

Sensitivity Analysis

Andrea Saltelli

June 2023

Statistical Data Analysis Module B

&

Project i4Driving



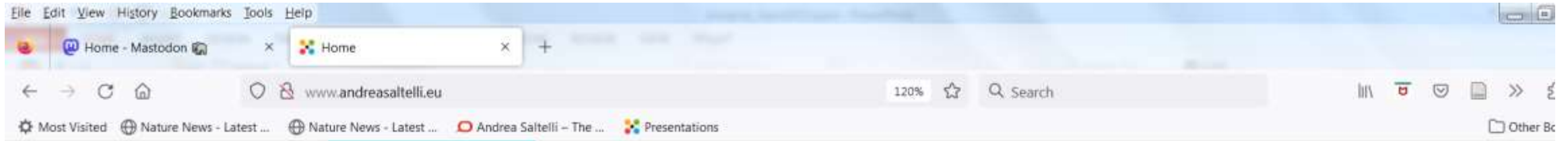
**Integrated 4D driver
modelling under
uncertainty**

Project started on October 1st, 2022

i4Driving

a EU Horizon Europe R&I project

Where to find this talk: www.andreasaltelli.eu



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Coming Out Soon: The politics of modelling



Praise for the volume

"A long awaited examination of the role —and obligation —of modeling."

Nassim Nicholas Taleb, Distinguished Professor of Risk Engineering, NYU Tandon School of Engineering. Author, of the 5 -volume series *Incerto*.

"A breath of fresh air and a much needed

Mastodon Toots by

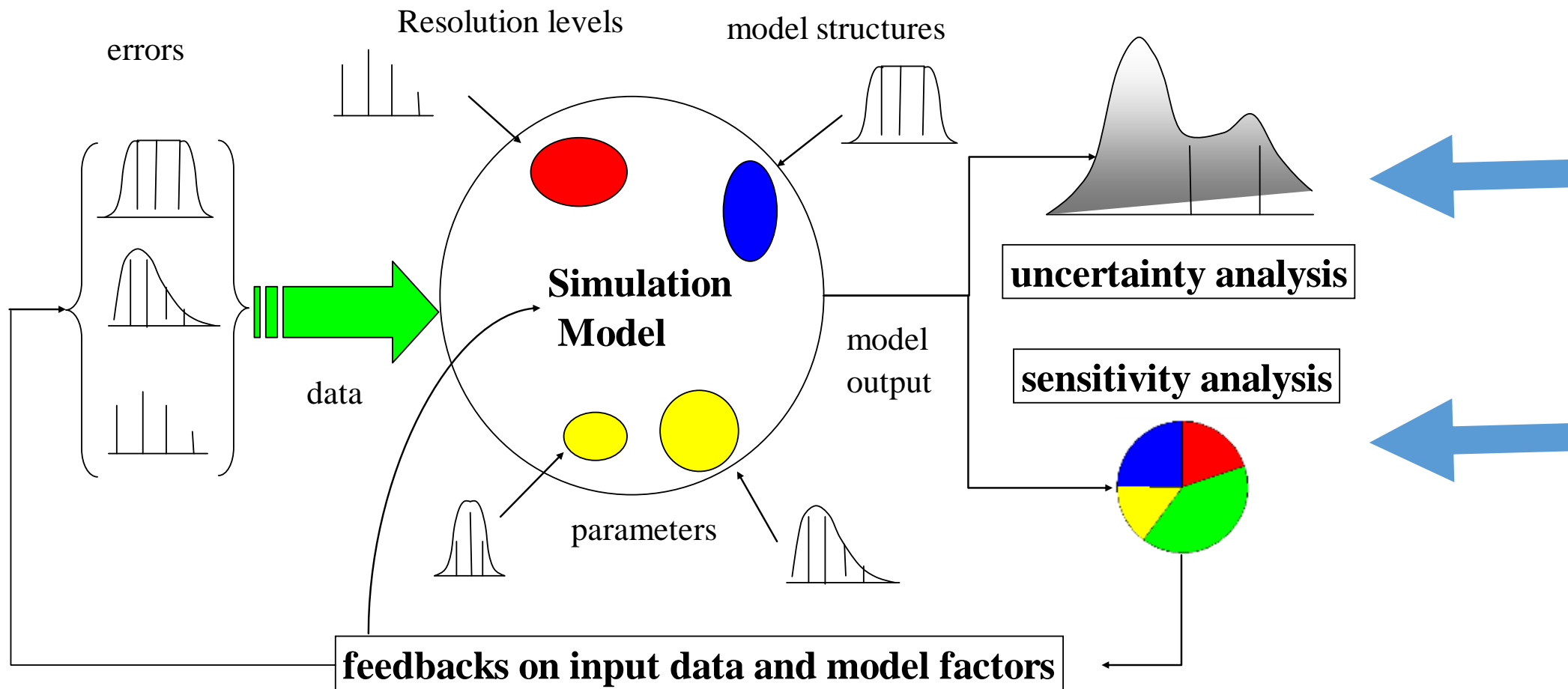
@AndreaSaltelli



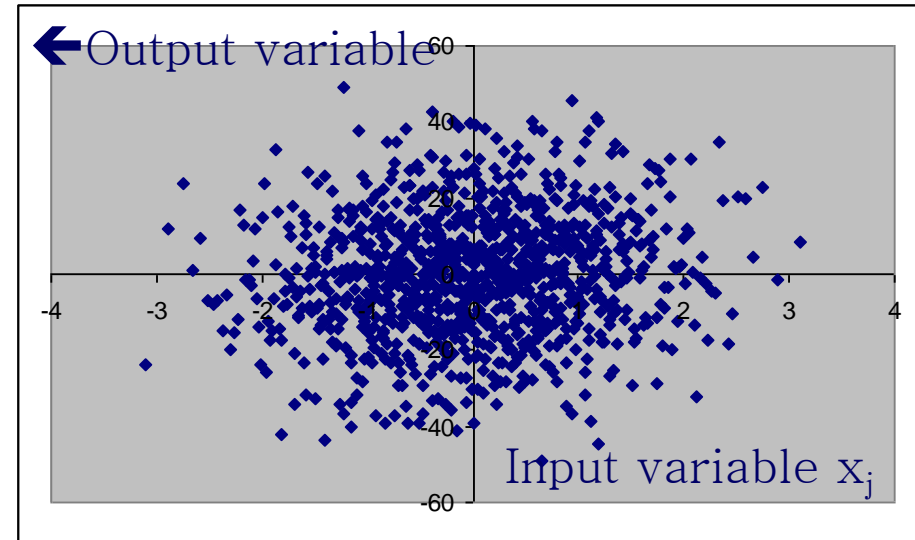
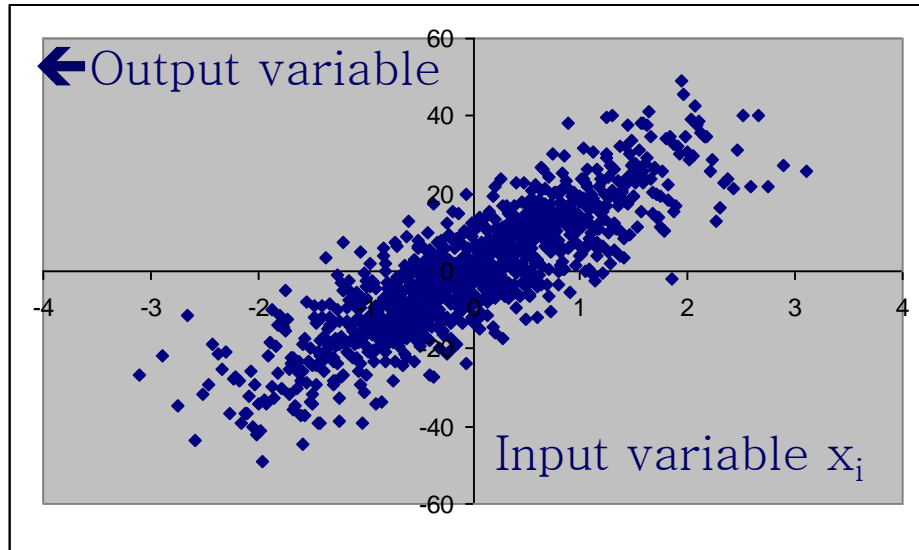
AndreaSaltelli

