



Sensitivity Analysis

Andrea Saltelli

Open Evidence Research, Open University of Catalonia



Virtual seminar for Applied BioMath LLC,
December 4, 2020



Where to find this talk: www.andreasaltelli.eu

The logo for Andrea Saltelli, featuring the name "Andrea Saltelli" in white text on a teal square background.[HOME](#)[ABOUT ME](#)[PUBLICATIONS](#)[NEWS & VIDEOS](#)[RESOURCES](#)A large background image of terraced rice fields in a valley, with mountains in the background under a hazy sky. The text "CAETERIS ARE NEVER PARIBUS" is overlaid on the left side of the image.

CAETERIS ARE
NEVER PARIBUS

Tweets by @AndreaSaltelli

andrea saltelli Retweeted

 **I-site ULNE**
@isiteULNE

#statistiques #probabilités #modélisation
#prédiction Isabelle Bruno du #CERAPS
@univ_lille @CNRS_HdF @ScPoLille nous parle
des dérives de la #quantophrénie dans un article à
lire sur le media @FR_Conversation
https://twitter.com/FR_Conversation/status/1302651033164881920



Sep 7, 2020

 **andrea saltelli**
@AndreaSaltelli

Pour mes amis francophones. Honoured to be co-
author of a statactivist like Isabelle Bruno du
#CERAPS @univ_lille @CNRS_HdF @ScPoLille
@OpenEvidence @UOCNews
Statistiques et modèles mathématiques : doit-on

Embed

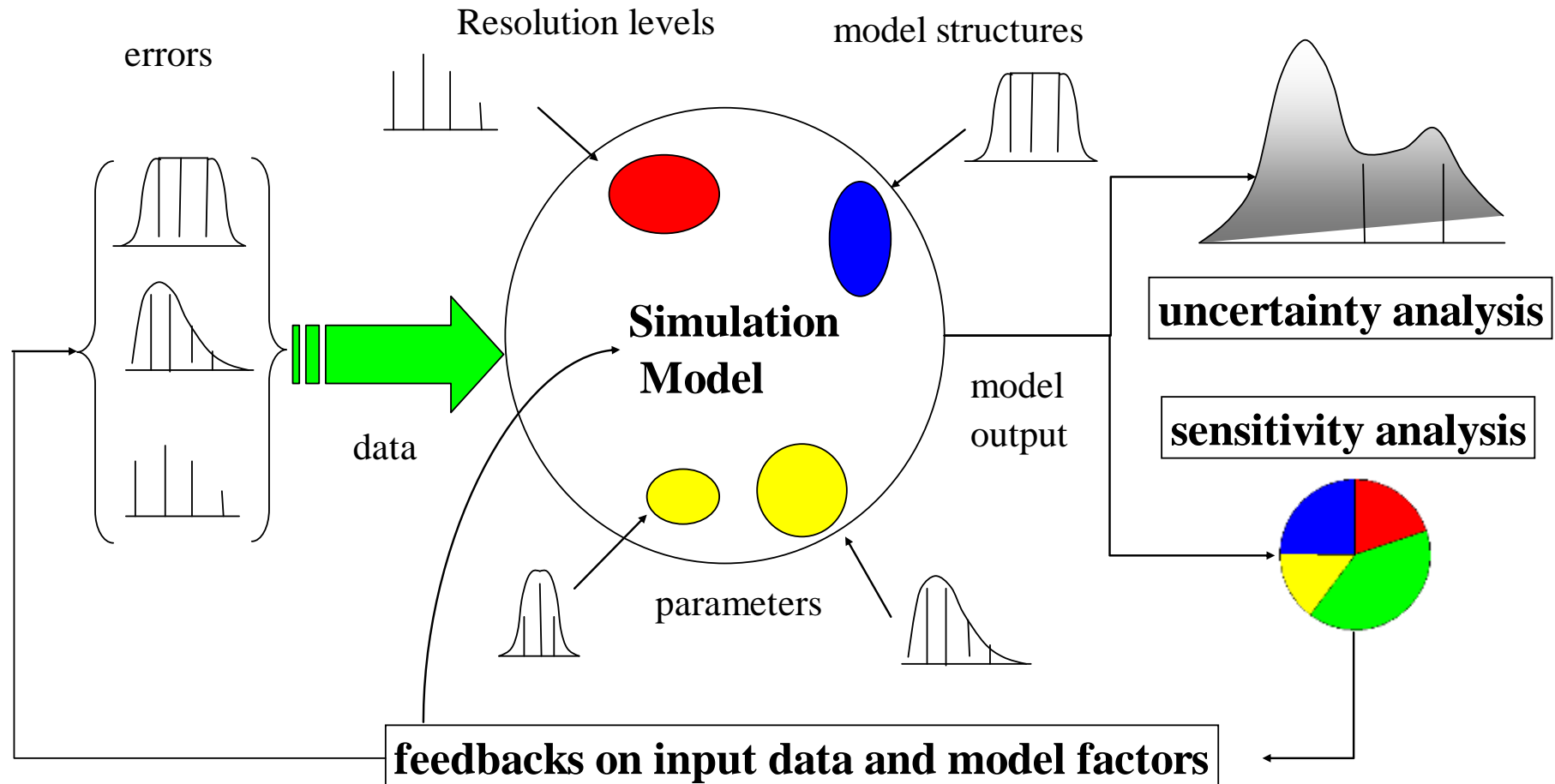
[View on Twitter](#)

Definitions

Uncertainty analysis: Focuses on just quantifying the uncertainty in model output

Sensitivity analysis: The study of the relative importance of different input factors on the model output

An engineer's vision of UA, SA

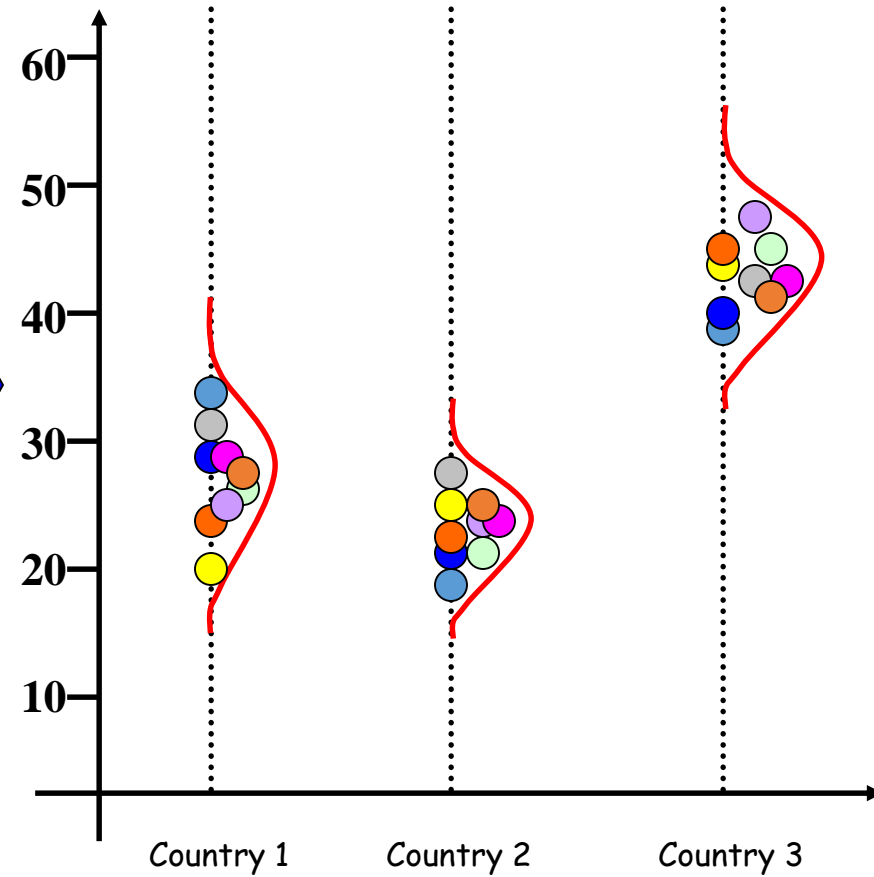
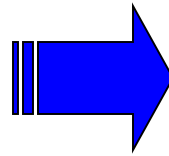
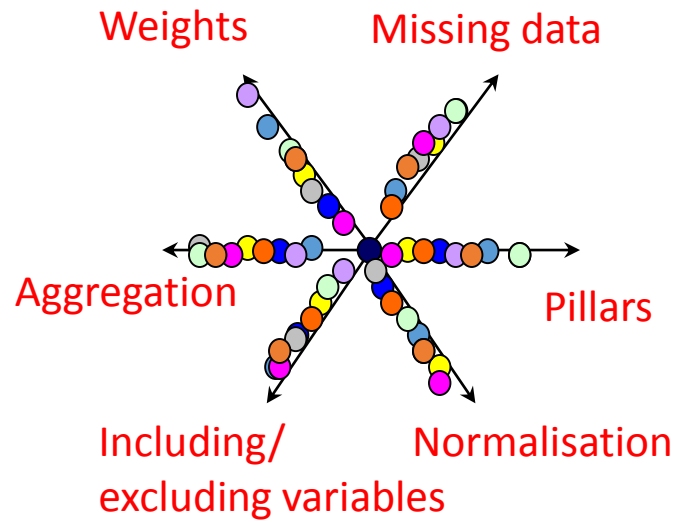


One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios ...

Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

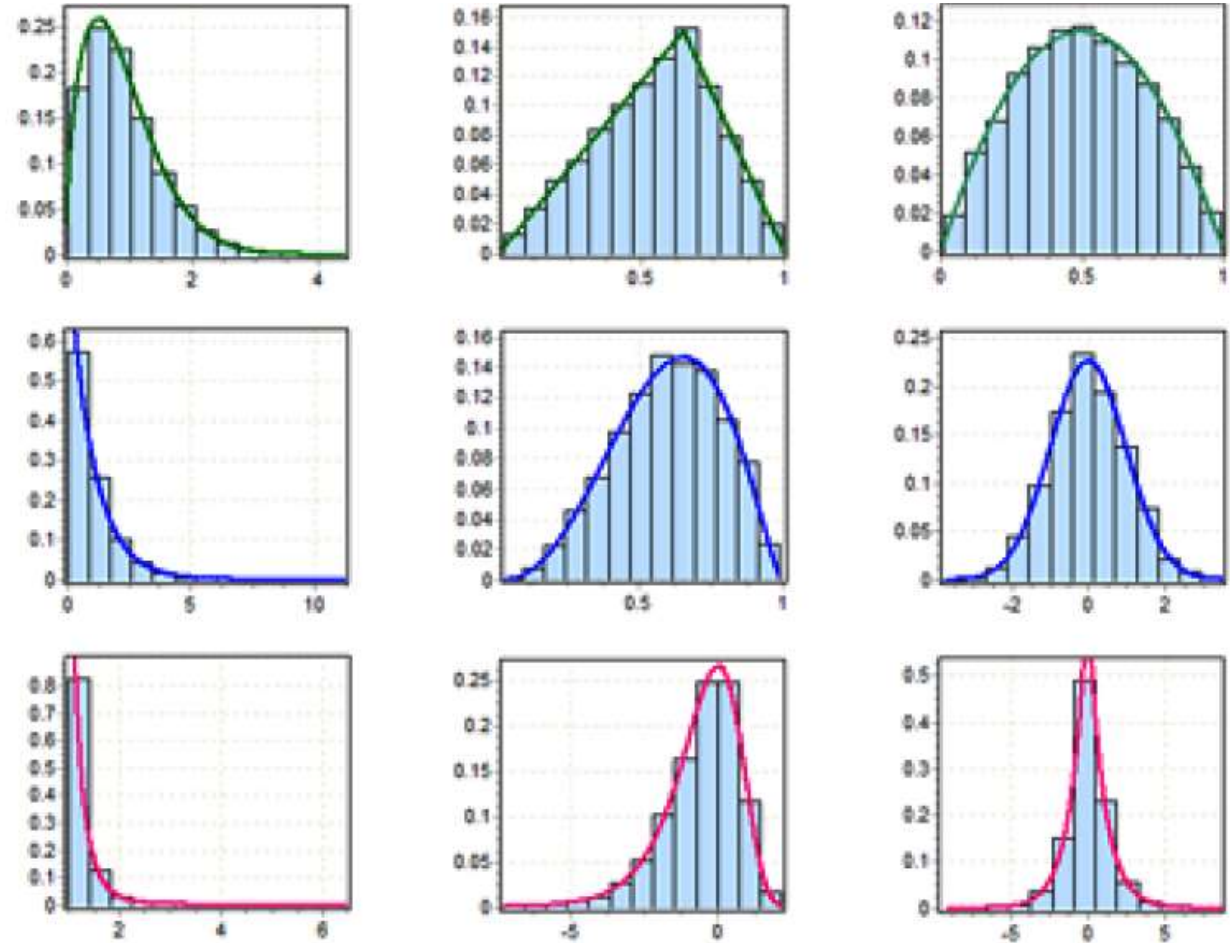
Space of alternatives



x_{11}	x_{12}	...	x_{1k}
x_{21}	x_{22}	...	x_{2k}
...
x_{N1}	x_{N2}	...	x_{Nk}

Each column is a sample from the distribution of a factor

Each row is a sample trial to generate a value of y

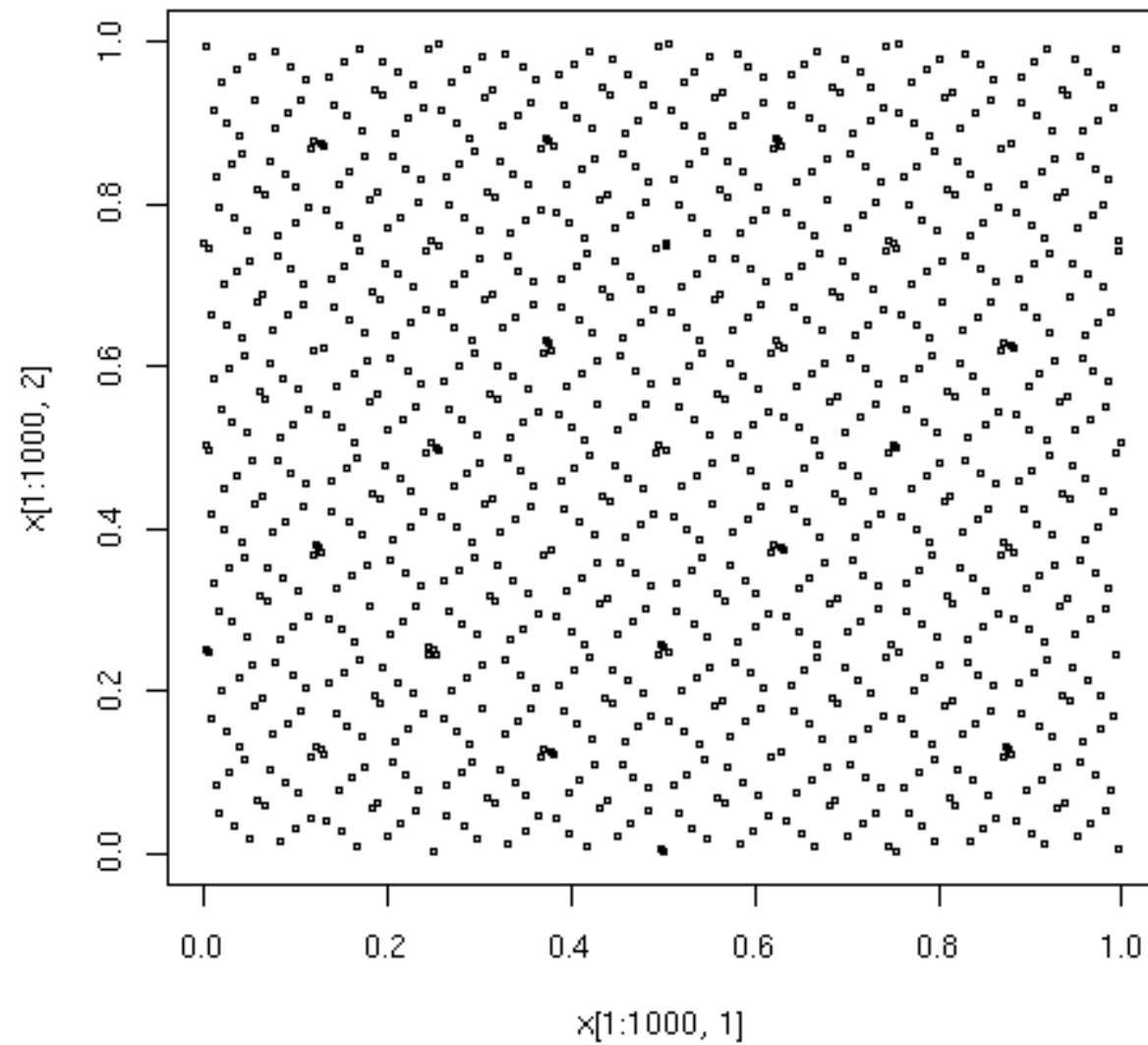


Examples of distributions of input factors

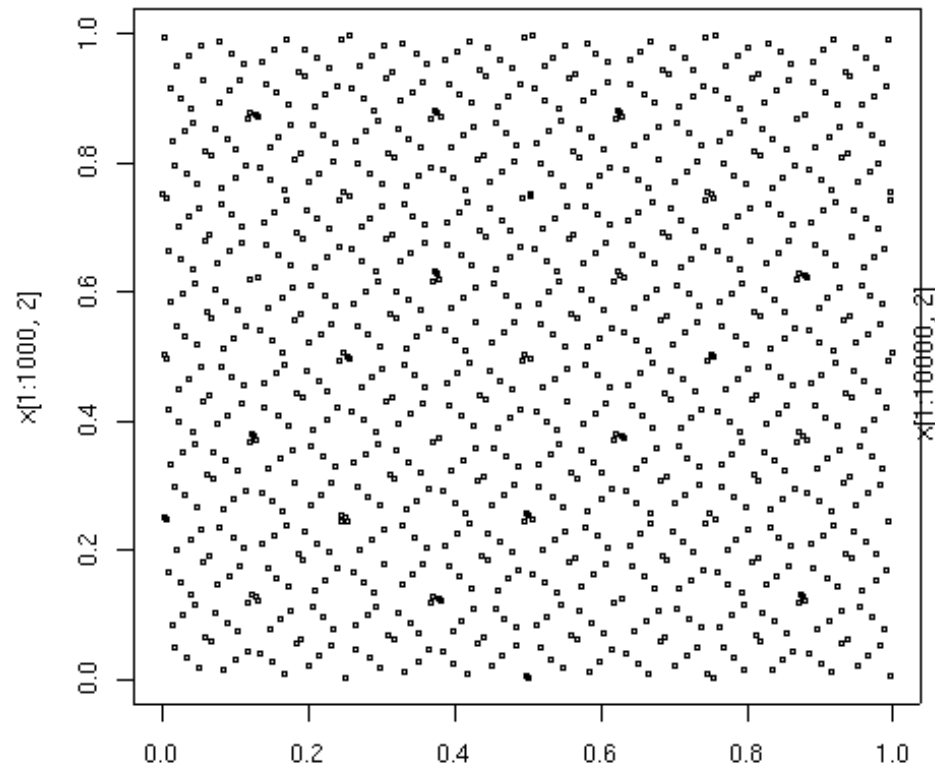


How to generate
the random sample?

