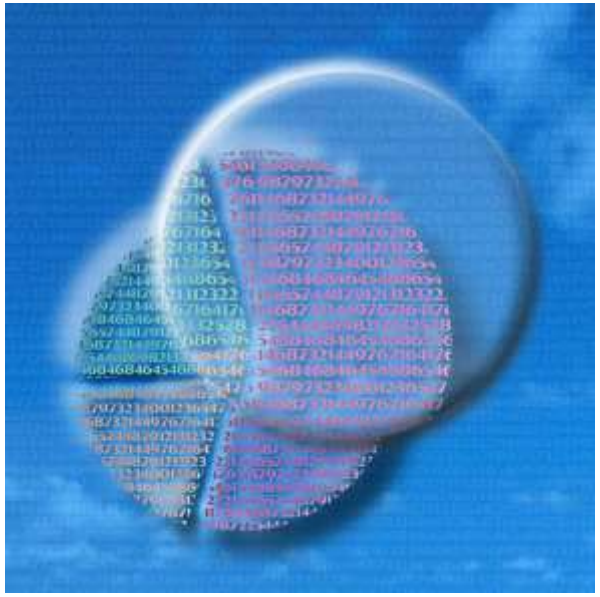


Sensitivity Analysis

Andrea Saltelli
Centre for the Study of the Sciences and the Humanities (SVT), University of Bergen (UIB), and Open Evidence Research, Open University of Catalonia

MNF990 / Theory of Science and Ethics, Bergen,
HIB Seminar rom 520B1 February 17

Why something technical in this course

Environmental Science & Policy
Volume 106, April 2020, Pages 87-98

The technique is never neutral. How methodological choices condition the generation of narratives for sustainability

Andrea Saltelli [a, b], Lorenzo Benini [c], Silvio Funtowicz [a], Mario Giampietro [d, e], Matthias Kaiser [a], Erik Reinert [a, f], Jeroen P. van der Sluijs [a, g, h]

Something general about mathematical modelling

Caeteris are never paribus

Ceteris paribus or caeteris paribus is a Latin phrase meaning "all other things being equal" or "other things held constant" or "all else unchanged“ (Wikipedia)

The case of DSGE, dynamic stochastic general equilibrium models

Rational expectations of agents
Efficient market hypothesis

Philip Mirowski

Philip Mirowski, 2013, Never let a serious crisis go wasted, Verso Books.

The US senate and Queen Elisabeth perplexed…

Philip Mirowski, 2013, Never let a serious crisis go wasted, Verso Books.

An ethical problem in the use of models in economics?

E

Paul Romer's Mathiness = use of mathematics to veil normative stances

Erik Reinert: scholastic tendencies in the mathematization of economics

P. M. Romer, "Mathiness in the Theory of Economic Growth," Am. Econ. Rev., vol. 105, no. 5, pp. 89–93, May 2015.

E. S. Reinert, "Full circle: economics from scholasticism through innovation and back into mathematical scholasticism," J. Econ. Stud., vol. 27, no. 4/5, pp. 364–376, Aug. 2000.

