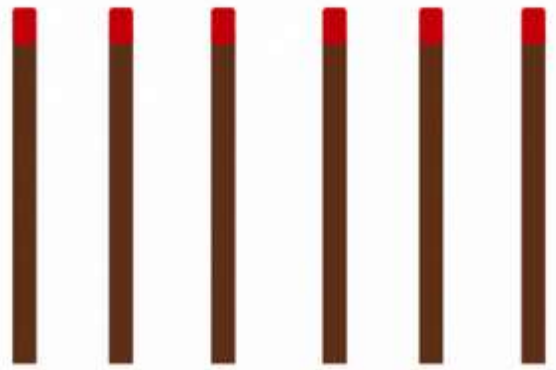
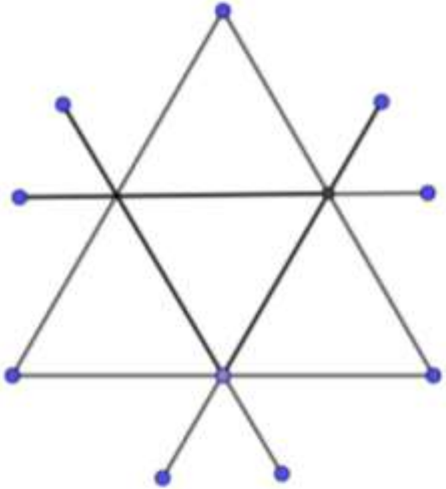

D
A
C
B



Using six stitches make four equal equilateral triangles



Which of the previous three games had unstated pre-analytic assumptions ‘broken’ by the solution?

Source: https://imgflip.com/

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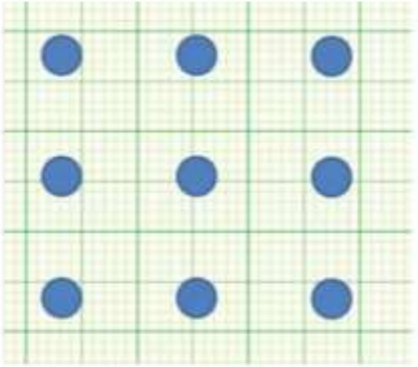
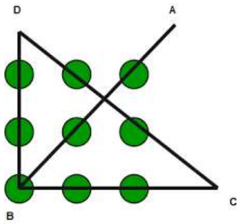
Which of the previous three games had unstated pre-analytic assumptions ‘broken’ by the solution?



Without lifting your pencil, connect the nine points with four consecutive strokes

Source: https://imgflip.com/

Staying in the square was not a specification of the problem but we normally assume it



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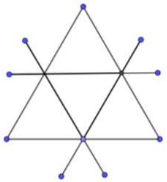
Which of the previous three games had unstated pre-analytic assumptions ‘broken’ by the solution?

Source: https://imgflip.com/

Using six stitches make four equal equilateral triangles



Staying in a plane was not a specification of the problem, nor that the size of the stitch was to be the size of the triangle, but we likely assumed both as true

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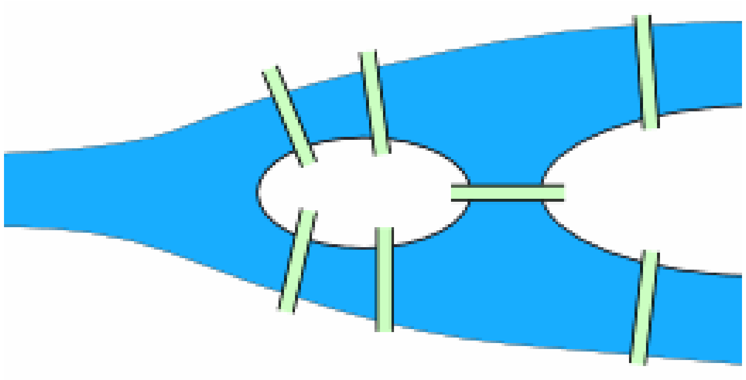
The Seven Bridges of Königsberg



Walk through the city crossing each of those bridges once and only once

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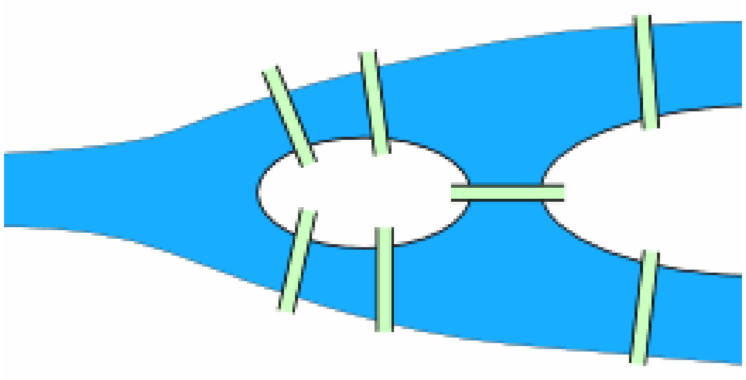

?

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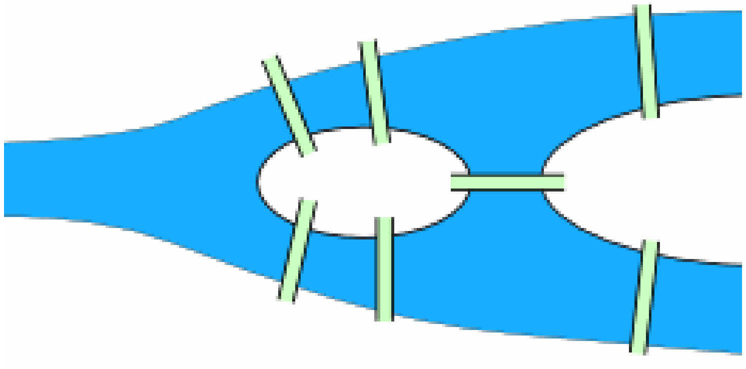
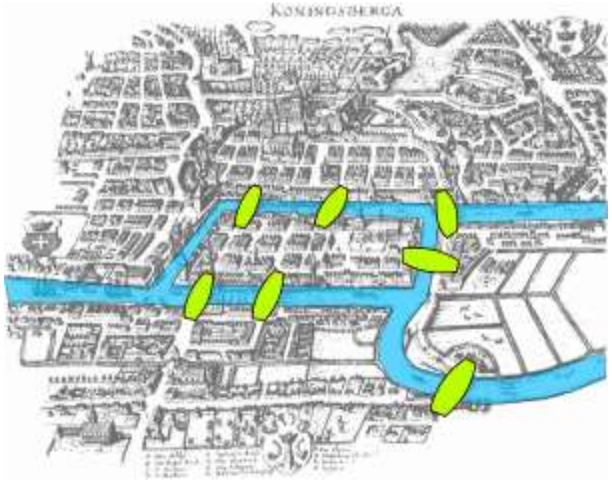


Leonhard Euler in 1736 posed this puzzle anticipating what is today graph theory



Map of Königsberg in Euler’s time.
Source: Wikipedia commons

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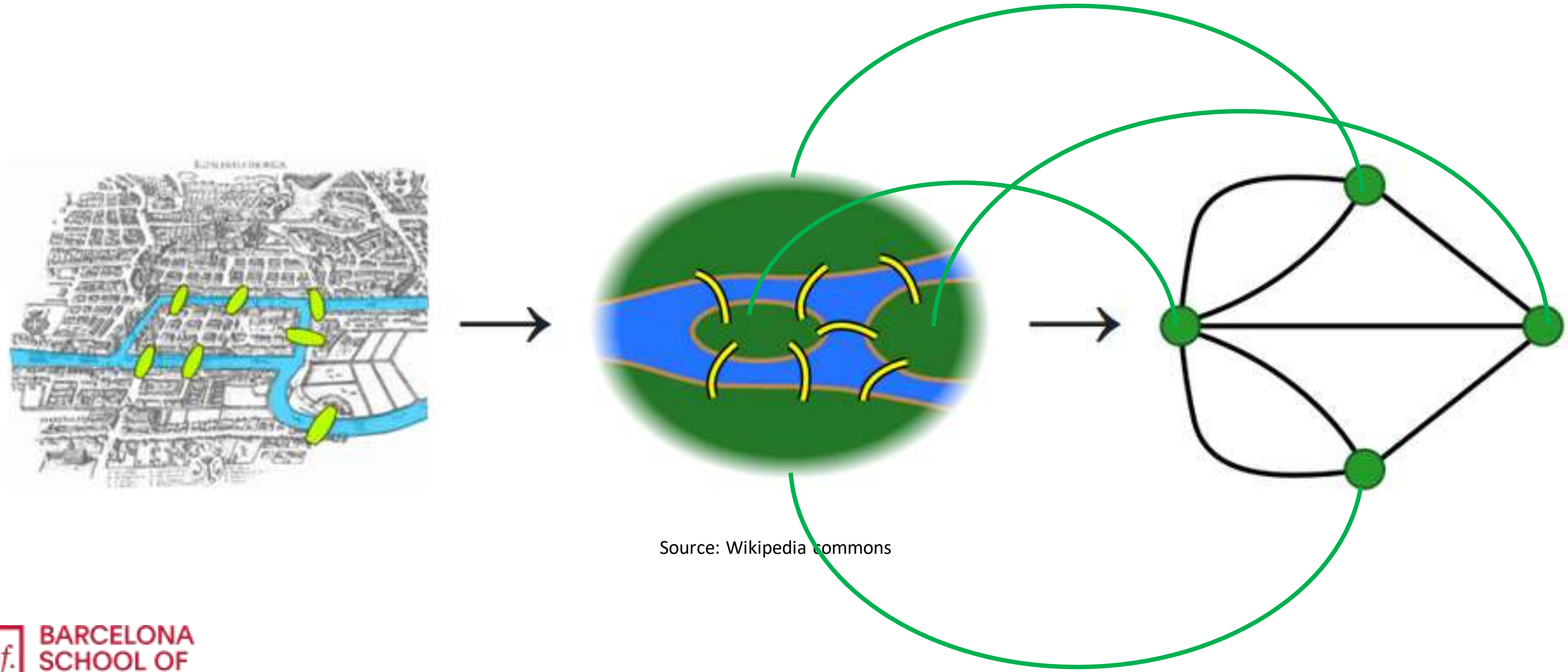
Euler proceeded by successive abstraction; a landmass is a node and a bridge is an edge … can you recognize the landmasses and nodes?



Source: Wikipedia commons

He noted that nodes with odd number of edges are problematic – how many nodes have odd edges here?