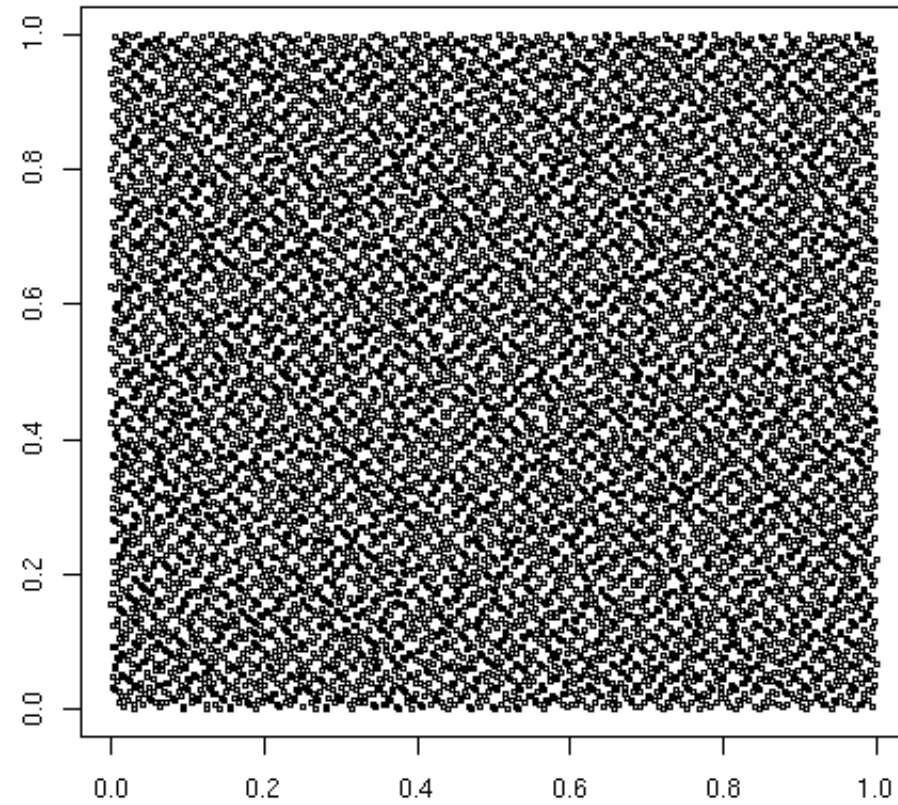
Quasi random
sequences
developed by I.M.
Sobol'



An LP_τ sequence

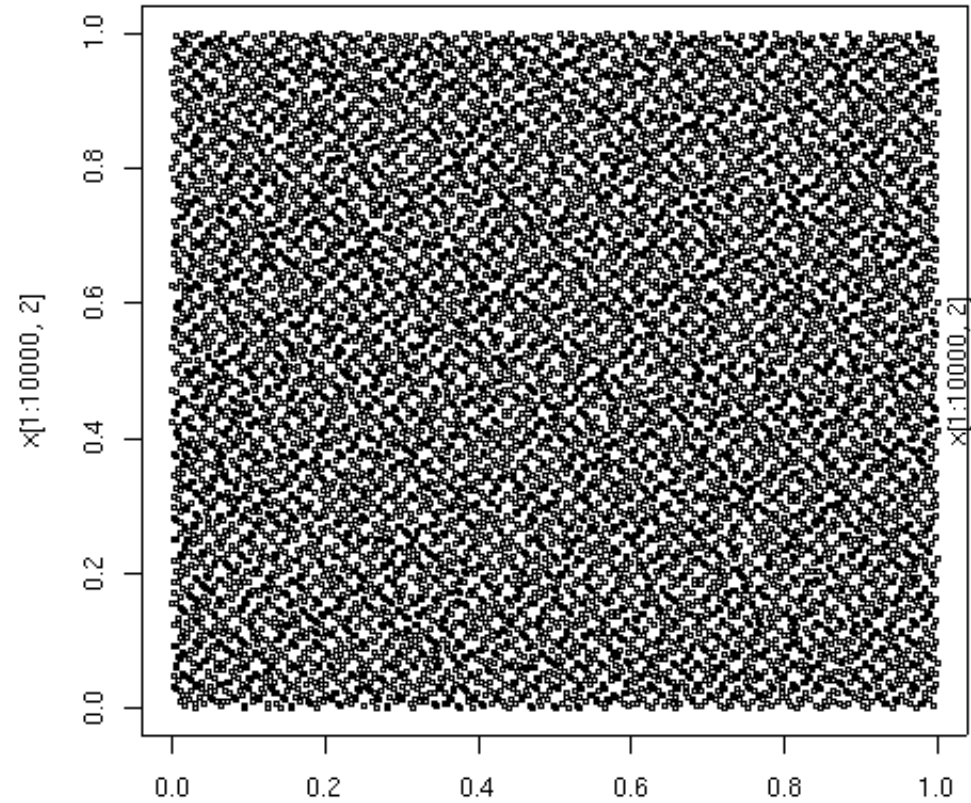


X_1, X_2 plane, 1000 Sobol' points

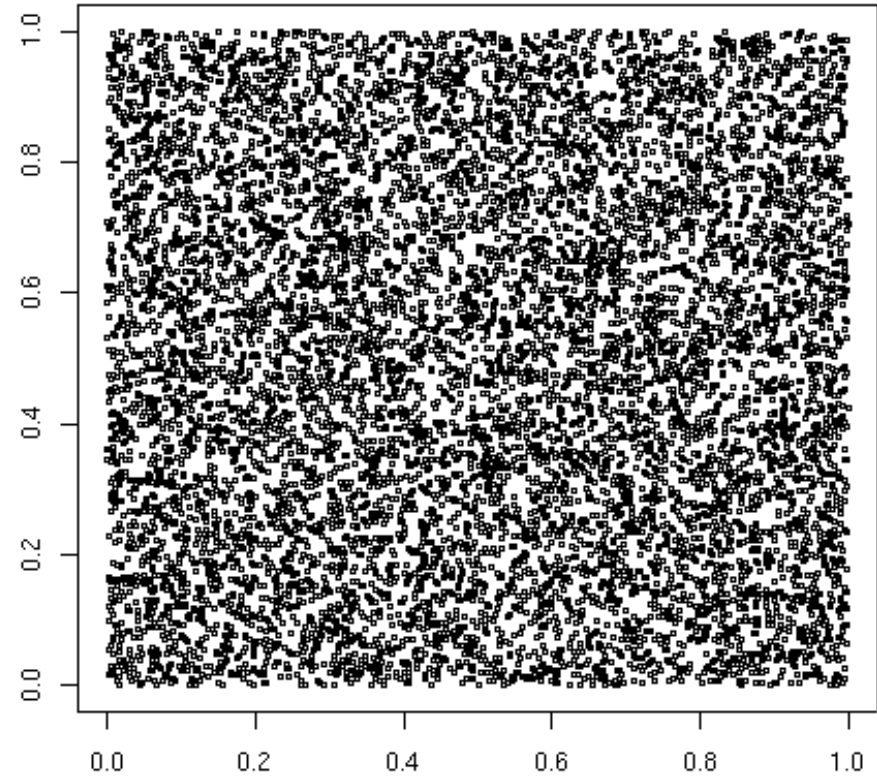


X_1, X_2 plane, 10000 Sobol' points

Sobol' sequences of quasi-random points



X1,X2 plane, 10000 Sobol' points



X1,X2 plane, 10000 random points

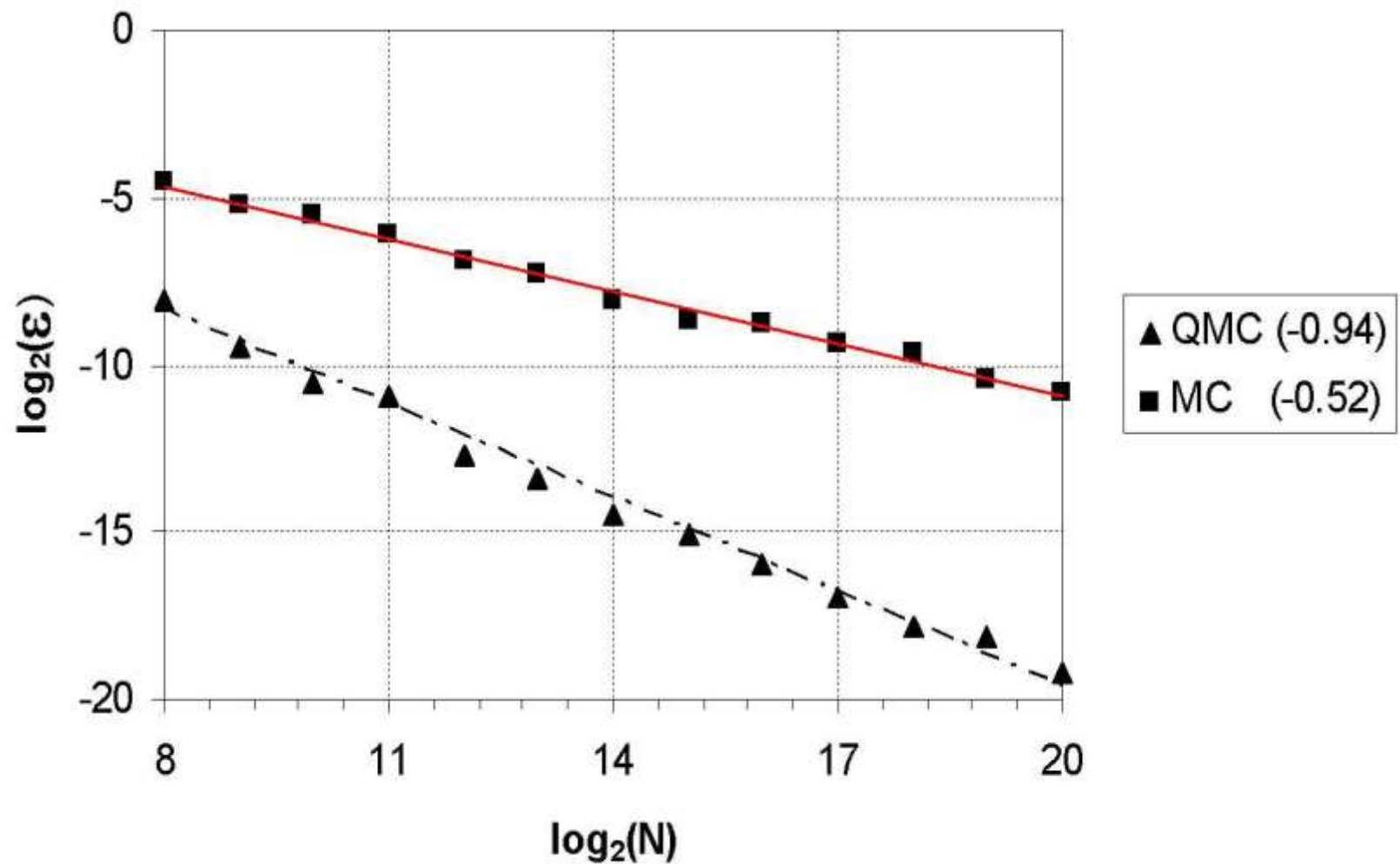
Sobol' sequences of quasi-random points
against random points

Why quasi-random: they have faster convergence



Sergei Kucherenko,
Imperial College London

Kucherenko S., Feil B., Shah N., Mauntz W. The identification of model effective dimensions using global sensitivity analysis Reliability Engineering and System Safety 96 (2011) 440–449.



$$\varepsilon = \left(\frac{1}{K} \sum_{k=1}^K (I[f] - I_k[f])^2 \right)^{1/2}$$

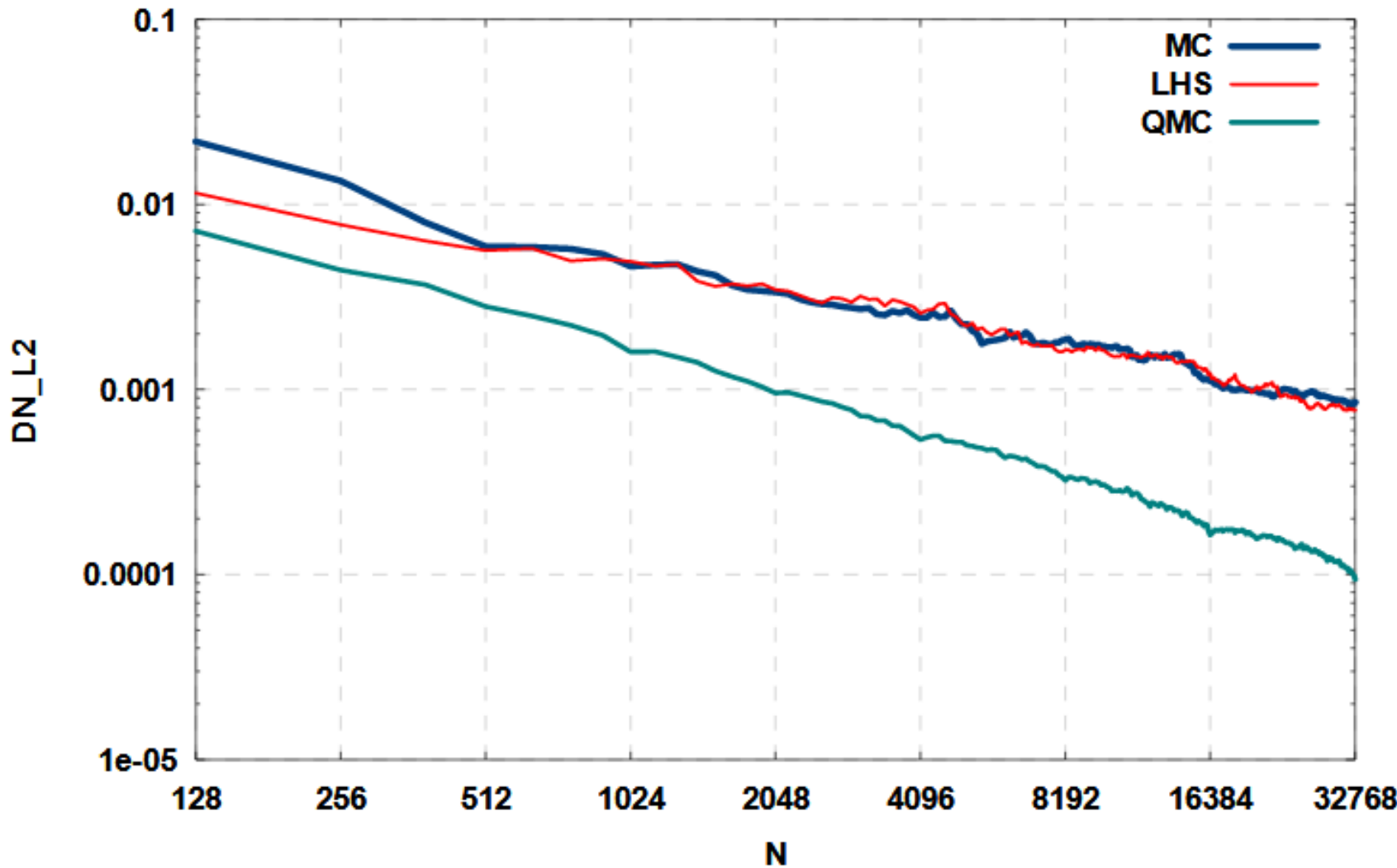
$$\sum_{i=1}^n (-1)^i \prod_{j=1}^i x_j$$

Error=numeric-
versus-analytic
value the integral
of the function (for
n=360) over its
dominion.

Root mean square error over K=50 different trials.

Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

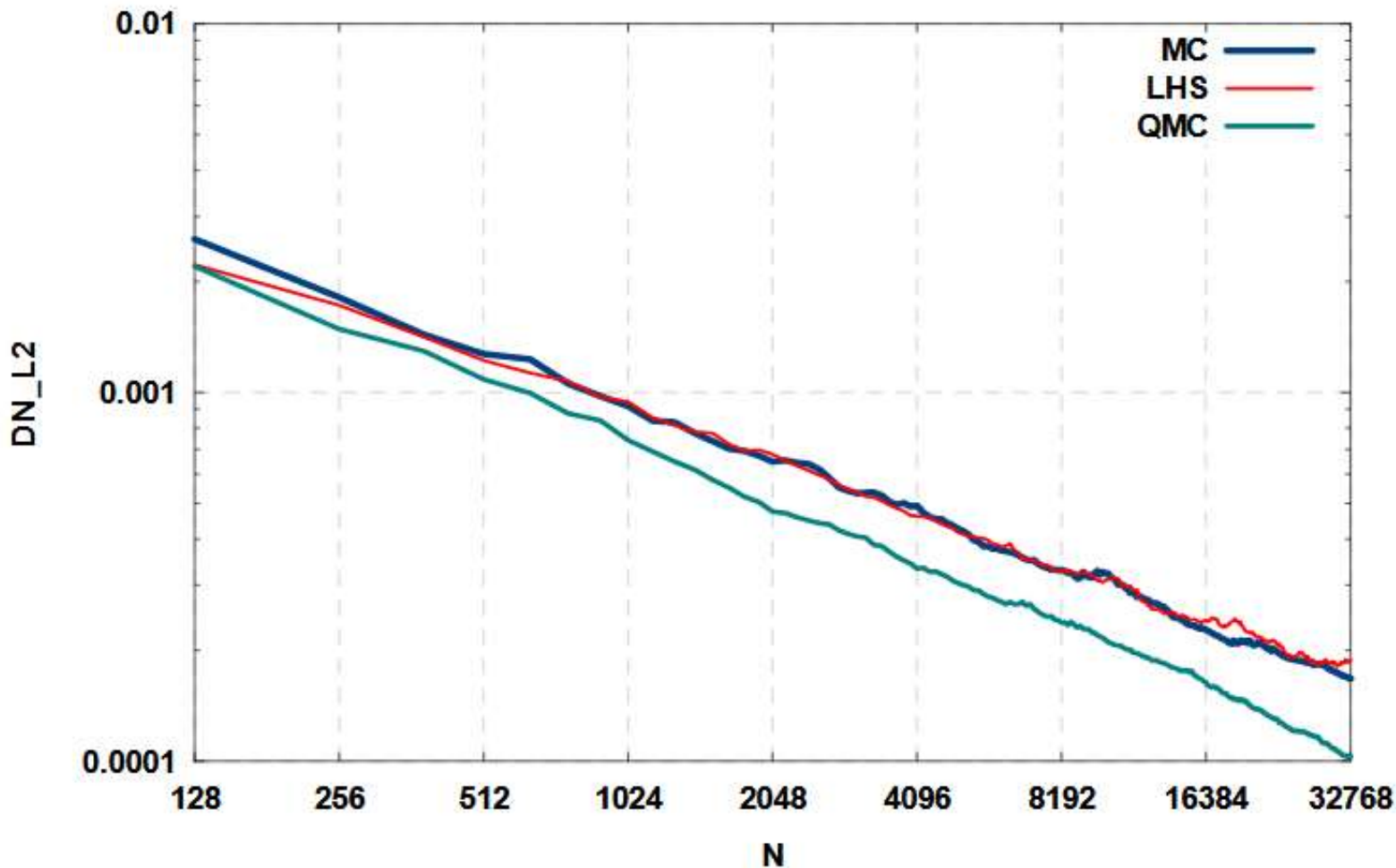
Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli



Discrepancy
in five
dimensions
as function
of sample
size

Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

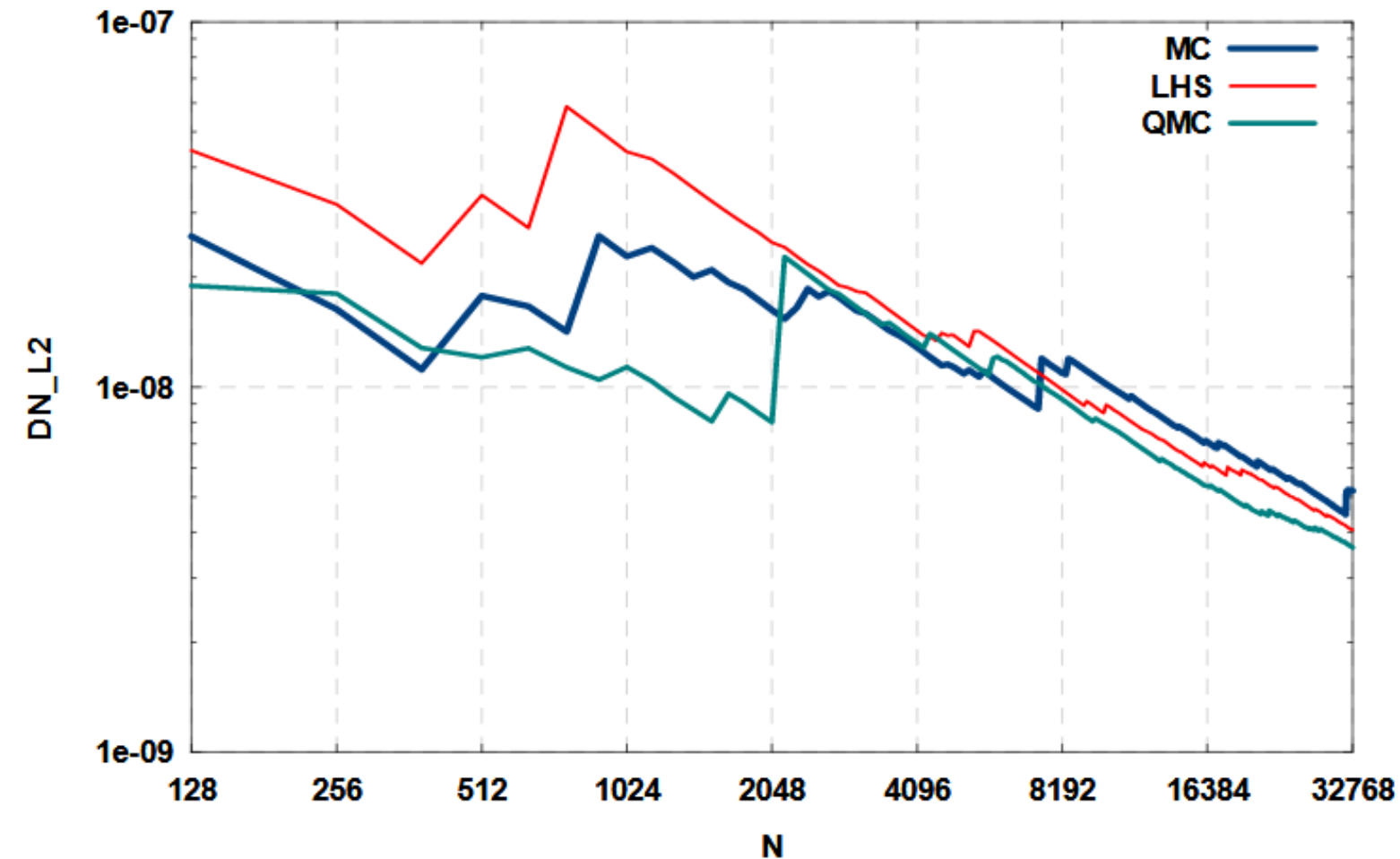
Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli



Discrepancy
in **ten**
dimensions
as function
of sample
size

Exploring multi-dimensional spaces: a Comparison of Latin Hypercube and Quasi Monte Carlo Sampling Techniques

Sergei Kucherenko, Daniel Albrecht, Andrea Saltelli



Discrepancy
in **forty**
dimensions
as function
of sample
size

Why Sensitivity analysis?