2023/4/18 17:24

"Kaufman, who was until recently a senior climate economist in the White House, questioned the need for the government to set a single price. Regulators should stop "pretending we

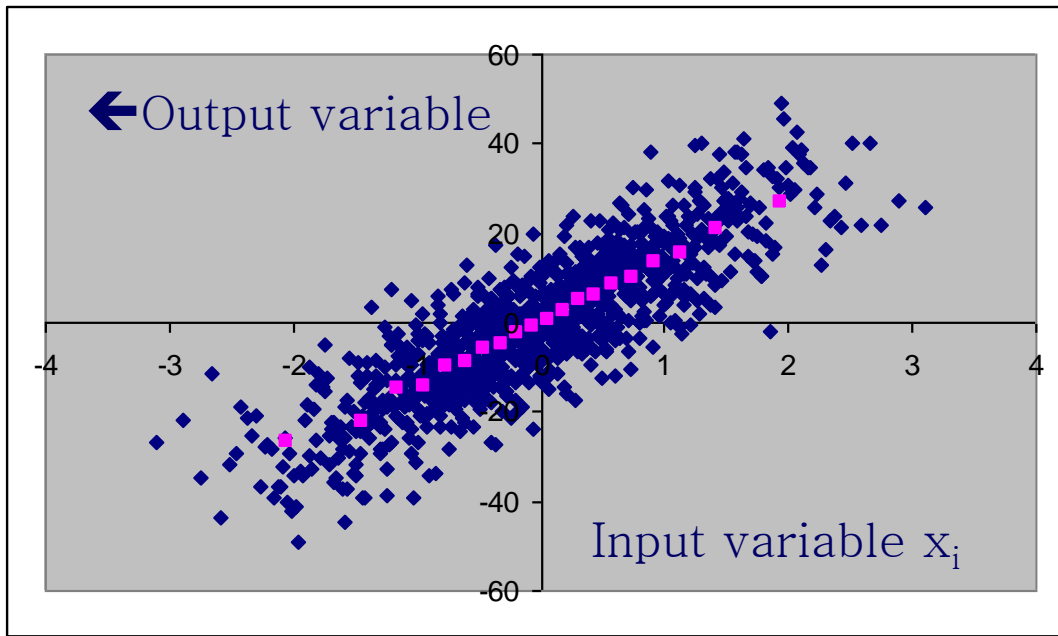


An introduction to variance based methods



Plotting the output as a function of two different input factors

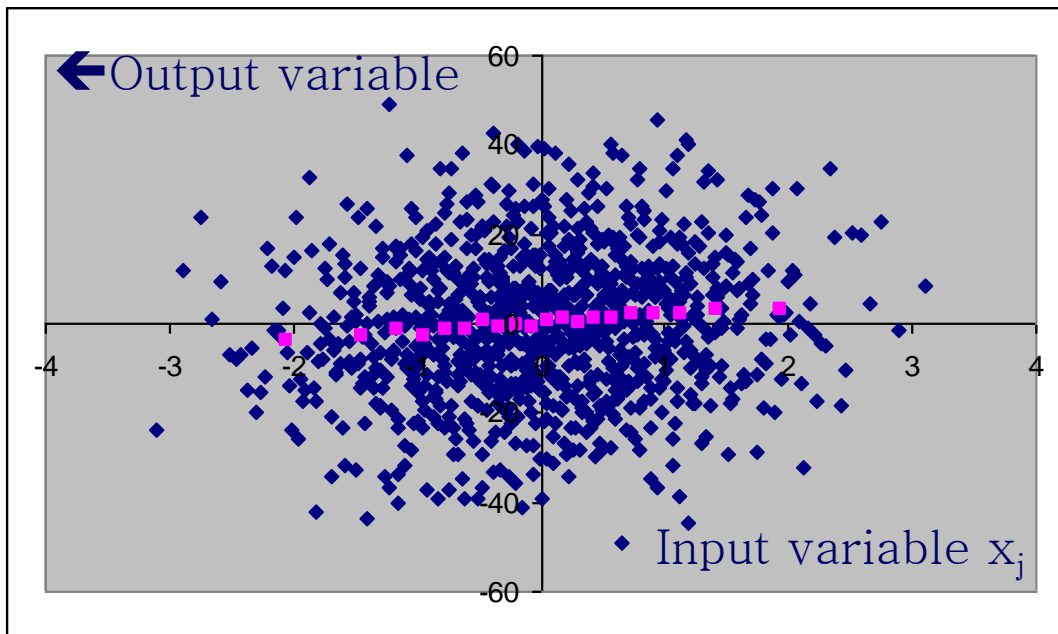
Which factor is more important?

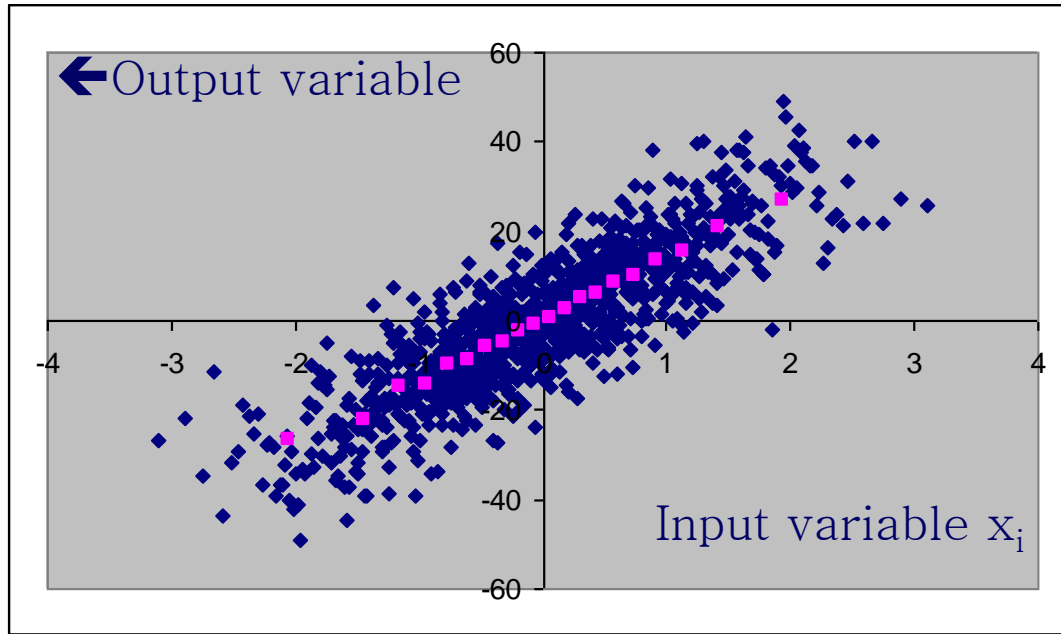


~1,000 blue points

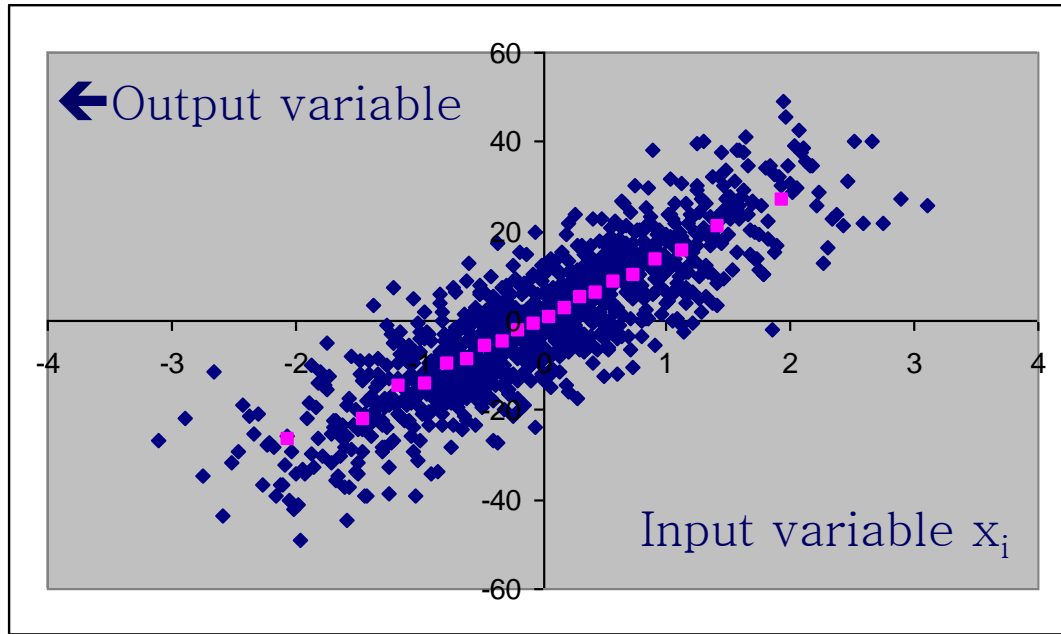
Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