Source: Wikipedia commons

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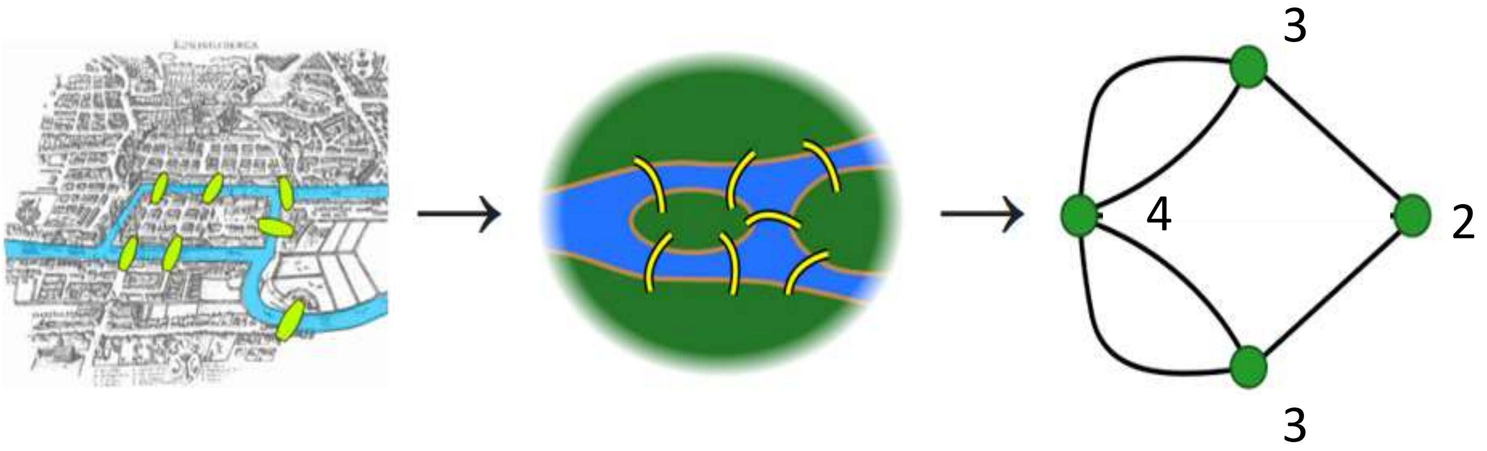
He concluded that the problem is soluble if either two or zero nodes have an odd number of edges. Try it out here.

Source: Wikipedia commons

… soluble if either two or zero nodes have an odd number of edges. Try to see if cutting off one edge does the job.



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The problem of Luca (the problem of points): two players interrupt a game. How should they split the money left on the table as a function of where they are in the game?

Luca Pacioli (1445 - 1517)

(1499-1557)

Blaise Pascal (1623-1662)

Pierre de Fermat (1601-1665)

Source for the images: Wikipedia Commons

Imagine a game where who scores six victories is the winner. The game is interrupted when player A is at 5 victories and player B is at 3 victories.

Luca Pacioli (1445 - 1517)

(One of) Luca Pacioli's solution:
Share in proportion to the number of games won

→ A=5/(5+3), B=3/(5+3)
A=5/8, B=3/8 (B=.375)

Imagine a game where who scores six victories is the winner. The game is interrupted when player A is at 5 victories and player B is at 3 victories.

(1499-1557)

Niccolò Tartaglia's solution:
Share in proportion to the lead divided the length of the game; lead of A=2, ratio 2/6=1/3, then A get his ½ money plus 1/3 of the ½ money of the opponent

➔ A=1/2+1/6=2/3, B=1/3 (B=.33)

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Imagine a game where who scores six victories is the winner. The game is interrupted when player A is at 5 victories and player B is at 3 victories.

Blaise Pascal (1623-1662)

Pierre de Fermat (1601-1665)

Their solution: ‘count’

A(5) B(3)

A(6) wins B(4) p=1/2

A(6) wins B(5) p=1/4

A(6) wins B(6) wins p=1/8

A=1/2+1/4+1/8=7/8, B=1/8 (B=.125)

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Why do we say that Pascal and Fermat were 'better' than Pacioli and Tartaglia?

Luca Pacioli (1445 - 1517)

Luca Pacioli's solution (split proportional to number of wins) becomes absurd when, for example, the game stops at the fist win of A, in which case A takes all

Why do we say that Pascal and Fermat were ‘better’ than Pacioli and Tartaglia?

(1499-1557)

Niccolò Tartaglia’s solution (split in proportion to the lead) becomes absurd when, for example, a lead of one at the beginning of the game gives the same split than a lead of one toward its end

Pascal and Fermat are ‘better’, in the sense of ‘fairer’. The fairness of a quantification is an important point, discussed more at the end of the course

More reading: Cantillo, Andres. 2011. ‘The Problem of Points’. 50831. Munich Personal RePEc Archive (MPRA). https://mpra.ub.uni-muenchen.de/50831/.

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2.

A brief recap of probability

When is a bet fair? Some element of probability calculus that opens the door to more games and examples. St. Petersburg paradox. Bayes' theorem (introduction). Frank Knight's risk and uncertainty. Fisher's discerning tea drinker. Permutations and combinations. Binomial distribution. Some computation with Excel. Based on Mann (2010) and other sources.

Possible introductory reading for statistics

Fair games: is it fair to play a 7 against a 3?

Photo: Creator: Bob Adelman | Credit: © Bob Adelman/Corbis. Dices https://creazilla.com/

Fair games: is it fair to play a 7 against a 3?

How can a 7 come about?						
6	5	4	3	2	1	
1	2	3	4	5	6	

Fair games: is it fair to play a 7 against a 3?

How can a 3 come about?		
2	1	
1	2	

How can a 7 come about?						
6	5	4	3	2	1	
1	2	3	4	5	6	

How can a 3 come about?		
2	1	
1	2	

How can a 7 come about?					
6	5	4	3	2	1
1	2	3	4	5	6

How can a 3 come about?	
2	1
1	2

How could the game be made fear?

3 against 7 should be given 3 to one:

- in case of victory the player betting 3 get $3,
- in case of victory the player playing 7 gets one dollar

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Bad luck: strike and rain

Photo: Toronto Public Library Archive. Drawing: Shutterstock

Strikes on average once every 100 days

Rains on average once every 10 days

What is the probability of rain and strike in any given day?

Source: https://imgflip.com/

Strikes on average once every 100 days

Rains on average once every 10 days

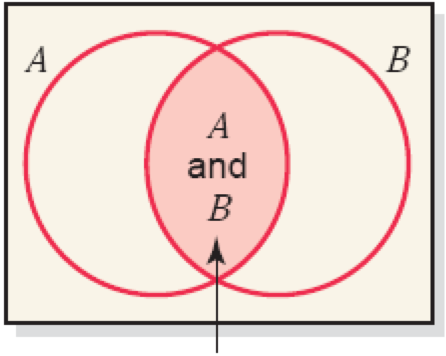
Definition

Joint Probability The probability of the intersection of two events is called their *joint probability*. It is written as

$$P(A \text{ and } B)$$

P(A)=P(strike)=1/100
B(B)=P(rain)=1/10

A and B are independent



Intersection of *A* and *B*

P(A and B)=P(A)P(B)=
(1/10)/(1/100)=1/1000

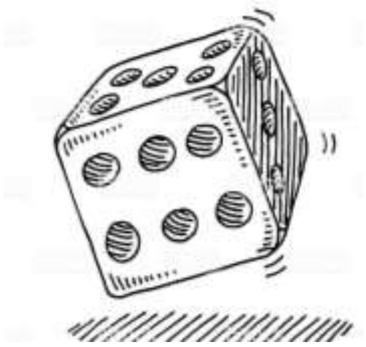
Some definitions

Definition

Experiment, Outcomes, and Sample Space An *experiment* is a process that, when performed, results in one and only one of many observations. These observations are called the *outcomes* of the experiment. The collection of all outcomes for an experiment is called a *sample space.*

Source: Mann, Prem S. 2010. Introductory Statistics. Wiley.

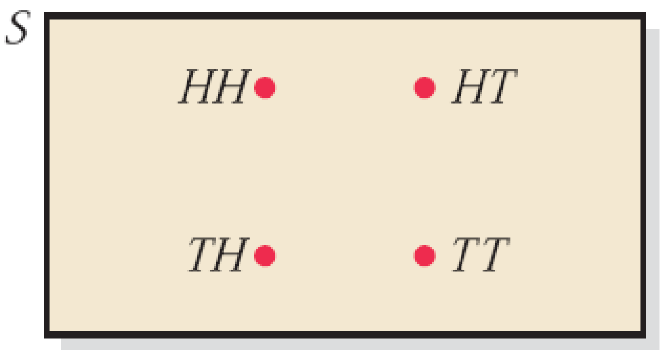
Experiment = tossing a coin
Outcomes = H,T
Sample space = {H,T}



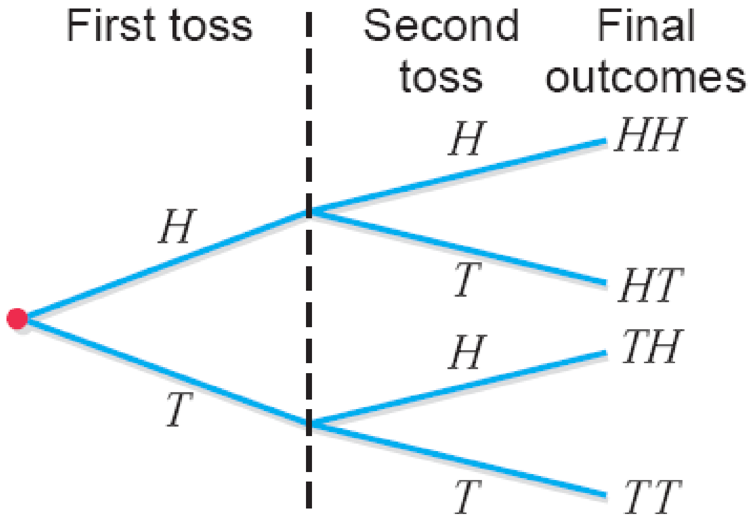
iStock by Getty images

Experiment = rolling a dice
Outcomes = 1,2,3,4,5,6
Sample space = {1,2,3,4,5,6}

Venn diagram representing the sample space when experiment = tossing two coins



Tree diagram for the same experiment



The probability is a number between 0 and 1

$$P(\text{Head}) = P(\text{Tail}) = 1/2$$

$$P(\text{Head}) + P(\text{Tail}) = 1$$

The sum of the probabilities of all possible outcomes is one

Definition

Law of Large Numbers If an experiment is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual or theoretical probability.

Source: Mann, Prem S. 2010. Introductory Statistics. Wiley.

The theoretical probability of a head in a toss of a coin is $\frac{1}{2}$

True or false?

Source: https://imgflip.com/

Definition

Law of Large Numbers If an experiment is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual or theoretical probability.

Source: Mann, Prem S. 2010. Introductory Statistics. Wiley.

The theoretical probability of a head in a toss of a <u>fair</u> coin is $\frac{1}{2}$

↑

<u>assumption</u>

Definition

Law of Large Numbers If an experiment is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual or theoretical probability.

Law published posthumously in 1713

The same Bernoulli of the Euler number *e* , named so after the Swiss mathematician Leonhard Euler

Jacob Bernoulli
(1655-1705)

How did Bernoulli get there? Studying compound interest

If the yearly interest is p, after a year your capital C becomes $C_1=C_0*(1+p)$, after the second year $C_0*(1+p)*(1+p)$ ⋯ and after n years $C_0*(1+p)^n$

If the bank decides to favour you paying the interest every month this becomes:

$C_0*(1+p)^n \rightarrow C_0*(1+p/12)^{n*12}$

(Instead of p every year you get p/12 every month)

Jacob Bernoulli
(1655-1705)

How about paying the interest every day?