It can answer interesting
questions

Global Environmental Change 20 (2010) 298–302



Contents lists available at ScienceDirect

Global Environmental Change

journal homepage: www.elsevier.com/locate/gloenvcha



Sensitivity analysis didn't help. A practitioner's critique of the Stern review

Andrea Saltelli^{*}, Beatrice D'Hombres

Joint Research Centre, Institute for the Protection and Security of the Citizen, Ispra, Italy

The case of Stern's Review – Technical Annex to postscript



William Nordhaus,
University of Yale
Nobel 'Economics'
2018



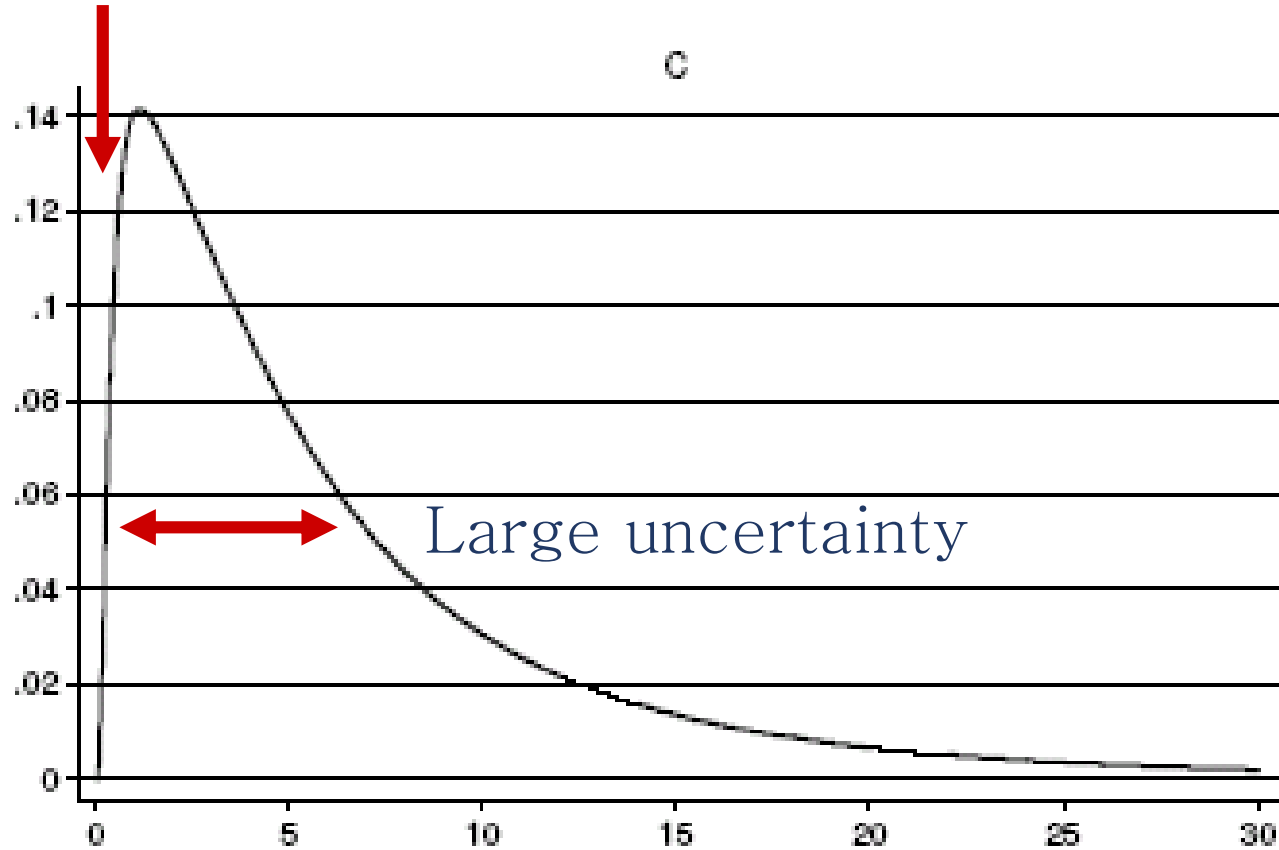
Nicholas Stern, London School
of Economics

Stern, N., Stern Review on the Economics of Climate Change. UK Government Economic Service, London, www.sternreview.org.uk.

Nordhaus W., Critical Assumptions in the Stern Review on Climate Change, SCIENCE, 317, 201–202, (2007).

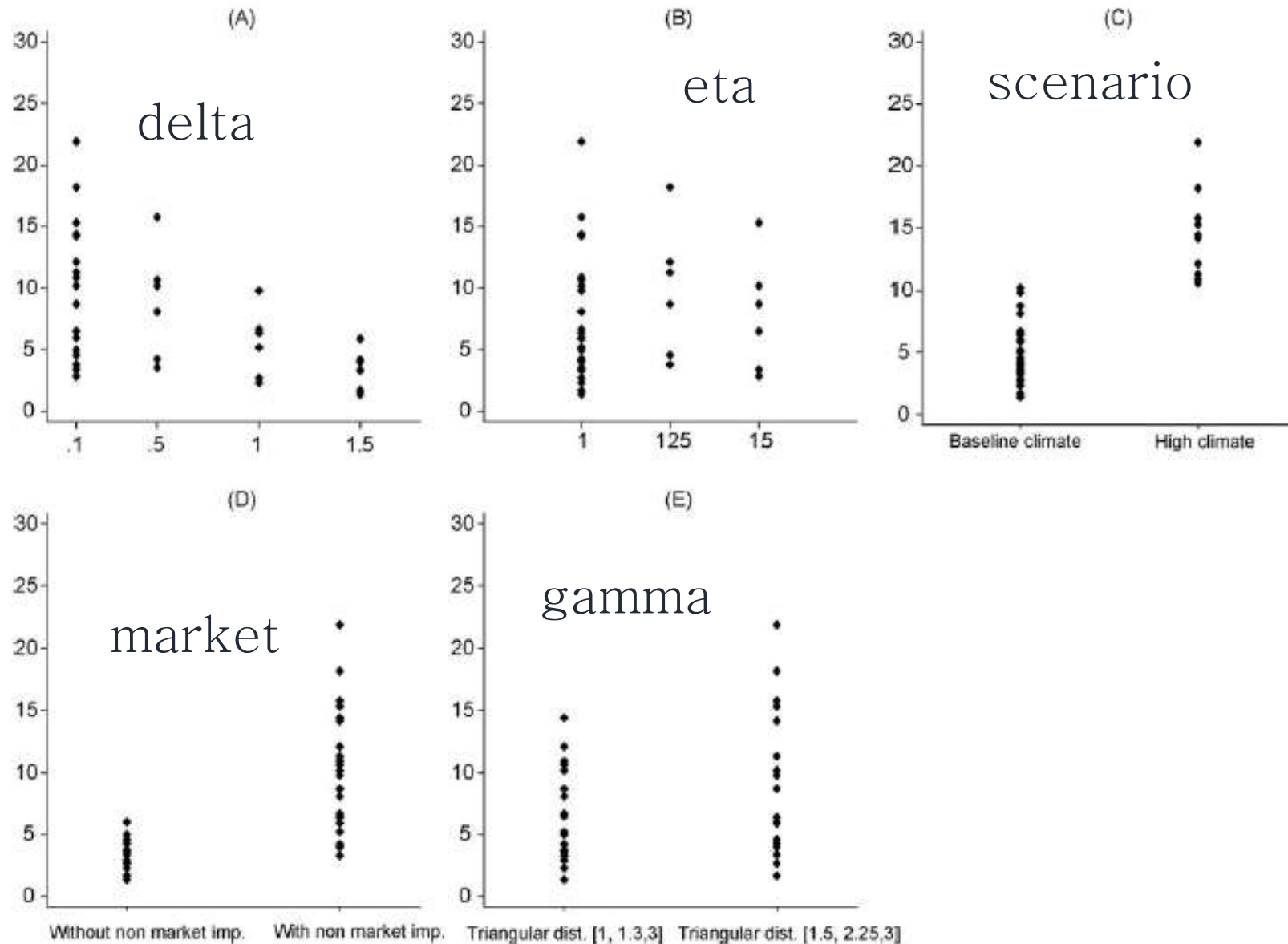
How was it done? A reverse engineering of the analysis

Missing points



% loss in GDP per capita

Sensitivity analysis here (by reverse engineering)



Why sensitivity analysis?

It allows interesting discoveries

J. R. Statist. Soc. A (2013)
176, Part 3, pp. 609–634

Ratings and rankings: voodoo or science?

Paolo Paruolo

University of Insubria, Varese, Italy

and Michaela Saisana and Andrea Saltelli

European Commission, Ispra, Italy

Why sensitivity
analysis

University rankings such as
ARWU and THES are
technically unsound

Ratings and rankings: voodoo or science?

Paolo Paruolo

University of Insubria, Varese, Italy

and Michaela Saisana and Andrea Saltelli

European Commission, Ispra, Italy

~~THE WORLD
UNIVERSITY
RANKINGS~~

Why sensitivity
analysis

Geophysical Research Letters / Volume 47, Issue 8

Research Letter

 Open Access



Current Models Underestimate Future Irrigated Areas

A. Puy✉, S. Lo Piano, A. Saltelli

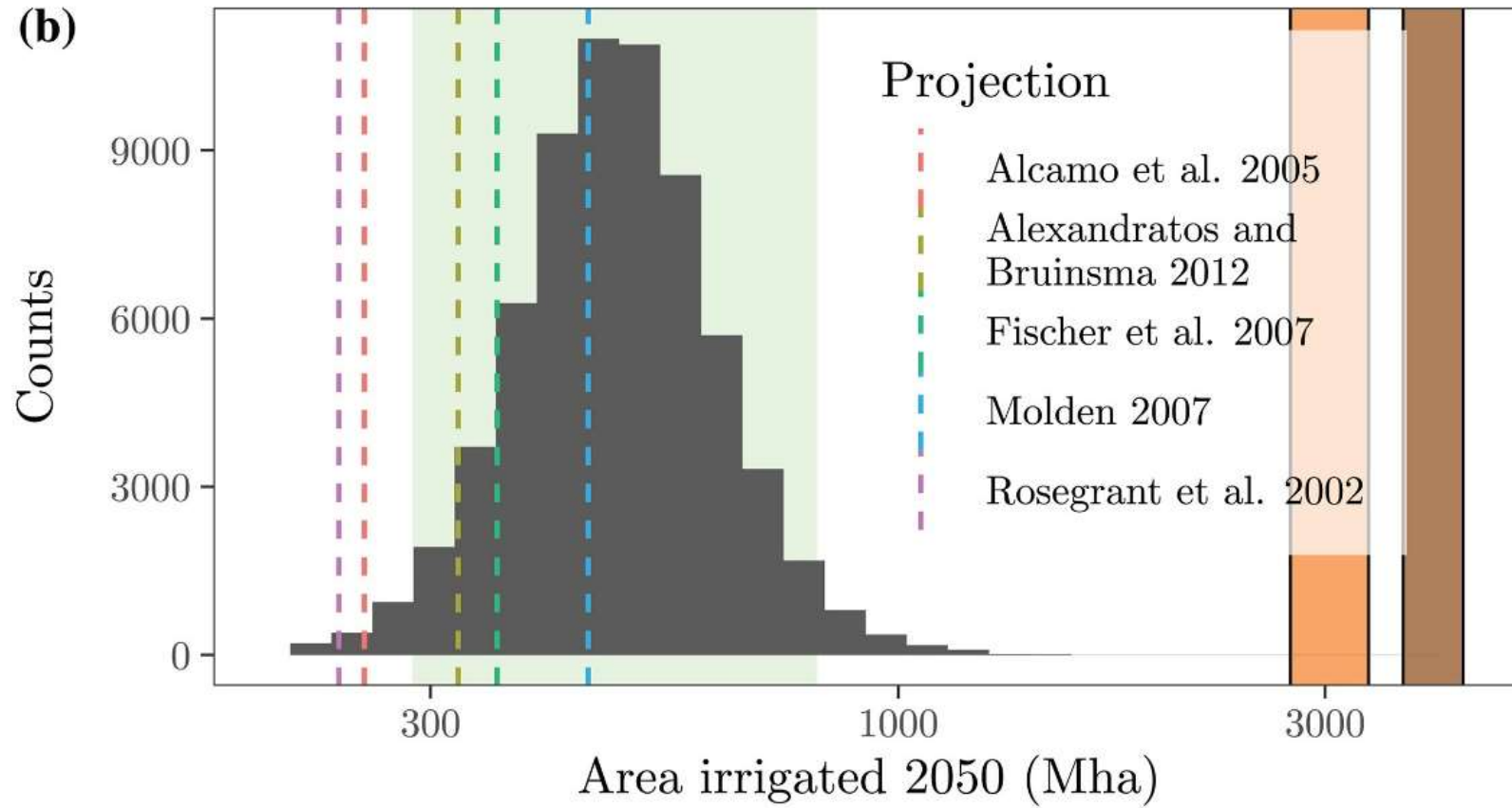
First published: 17 April 2020

<https://doi.org/10.1029/2020GL087360>

Citations: 2

Why sensitivity
analysis

(b)



Geophysical Research Letters / Volume 47, Issue 8

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Current Models Underestimate Future Irrigated Areas

A. Puy , S. Lo Piano, A. Saltelli

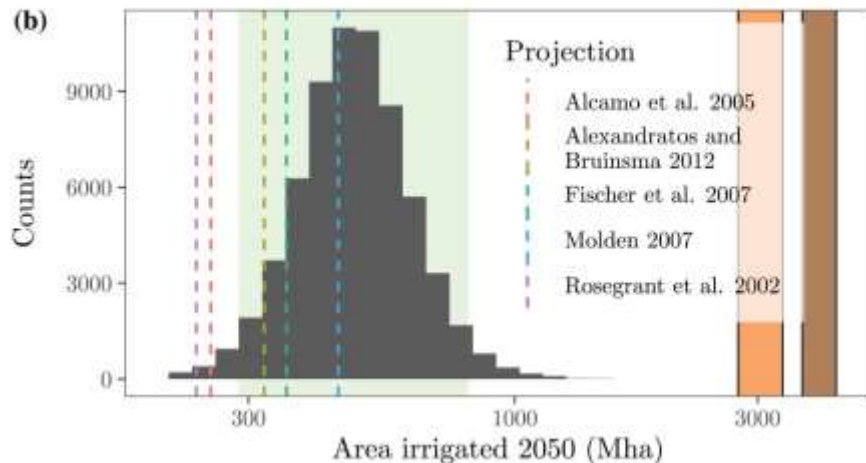
First published: 17 April 2020

<https://doi.org/10.1029/2020GL087360>

Citations: 2

Why sensitivity
analysis

Predictions underestimate the potential extension of irrigation ... uncertainty is mostly irreducible as it is largely caused by either population-related parameters or the assumptions behind the model design



Geophysical Research Letters / Volume 47, Issue 8

Research Letter | [Open Access](#) |

Current Models Underestimate Future Irrigated Areas

A. Puy , S. Lo Piano, A. Saltelli

First published: 17 April 2020

<https://doi.org/10.1029/2020GL087360>

Citations: 2

Why sensitivity analysis

Five ways to ensure that models serve society: a manifesto

Pandemic politics highlight how predictions need to be transparent and humble to invite insight, not blame.

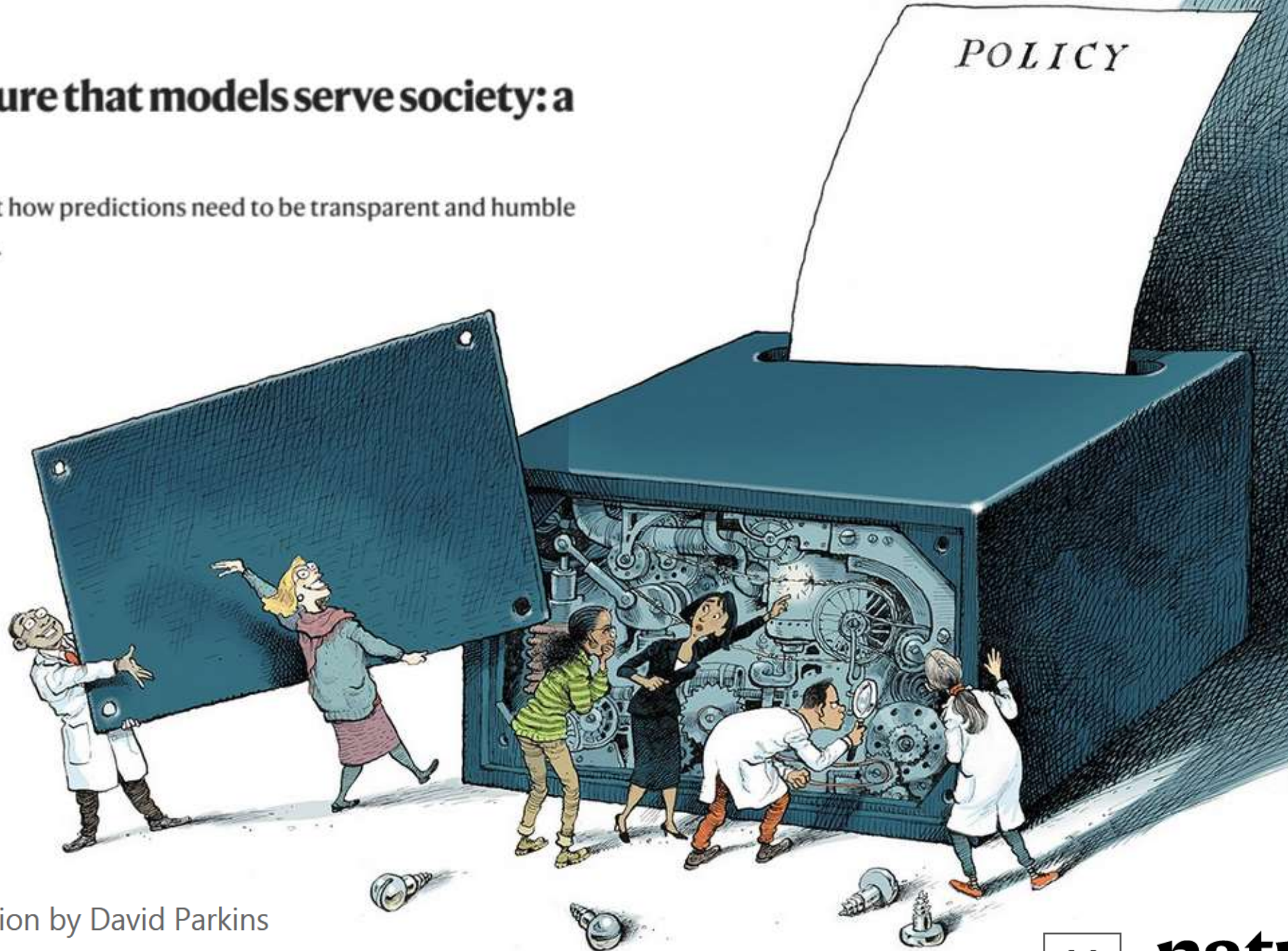


Illustration by David Parkins

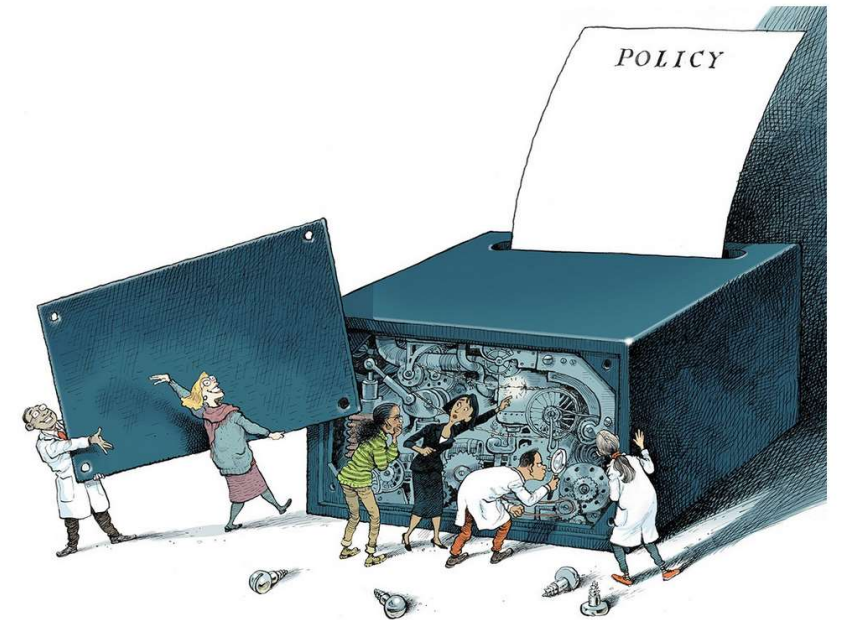


nature

Five ways to ensure that models serve society: a manifesto

Pandemic politics highlight how predictions need to be transparent and humble to invite insight, not blame.