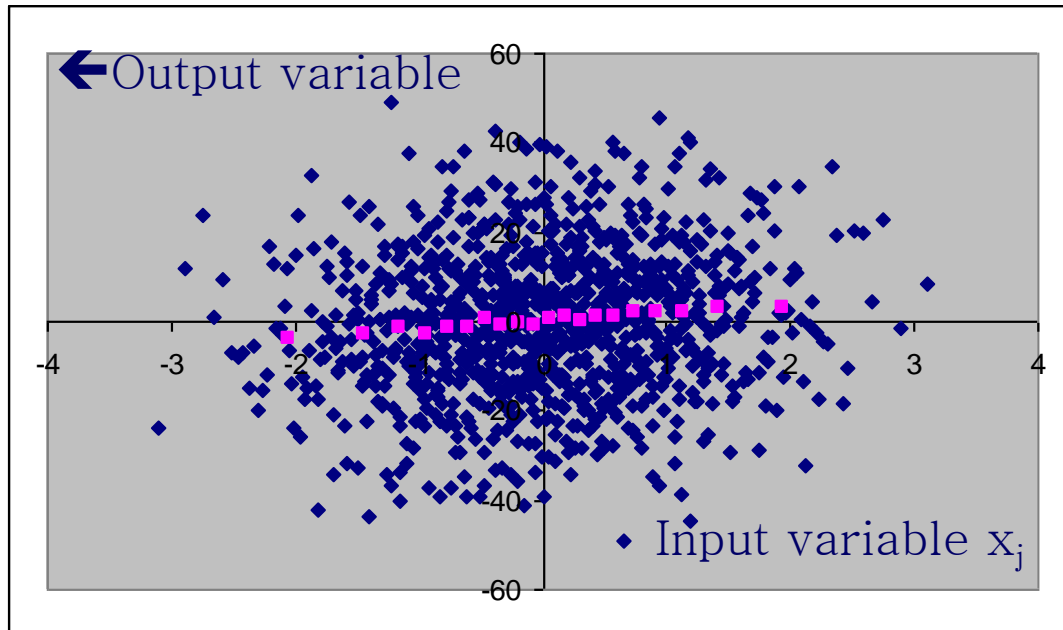
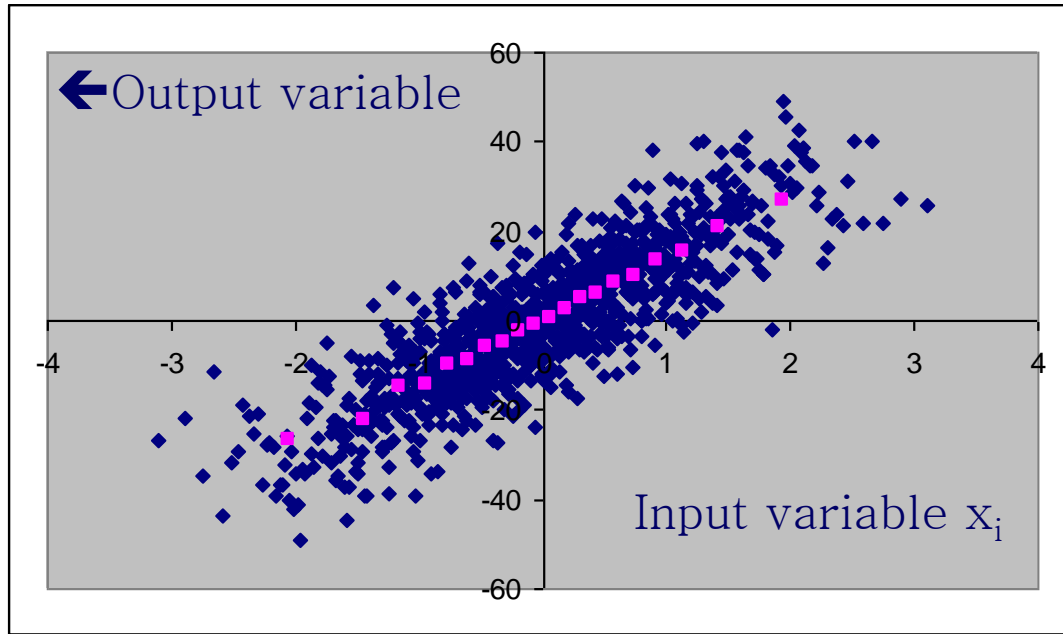


Each pink point is $\sim E_{\mathbf{X}_{\sim i}}(Y|X_i)$



Taking the variance
of the pink points one
obtains a sensitivity
measure

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

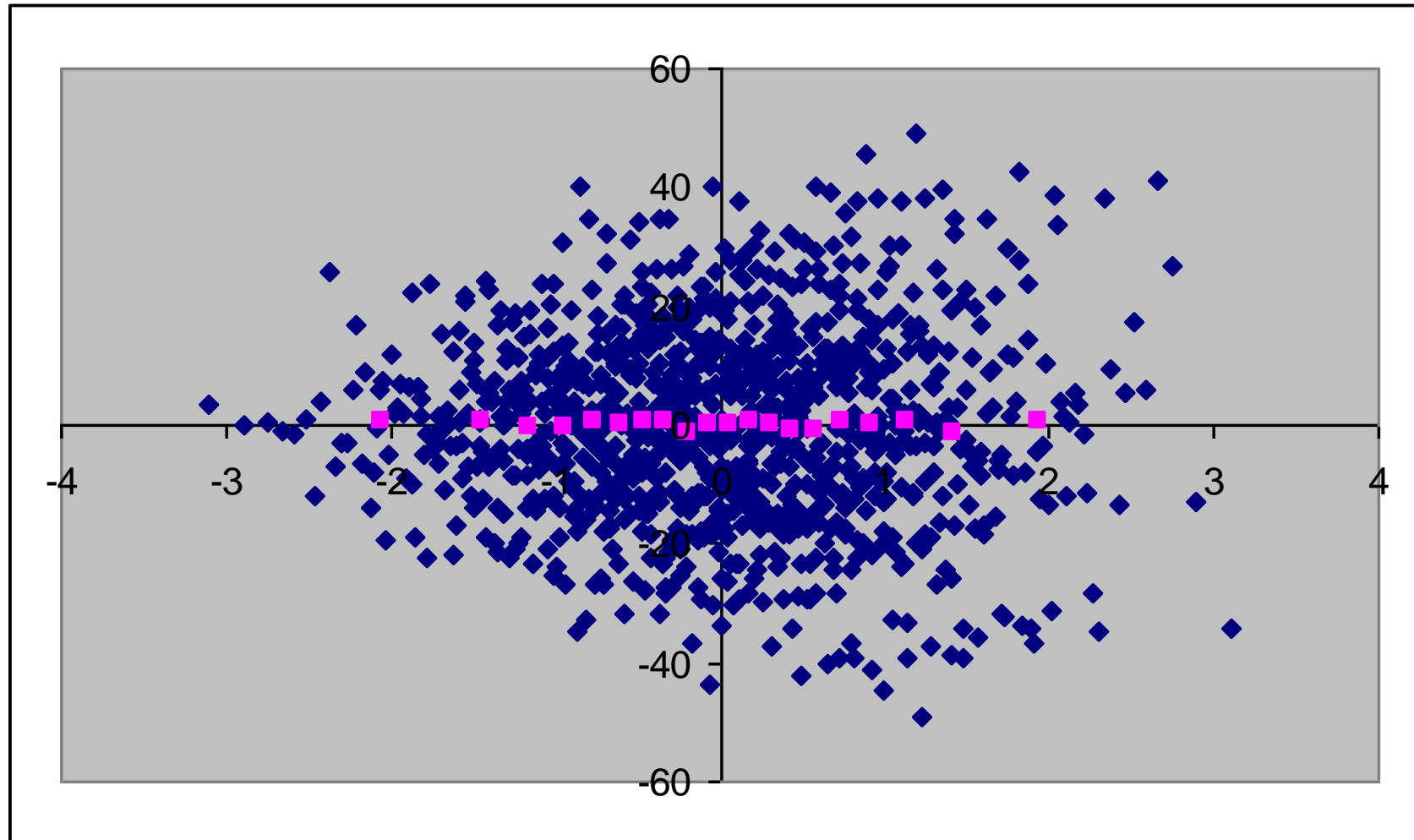


Which factor
has the highest
 $V_{X_i} (E_{\mathbf{X}_{\sim i}} (Y | X_i))$?

$$S_i = \frac{V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)}$$

The partial variance divided by the total variance is the so-called sensitivity index of the first order

Is this factor non-important?