$C_0*(1+p)^n$ ➔ $C_0*(1+p/12)^{n*12}$ ➔ $C_0*(1+p/365)^{n*365}$

(Instead of p every year you get p/365 every day)

If we rewrite **x=n*365 w**e can write this as

➔ $C_0*(1+pn/x)^x$

… you see where this is leading:

$$C_n = C_0 \lim_{x\to\infty}\left(1+\frac{pn}{x}\right)^x = C_0 e^{pn}$$

'Famous' limit: $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

Jacob Bernoulli
(1655-1705)

We cannot leave the Bernoulli family before mentioning the first appearance of utility theory, discussed by Daniel Bernoulli and his cousin Nicolas

This is the story the St. Peterburg paradox (another game!)

But first ··· Would you accept one million dollars with certainty or one chance in ten of winning 20 millions?

Daniel Bernoulli
(1700-1782)

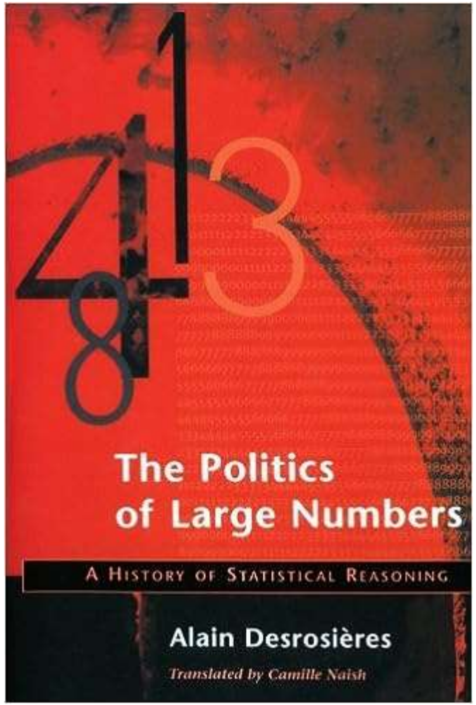
St. Petersburg paradox

"Peter throws "heads or tails" until heads appears once. He gives one dollar to Jack if the first throw is heads; two dollars if heads appears only on the second throw, four dollars if it appears on the third throw . . . and 2^{n-1} dollars if heads first appears only at the n^{th} throwing. Jack's expectation of winning is:

$$\frac{1}{2} + 2\left(\frac{1}{2}\right)^2 + 2^2\left(\frac{1}{2}\right)^3 + \cdots\ 2^{n-1}\left(\frac{1}{2}\right)^n$$

It is therefore infinite" (Desrosières, p. 54)

How much should Jack pay to enter the game?



While Nicolas would have taken the bet at any sum, Daniel retorted that no 'prudent' man would have done that – anticipating the concept of risk aversion of modern utility theory

Nicolas used the expected value, logical choice over a repeated, possibly infinite, series of games, but absurd otherwise

Nicolas Bernoulli
(1687-1759)

Daniel Bernoulli
(1700-1782)

Definition

Law of Large Numbers If an experiment is repeated again and again, the probability of an event obtained from the relative frequency approaches the actual or theoretical probability.

John Edmund Kerrich: tossing a coin 10,000 times ➔ 5067 heads (1946, before large computers)

Source: https://en.wikipedia.org/wiki/John_Edmund_Kerrich

Thomas Bayes and his theorem

Thomas Bayes, 1701–1761

Two ways of looking at probabilities

Bayesian, reason to believe, degree of conviction, 'subjective', inductive…

Frequentist, the limit of a sequence of equiprobable trials; also called 'objective', statistical…

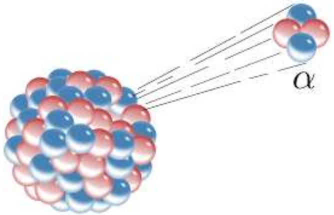
Strong beliefs (even wars) in both positions

Bayesian, XVIII century, a comeback in the XX

Frequentist, XIX century, mainstream e.g. in teaching

Other distinguish not two but three ways of defining probabilities

Equal possibilities based
on physical symmetry
(coins, dices)

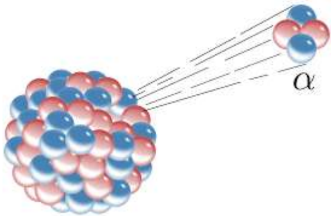


Observed frequencies (decay)

Degree of conviction (belief!)
Experiments

Equal possibilities

Frequencies



Degrees of belief

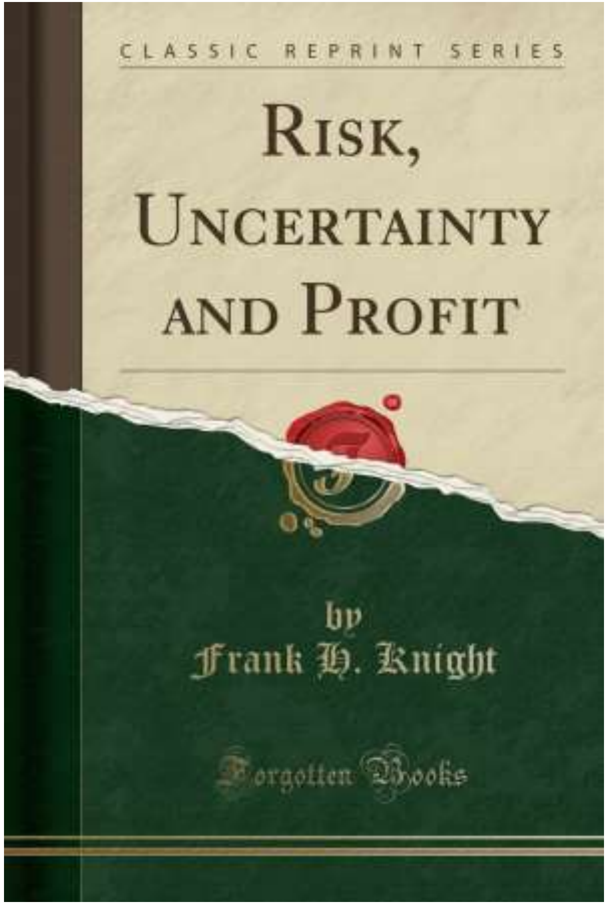
Are these separate domains?

Source: https://imgflip.com/

Frank Knight (1921) distinguished risk from uncertainty

Risk = know outcomes & probabilities; roulette game

Uncertainty = unsure about the probabilities; starting a business



Frank H. Knight
1885-1972

Quote:

"We live in a world of contradiction and paradox, a fact of which perhaps the most fundamental illustration is this: that the existence of a problem of knowledge depends on the future being different from the past, while the possibility of the solution of the problem depends on the future being like the past."

Frank H. Knight
1885-1972

Probability of A given B: $P(A|B)$

Probability of B given A: $P(B|A)$

Example:

A=I have disease COVID-19
B=My clinical test for disease COVID-19 is positive

Example:

A=I have disease X
B=My clinical test for disease X is positive

In general $P(A|B)$ is neither 1 nor $P(A)$,
nor in general is $P(B|A)$ 1 or $P(B)$ because …

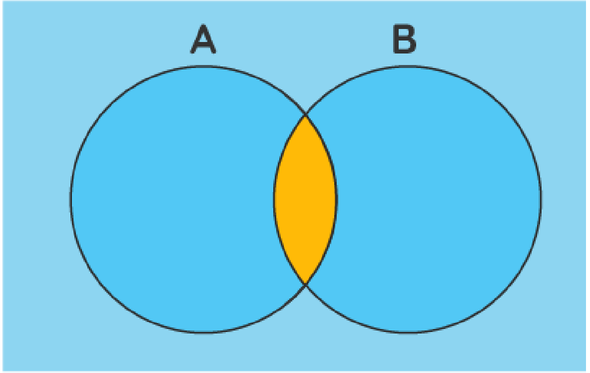
The fact that the test is positive does not guarantee that I have the disease, nor the fact that I have the disease guarantees that the test will be positive (more soon)

Source: https://www.123rf.com/photo_30337574_sick-boy-lying-in-bed.html

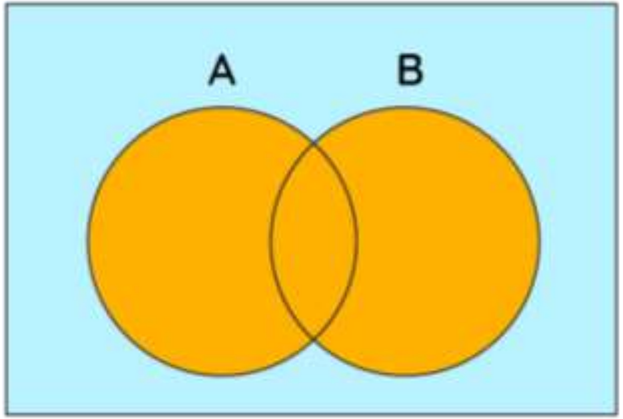
Probability of A given B: $P(A|B)$

Probability of B given A: $P(B|A)$

Probability and A **and** B being simultaneously true $P(A \cap B)$



Probability and A **or** B being true P(A**U**B)



In general $P(A|B) \neq P(B|A)$ but

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

If A and B are independent

$P(A|B)=P(A)$ and $P(B|A)=P(B)$

So that

$$P(A\cap B)= P(A|B)P(B)=P(B|A)P(A)=P(A)P(B)$$

(example strike and rain)

A classic exercise in screening

You test positive for AIDS (one test only). Time for despair?

Only one 1 in 100,000 has AIDS in your population

The test has a 5% false positive rate

Already one can say: in a population of say 100,000 one person will have AIDS and 5,000 (5% of 100,000) will test positive

➔ Don’t despair (yet) … why?

In a population of say 100,000 one person will have AIDS and 5,000 (5% of 100,000) will test positive; so your chance of having AIDS is
1/5000 = 0.0002 = 0.02%

… but we can use Bayes instead

$$P(AIDS|TestPositive)P(TestPositive) = P(TestPositive|AIDS)P(AIDS)$$

$$P(AIDS|TestPositive) = \frac{P(TestPositive|AIDS)P(AIDS)}{P(TestPositive)}$$

The values to plug in

P(Test Positivo)=0.05 [approximation]
P(AIDS)=0.00001 [prevalence]
P(Test Positivo|AIDS)= 1 [assumption of no false negative]

$$P(AIDS|TestPositive) = \frac{1*0.00001}{0.05} = 0.0002 = 0.02\%$$

as before

The power of Bayesian statistics is foremost in saying something about unknown causes given known events

Theory Hi is under discussion and an experiment is performed that gives the outcome E

What is **now** the probability of Hi?

If the experiment gives the outcome E, how does the probability of Hi change?