Andrea Saltelli , Gabriele Bammer, Isabelle Bruno, Erica Charters, Monica Di Fiore, Emmanuel Didier, Wendy Nelson Espeland, John Kay, Samuele Lo Piano, Deborah Mayo, Roger Pielke Jr, Tommaso Portaluri, Theodore M. Porter, Arnald Puy, Ismael Rafols, Jerome R. Ravetz, Erik Reinert, Daniel Sarewitz, Philip B. Stark, Andrew Stirling, Jeroen van der Sluijs & Paolo Vineis



Mind the assumptions

Assess uncertainty and sensitivity



Mind the hubris

Complexity can be the enemy of relevance

Mind the framing

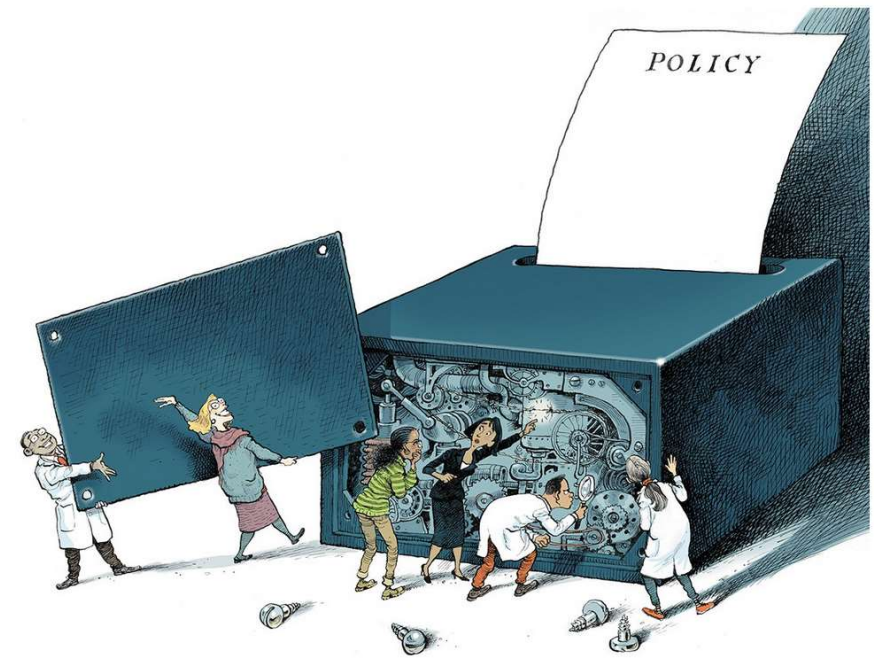
Match purpose and context

Mind the consequences

Quantification can backfire.

Mind the unknowns

Acknowledge ignorance



SUPPLEMENTARY INFORMATION

1. Additional information and references

>260 references

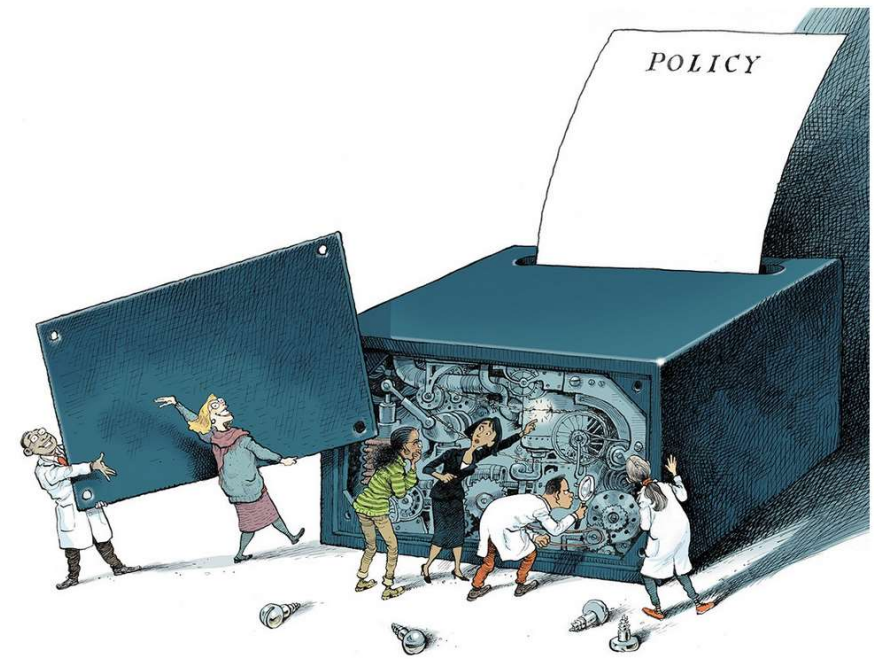
Mind the assumptions

Assess uncertainty and sensitivity



... assumptions that are reasonable in one situation can become nonsensical in another...

... models require input values for which there is no reliable information.



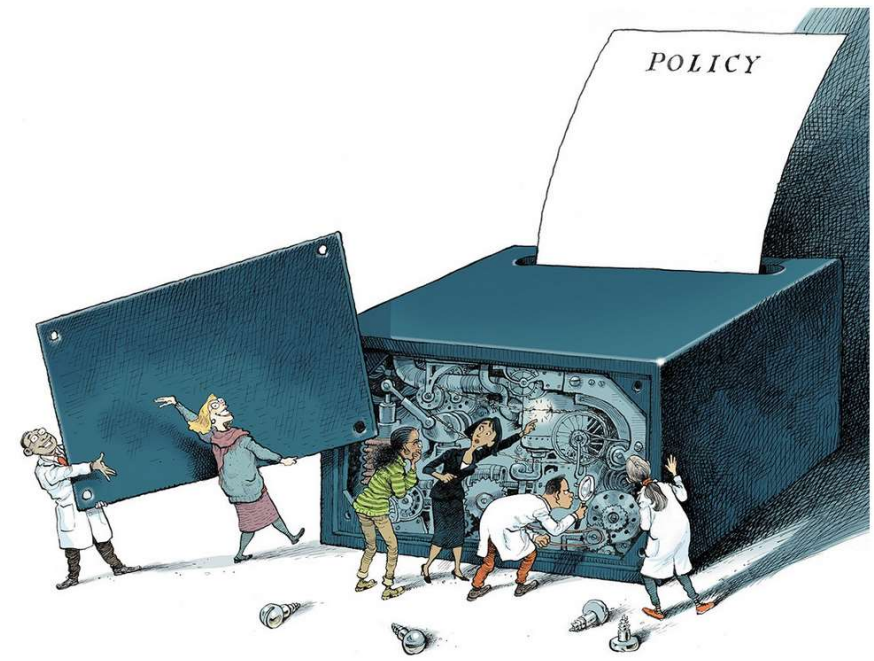
Mind the assumptions

Assess uncertainty and sensitivity



... to mitigate these issues:
perform global uncertainty and
sensitivity analyses ...

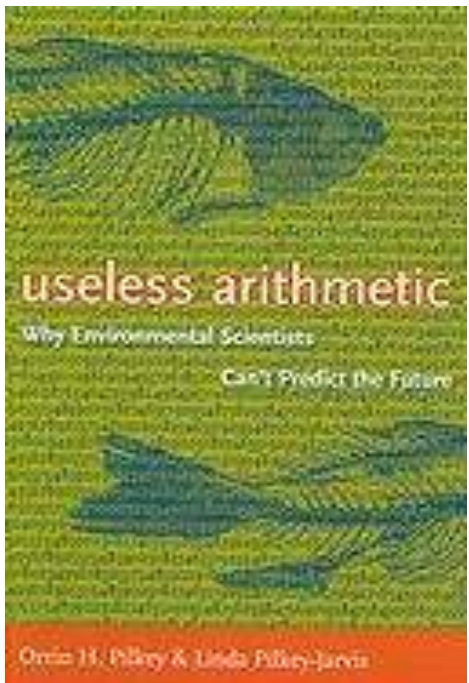
This often reveals that the uncertainty in
predictions is substantially larger than originally
asserted



Limits of sensitivity analysis

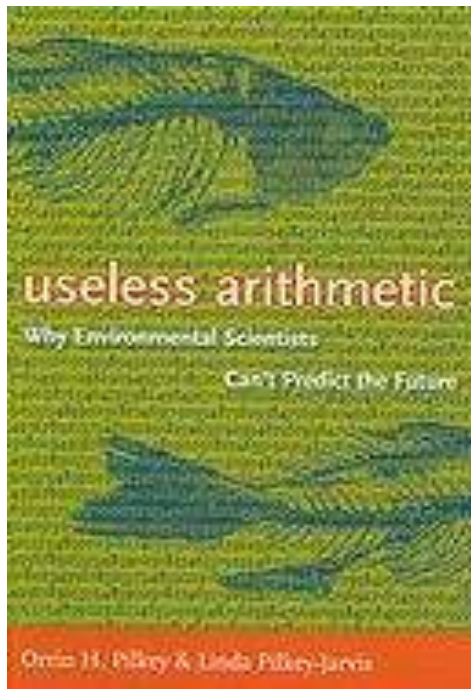


Orrin H.
Pilkey



Useless Arithmetic: Why Environmental Scientists Can't Predict the Future
by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

The map is not the
territory

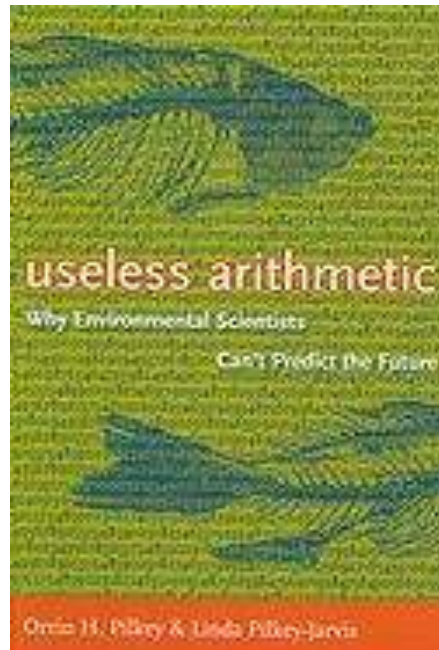


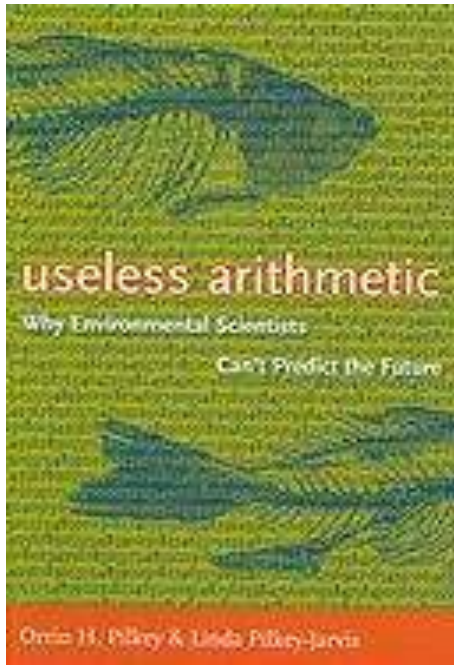
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

[...] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

One of the examples discussed concerns the **Yucca Mountain** repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

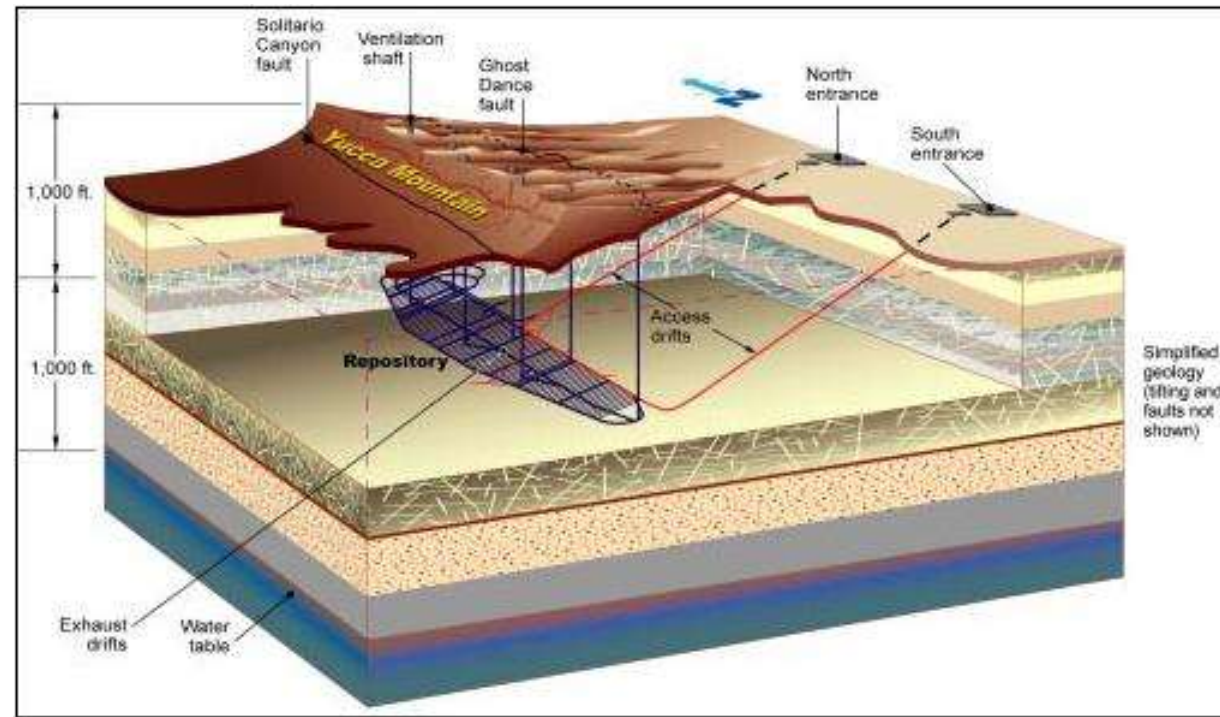
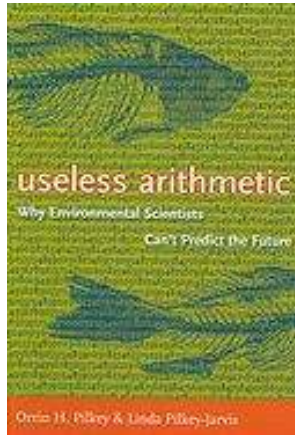
TSPA is Composed of 286 sub-models.





TSPA (like any other model) **relies on assumptions** → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.



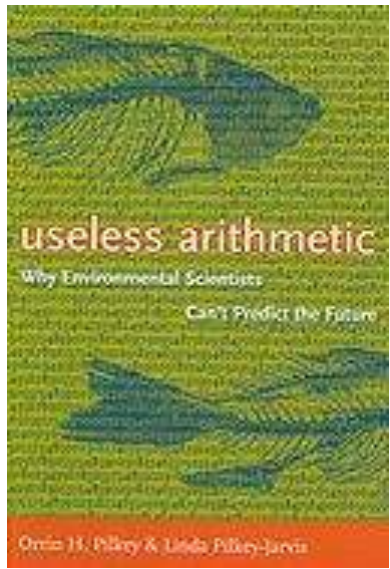


The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter
(the ^{36}Cl story)

Type III error in sensitivity: Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→... SA useless if it is instead ~ 3,000 millimetres per year.



“Scientific mathematical modelling should involve constant efforts to falsify the model”

➔ Organized skepticism (as per CUDOS)

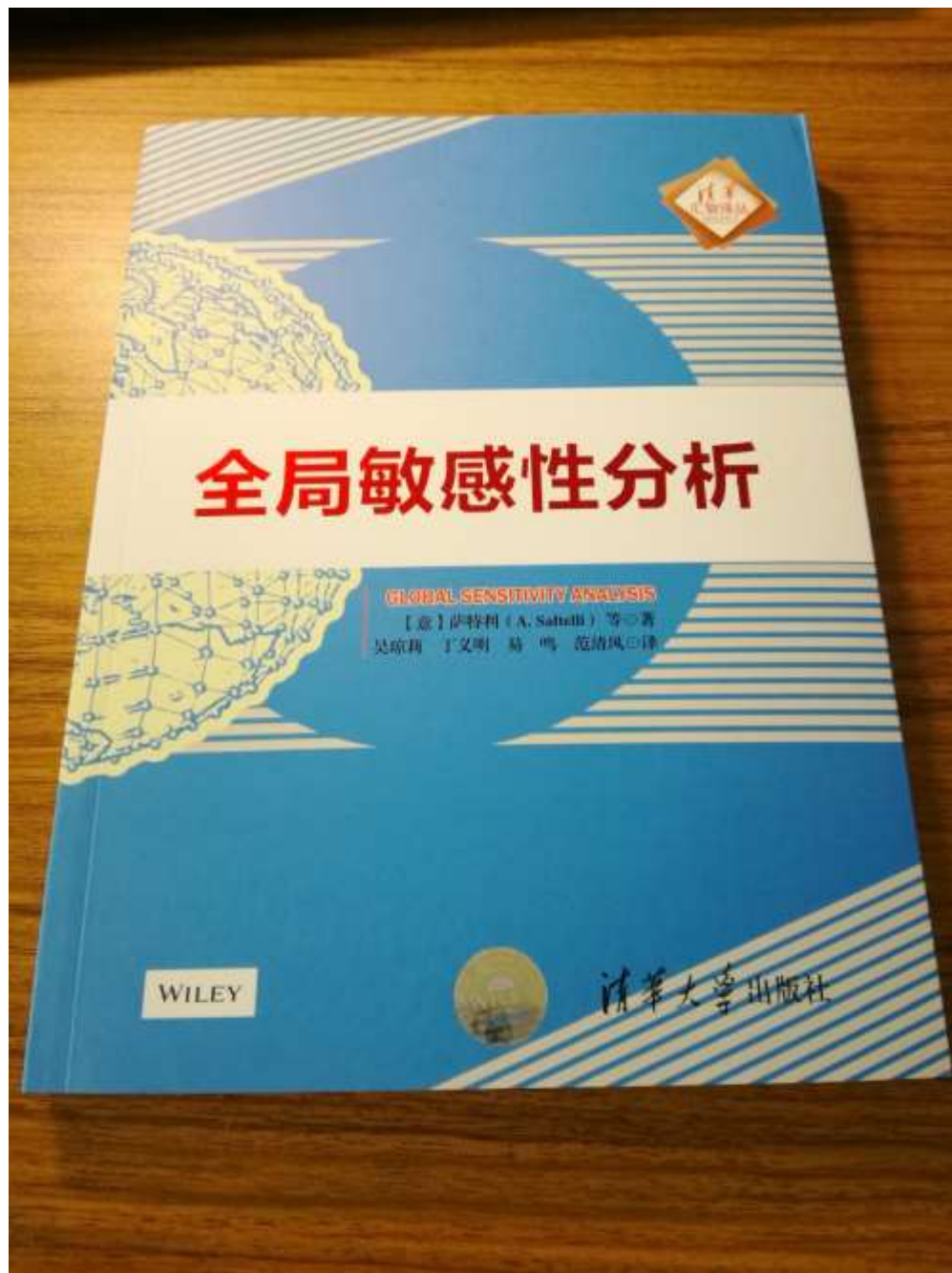
Where to study
sensitivity analysis?