For cases where S_i is zero but the variable is still important we need to compute 'total effect' sensitivity indices

The measures and their ‘settings’
= when to use them

First order effect	Factor prioritization (orienting research)
Total effect	Factor fixing (model simplification)

Plenty of code available in R, MATLAB, and Python



<https://cran.r-project.org/web/packages/sensitivity/sensitivity.pdf>

<https://cran.rstudio.com/web/packages/sensobol/index.html>



<https://www.uqlab.com/> (in MatLab, by Bruno Sudret and his team)

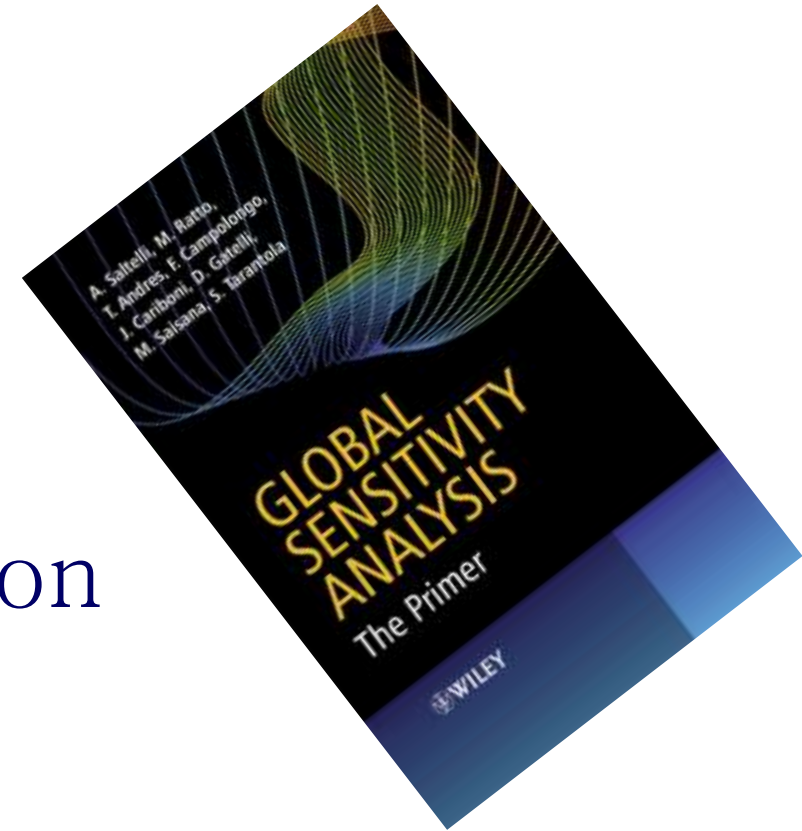


SALib <https://salib.readthedocs.io/en/latest/>

...but there is more,
such as Sensobol in R,
SALib in Python ...

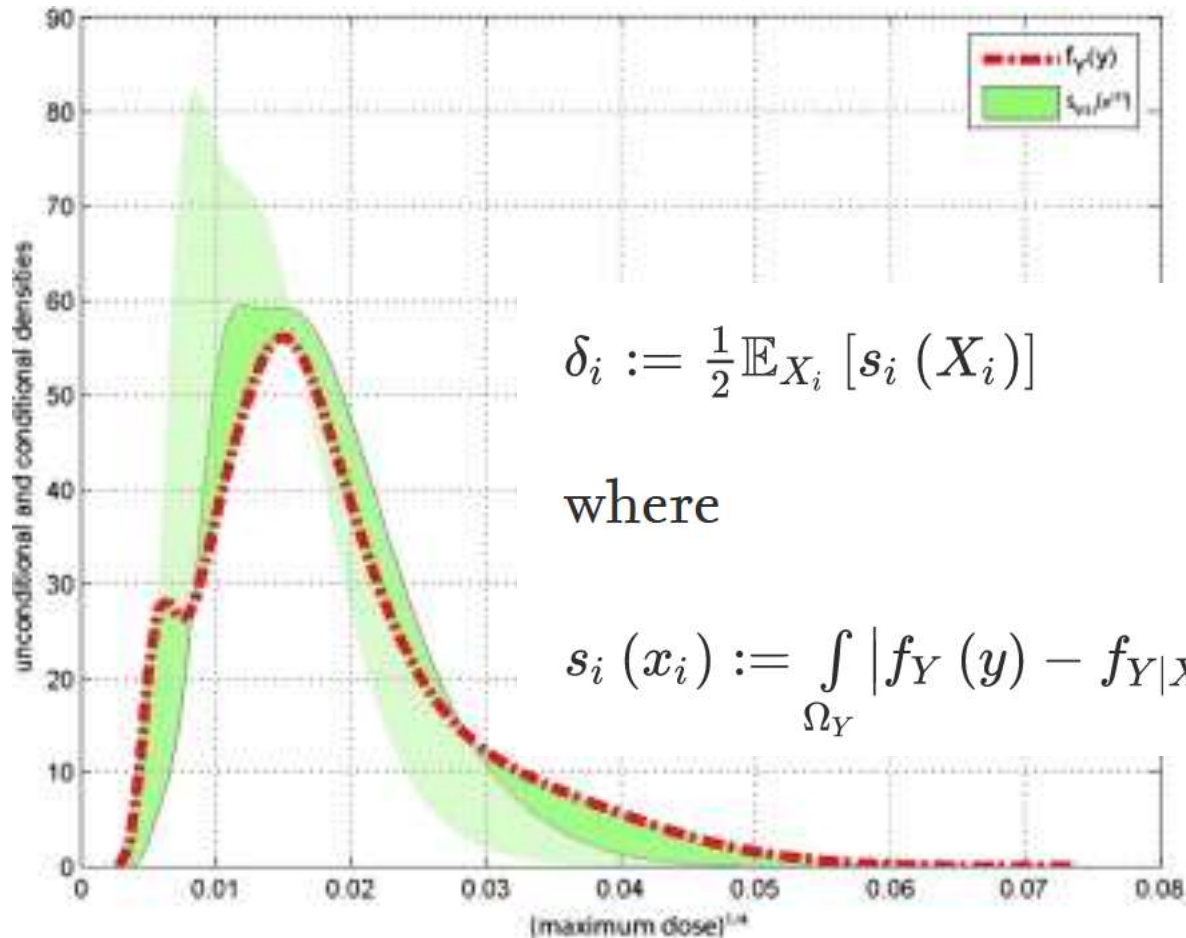
Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization
- give the effective dimension



Chapter 1 its
exercises

... but there are other methods that can be used for different settings, e.g. moment independent methods, Shapley coefficients, reduced spaces, VARS ...



$$\delta_i := \frac{1}{2} \mathbb{E}_{X_i} [s_i (X_i)]$$

where

$$s_i (x_i) := \int_{\Omega_Y} |f_Y (y) - f_{Y|X_i=x_i} (y)| dy$$



Environmental Modelling & Software

Volume 34, June 2012, Pages 105-115



Model emulation and moment-independent sensitivity analysis: An application to environmental modelling

E. Borgonovo ^a, W. Castaings ^{b, c}, S. Tarantola ^d  

Don't use One factor At a
Time (OAT)