- If I know the probability of Hi before the experiment P(Hi), known as 'the prior'

- And I also know how probable event E would be if Hi were true, then

$$P(H_i|E) = \frac{P(E|H_i)P(H_i)}{P(E)}$$

If there are alternative theories that would result in outcome E, H1,H2,⋯

$$P(H_i|E) = \frac{P(E|H_i)P(H_i)}{P(E|H_1) + P(E|H_2 \ldots)}$$

$$P(H_i|E) = \frac{P(E|H_i)P(H_i)}{P(E|H_1) + P(E|H_2 \ldots)}$$

"the probability of the existence of each cause is equal to the probability of the event given that cause, divided by the sum of all the probabilities of the event given each of these causes"

(Laplace, 1774)

Pierre Simon Laplace
1749-1827

Inferring the causes from observed events is "the way of the historian, the policemen, and a doctor, who suggest a diagnosis based on symptoms" (Desrosières 1993)

Arthur Conan Doyle was a medical doctor …

Source: https://www.pngwing.com/en/free-png-nsxet

$$P(H_i|E) = \frac{P(E|H_i)P(H_i)}{P(E|H_1) + P(E|H_2 \ldots)}$$

The probability of theory Hi given outcome E also depends upon the probability of E for all possible theories

$$P(H_i|E) = \frac{P(E|H_i)}{P(E)} P(H_i)$$

posterior

likelihood

prior

A discerning tea drinker

From Mary Evans picture library
https://www.maryevans.com/search.php?prv=preview&job=5382258&itm=2&pic=10029395&row=1

Claims she can distinguish a cup of tea where milk has been added before pouring the tea or after

Eight cups of tea are given to her where four times the milk is added before, and four after. She can tell the difference every time.

Luck?

https://www.twinkl.es/resource/teacup-display-cut-outs-t-m-3640

The problem is hence to pick the four ‘right’ cups / colors out of eight ···

Solving this demands some combinatorial calculus···

Combinations and permutations

ABC
ACB
BAC
BCA
CAB
CBA

Permutations of three elements (A,B,C) in groups of three

Combinations and permutations

ABC
ACB
BAC
BCA
CAB
CBA

··· they are six because one has three ways to select the fist letter, two ways to select the second, and only one way to select the third. Permutations = $3*2*1=6$

In the general case of n items to be compared in groups of k the number of permutations is

$$n*(n-1)*(n-2)\cdots(n-k+1)$$

Combination and permutations

ABC
ABD
ACD
BCD

Combinations of four elements in groups of three;

If these were instead permutations they would be 4*3*2=24 because each of the four combinations above would give more permutations e.g.

ABC=>ABC,ACB,BAC,BCA,CAB,CBA

(three elements in group of three=3*2*1)

ABC
ABD
ACD
BCD

Four combinations of A,B,C,D,
each of which gives six permutations

ABC=>ABC,ACB,BAC,BCA,CAB,CBA

$$\text{Permutations} = n*(n-1)*(n-2)\cdots(n-k+1)$$

$$\text{Combinations} = \frac{n*(n-1)*(n-2)\cdots(n-k+1)}{k*(k-1)*(k-2)\cdots(k-k+1)}$$

If we call $k! = k*(k-1)*(k-2)\cdots(k-k+1)$ (or k factorial, where $0!=0$), then the number of combinations of n elements in groups of k is

$$n*(n-1)*(n-2)\cdots(n-k+1)/k!$$

e.g. four elements in groups of three $4*3*2/(3*2)=4$ as in the example

ABC
ABD
ACD
BCD

If $k! = k*(k-1)*(k-2)\cdots*1$

Then the number of combinations of n elements in groups of k is

$n*(n-1)*(n-2)\cdots(n-k+1)/k!$ →

$$\binom{n}{k}$$

Written as

e.g. four elements in groups of three
$4*3*2/(3*2)=4$ as in the example

ABC
ABD
ACD
BCD

And computed as

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Back to the discerning tea drinker

How many ways are there to pick four cups out of eight?

Are these combinations or permutations?

Combinations; how many then?

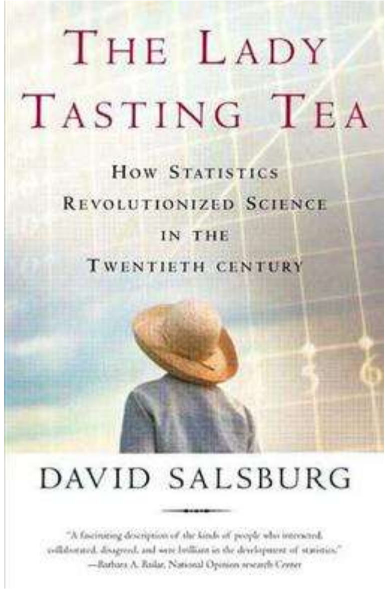
$$\binom{8}{4} = \frac{8!}{4!(4!)} = \frac{8*7*6*5}{4*3*2} = 2*7*5 = 70$$

➔ There are 70 ways to select 4 cups out of eight; the odds of getting it right by chance are 1/70 ≈0.0143 or 1.43%

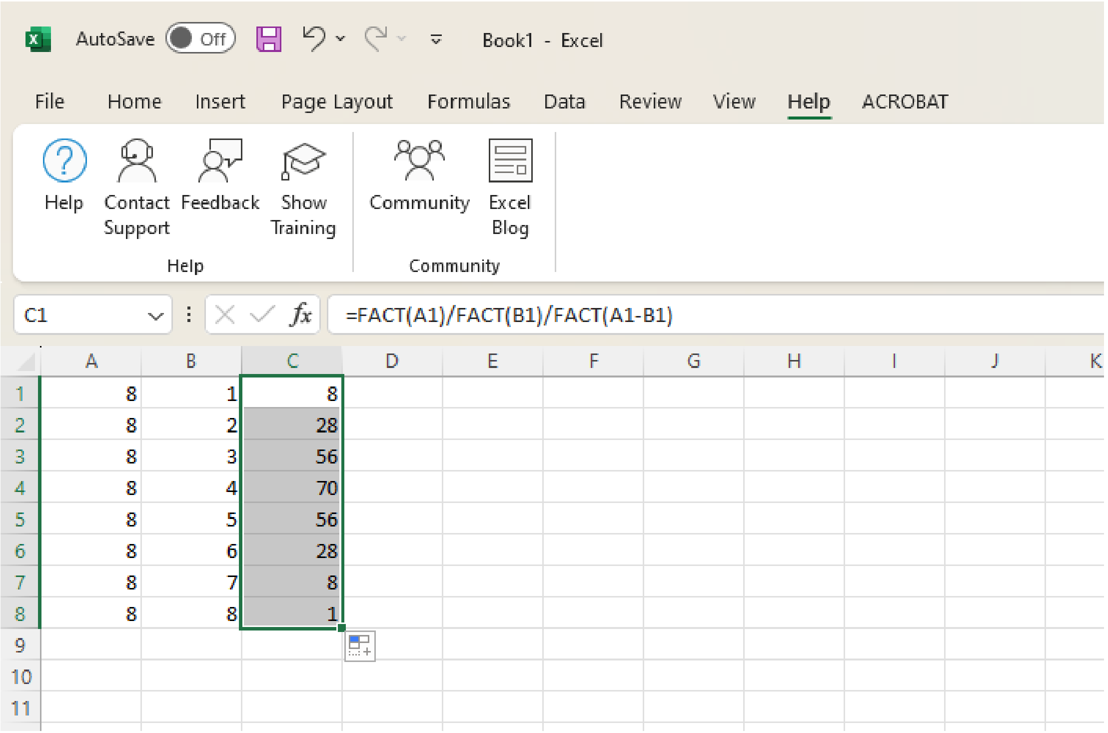
This was in fact a real experiment run by statistician Ronald Fisher and reported in 1935

The thinking behind the experiment is used in what we now call the p-test which is ubiquitous in experimental sciences

Ronald Fisher
(1913-1890)
Source: Wikipedia Commons



What if the test had involved still 8 cups, but the milk had been poured before the tea 2,3, or another number of times instead of four? What would be the odds to get them right by chance? Compute it using Excel



The so-called binomial distribution comes handy to compute probabilities for sequences of n independent experiments (coin dices…), each leading to either success (with probability p) or failure (with probability 1-p)

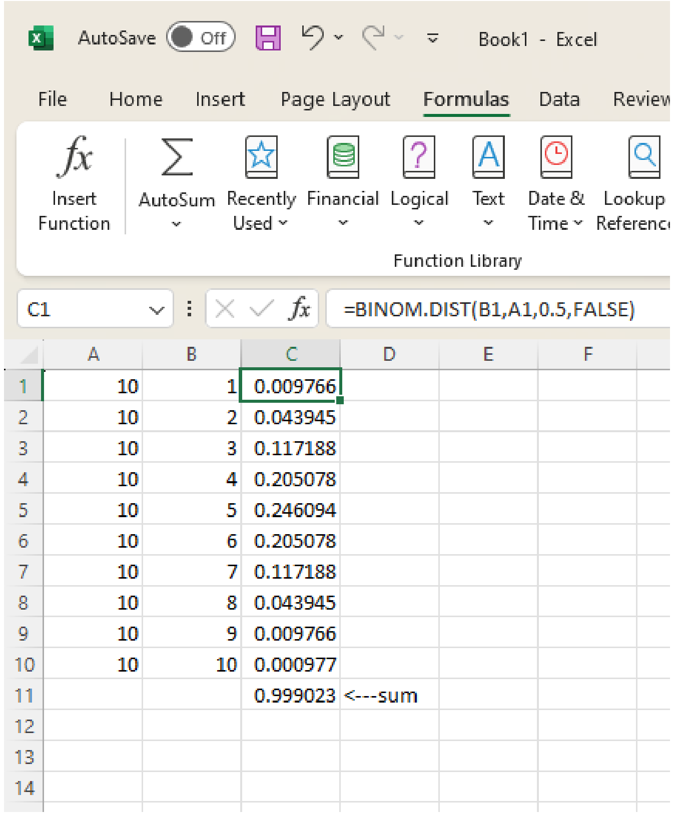
$$f(k,n,p) = Pr(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

Probabilities of getting exactly two heads launching a coin 5 times

$$\Pr(2;5,{}^{1}/{}_{2}) = \binom{5}{2}\left(\frac{1}{2}\right)^2 (1-\frac{1}{2})^{5-2}$$

$$= \binom{5}{2}\left(\frac{1}{2}\right)^5 = \frac{10}{32}$$

If instead I launch a coin ten times what are the probabilities of getting exactly 1,2,⋯10 heads? Compute in Excel



	A	B	C	D	E	F
1	10	1	0.009766			
2	10	2	0.043945			
3	10	3	0.117188			
4	10	4	0.205078			
5	10	5	0.246094			
6	10	6	0.205078			
7	10	7	0.117188			
8	10	8	0.043945			
9	10	9	0.009766			
10	10	10	0.000977			
11			0.999023	<---sum		
12						
13						
14						

There are also combination and permutation with repetition ⋯ keep these formulae at hand

	Permutations n elements in classes of k (variations)	Combinations n elements in classes of k
No repetition	$\frac{n!}{(n-k)!}$	$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$
Repetition	n^k	$\binom{n+k-1}{k}$

There are also combinations and permutations with repetition: example **three objects ABC in groups of 2**