A. Saltelli, M. Ratto,
T. Andres, F. Campolongo,
J. Cariboni, D. Gatelli,
M. Saisana, S. Tarantola

GLOBAL SENSITIVITY ANALYSIS

The Primer

 WILEY





A. Saltelli, M. Ratto,
T. Andres, F. Campolongo,
J. Cariboni, D. Gatelli,
M. Saisana, S. Tarantola

GLOBAL SENSITIVITY ANALYSIS

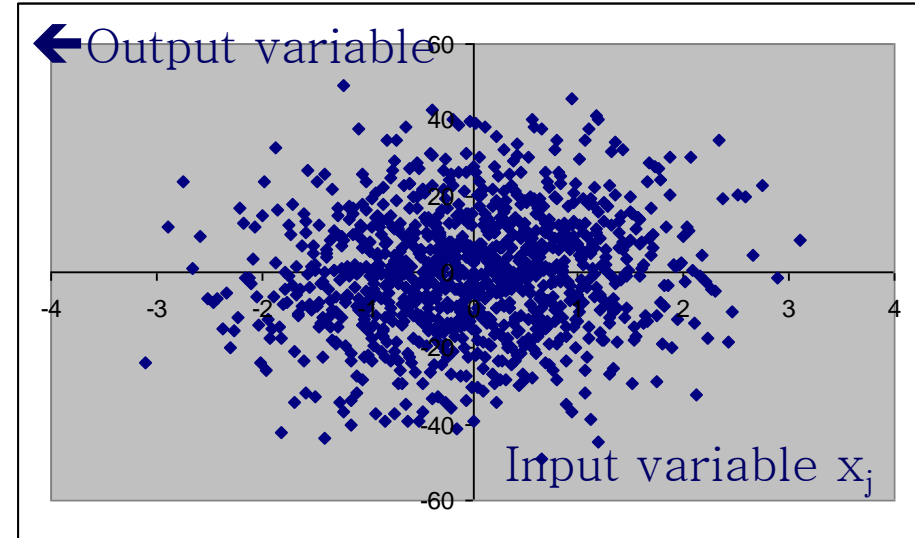
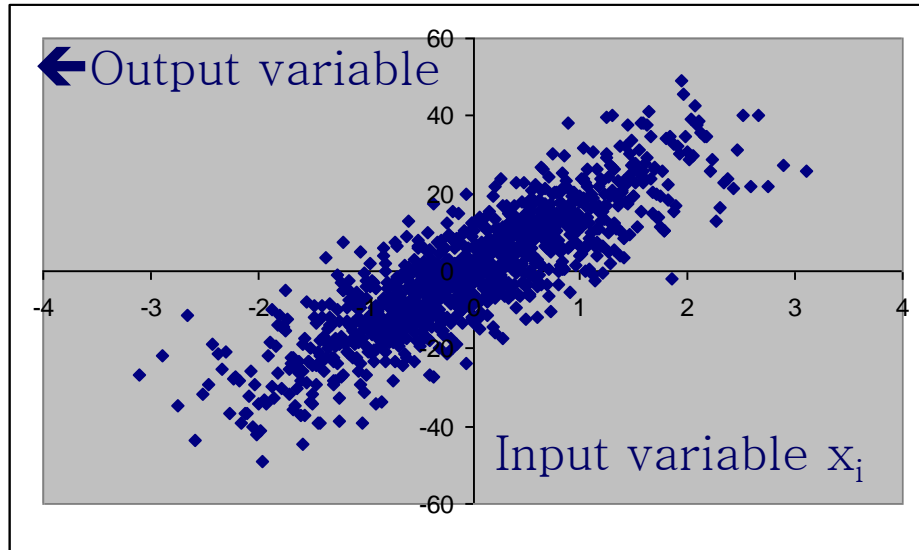
The Primer

 WILEY

Available for free at

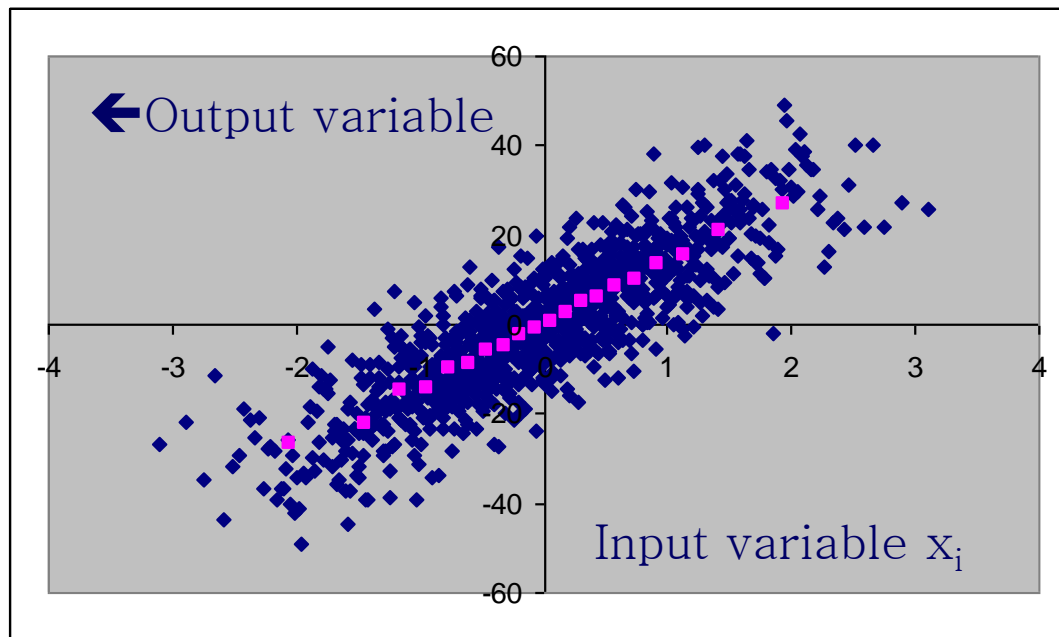
<http://www.andreasaltelli.eu>

How is it done in
practice?



Plotting the output as a function of two different input factors

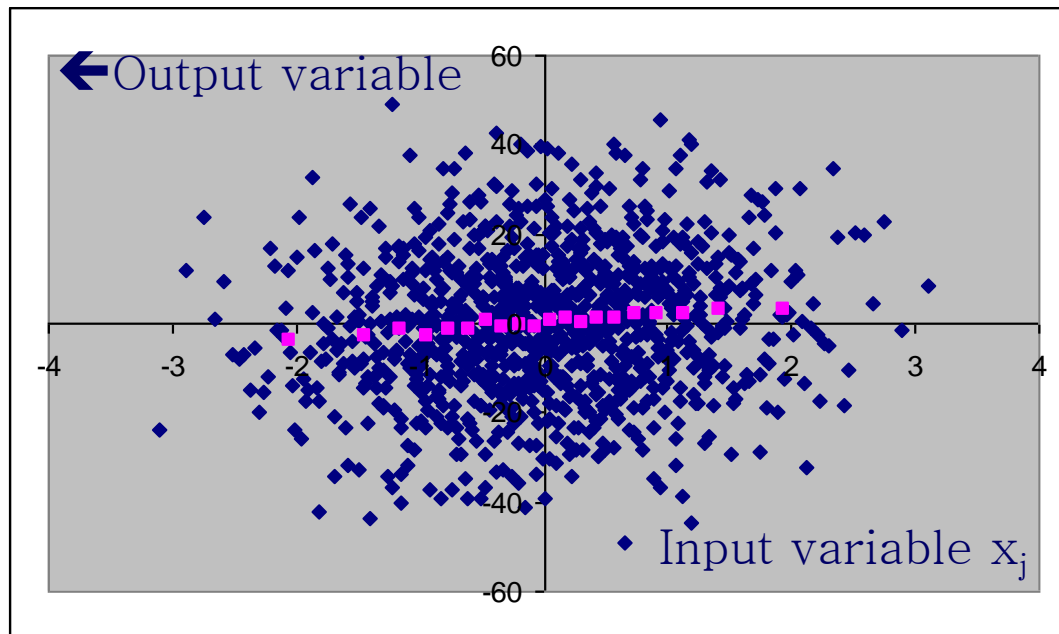
Which factor is more important?

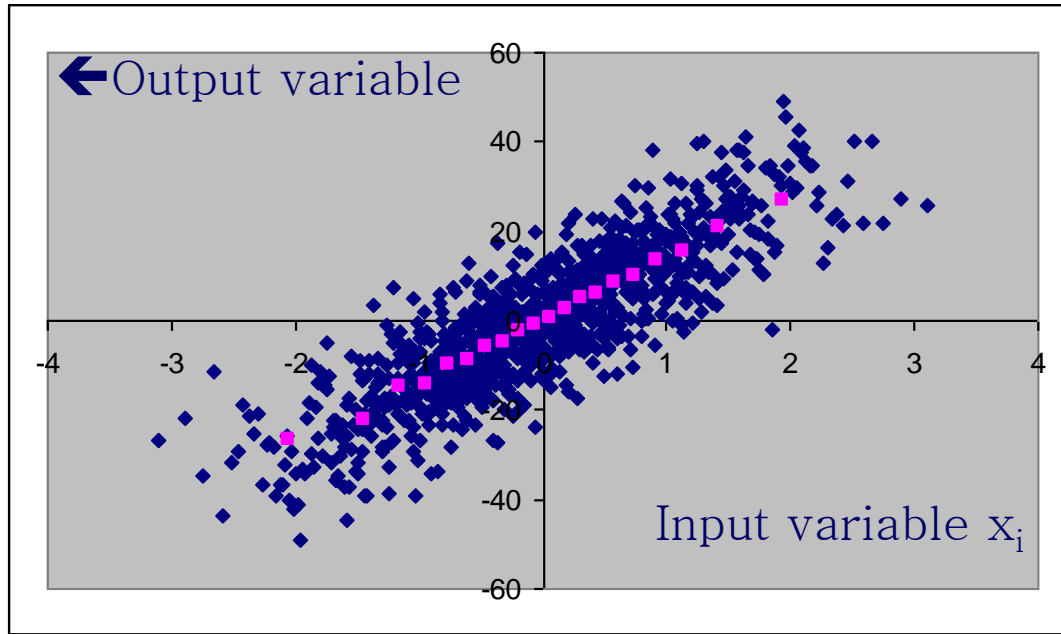


~1,000 blue points

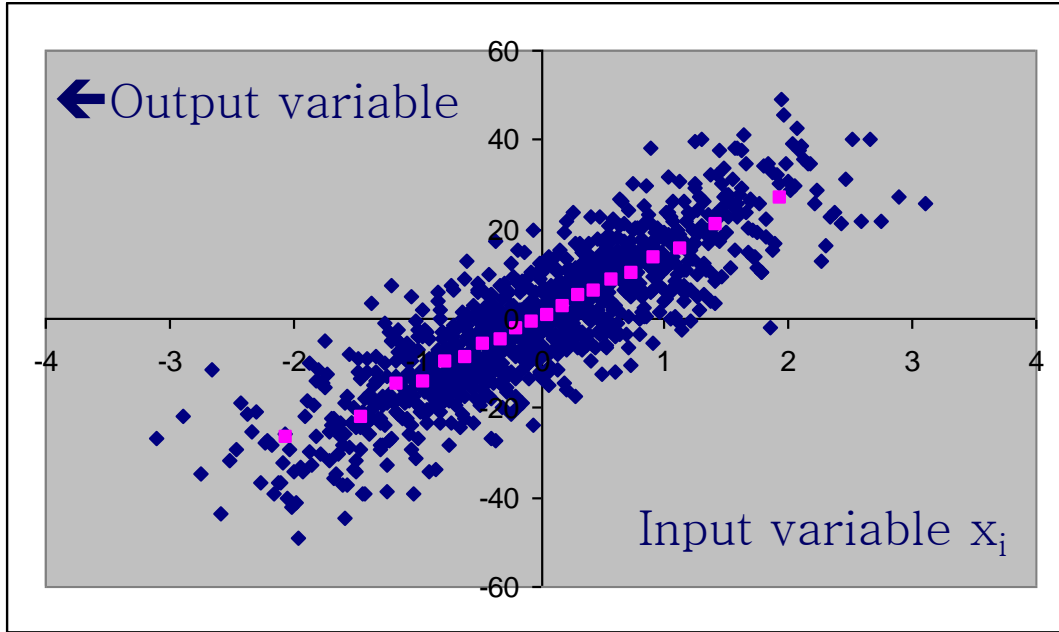
Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



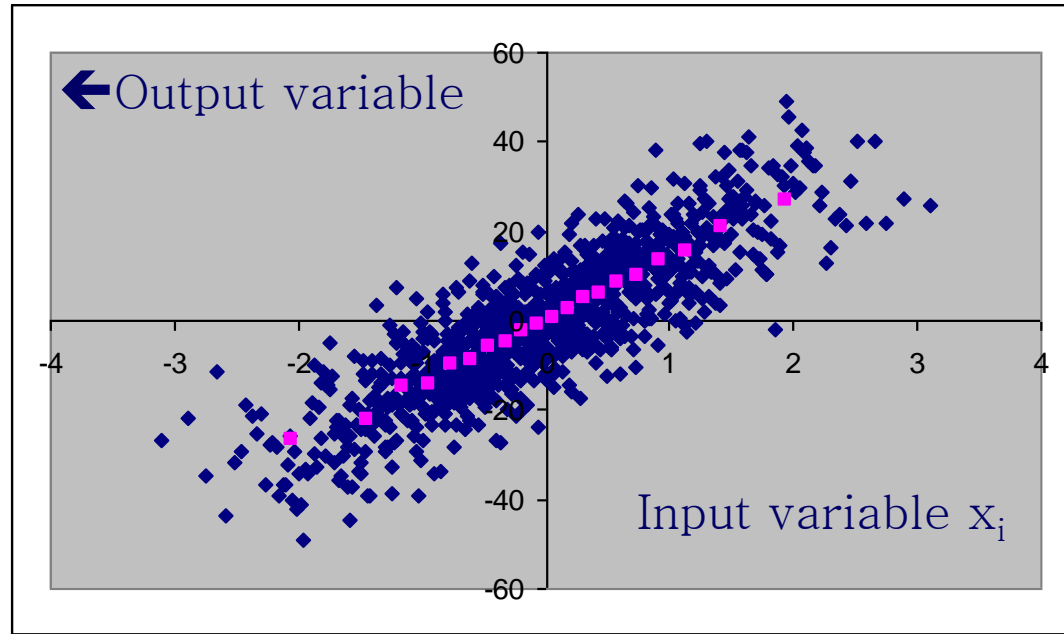


Each pink point is $\sim E_{\mathbf{x}_{\sim i}}(Y|X_i)$

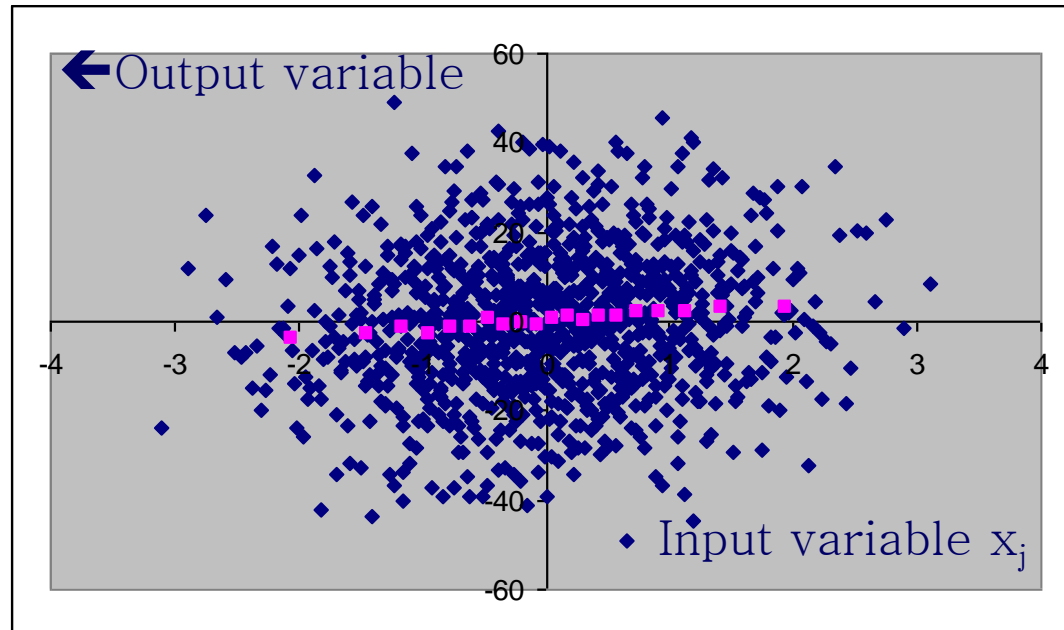


Take the variance of
the pink points one
obtains a sensitivity
measure

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$



Which factor
has the highest
 $V_{X_i} \left(E_{\mathbf{x}_{\sim i}} (Y | X_i) \right) ?$



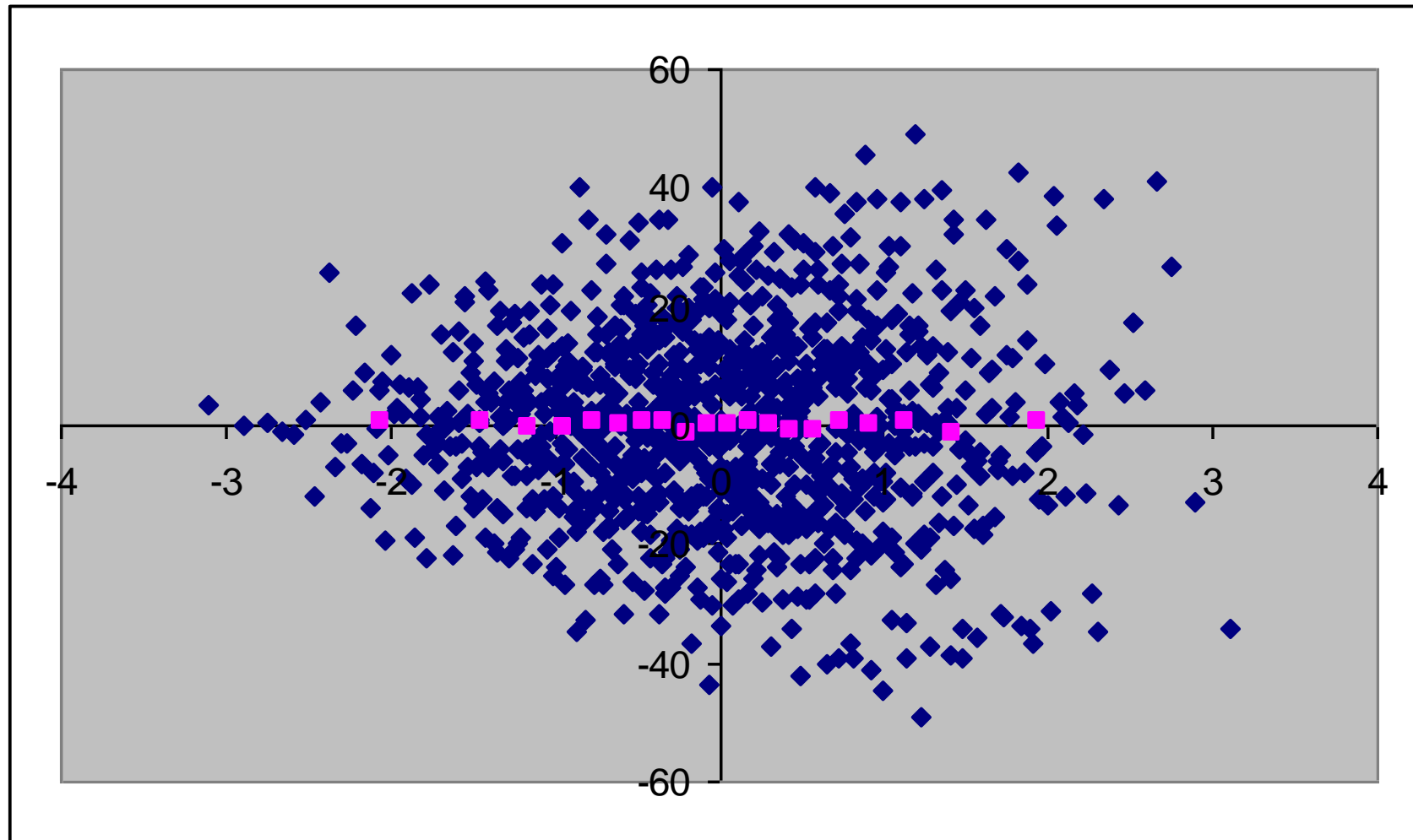
For additive models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) \approx V(Y)$$

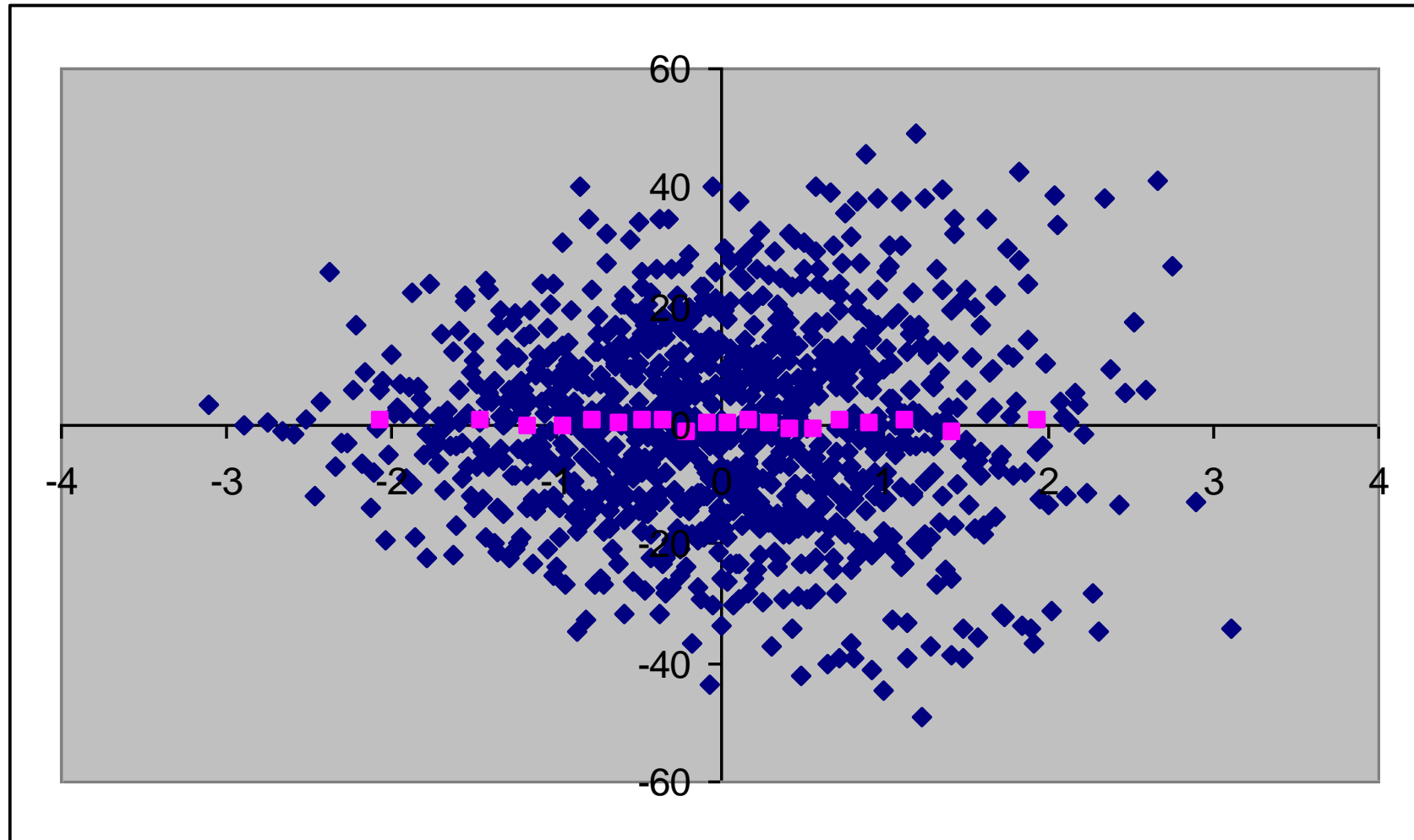
... which is also how additive models are defined

Non additive models

Is $S_i = 0$?