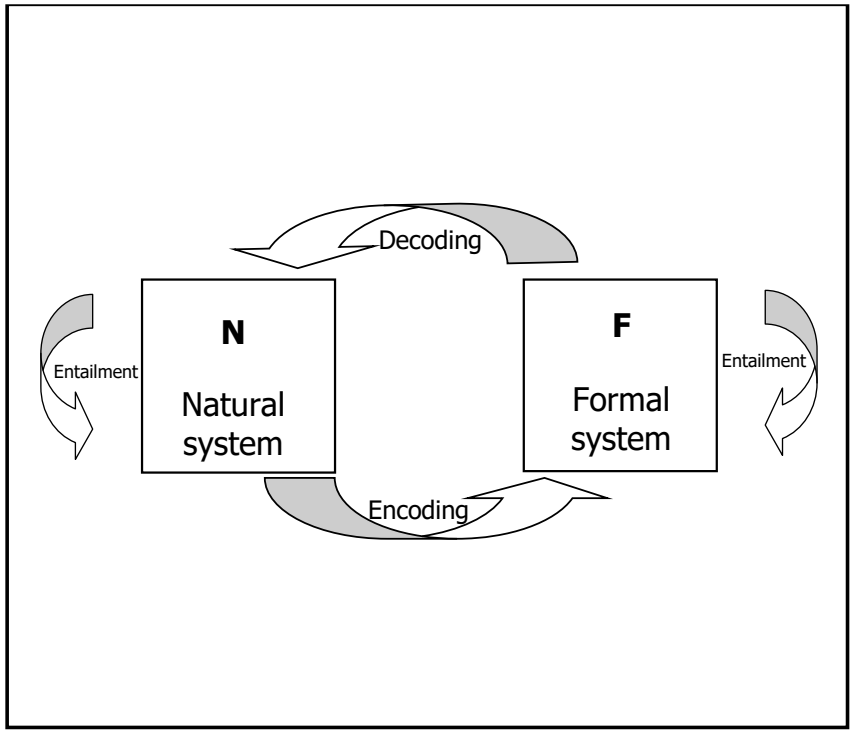
Models have little memory

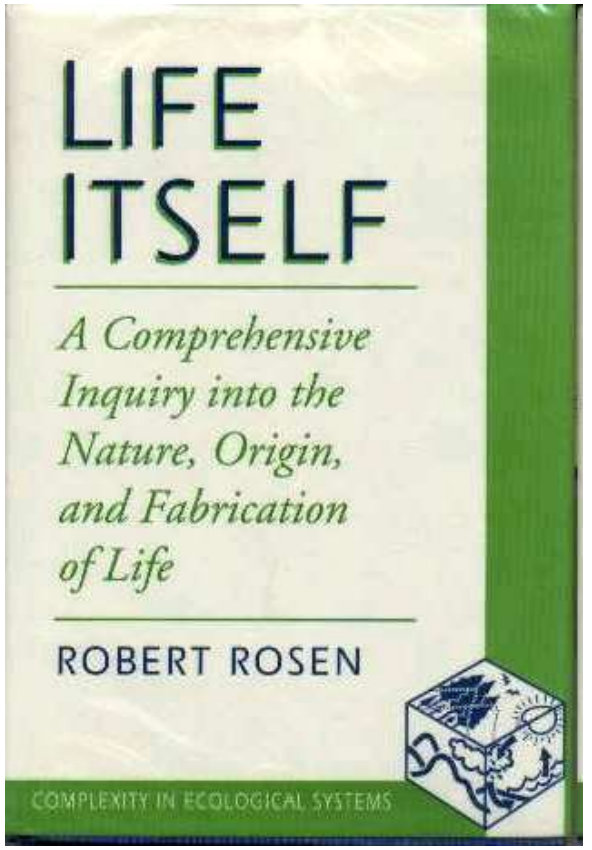
“[…] The process of constructing and validating [value-at risk] models is time consuming and detail oriented; normally even the people who produced the model will not remember many of the assumptions incorporated into it, short of redoing their work, which means that the client cannot simply ask then what went into it.”

E. Millgram The Great Endarkenment, p. 29

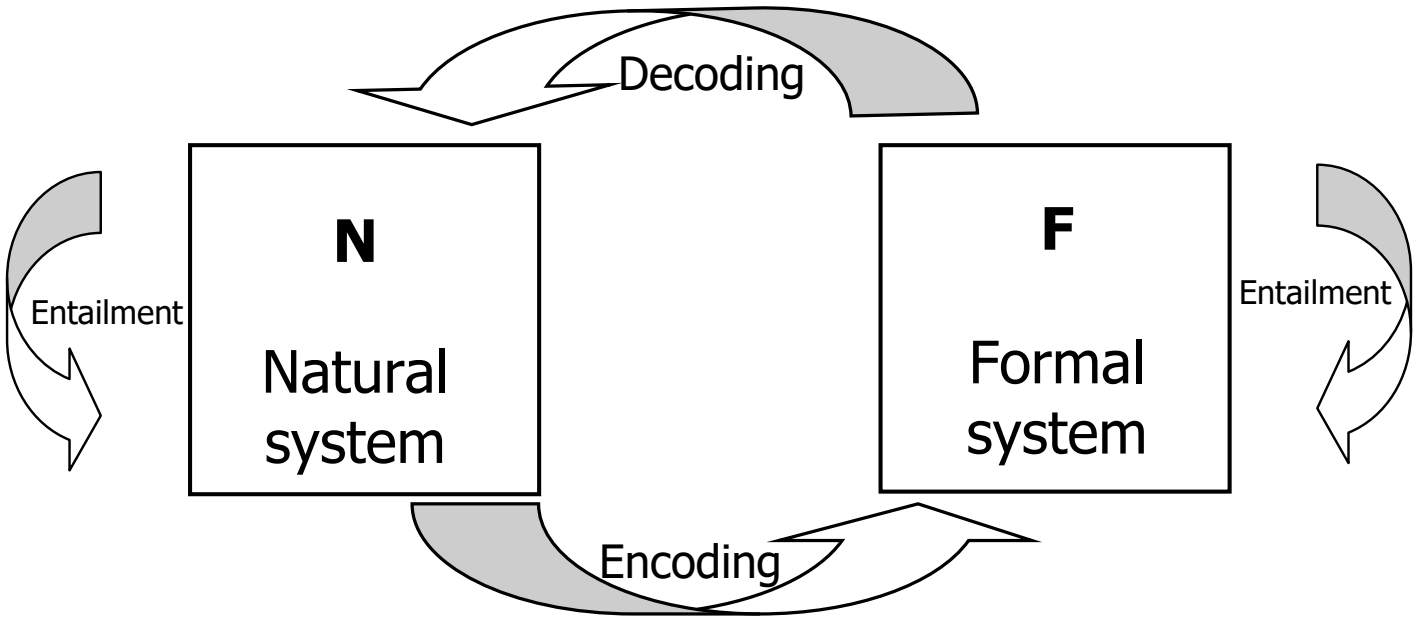
Modelling is a craft more than a science