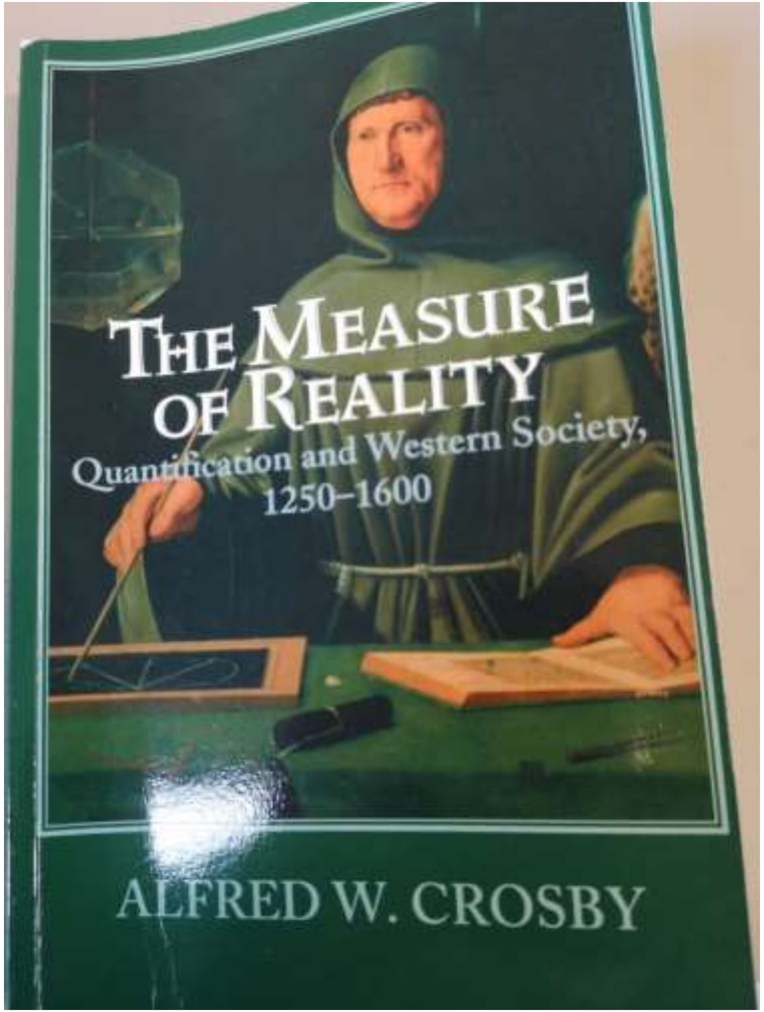
	Permutations 3 elements in classes of 2 (variations)	**Combinations 3 elements in classes of 2**
No repetition	AB,BA,AC,CA,BC,CB (3!/1!=6)	AB,AC,BC (3!/2!=3)
Repetition	AA,BB,CC,AB,BA,AC,CA,BC,CB (3^2=9)	AA,BB,CC,AB,AC,BC (4!/(2!2!)=6)

3.

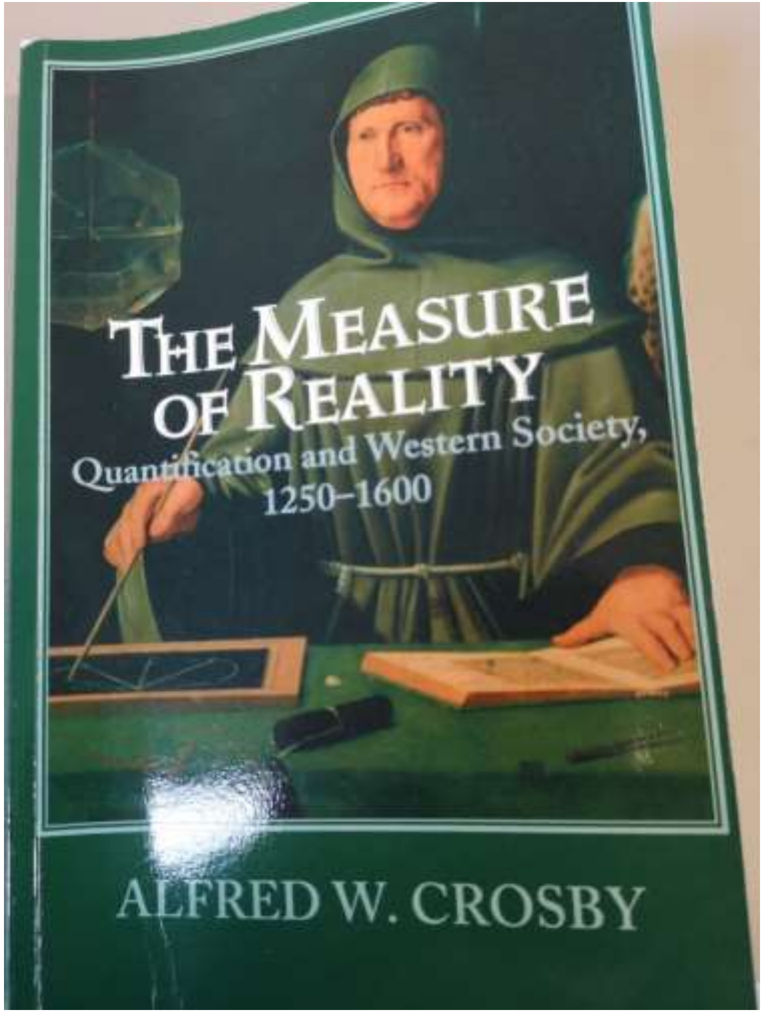
A mini history of quantification and operation research

When did we learn to quantify and plan using numbers? And how important is quantification for the our civilization? Mostly based on Gass, S.I. and Assad, A.A. (2006) An Annotated Timeline Of Operations Research: An Informal History. 2006 edition. New York: Springer-Verlag New York Inc. The prisoners dilemma and Nash equilibrium.

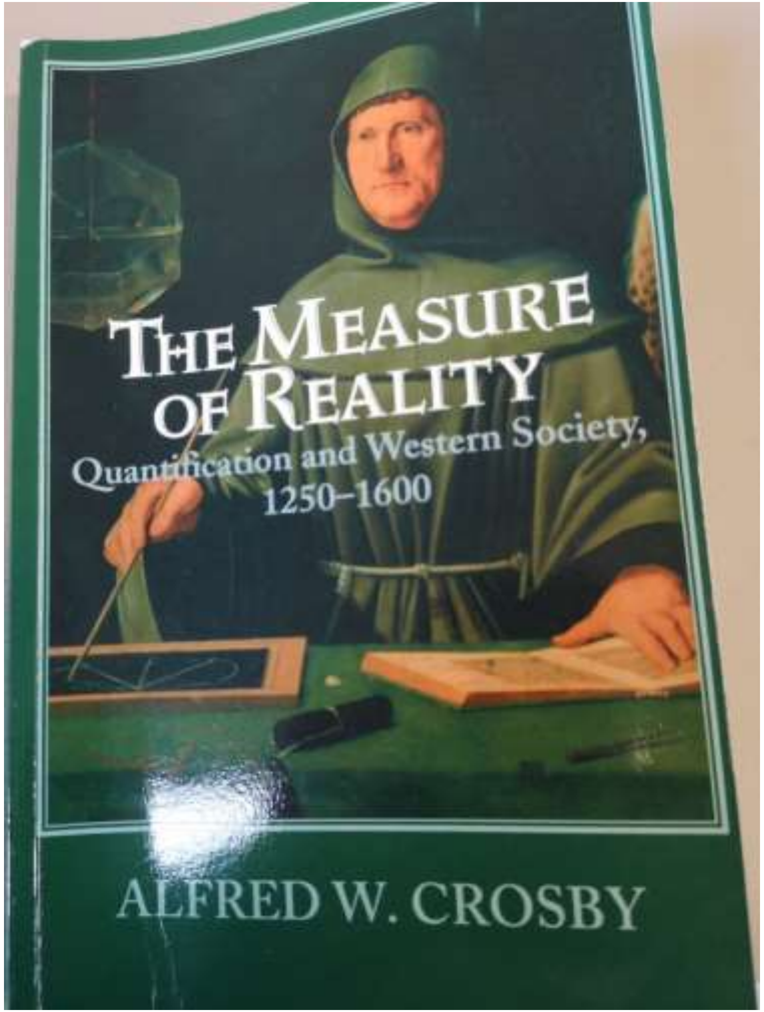
upf. BARCELONA SCHOOL OF MANAGEMENT



Were quantification and visualization the engine inside the engine of western success and domination?



Quantification and visualization of space and time gave rise in the XIV century to a true revolution, in music, painting, accounting, cartography, astronomy …



… a revolution that in the following two centuries XV–XVI ensured the epochal success of the West and its domination over the rest of the world

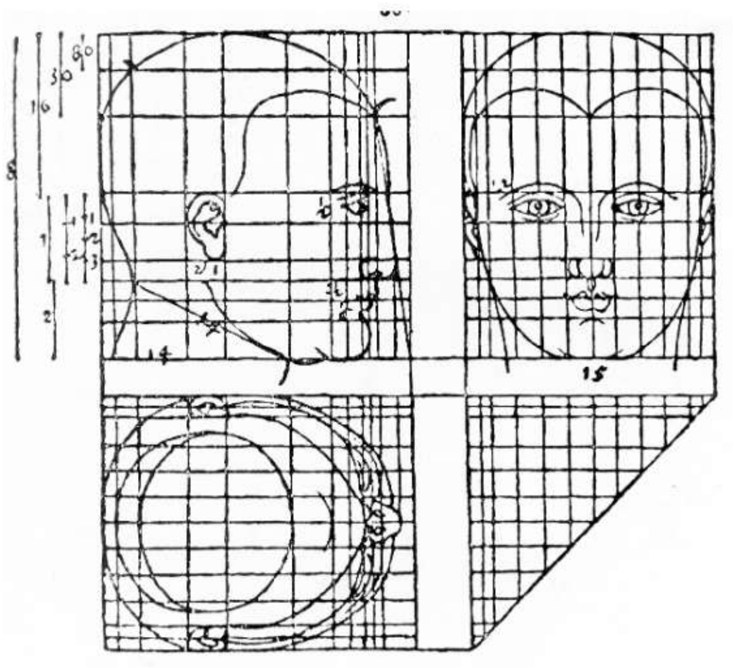
Pieter Bruegel the Elder, Temperance, 1560

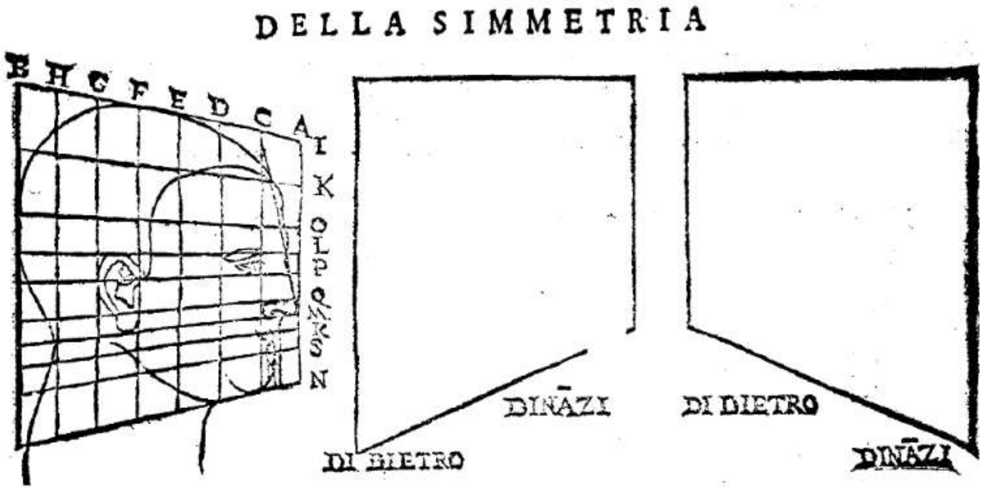
Measuring, military technology (math), dispute on a printed bible, learning, accounting, perspective, polyphonic music, the windmill, the watch …