Is this factor non-important?



There are terms which capture two-way, three way, \cdots interactions among variables.

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i,j>i} V_{ij} + \dots + V_{123\dots k}$$

Variance decomposition (ANOVA)

When the factors are independent the total variance can be decomposed into main effects and interaction effects up to the order k , the dimensionality of the problem.

If fact interactions terms are awkward to handle: **just the second order terms** for a model with k factors are as many as $k(k-1)/2 \dots$

(10 factors=45 second order terms)

Wouldn't it be handy to have just a single 'importance' terms for all effects, inclusive of first order and interactions?

In fact such terms exist and can be computed easily, without knowledge of the individual interaction terms

Thus given a model $Y=f(X_1,X_2,X_3)$

Instead of

$$\begin{aligned} V &= V_1 + V_2 + V_3 + \\ &+ V_{12} + V_{13} + V_{23} + \\ &+ V_{123} \end{aligned}$$

Or – divided by V

$$\begin{aligned} 1 &= S_1 + S_2 + S_3 + \\ &+ S_{12} + S_{13} + S_{23} + \\ &+ S_{123} \end{aligned}$$

We have:

$$S_{T1} = S_1 + S_{12} + S_{13} + S_{123}$$

(and analogue formulae for S_{T2} , S_{T3})
which can be computed without
knowing S_1 , S_{12} , S_{13} , S_{123}

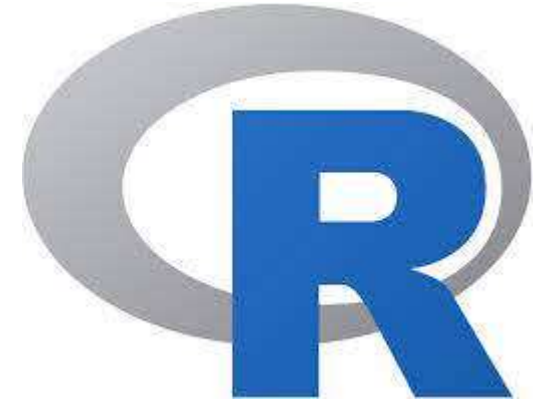
S_{T1} is called a total effect
sensitivity index

$$E_{\mathbf{X}_{\sim i}} \left(V_{X_i} \left(Y | \mathbf{X}_{\sim i} \right) \right)$$

Total effect, or bottom marginal variance=
 = the expected variance that would be left if
 all factors but X_i could be fixed (self evident
 definition)

$$S_{Ti} \equiv \frac{E\left(V\left(Y|\mathbf{X}_{\sim i}\right)\right)}{V_Y}$$

Plenty of code available in MATLAB, R and Python



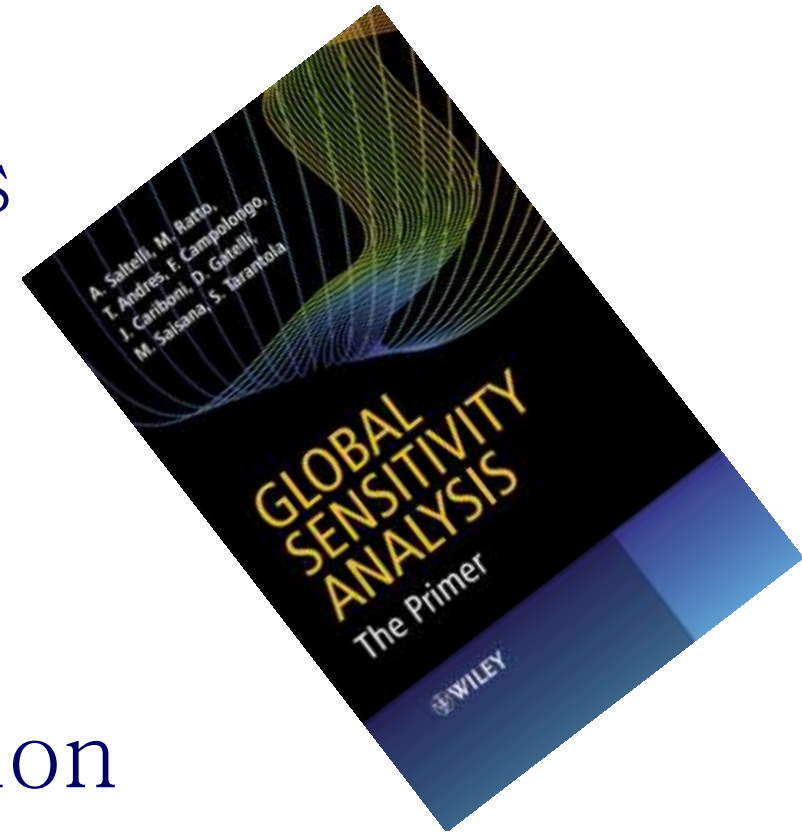
<https://cran.r-project.org/web/packages/sensitivity/sensitivity.pdf>

<https://cran.rstudio.com/web/packages/sensobol/index.html>

Why using variance-based
sensitivity analysis methods

Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization



Why not using correlation- regression based techniques? PCC, PRCC, SRC, SRRC

Reliability Engineering and System Safety **28** (1990) 229–253

Non-parametric Statistics in Sensitivity Analysis for Model Output: A Comparison of Selected Techniques

A. Saltelli

Commission of the European Communities, Joint Research Centre—Ispra Establishment,
21020 Ispra (Varese), Italy

&

J. Marivoet

Belgian Nuclear Research Establishment SCK/CEN,
Boeretang 200, B-2400, Belgium

(Received 26 May 1989; accepted 3 August 1989)

Computational Statistics & Data Analysis **13** (1992) 73–94
North-Holland

Sensitivity analysis for model output

**Performance of black box techniques on three
international benchmark exercises**

A. Saltelli

Commission of the European Communities, Joint Research Centre, Ispra, Italy

T. Homma

*Japan Atomic Energy Research Institute, Tokai Research Establishment, Department of Environmental
Safety Research, Tokai-Mura, Ibaraki, Japan*

Received February 1990

Revised October 1990

They assume linearity (PCC) or
monotonicity (PRCC), which is
difficult to know *ex-ante*

Secrets of sensitivity analysis

Why should one
ever run a model
just once?

EC impact assessment guidelines: sensitivity analysis & auditing



http://ec.europa.eu/smart-regulation/guidelines/docs/br_toolbox_en.pdf

First secret: The most important question is the question.

Or: sensitivity analysis is not “run” on a model but on a model once applied to a question

Second secret: Sensitivity analysis should not
be used to hide assumptions
[it often is]



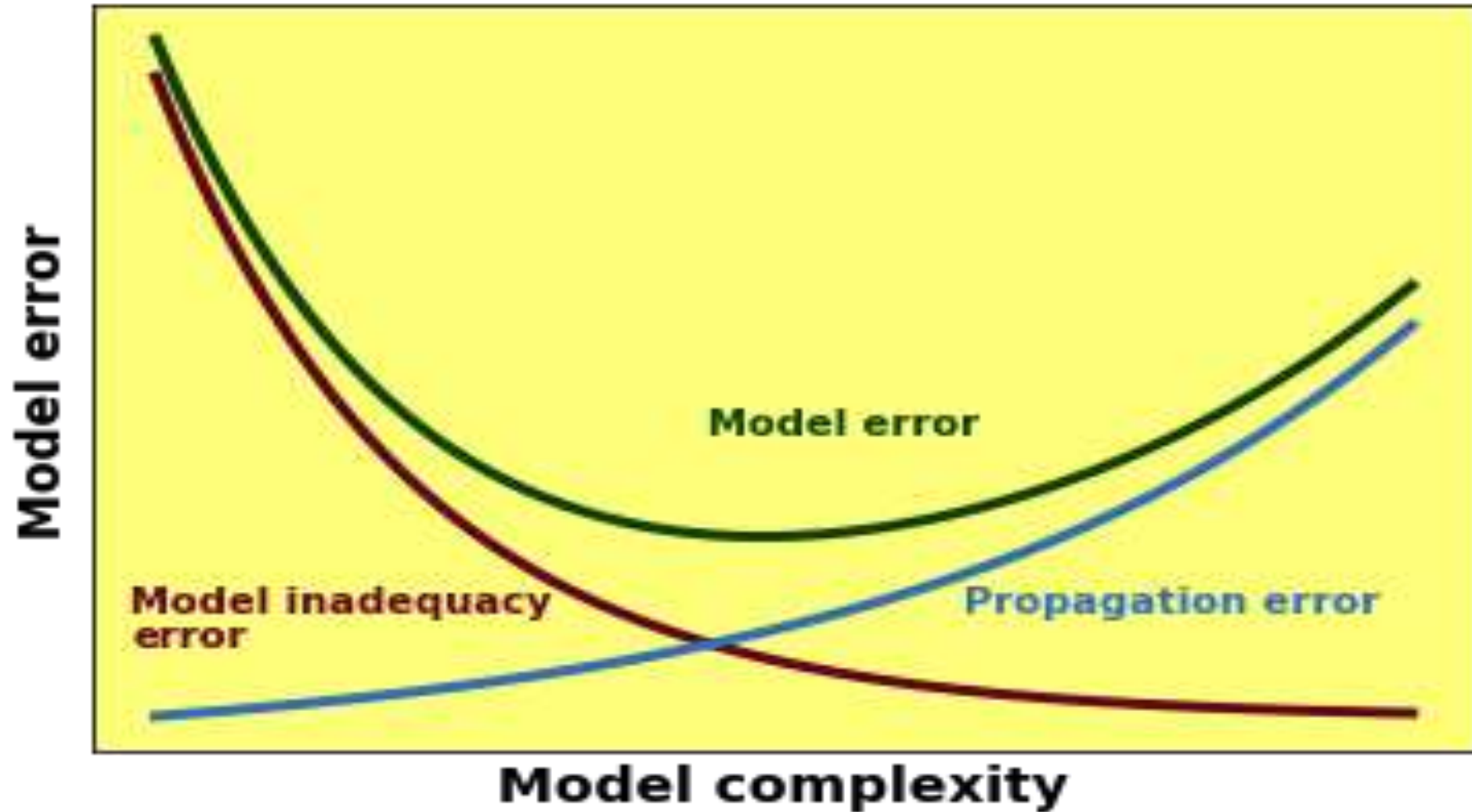
Third secret: If sensitivity analysis shows that a question cannot be answered by the model one should find another question or model

[Often the love for one's own model prevails]

Fourth (badly kept) secret:
There is always one more bug!
=Lubarsky's Law of Cybernetic Entomology

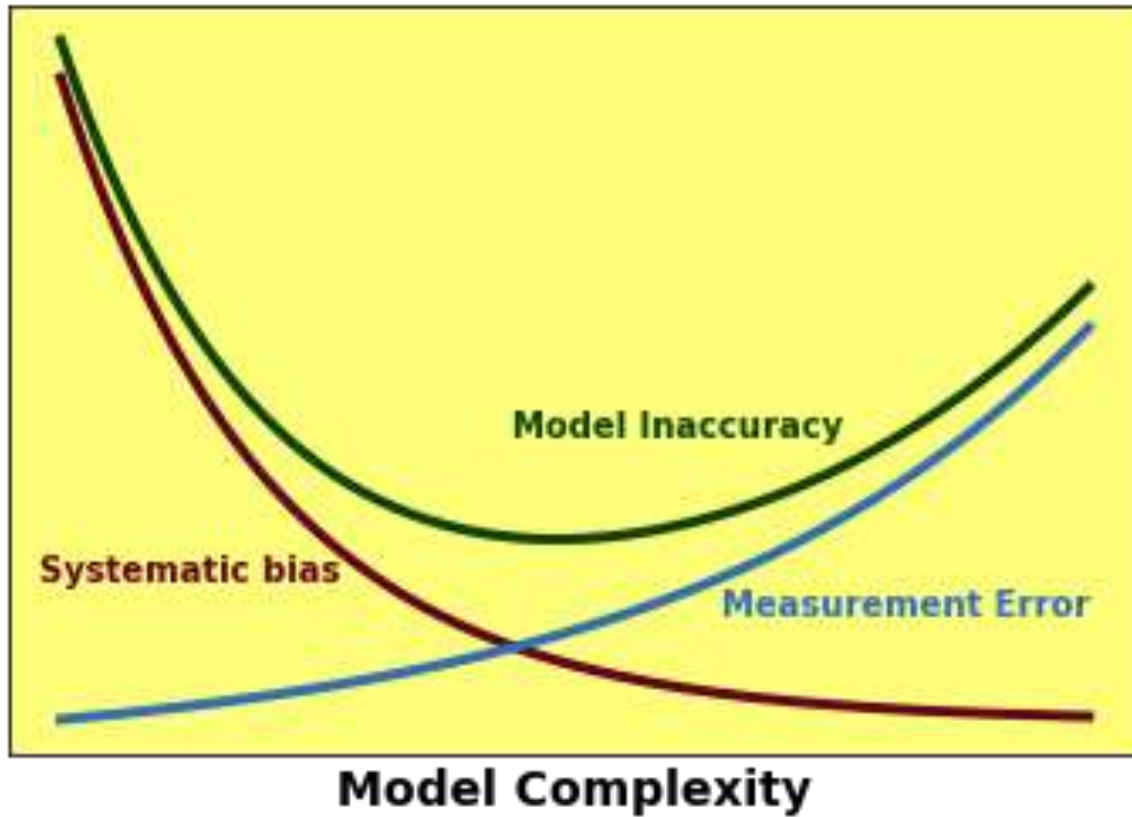


Fifth secret: use SA to calibrate complexity



Presented as ‘Conjecture by O’Neill’

In M. G. Turner and R. H. Gardner,
“Introduction to Models” in Landscape
Ecology in Theory and Practice, New
York, NY: Springer New York, 2015, pp.
63–95.



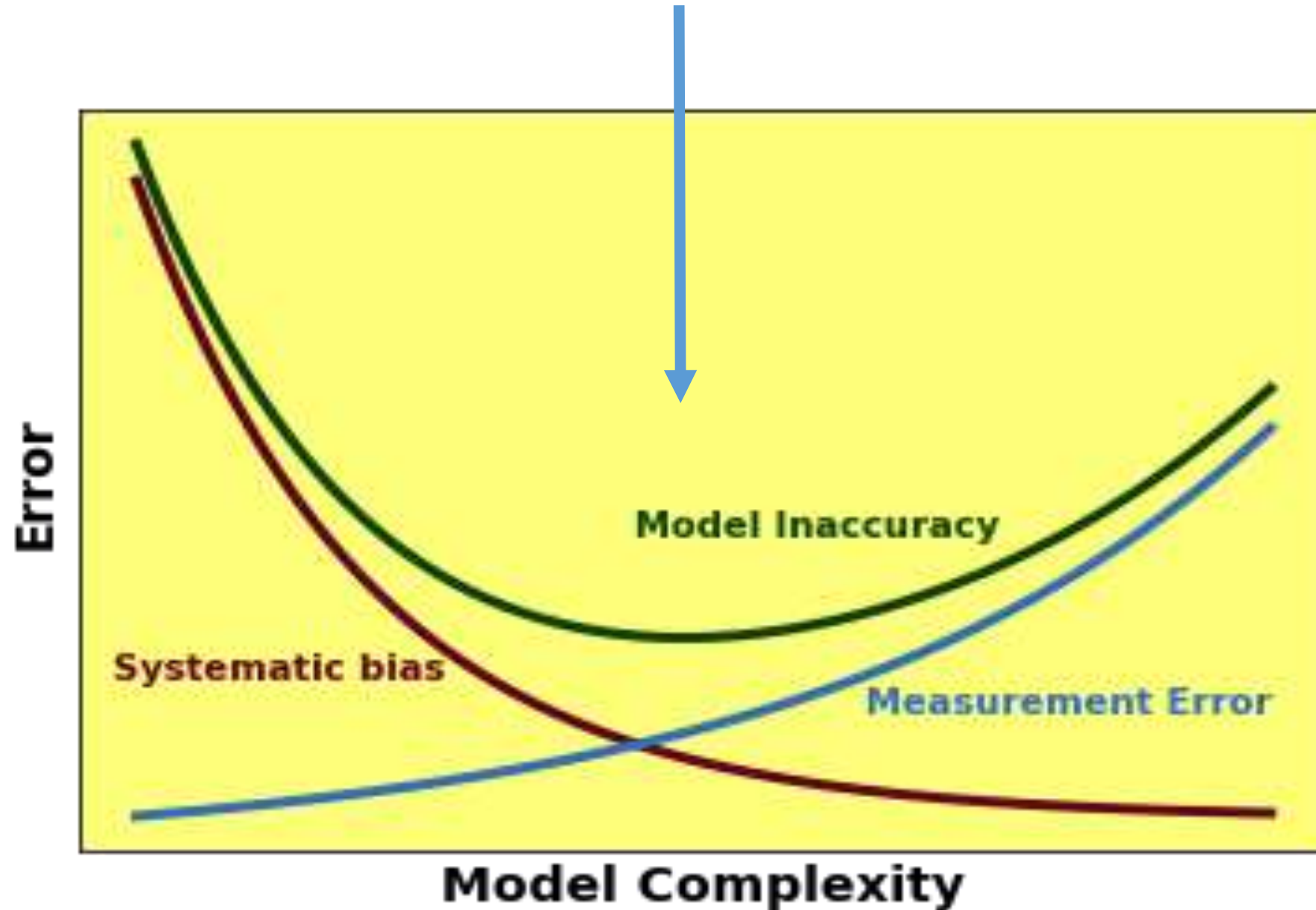


Lofti Aliasker Zadeh

Also known as Zadeh's principle of incompatibility, whereby as complexity increases “precision and significance (or relevance) become almost mutually exclusive characteristics”

L. Zadeh, “Outline of a New Approach to the Analysis of Complex Systems and Decision Processes,” *IEEE Trans. Syst. Man. Cybern.*, vol. 3, no. 1, pp. 28–44, 1973.