Modelling as a craft rather than as a science for Robert Rosen





R. Rosen, Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life. Columbia University Press, 1991.



What is a model ?

Robert Rosen

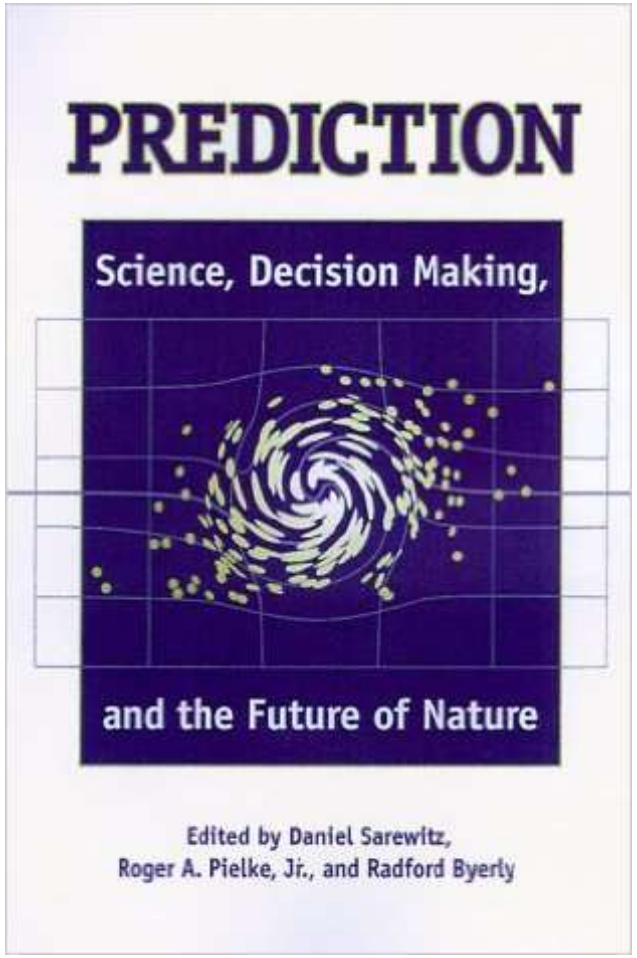
Can models be falsified?

“models are most useful when they are used to challenge existing formulations, rather than to validate or verify them”