From the abacus to Arabic numerals

The Annunciation, Carlo Crivelli (1435, 1495)

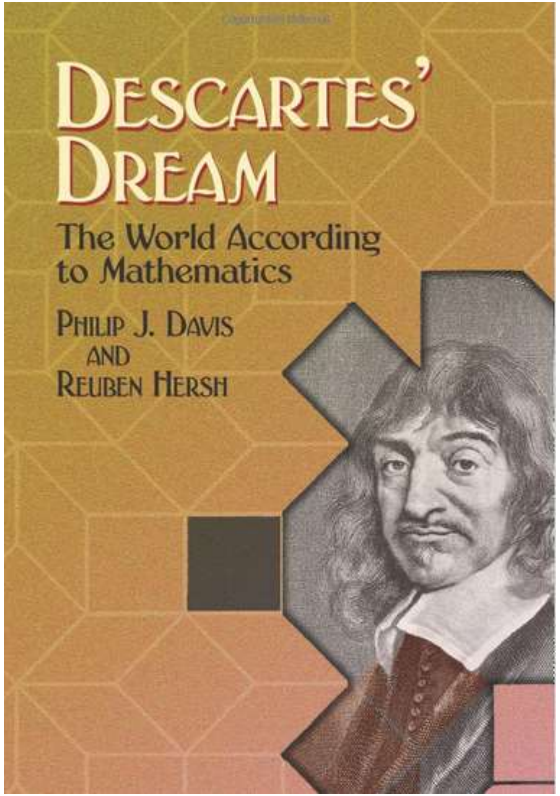
Draftsman Drawing a Reclining Nude
Albrecht Dürer (1471-1528)

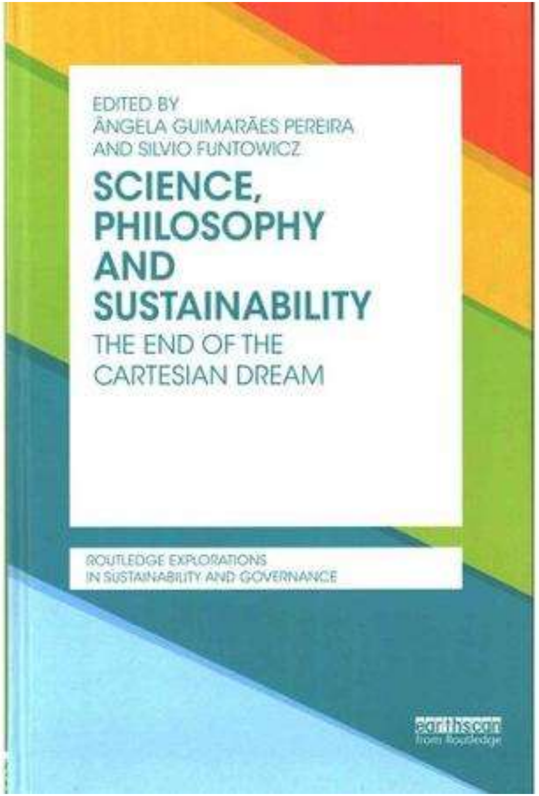




From “De Varietate figurarum”
Albrecht Dürer (1471-1528)

Quantification as a key element of the Cartesian Dream





Quantifications and the roots of the Cartesian dream

Separate but related stories

Cartesian dream:
possess and domination
of nature

Achieving progress with
calculations

We call Cartesian dream the idea of man as master and possessor of nature, of prediction and control, of Bacon's wonders of science and of Condorcet's mathematique sociale…

Francis Bacon
(1561-1626)

René Descartes
(1596-1650)

Nicolas de Caritat, marquis de Condorcet
(1743- 1794)

Francis Bacon
(1561-1626)

Magnalia Naturae, in the New Atlantis (1627),
'Wonders of nature, in particular with respect to human use'

The prolongation of life; The restitution of youth in some degree; The retardation of age; The curing of diseases counted incurable; The mitigation of pain; More easy and less loathsome purgings; The increasing of strength and activity; The increasing of ability to suffer torture or pain; The altering of complexions, and fatness and leanness; The altering of statures; The altering of features; The increasing and exalting of the intellectual parts; Versions of bodies into other bodies; Making of new species; Transplanting of one species into another; Instruments of destruction, as of war and poison; Exhilaration of the spirits, and putting them in good disposition; Force of the imagination, either upon another body, or upon the body itself; Acceleration of time in maturations; Acceleration of time in clarifications; Acceleration of putrefaction; Acceleration of decoction; Acceleration of germination; Making rich composts for the earth; Impressions of the air, and raising of tempests; Great alteration; as in induration, emollition, &c; Turning crude and watery substances into oily and unctuous substances; Drawing of new foods out of substances not now in use; Making new threads for apparel ; and new stuffs, such as paper, glass, &c; Natural divinations; Deceptions of the senses; Greater pleasures of the senses; Artificial minerals and cements.

Francis Bacon
(1561–1626)

Magnalia Naturae, in the New Atlantis (1627),
'Wonders of nature, in particular with respect to human use'

The prolongation of life; The restitution of youth in some degree; The retardation of age; The curing of diseases counted incurable; The mitigation of pain; More easy and less loathsome purgings;

…

Natural divinations; Deceptions of the senses; Greater pleasures of the senses; Artificial minerals and cements.

“I perceived it to be possible to arrive at knowledge highly useful in life; and in room of the Speculative Philosophy […]

http://www.bartleby.com/34/1/6.html

René Descartes (1596–1650)

Discourse on Method (1637)

"… to discover a Practical, by means of which, knowing the force and action of fire, water, air, the stars, the heavens, and all the other bodies that surround us, […]we might also apply them […]
and thus render ourselves the lords and possessors of nature."

René Descartes (1596–1650)

Discourse on Method (1637)

http://www.bartleby.com/34/1/6.html

In the formulation of Condorcet: “All the errors in politics and in morals are founded upon philosophical mistakes, which, themselves, are connected with physical errors” (Ninth Epoch)

Nicolas de Caritat, marquis de Condorcet
(1743- 1794)

‘Sketch for a Historical Picture of the Progress of the Human Spirit’

http://oll.libertyfund.org/titles/1669

‘Mathématique sociale’: We still use today terms such as ‘Condorcet method’, ‘Condorcet winner’, ‘Condorcet-ranking procedure’

Nicolas de Caritat,
marquis de Condorcet
(1743- 1794)
‘

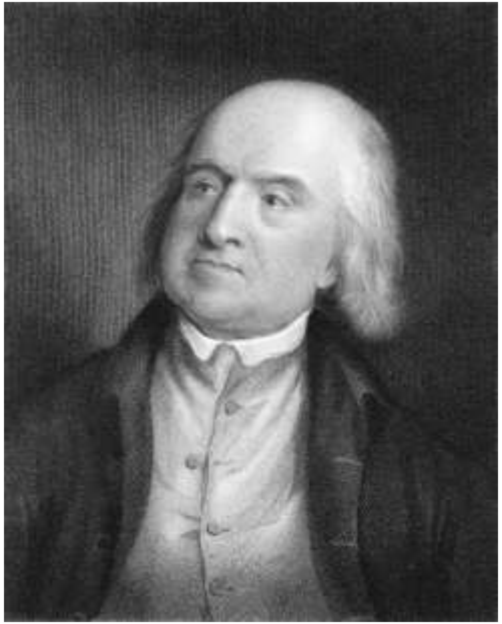
Feldman, J., 2005, Condorcet et la mathematique sociale: enthousiasmes et bemols, Mathematics and Social Sciences, 172(4), 7-41, http://www.ehess.fr/revue-msh/pdf/N172R955.pdf

Munda G. (2007) - Social multi-criteria evaluation, Springer-Verlag, Heidelberg, New York, Economics Series

Condorcet's algorithms and Descartes' Geometry at the root of our present ways of being

Condorcet's Mathématique sociale had its continuation in Jeremy Bentham's utilitarianism

Marquis de Condorcet (1743- 1794)

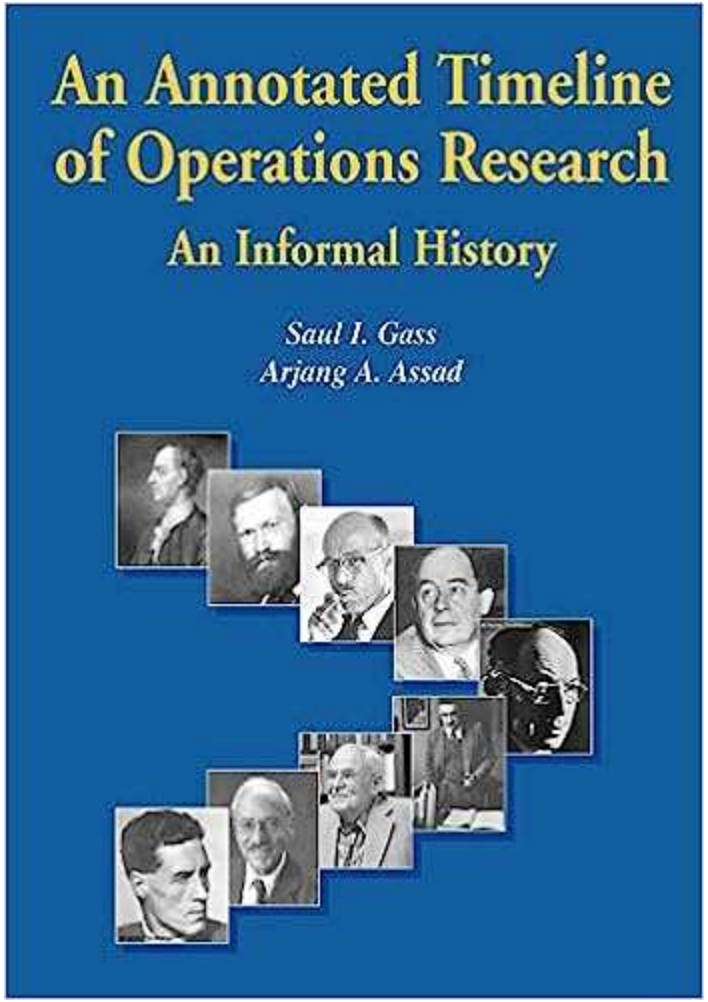


Felicific calculus: 'The greatest good for the greatest number'
(**utility** or hedonistic calculus)

Jeremy Bentham (1748-1832)

- Intensity: How strong is the pleasure?
- Duration: How long will the pleasure last?
- Certainty or uncertainty: How likely or unlikely is it that the pleasure will occur?
- Propinquity or remoteness: How soon will the pleasure occur?
- Fecundity: The probability that the action will be followed by sensations of the same kind.
- Purity: The probability that it will not be followed by sensations of the opposite kind.
- Extent: How many people will be affected?

Jeremy Bentham



Gass, Saul I., and Arjang A. Assad. 2006. An Annotated Timeline Of Operations Research: An Informal History. 1st Corrected ed. 2005. Corr. 2nd printing 2006 edition. New York: Springer-Verlag New York Inc.