SA can help to find this minimum





Comment

Open Access

Published: 27 August 2019

A short comment on statistical versus mathematical modelling

Andrea Saltelli 

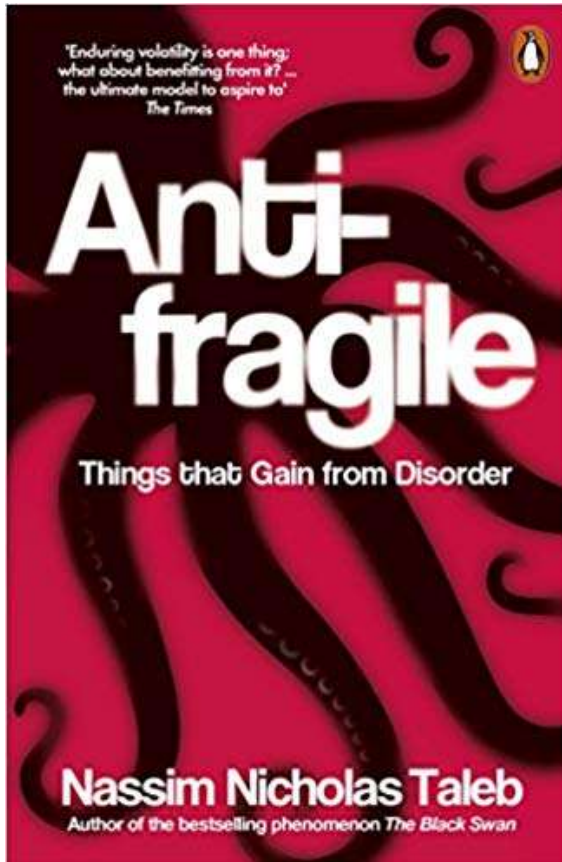
Sixth secret:

With SA it is easier to disprove than to prove; use
SA 'via negativa':

Doing the right thing

or

Avoiding something wrong?



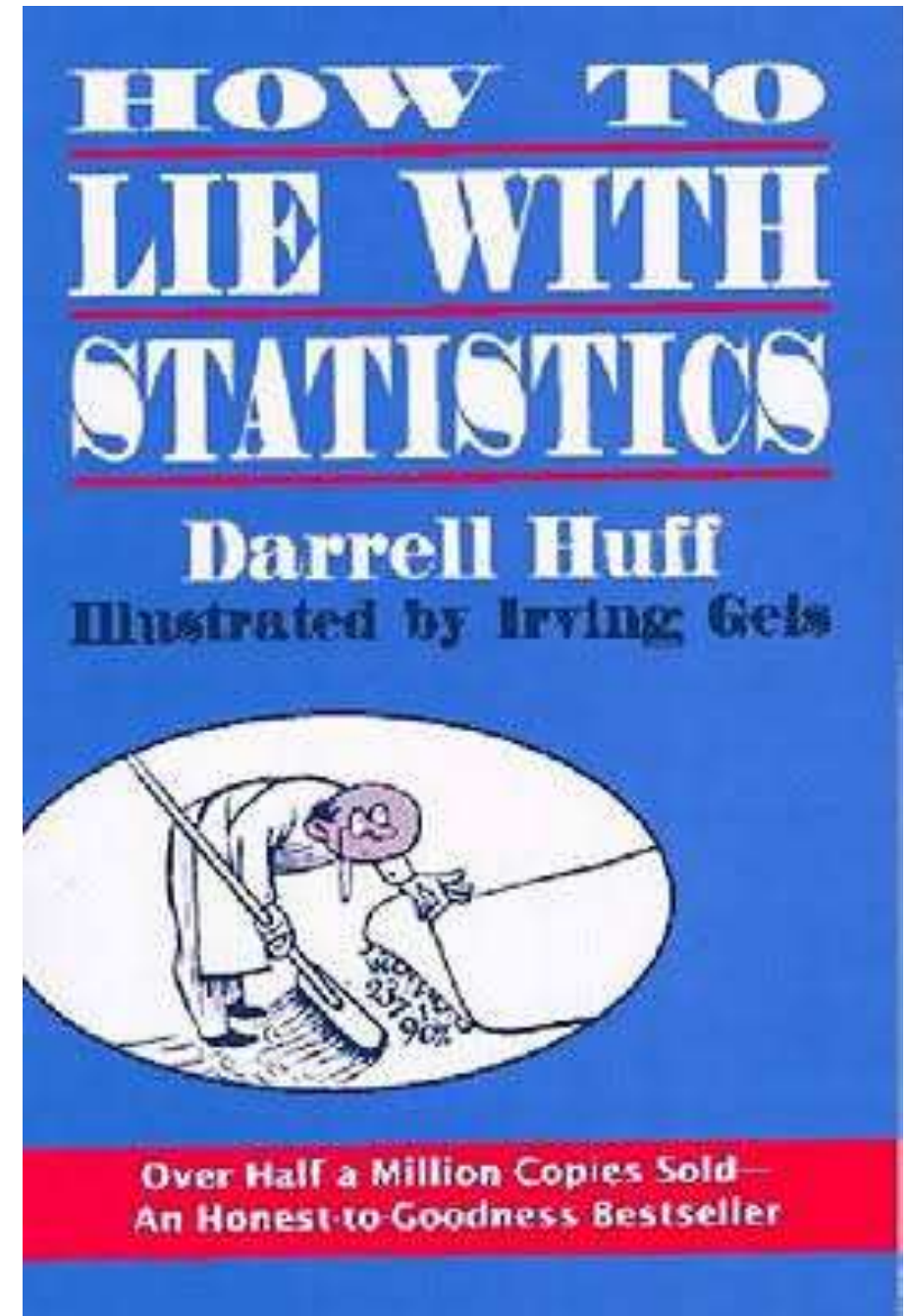
And of course please don't run a sensitivity analysis where each factors has a 5% uncertainty





Why?

Can we say that one lies with sensitivity analysis as one can lie with statistics?



Limit of SA: Often no SA (SA
conflated with UA e.g. in economics) or
one-factor-at-a-time SA

Why is OAT (one-factor-at-a-time) SA so bad?



Contents lists available at ScienceDirect

Environmental Modelling & Software

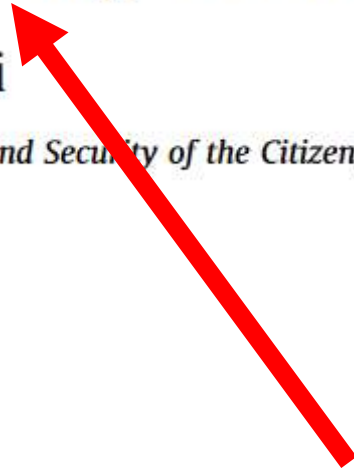
journal homepage: www.elsevier.com/locate/envsoft



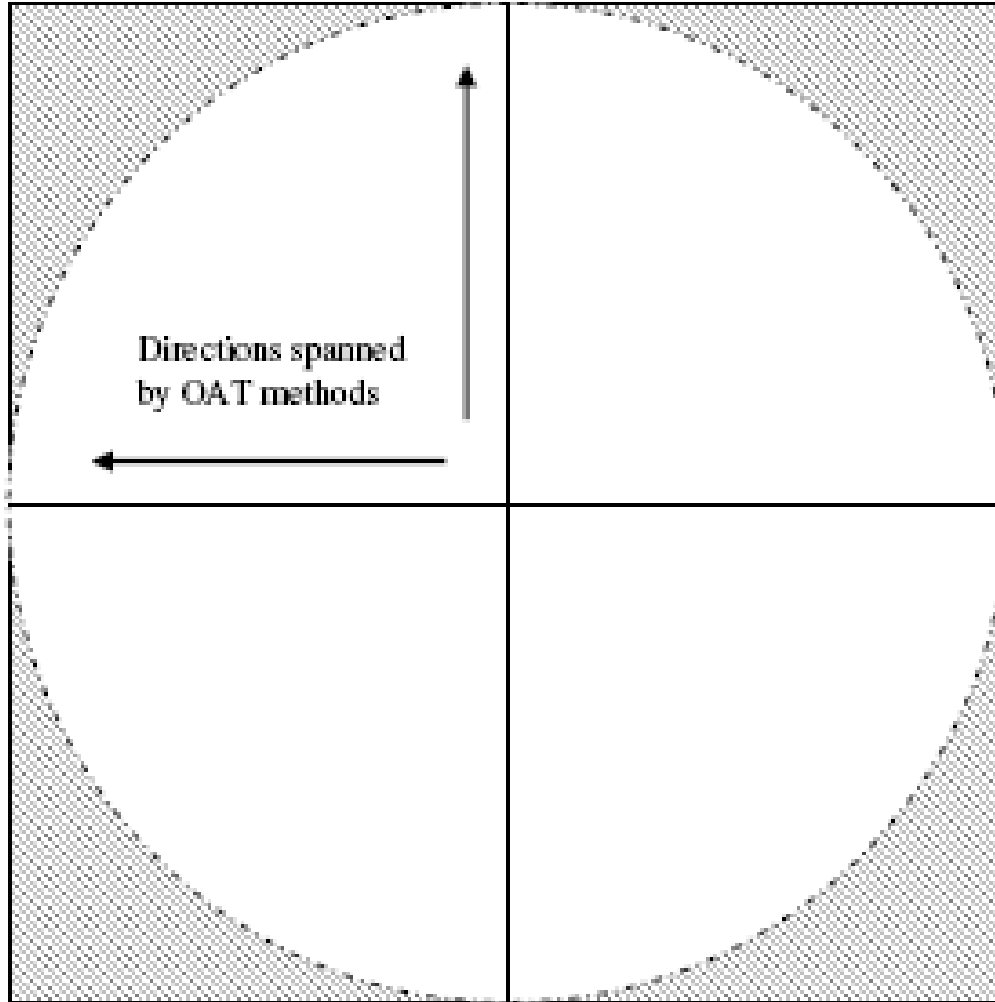
How to avoid a perfunctory sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy



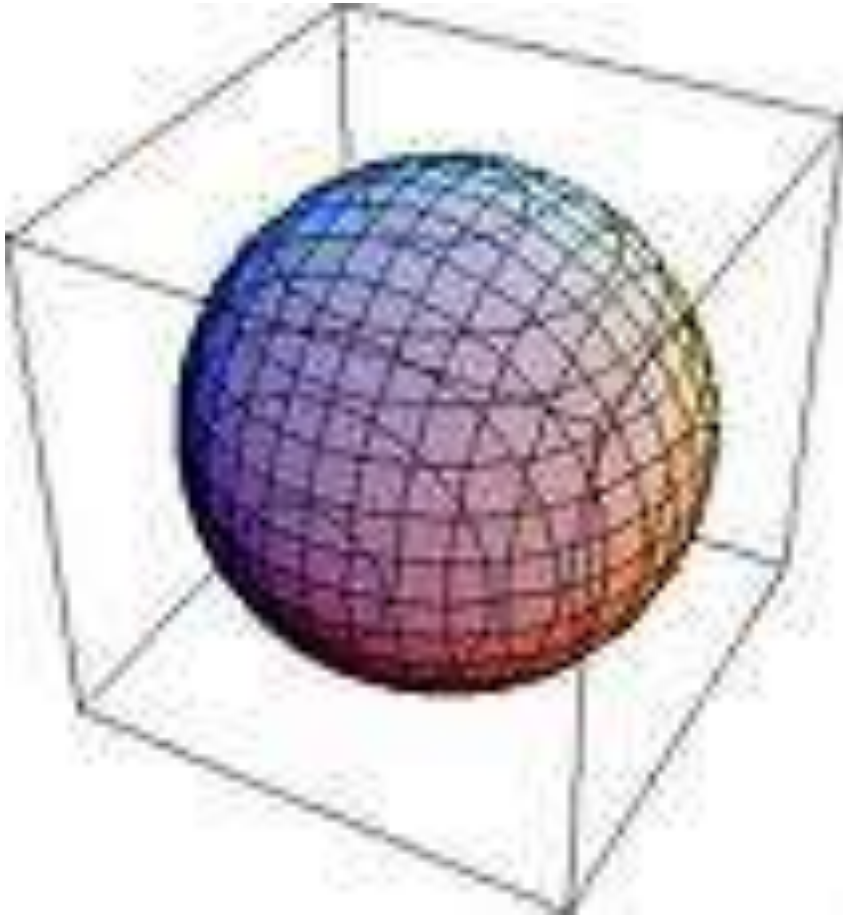
OAT in 2 dimensions



Area circle
/ area
square =?

~ 3/4

OAT in 3 dimensions



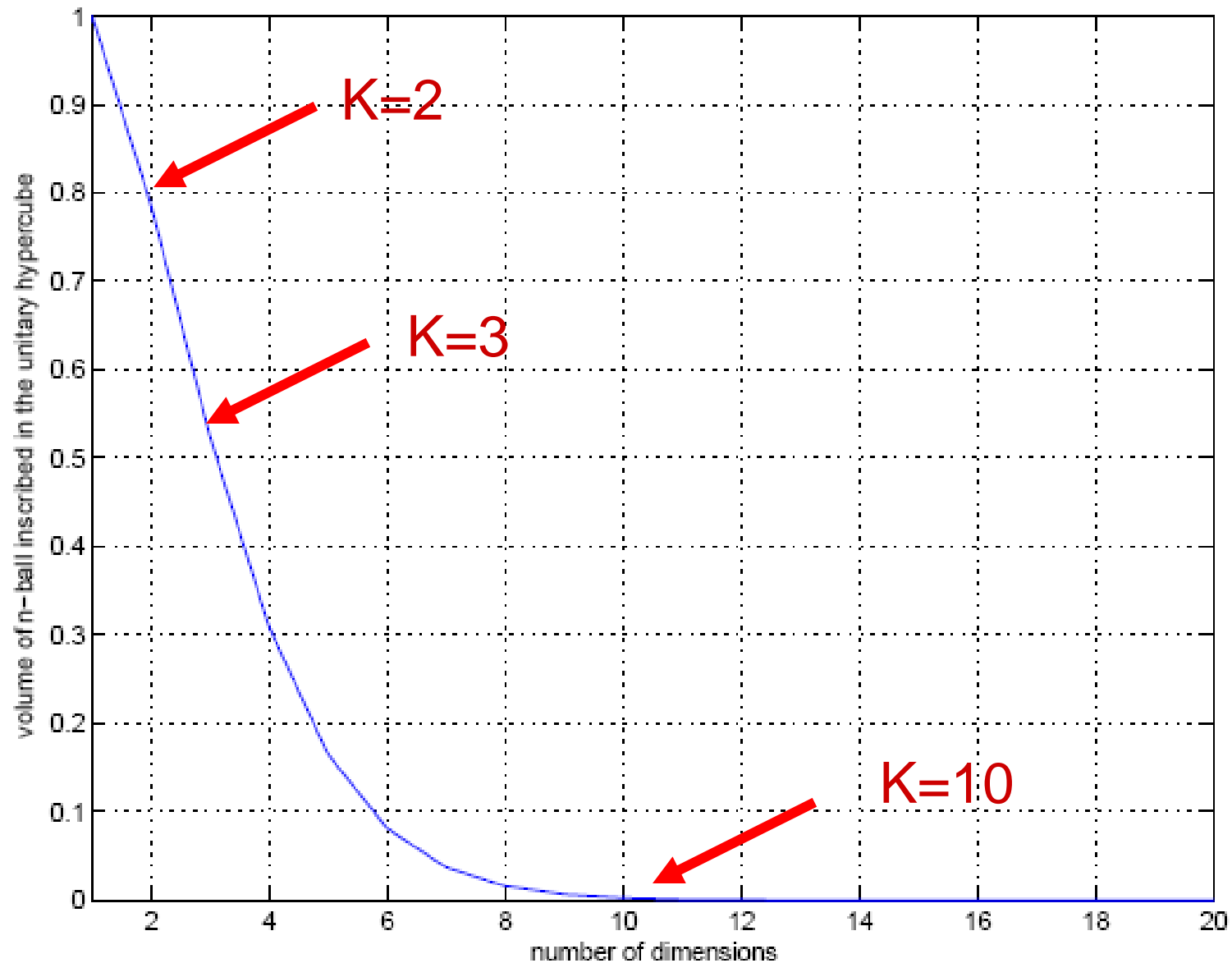
Volume sphere /
volume cube = ?

~ 1/2

OAT in 10 dimensions; Volume
hypersphere / volume ten dimensional
hypercube =? ~ 0.0025



OAT in k dimensions

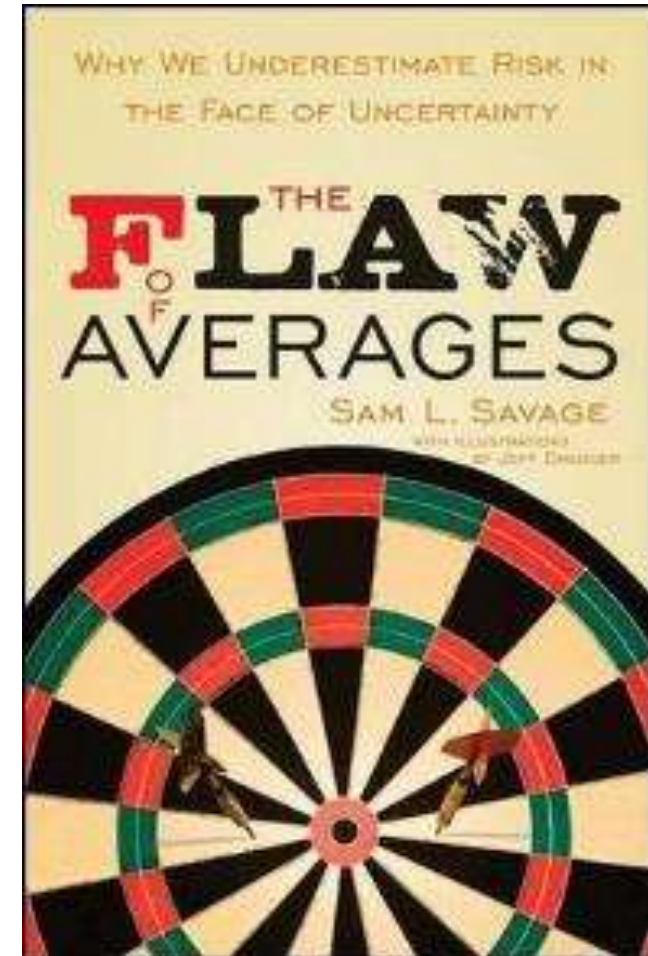
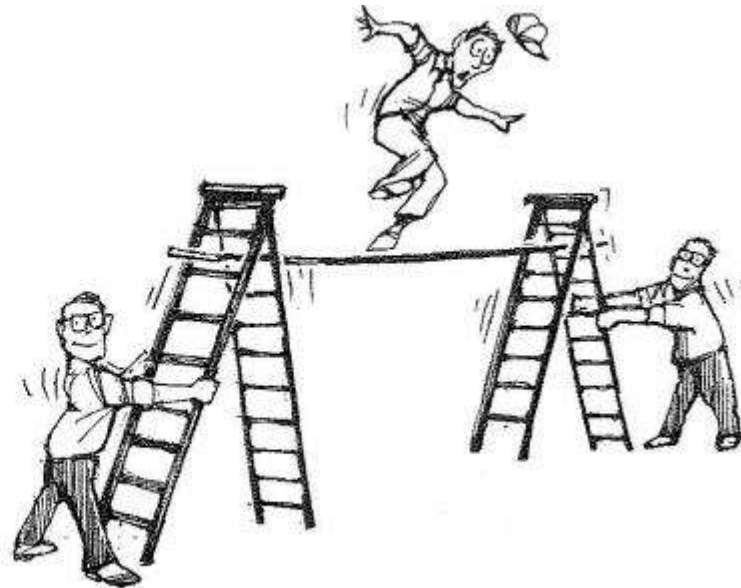


How would you test the scaffolding?