Naomi Oreskes

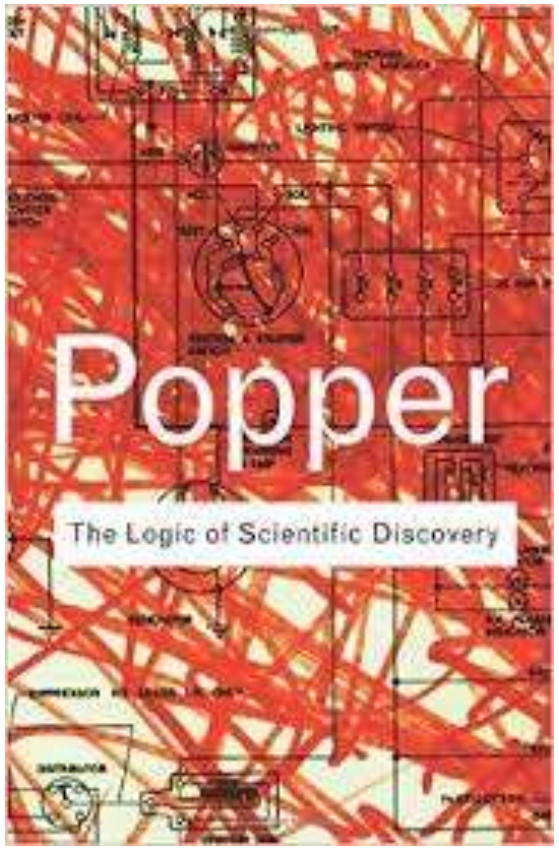
N. Oreskes, K. Shrader-Frechette, and K. Belitz, “Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences,” Science, 263, no. 5147, 1994.

Models are not physical laws



Oreskes, N., 2000, Why predict? Historical perspectives on prediction in Earth Science, in Prediction, Science, Decision Making and the future of Nature, Sarewitz et al., Eds., Island Press, Washington DC

“[…] to be of value in theory testing, the predictions involved must be capable of refuting the theory that generated them”
(N. Oreskes)



“In many cases, these temporal predictions **are treated with the same respect** that the hypothetic-deductive model of science accords to logical predictions. But this respect is largely misplaced”

"[…] models are complex amalgam of theoretical and phenomenological laws (and the governing equations and algorithms that represent them), empirical input parameters, and a model conceptualization […] When a model generates a prediction, of what precisely is the prediction a test? The laws? The input data? The conceptualization? Any part (or several parts) of the model might be in error, and there is no simple way to determine which one it is"

Model-based knowing is conditional

When models need as input information which we don't have

John Kay

J. A. Kay, "Knowing when we don't know," 2012, https://www.ifs.org.uk/docs/john_kay_feb2012.pdf

WebTAG: Annual Percentage Change in Car Occupancy (% pa) up to 2036

Journey Purpose	Weekday					Weekend	All Week
	7am-10am	10am-4pm	4pm-7pm	7pm-7am	Weekday Average		
Work	-0.48	-0.4	-0.62	-0.5	-0.44	-0.48	-0.45
Non - Work (commuting and other)	-0.67	-0.65	-0.53	-0.47	-0.59	-0.52	-0.56

Models and their data

“[in climate modelling] it looks very little like our idealized image of science, in which pure theory is tested with pure data. [impossible to] eliminate the model-dependency of data or the data-ladenness of models”

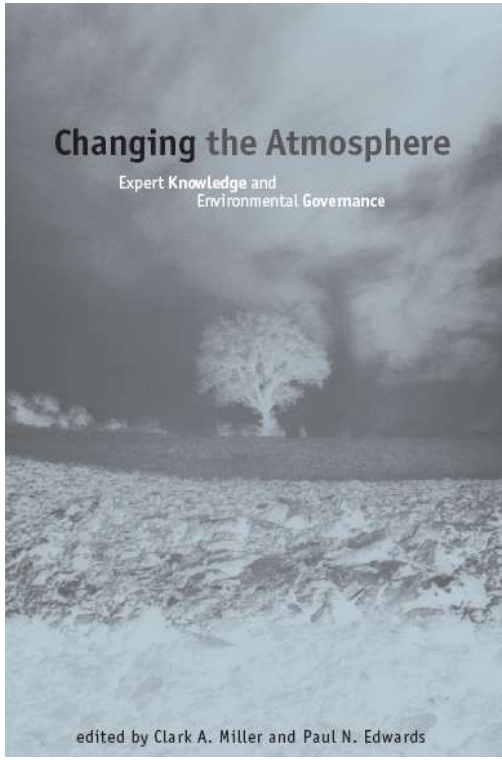
Paul N. Edwards, 1999, Global climate science, uncertainty and politics: Data-laden models, model-filtered data.