A geometric proof



Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

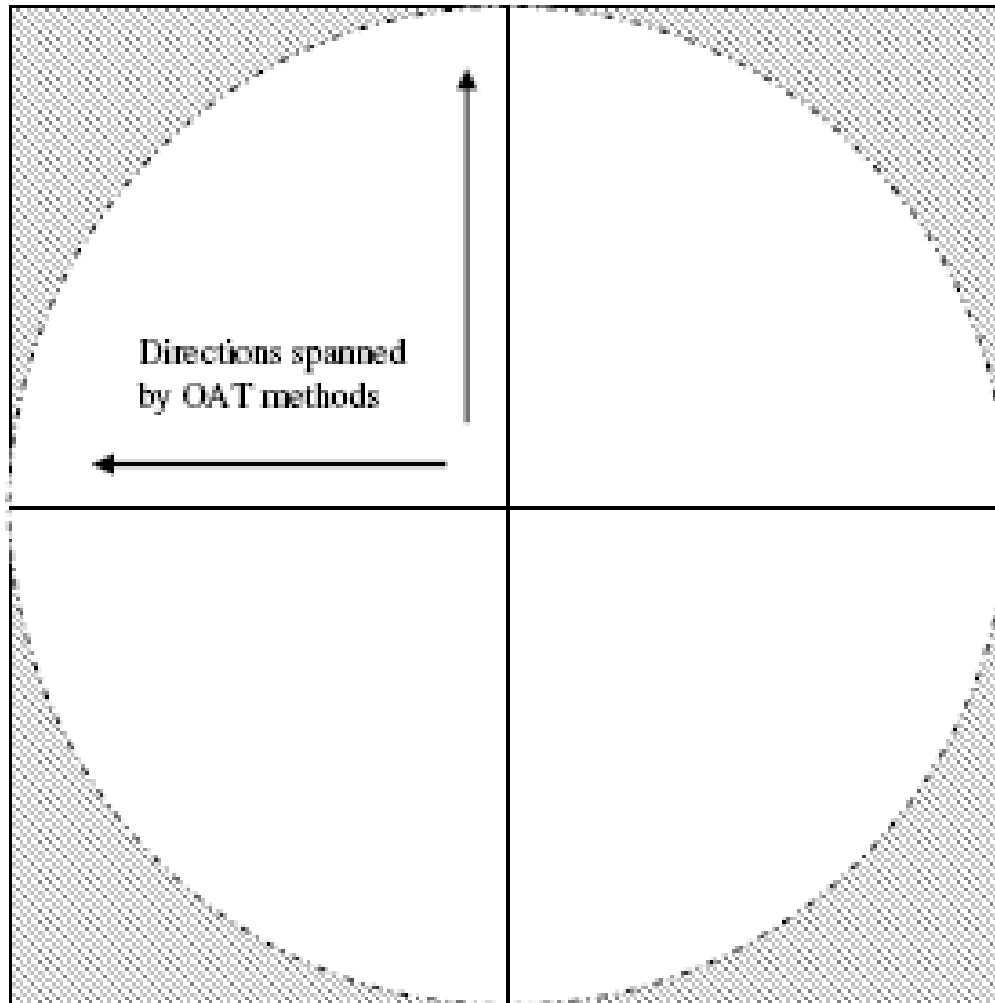


How to avoid a **perfunctory** sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy

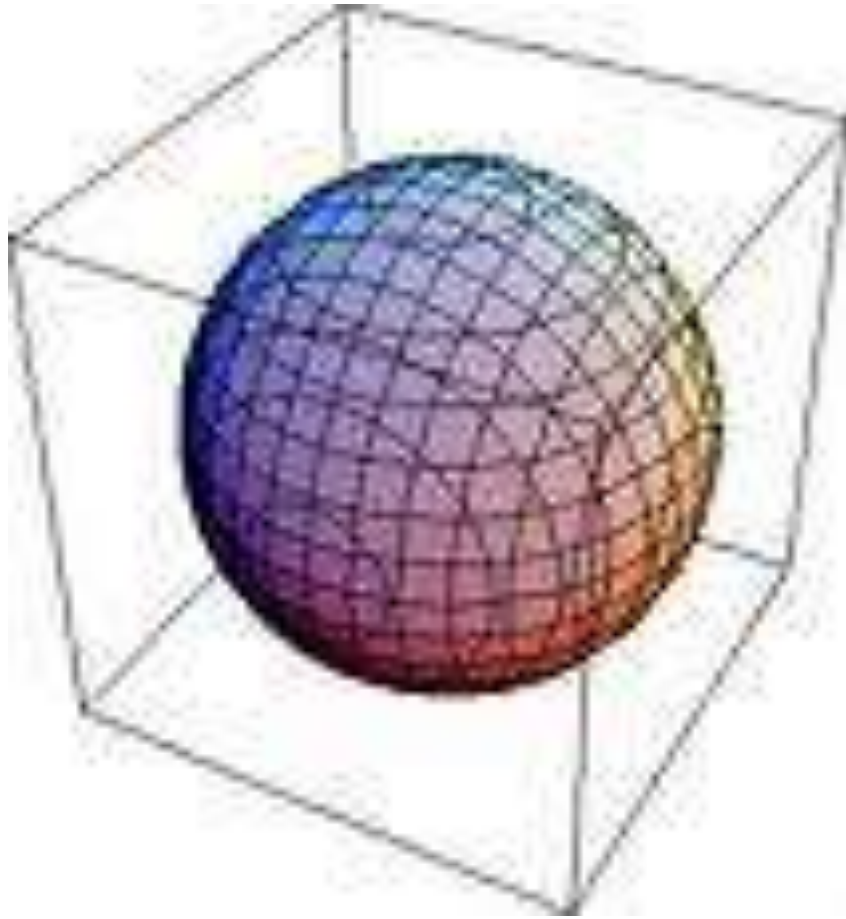
OAT in 2 dimensions



Area circle
/ area
square = ?

$\sim 3/4$

OAT in 3 dimensions



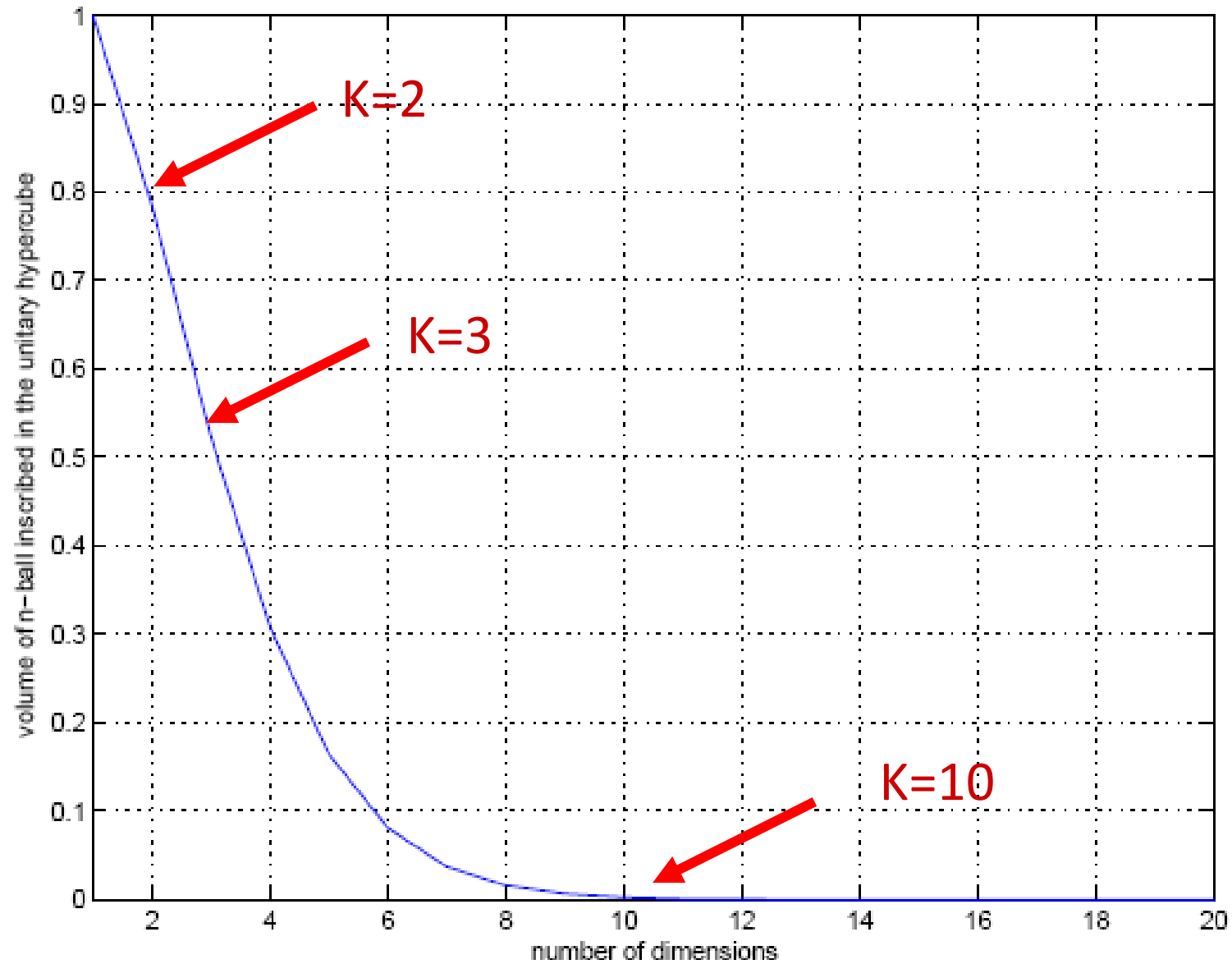
Volume sphere /
volume cube = ?