How coupled ladders are shaken in most of available literature



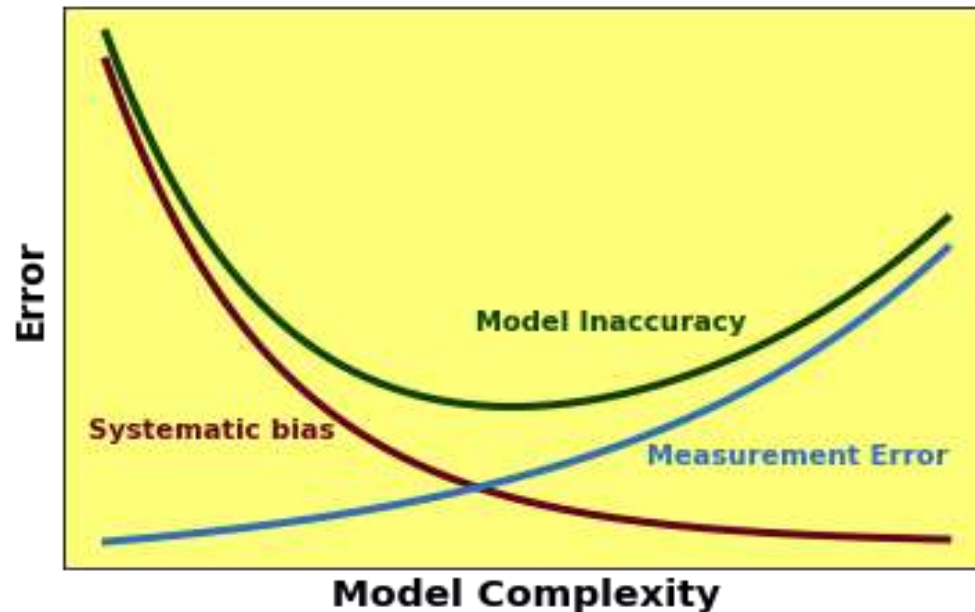
How to shake coupled ladders



Lessons from sensitivity analysis

- Global SA
- UA and SA coupled
- Purpose- & context-specific
- The map is not the territory

- Memento

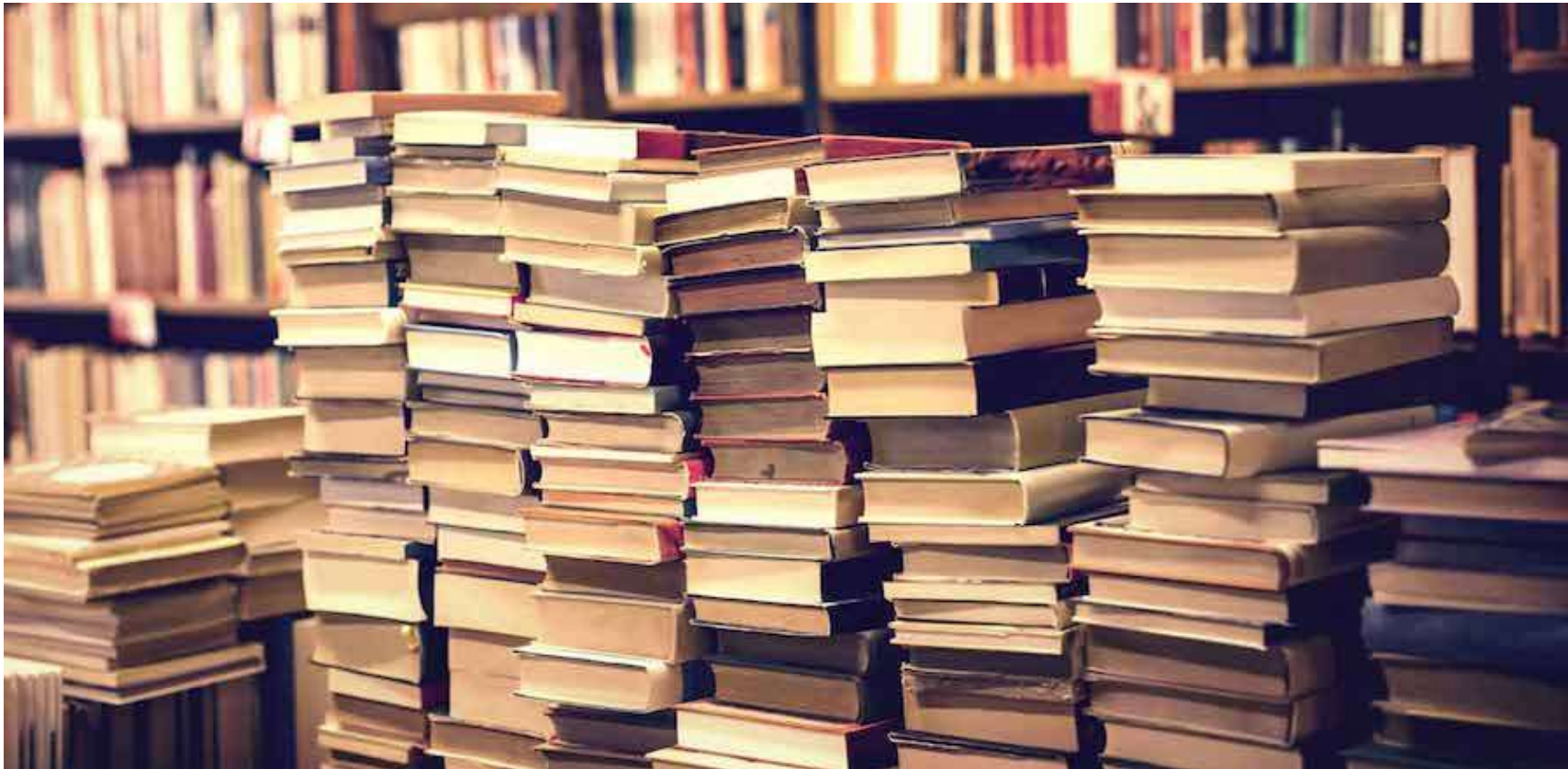


The End

@andreasaltelli



Extra slides







Environmental Modelling & Software

Volume 114, April 2019, Pages 29-39



Why so many published sensitivity analyses are false: A systematic review of sensitivity analysis practices

Andrea Saltelli ^{a, b}  , Ksenia Aleksankina ^c, William Becker ^d, Pamela Fennell ^e, Federico Ferretti ^d, Niels Holst ^f, Sushan Li ^g, Qiongli Wu ^h

Literature search in Scopus

Query: “sensitivity analysis” & “model/modelling”
& “uncertainty”; years 2012–2017; journal
articles; in English

➔ 6000 articles

- AgrBioSci (Agricultural and Biological Sciences)
- BiochemGenMBio (Biochemistry, Genetics and Molecular Biology)
- BusManAcc (Business, Management and Accounting)
- Chemi (Chemistry)
- ChemEng (Chemical Engineering)
- CompSci (Computer Science)
- DecSci (Decisional Science)
- EarthSci (Earth and Planetary Sciences)
- EconFin (Economy and Finance)
- Energy (Energy)
- Engineering (Engineering)
- EnvSci (Environmental Science)
- ImmunMicrobio (Immunology and Microbiology)
- MatSci (Material Science)
- Math (Math)
- Medicine (Medicine)
- PharTox (Pharmacology and Toxicology)
- PhysAstro (Physics and Astronomy)
- SocSci (Social Science)

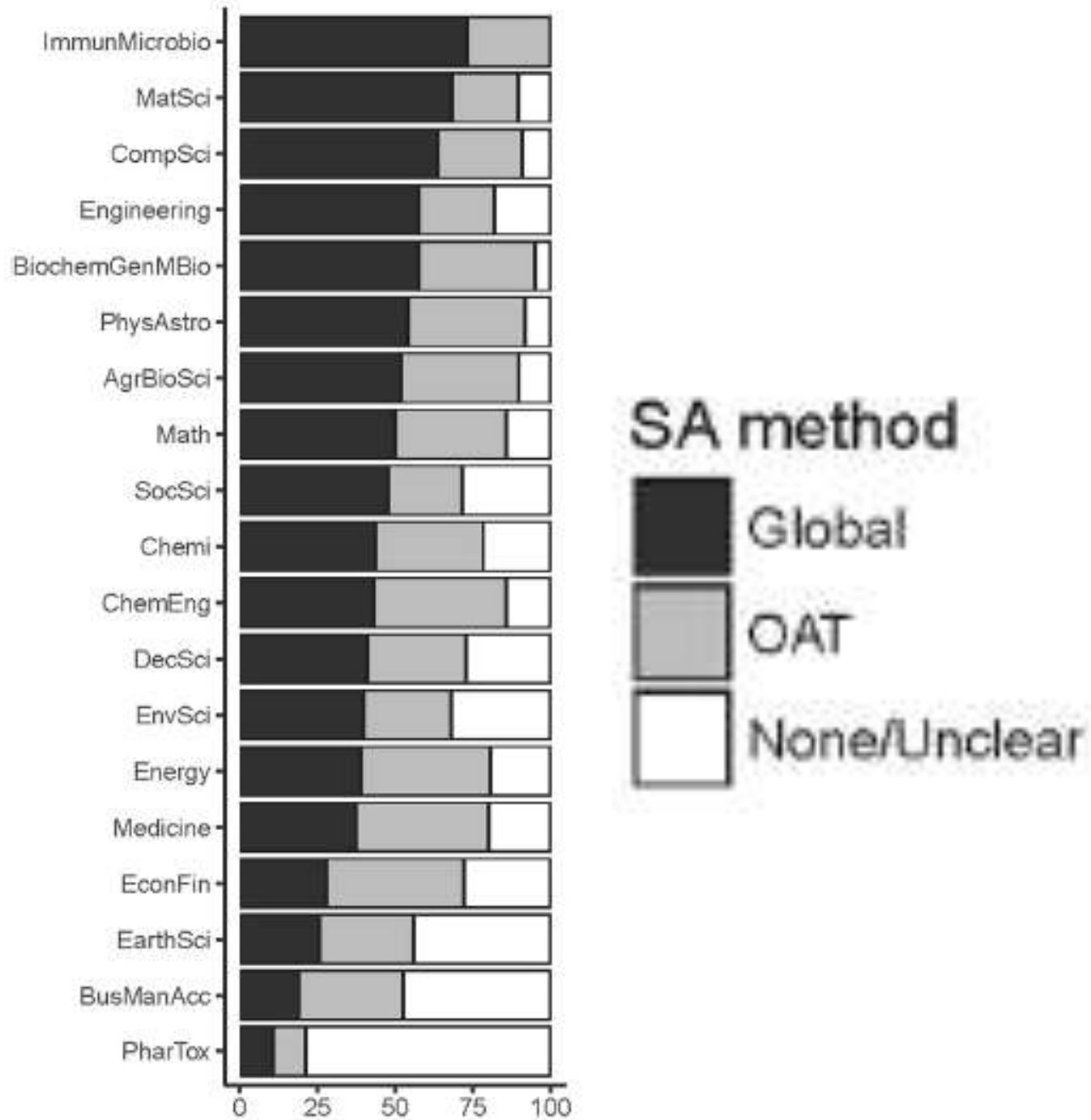
subject areas >100 articles

Taking the top twenty most-cited papers in each subject area:

➔ 324 articles, divided among authors

Cleansing manually irrelevant articles:

➔ 280 articles



Still many papers
apply an OAT SA:
65%

What if the model is truly linear?

Linear	7%
Nonlinear	61%
Unclear	32%

Linear	7%
Nonlinear	61%
Unclear	32%

65% highly cited articles are OAT

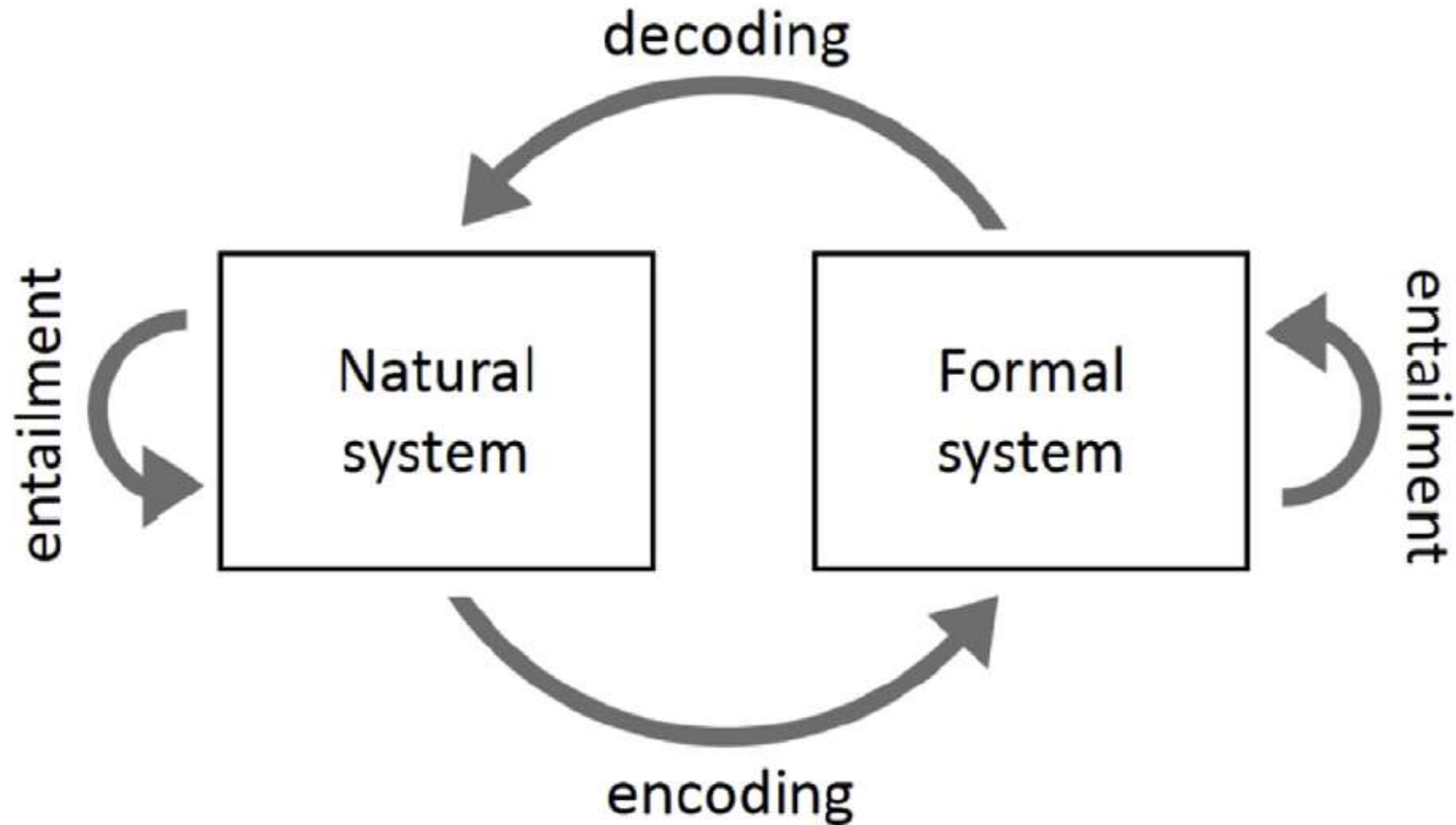
Taking all unclear = linear → still
over 20% of papers wrong
(OAT & non-linear model)

Why?

5. Discussion

5.1. Reasons for bad practice

Why? ➔ 1. Modelling as a craft



Why? ➔ 2. Each discipline going about modelling on its own separate way; pockets of SA practitioners (out of our 280 papers, 35 were methodological, of which 24 suggest global SA)

Why? ➔ 3. Mathematical modelling is not
a discipline

Based on a survey of modellers: “there is no dominating paradigm in modelling and simulation... simulation verification is mostly a trial and error activity ➔ challenges model/simulation validity”

Padilla, J. J., Diallo, S. Y., Lynch, C. J., & Gore, R. (2018). Observations on the practice and profession of modeling and simulation: A survey approach. *SIMULATION*, 94(6), 493–506.

... mathematical modelling cannot do this:



**AMERICAN STATISTICAL ASSOCIATION RELEASES STATEMENT ON
STATISTICAL SIGNIFICANCE AND *P*-VALUES**

*Provides Principles to Improve the Conduct and Interpretation of Quantitative
Science*

March 7, 2016

Wasserstein, R.L. and Lazar, N.A., 2016. 'The ASA's statement on p-values: context, process, and purpose', *The American Statistician*, Volume 70, 2016 – Issue 2, Pages 129–133.

Why? ➔ 4. Good practices require
training in statistics

Why? ➔ 5. More time is needed; though mature global sensitivity analysis methods around for more than 25 years researchers tend to emulate methods found in highly cited papers assuming that they are best practice

Why? ➔ 6. Strategic reasons: global SA is bad if one wants to play the uncertainty game, inflating or deflating uncertainties instrumentally

Solutions? 1. Statistics as a discipline
takes responsibility for statistical
methods for
model validation and verification

Example: who can authoritatively suggest
to modellers not to overinterpret results
from multi-model ensembles?

Climate Models as Economic Guides: Scientific Challenge or Quixotic Quest?

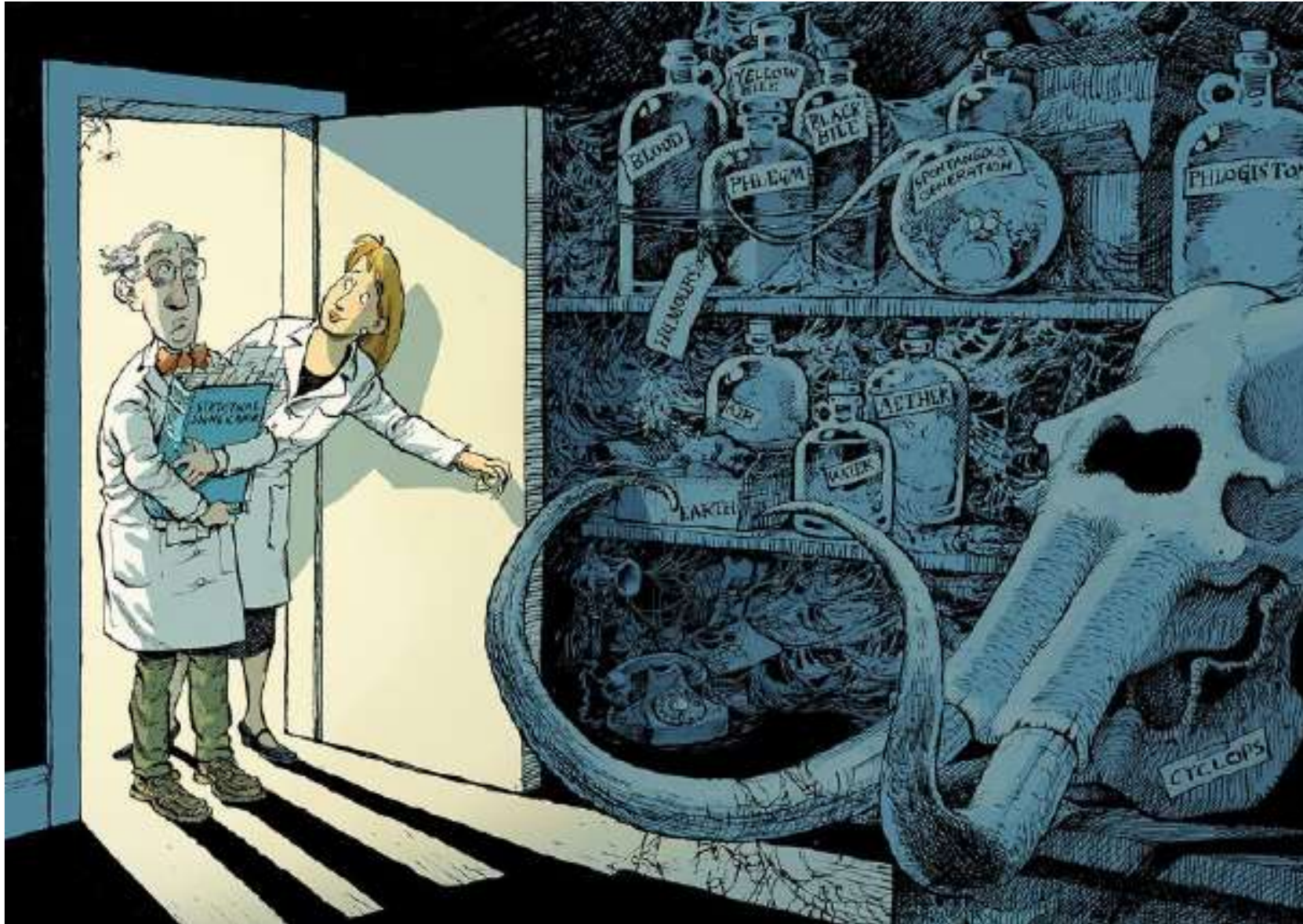
BY ANDREA SALTELLI, PHILIP B. STARK, WILLIAM BECKER, PAWEL STANO

Climate Models as Economic Guides: Scientific Challenge or Quixotic Quest?

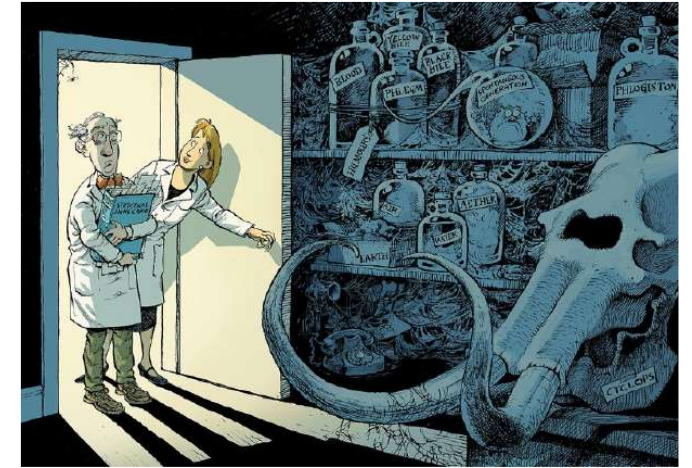
BY ANDREA SALTELLI, PHILIP B. STARK, WILLIAM BECKER, PAWEL STANO

A plea against audacious risk or cost-benefit analysis running over centennial time scales; example: crime rate as modified by climate change at US county level in 2100

Solutions? 2. Learn from what happens in statistics where the p-test crisis is being tackled head on



Throw away
the concept of
statistical
significance?



COMMENT • 20 MARCH 2019

Scientists rise up against statistical significance

Valentin Amrhein, Sander Greenland, Blake McShane and more than 800 signatories call for an end to hyped claims and the dismissal of possibly crucial effects.

Valentin Amrhein , Sander Greenland & Blake McShane

See the discussion on the blog of Andrew Gelman <https://statmodeling.stat.columbia.edu/>



Cargo-cult statistics and scientific crisis

The mechanical, ritualistic application of statistics is contributing to a crisis in science. Education, software and peer review have encouraged poor practice – and it is time for statisticians to fight back. By **Philip B. Stark** and **Andrea Saltelli**

The End

@andreasaltelli