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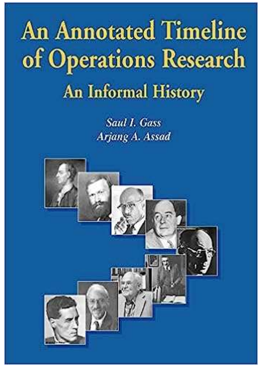
We already met some precursors of OR (Fermat, Cardano, Condorcet, Bayes, Bentham, …)

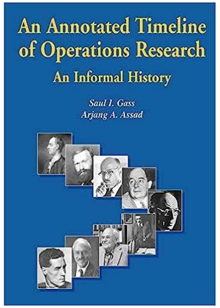
For Gass and Assad the oldest OR person was a woman, queen Dido in Virgilio Aenead



Dido and Aeneas, from a Roman fresco, Pompeian Third Style (10 BC – 45 AD), Pompeii, Italy. Source: Wikipedia Commons

Told by a Berber king that she could keep as much land as could be encircled in a bull hide, Dido ⋯ did what?

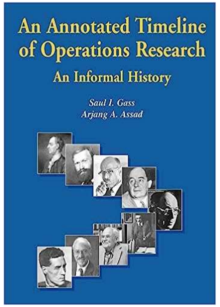
Cut the hide in very thin stripes so that an entire hill could be had, anticipating what we call today the isoperimetric problem: enclosing the maximum area within a fixed boundary



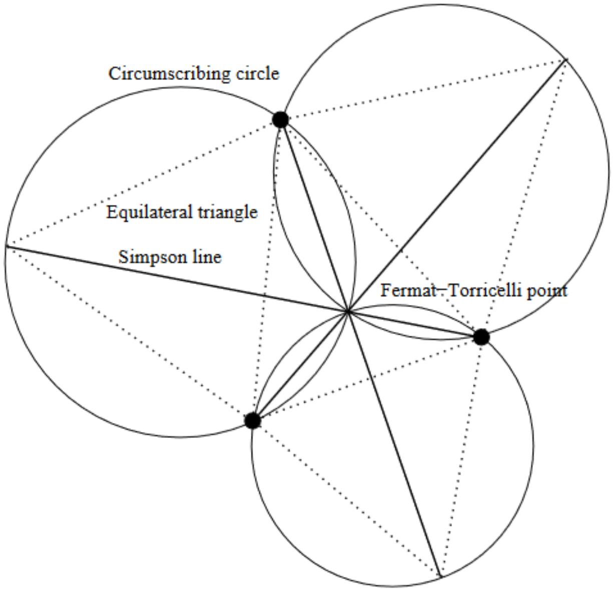
Dido and Aeneas, from a Roman fresco, Pompeian Third Style (10 BC – 45 AD), Pompeii, Italy. Source: Wikipedia Commons

Another famous OR-like problem, formulated by Pierre de Fermat: given three point in a plane find a fourth point such that the sum of the distance of it from the three points is minimum. Solved by Evangelista Torricelli

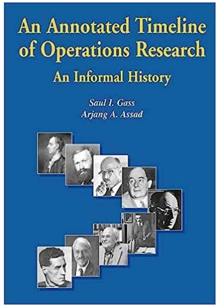
Evangelista Torricelli (1608-1647)
painting by Lorenzo Lippi.
Source: Wikipedia Commons



- Construct an equilateral triangle on each of the sides
- From each of the farmost vertex draw a line the opposite vertex of the original triangle.
- Where the three lines intersect is the Torricelli–Fermat point



Evangelista Torricelli (1608-1647)
painting by Lorenzo Lippi.
Source: Wikipedia Commons

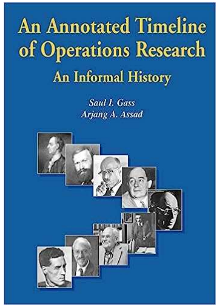


Vilfredo Pareto and its optimal solution

An optimal solutions affecting a group of actors is optimal if no actor can see his satisfaction increased without making other actor(s) worse off.

You cannot Rob Peter to pay Paul

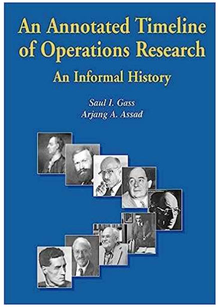
Vilfredo Pareto (1848-1923)
Source: Wikipedia Commons



Pareto is perhaps even best known for its famous distribution (originally to describe wealth distribution); the so called Pareto-principle[1] or "80-20 rule" – e.g. invoked when one says

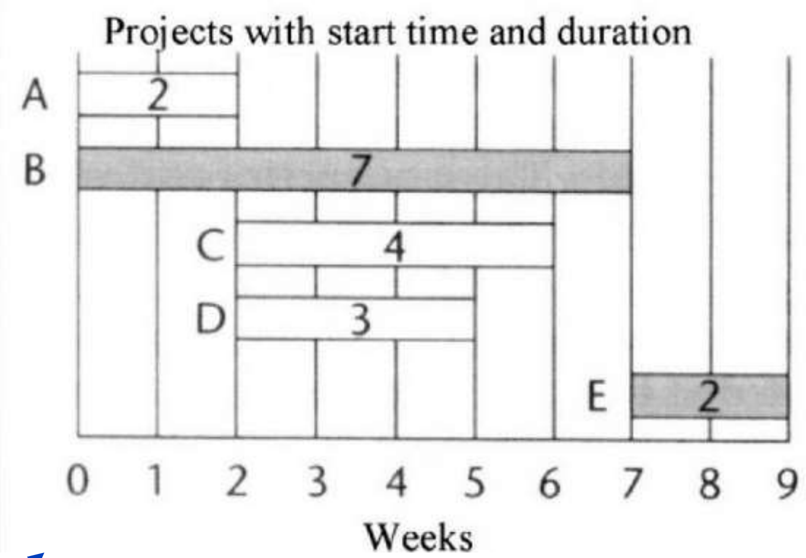
> *80% of problems/resources/outputs are caused / used / dominated by 20% of people / activities / factors*

Vilfredo Pareto (1848-1923)
Source: Wikipedia Commons

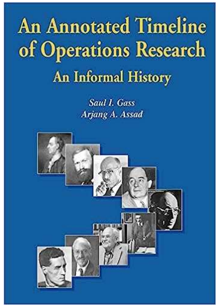


[1] This is a generalization and only holds for specific values of Pareto's power law distribution.

Gantt and his charts



Henry L. Gantt (1861-1919)
Source: Wikipedia Commons



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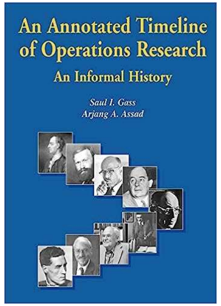
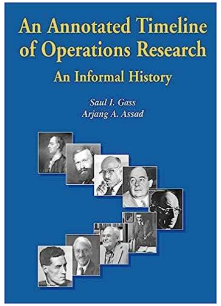


1936: Time zero: British military application of OR

The establishment of the Bawdsey Manor Research Station near London in 1936 to study how to best deploy the new radar technology (operated by RAF and civilians)

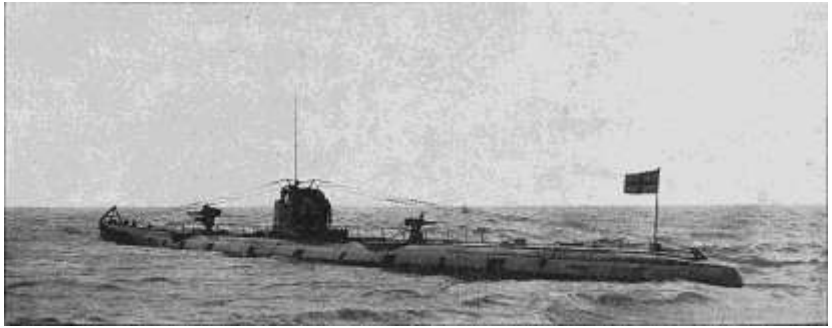
Source: https://www.pgl.co.uk/en-gb/school-trips/secondary-schools/centres/bawdsey-manor



Blackett's Circus

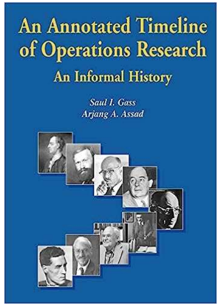
Established in 1940 under the direction of physicist Patrick Blackett to apply OR to radar as well as other war technologies – optimal depth for explosive charges in anti-submarine warfare, optimal size of sea convoys …

- Prescient in his critique of strategic bombing (he argued that fighting U-boats should have been given priority)
- 1948 Nobel Prize in Physics for his cloud chamber and cosmic rays investigation



Source: Wikipedia Commons

Patrick Blackett
(1897-1974)
Source: Wikipedia Commons



→ The 40's also saw the birth of modern utility, theory of games and economic behaviour, and the establishment in 1945, after the war, of Project RAND, (becoming a corporation in 1948)

Theory of games and economic behaviour; a very influential (and precious) work by John Von Neuman and Oskar Morgenstein, published in 1944

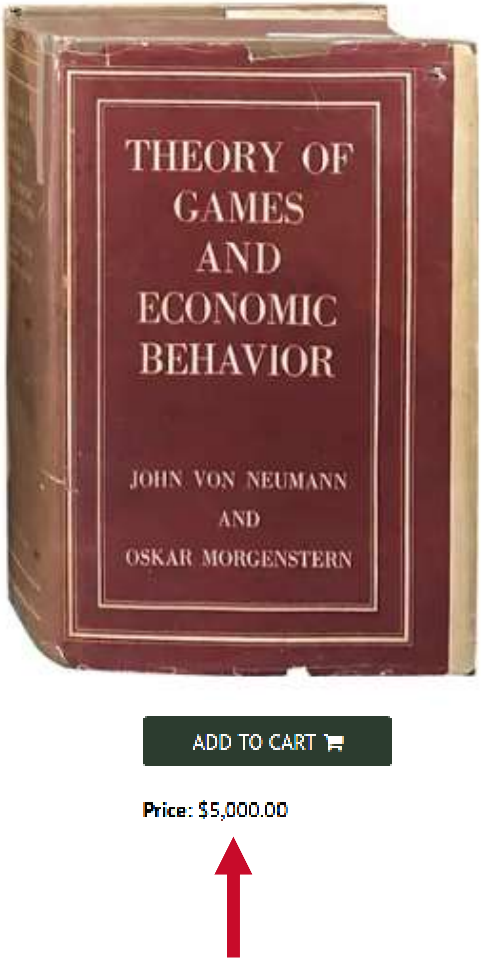
Introduced the expected utility hypothesis

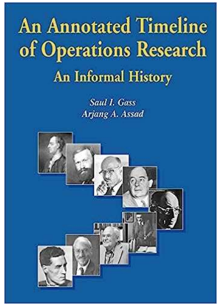
Rational agents take decisions based of expected payoffs and preferences

$$U(p) = \sum u(x_k)p(x_k)$$

Where $u(x_k)$ is the utility of choice/payoff x_k and $p(x_k)$ is its probability