For philosophers Frederick Suppe and Stephen Norton the blurry model/data relationship pervades all science

3

Why Atmospheric Modeling Is Good Science

Stephen D. Norton and Frederick Suppe



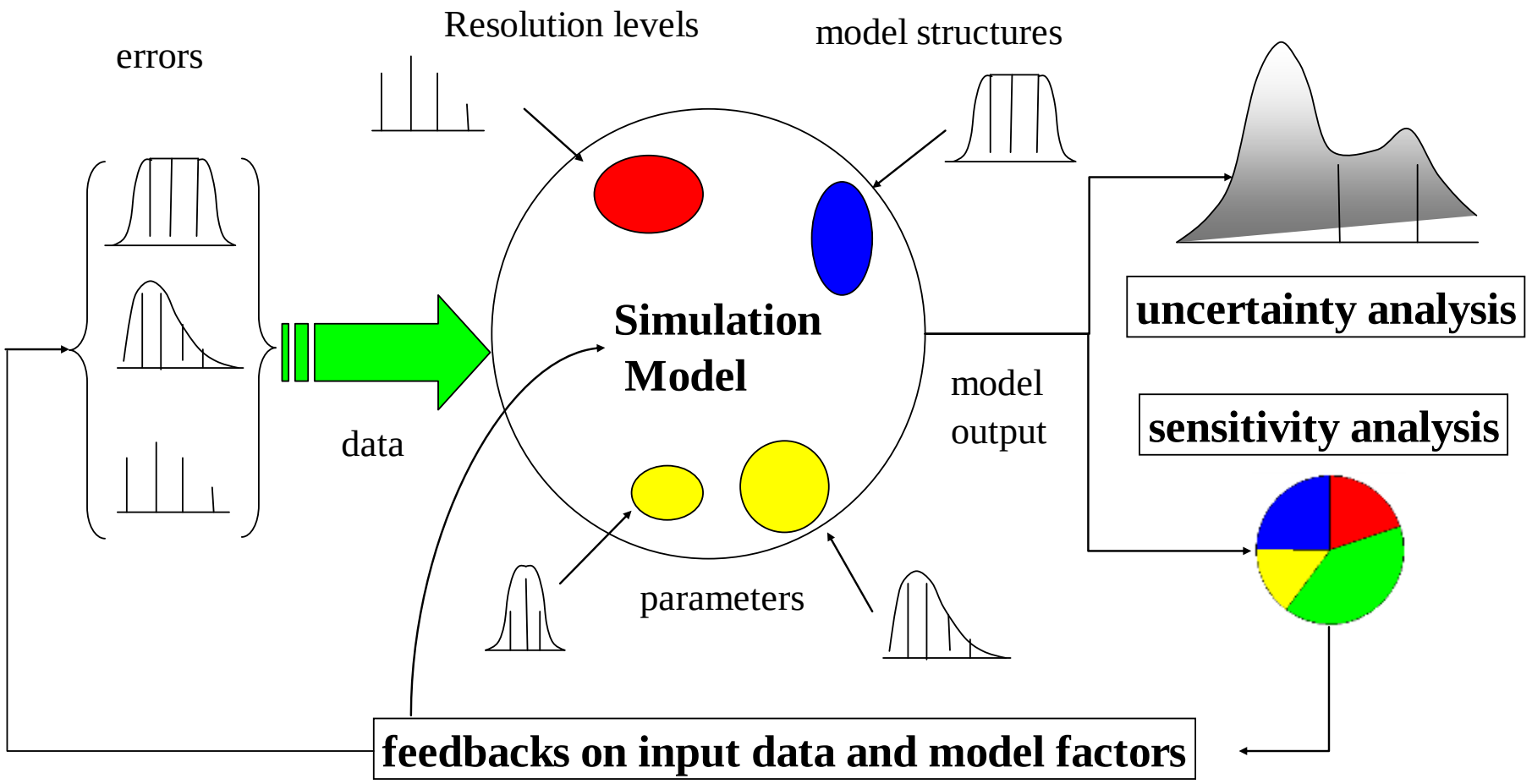
Changing the Atmosphere: Expert Knowledge and Environmental Governance, edited by Clark A. Miller, Paul N. Edwards,

Definitions

Uncertainty analysis: Focuses on just quantifying the uncertainty in model output

Sensitivity analysis: The study of the relative importance of different input factors on the model output