$\sim 1/2$

OAT in 10 dimensions; Volume
hypersphere / volume ten dimensional
hypercube =? ~ 0.0025



OAT in k dimensions



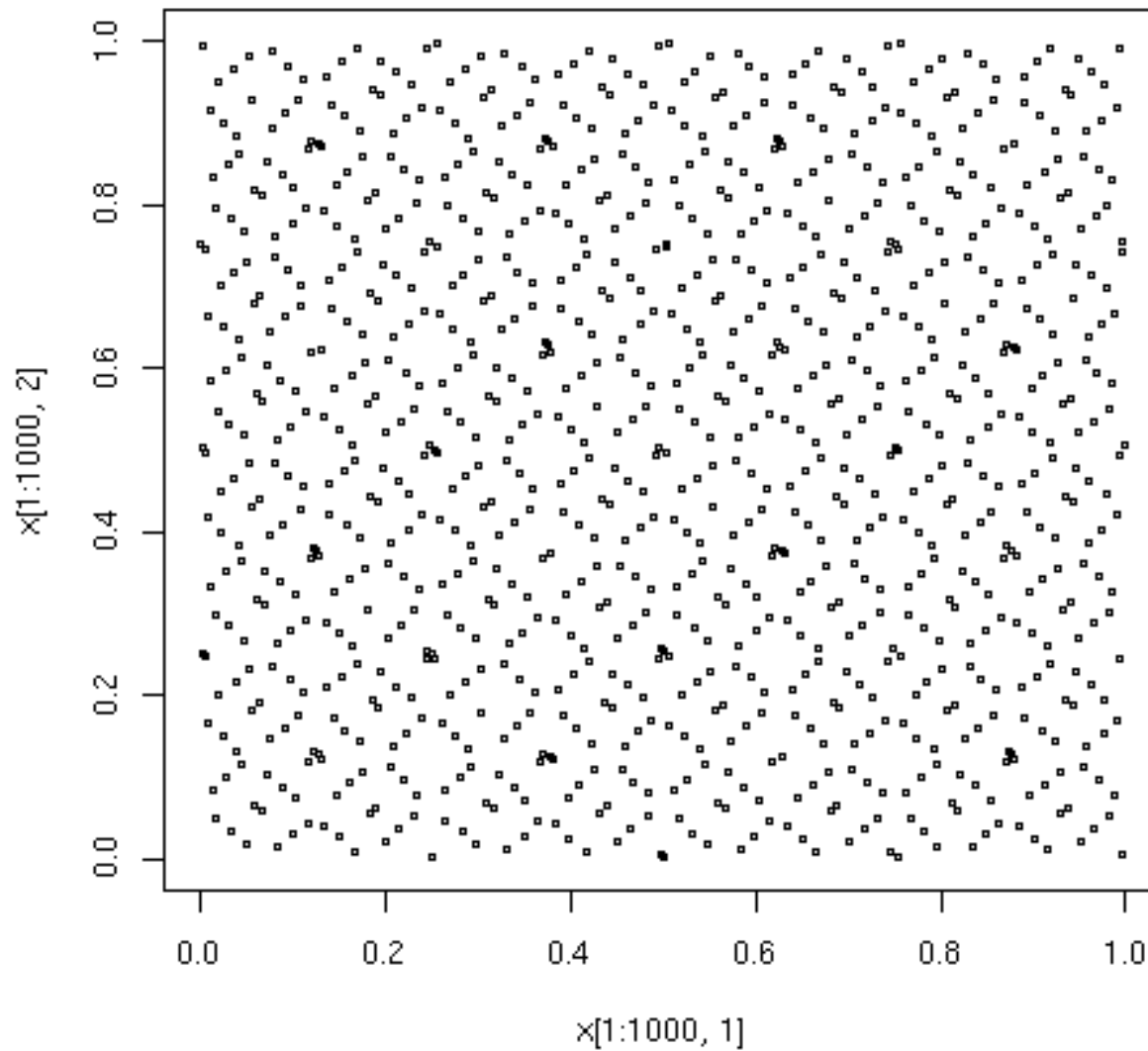
OAT does not capture interactions

➔ The resulting analysis is non conservative

Quasi random sequences



Ilya M. Sobol'