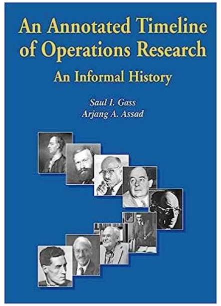
➔ Recall Daniel Bernoulli's discussion of the Saint Petersburg Paradox





→ The birth of linear programming and of its solution via the simplex method, due to George B. Dantzig in 1947

Maximize cx subject to $Ax = b, x \geq 0$, where c is a $1 \times n$ row vector, x is a $1 \times n$ column vector, A is a $\mathrm{m} \times n$ matrix and b is a $m \times 1$ column vector



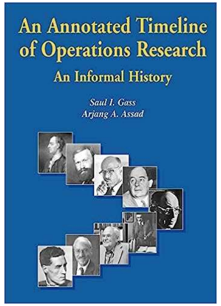
→ Perhaps better written explicitly as

Maximize $$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n,$$

Subject to:
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m, \end{aligned}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad \ldots, \quad x_n \geq 0.$$

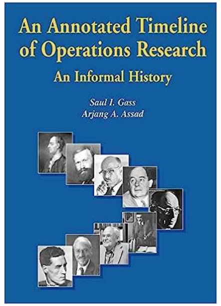
About which more soon



→ The birth of linear programming and of its solution via the simplex method, due to George B. Dantzig in 1947

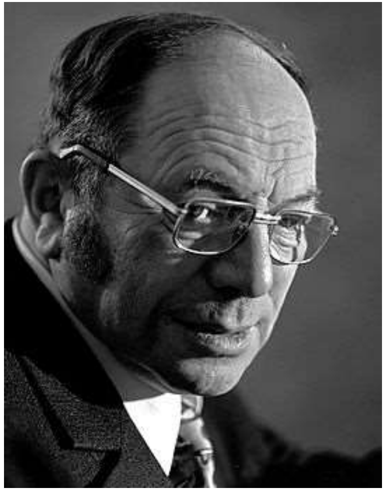
Koopman coined the term linear programming while Kantorovich had anticipated the theory in 1939' Soviet Union. Curiously, Koopman and Kantorovich shared a Nobel prize in 1975, but not Dantzig

Tjalling C. Koopman, George B. Dantzig and Leonid V. Kantorovich. Source: https://www.informs.org/

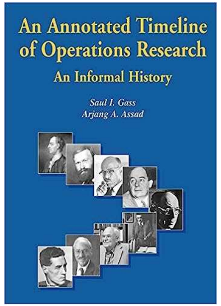


→ Another curiosity: Kantorovich efforts in the URSS were frustrated as the method of mathematics applied to economics were perceived as "anti-Marxist" and "apologist for capitalism" (1939)

Interesting parallel with relativity and quantum mechanic perceived as "Jewish science" in Nazi Germany



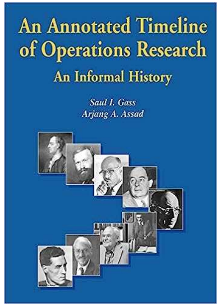
Leonid V. Kantorovich
(1912-1986)
Source: Wikipedia Commons



→ Von Neumann's minimax theorem about two-person zero sum finite game. The numbers $j = 1, 2, \cdots n$ and $i = 1, 2, \cdots m$ represent the choices of A and B respectively.

Let a_j^i represents the payment to A if A choses payment i and B payment j. Since the game is zero sum, if A gets a_j^i then B gets $-a_j^i$.


John von Neumann
(1903-1957)
Source: Wikipedia Commons



→ Let a_j^i represents the payment to A if A choses payment i and B payment j. Since the game is zero sum, if A gets a_j^i then B gets $-a_j^i$.

John von Neumann
(1903-1957)
Source: Wikipedia Commons

$$\begin{pmatrix} a_1^1 & a_2^1 & \cdots a_n^1 \\ a_1^2 & a_2^2 & \cdots a_n^2 \\ \cdots & \cdots & \cdots \\ a_1^m & a_2^m & \cdots a_n^m \end{pmatrix}$$

A wants to maximize a_j^i, but A only controls the choice of i

B wants to minimize a_j^i, but B only controls the choice of j

Try it yourself with some numbers

What would A choose?

$$\begin{pmatrix} .92 & .33 & .62 \\ .17 & .29 & .54 \\ \cdots & \cdots & \cdots \\ .82 & .67 & .50 \end{pmatrix}$$

What would B choose?

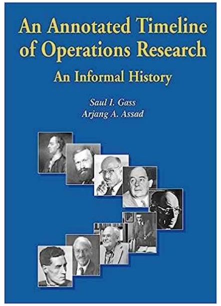
What would B choose?

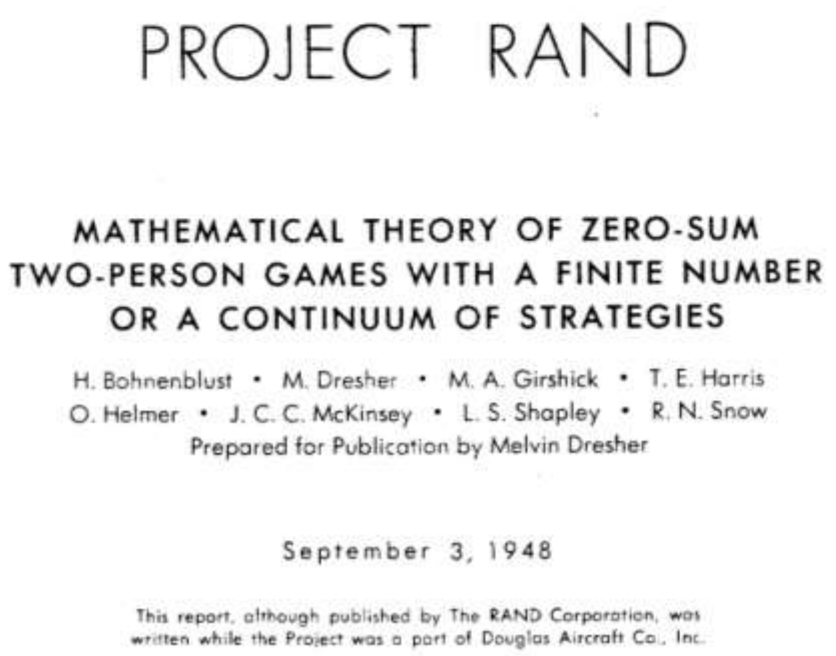
What would B choose?

A wants to maximize a_j^i, but A only controls the choice of i

B wants to minimize a_j^i, but B only controls the choice of j

If A choses payment i, B will choose payment j as to minimize a_j^i, so that A will get $\min_j a_j^i$; A should thus aim for $\max_i \left(\min_j a_j^i \right)$




PROJECT RAND

MATHEMATICAL THEORY OF ZERO-SUM TWO-PERSON GAMES WITH A FINITE NUMBER OR A CONTINUUM OF STRATEGIES

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Prepared for Publication by Melvin Dresher

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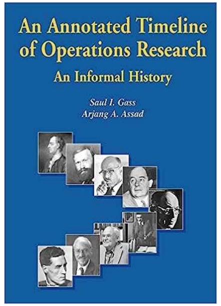
https://www.rand.org/content/dam/rand/pubs/reports/2018/R115.pdf



→ A last game: the prisoners' dilemma, formulated first by Albert W. Tucker

Two men, charged with a joint violation of law, are held separately by the police. Each is told that

(1) if one confesses and the other does not, the former will be given a reward of one unit and the latter will be fined two units,
(2) if both confess, each will be fined one unit.

At the same time each has good reason to believe that

(3) if neither confesses, both will go clear.

		II	
		confess	not confess
I	confess	$(-1, -1)$	$(1, -2)$
	not confess	$(-2, 1)$	$(0, 0)$

Source: Tucker, A. W. 1983. 'The Mathematics of Tucker: A Sampler'. The Two-Year College Mathematics Journal 14 (3): 228–32. https://doi.org/10.2307/3027092.

Please try it yourself in groups of two without revealing your decision to your partner (write it down)

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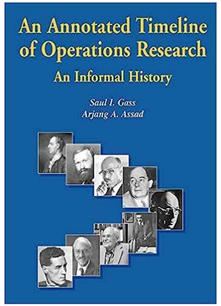
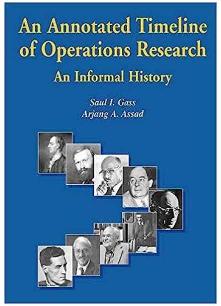


→ The game is widely used, e.g. to illustrate the concept of Nash equilibrium:

		II	
		confess	not confess
I	confess	(−1, −1)	(1, −2)
	not confess	(−2, 1)	(0, 0)

In this mixed strategy game the Nash equilibrium is reached when no player can improve her/his score by changing unilaterally strategy. Here both prisoners are bound to defect and get (−1,−1) ⋯ although as a group they would be better off cooperating (0,0)

John Forbes Nash
(1928–2015)
Credit: MIT museum

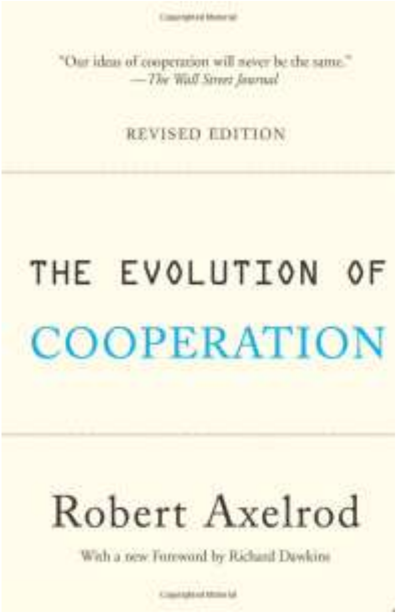


→ The setting of the game is that it is non-repeated

		II	
		confess	not confess
I	confess	$(-1, -1)$	$(1, -2)$
	not confess	$(-2, 1)$	$(0, 0)$

Optimal strategies are instead available if the game can be repeated.

Tit for Tat strategy, see Axelrod, 1984; how cooperation can emerge and become sustained



Homework (to be handed over at the next lesson – **handwritten**)

1) Compute by hand all possible outcomes of throwing two dices, write them down by hand, and compute their probabilities to check that they add to 1.
2) By hand compute the chance of having three times an 11 over a series of 5 throws of the two dices (showing all handwritten passages). Check this result using Excel. [Not mandatory: If you know how to use ChatGPT try it and compare its results with yours].
3) Try solving The Seven Bridges of Königsberg by cutting two edges instead of one (one edge was done in class already). Trying adding one or two edges instead. Show some of the handwritten results with few words of comment.
4) Read Chapter 1 of "Gass, S. I., & Assad, A. A. (2006). An Annotated Timeline Of Operations Research: An Informal History. Springer-Verlag New York", available at https://download.e-bookshelf.de/download/0000/0046/50/L-G-0000004650-0002369136.pdf, select a scholar described in this work, and write a handwritten page about this person by researching on the web.
5) Download the textbook we shall mostly use in the course: https://www.dropbox.com/sh/ddd48a8jguinbcf/AABF0s4eh1lPLVxdx0pes-Ofa?dl=0&preview=Introduction+to+Operations+Research+-+Frederick+S.+Hillier.pdf and read Chapter 1, pages 1-9.

Thank you

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