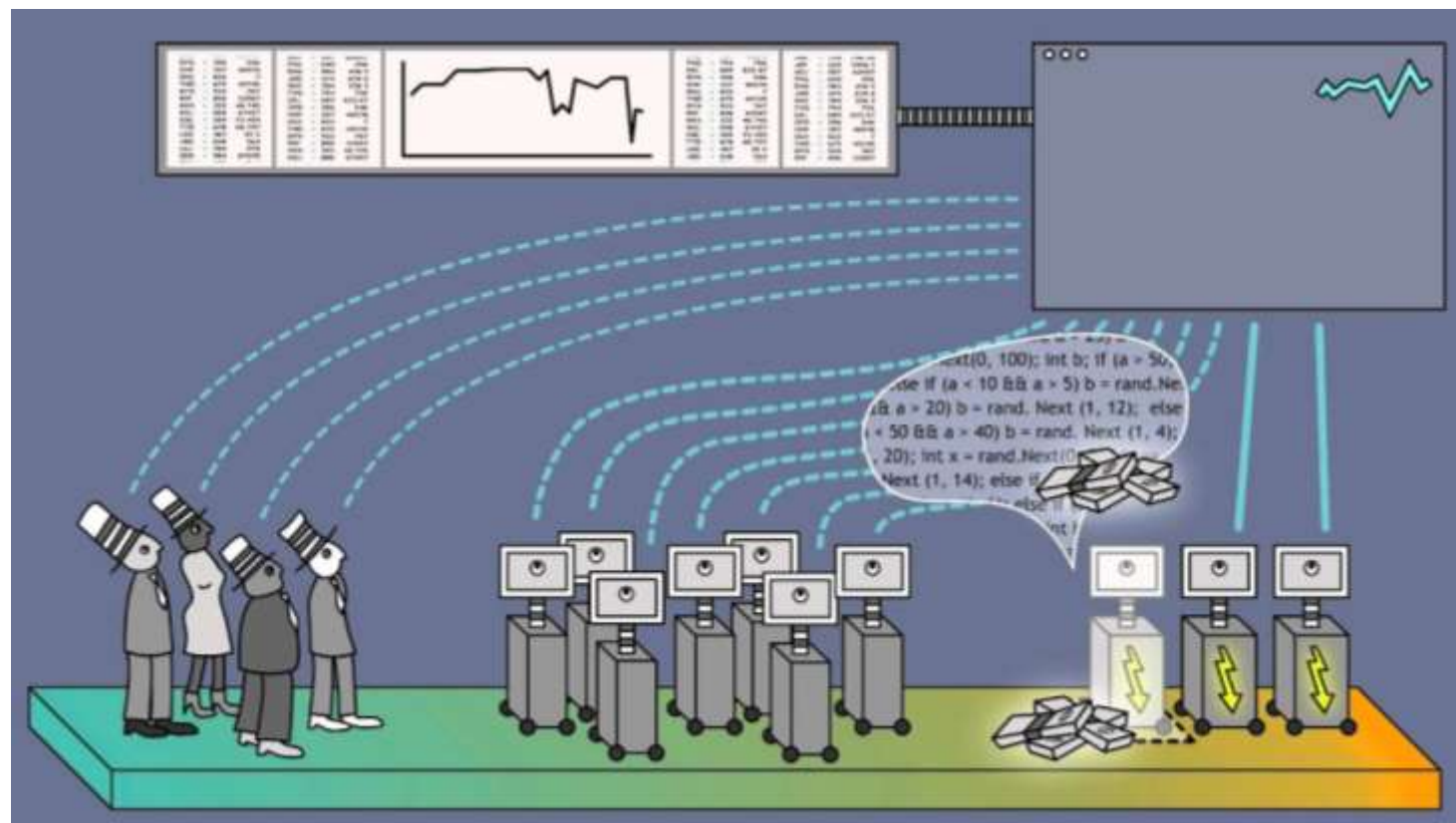


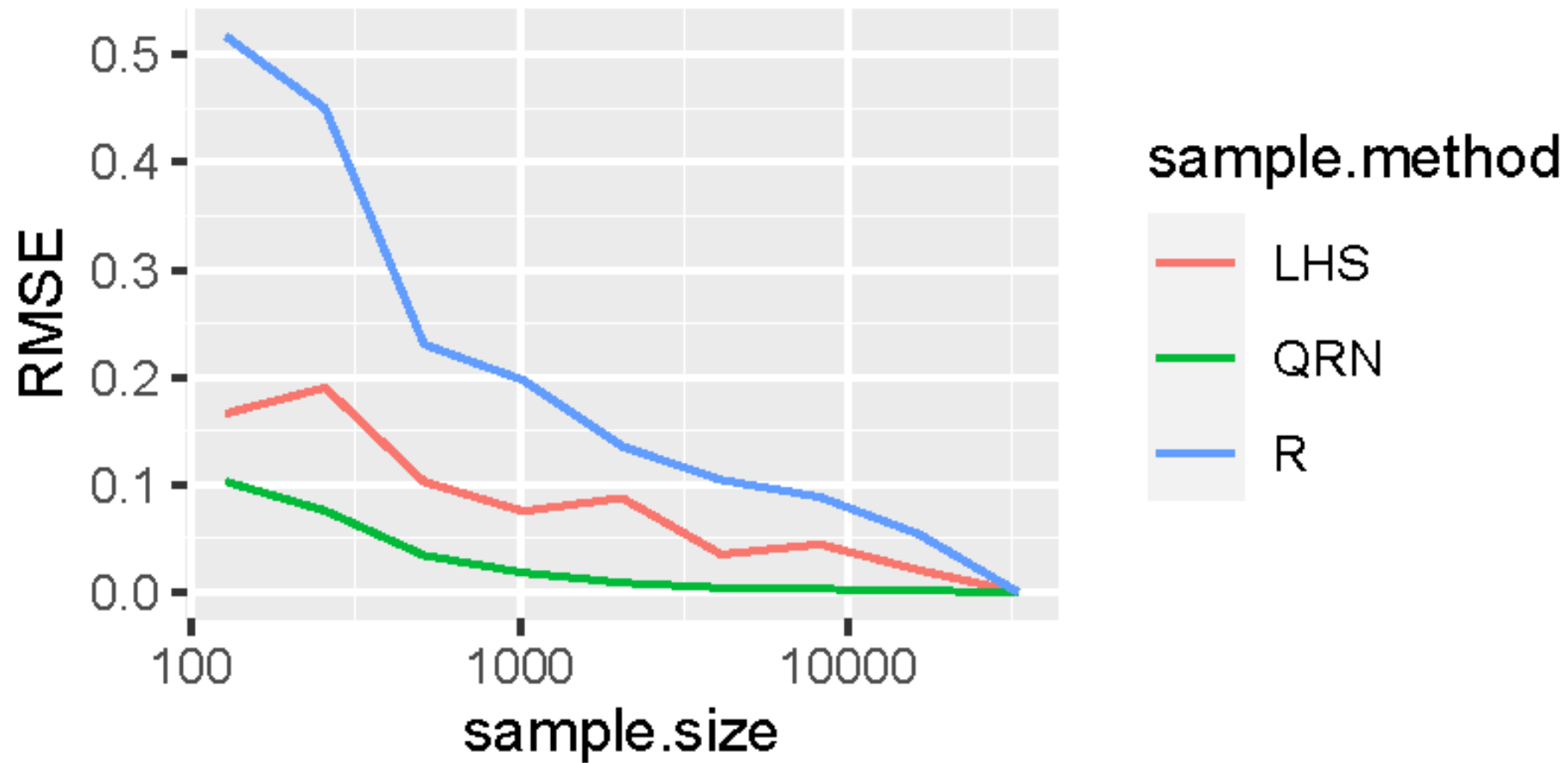
[Submitted on 10 May 2015]

Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli

Sobol' LP-TAU
are used in high
frequency trading





Root mean square error with different designs.

Don't run the model just once

There is much to learn by running the model a few times, especially during model building

Lubarsky's Law of Cybernetic Entomology:
there is always one more bug!



Model routinely used to produce point estimates may become non conservative when the uncertainty is plugged in

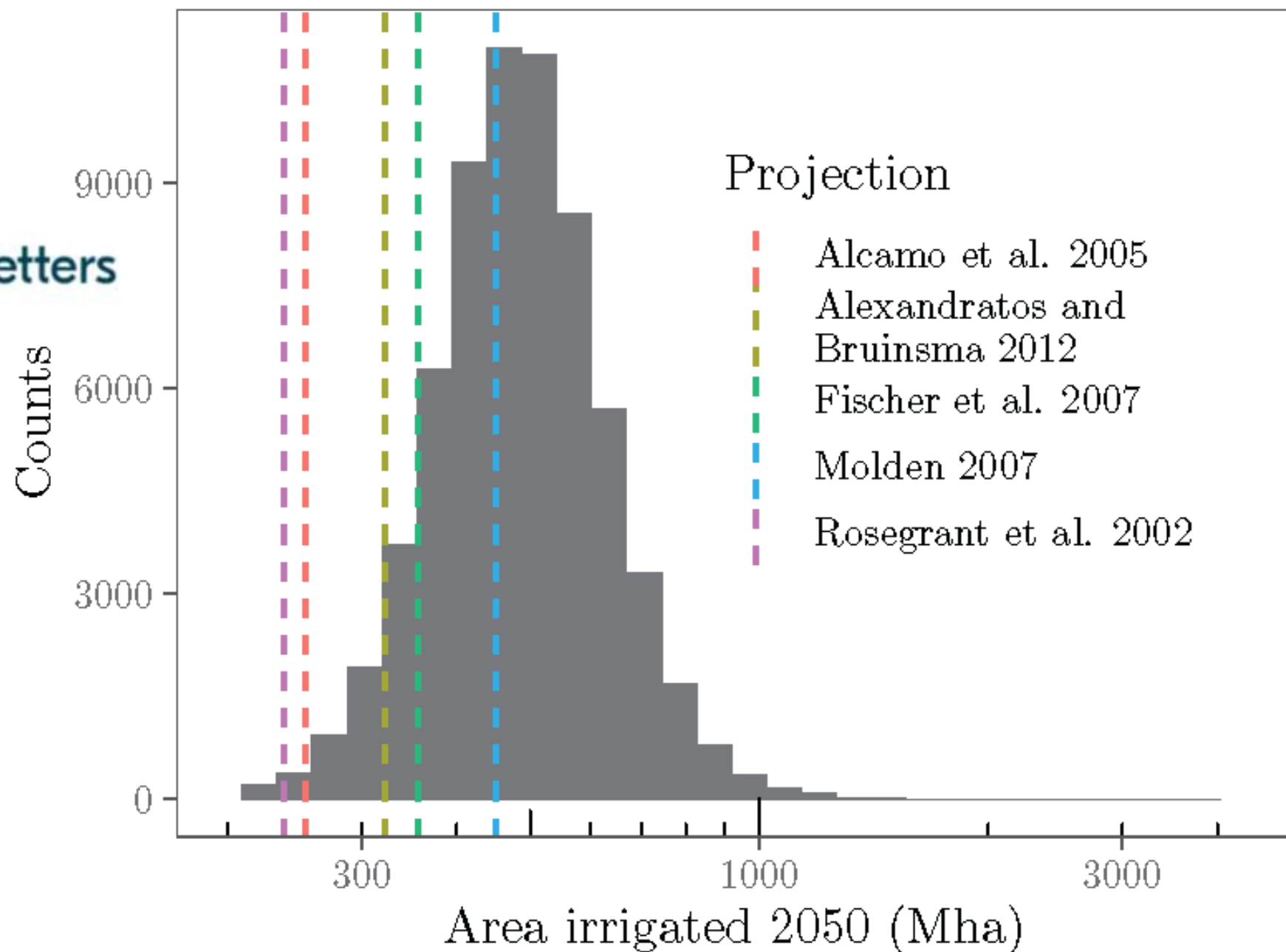
Current Models Underestimate Future Irrigated Areas

Geophysical Research Letters

A. Puy✉, S. Lo Piano, A. Saltelli

Citation:

Puy, A., Lo Piano, S., & Saltelli, A. (2020). Current models underestimate future irrigated areas. *Geophysical Research Letters*, 47, e2020GL087360. <https://doi.org/10.1029/2020GL087360>



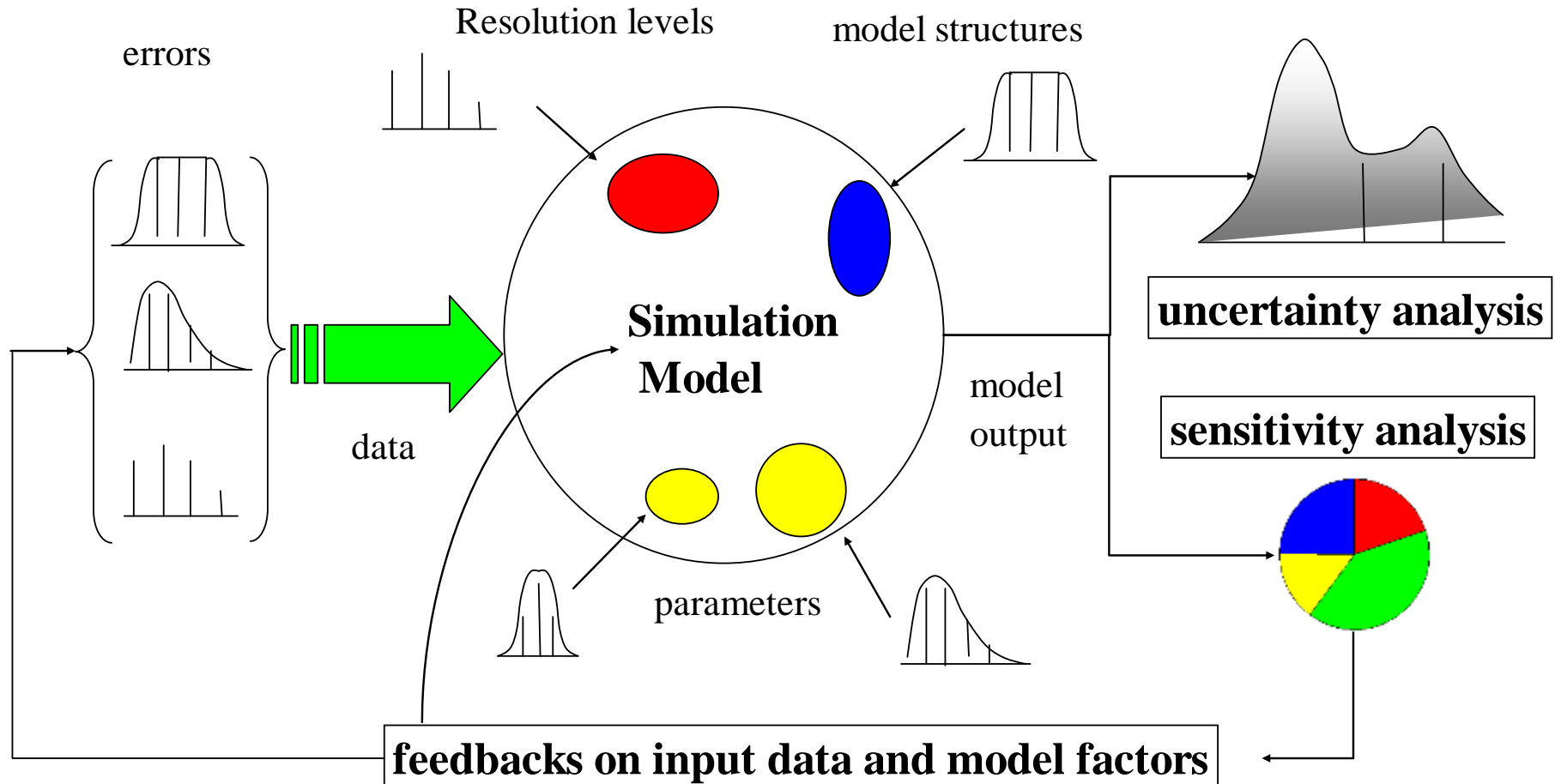
Solution? Modelling of
the modelling process by
taking 'all paths in the
garden'

Don't sample just
parameters and boundary
conditions

Explore thoroughly the space of the
assumptions

An engineer's vision of UA, SA

Remember?



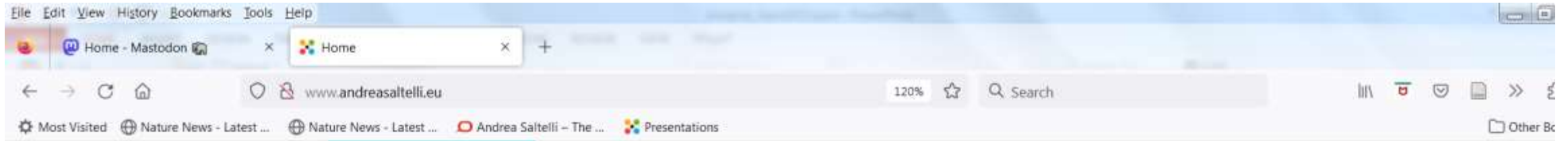
One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios ...

Reading material

- Borgonovo, E. 2007. “A New Uncertainty Importance Measure.” *Reliability Engineering & System Safety* 92 (6): 771–84. <https://doi.org/10.1016/J.RESS.2006.04.015>.
- Puy, Arnald, Pierfrancesco Beneventano, Simon A. Levin, Samuele Lo Piano, Tommaso Portaluri, and Andrea Saltelli. 2022. “Models with Higher Effective Dimensions Tend to Produce More Uncertain Estimates.” *Science Advances* 8 (eabn9450).
- Razavi, Saman, Anthony Jakeman, Andrea Saltelli, Clémentine Prieur, Bertrand Iooss, Emanuele Borgonovo, Elmar Plischke, et al. 2021. “The Future of Sensitivity Analysis: An Essential Discipline for Systems Modeling and Policy Support.” *Environmental Modelling & Software*, 104954–104954. <https://doi.org/10.1016/j.envsoft.2020.104954>.
- Saltelli, Andrea, Ksenia Aleksankina, William Becker, Pamela Fennell, Federico Ferretti, Niels Holst, Sushan Li, and Qiongli Wu. 2019. “Why so Many Published Sensitivity Analyses Are False: A Systematic Review of Sensitivity Analysis Practices.” *Environmental Modelling & Software* 114 (April): 29–39. <https://doi.org/10.1016/J.ENVSOFT.2019.01.012>.
- Saltelli, Andrea, and Paola Annoni. 2010. “How to Avoid a Perfunctory Sensitivity Analysis.” *Environmental Modelling & Software* 25 (12): 1508–17. <https://doi.org/10.1016/j.envsoft.2010.04.012>.
- Saltelli, Andrea, M. Ratto, T. H. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola. 2008. *Global Sensitivity Analysis : The Primer*. John Wiley. <https://doi.org/10.1002/9780470725184>.
- Sobol', I.M. 1993. “Sensitivity Analysis for Non-Linear Mathematical Models,.” *Mathematical Modelling and Computational Experiment* (Translated from Russian: I.M. Sobol', Sensitivity Estimates for Nonlinear Mathematical Models, *Matematicheskoe Modelirovanie* 2 (1990) 112–118) 1 (4): 407–14.

More material at www.andreasaltelli.eu



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