An engineer's vision of UA, SA

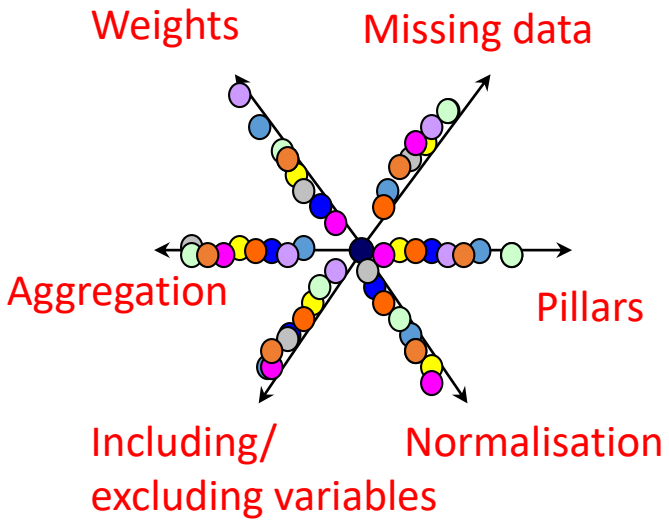


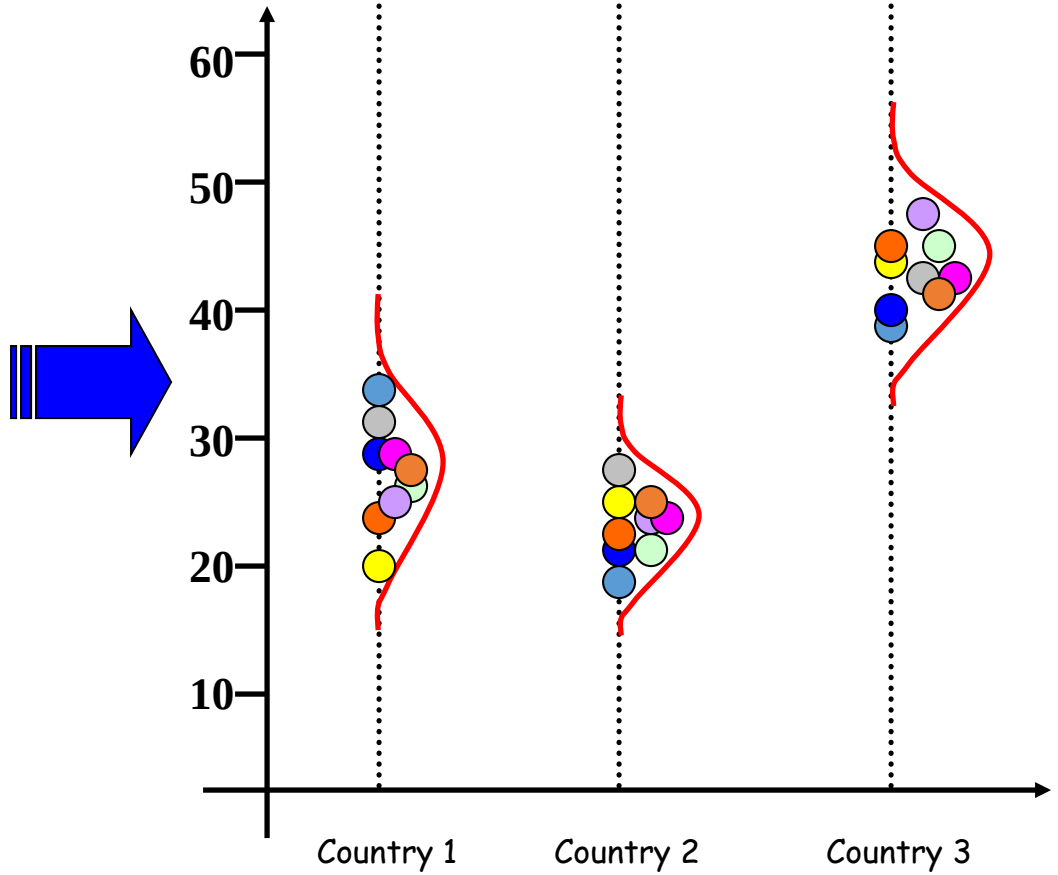
One can sample more than just factors:

- modelling assumptions,
- alternative data sets,
- resolution levels,
- scenarios …

Assumption	Alternatives
Number of indicators	▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	▪ original set of weights, ▪ factor analysis, ▪ equal weighting, ▪ data envelopment analysis
Aggregation rule	▪ additive, ▪ multiplicative, ▪ Borda multi-criterion

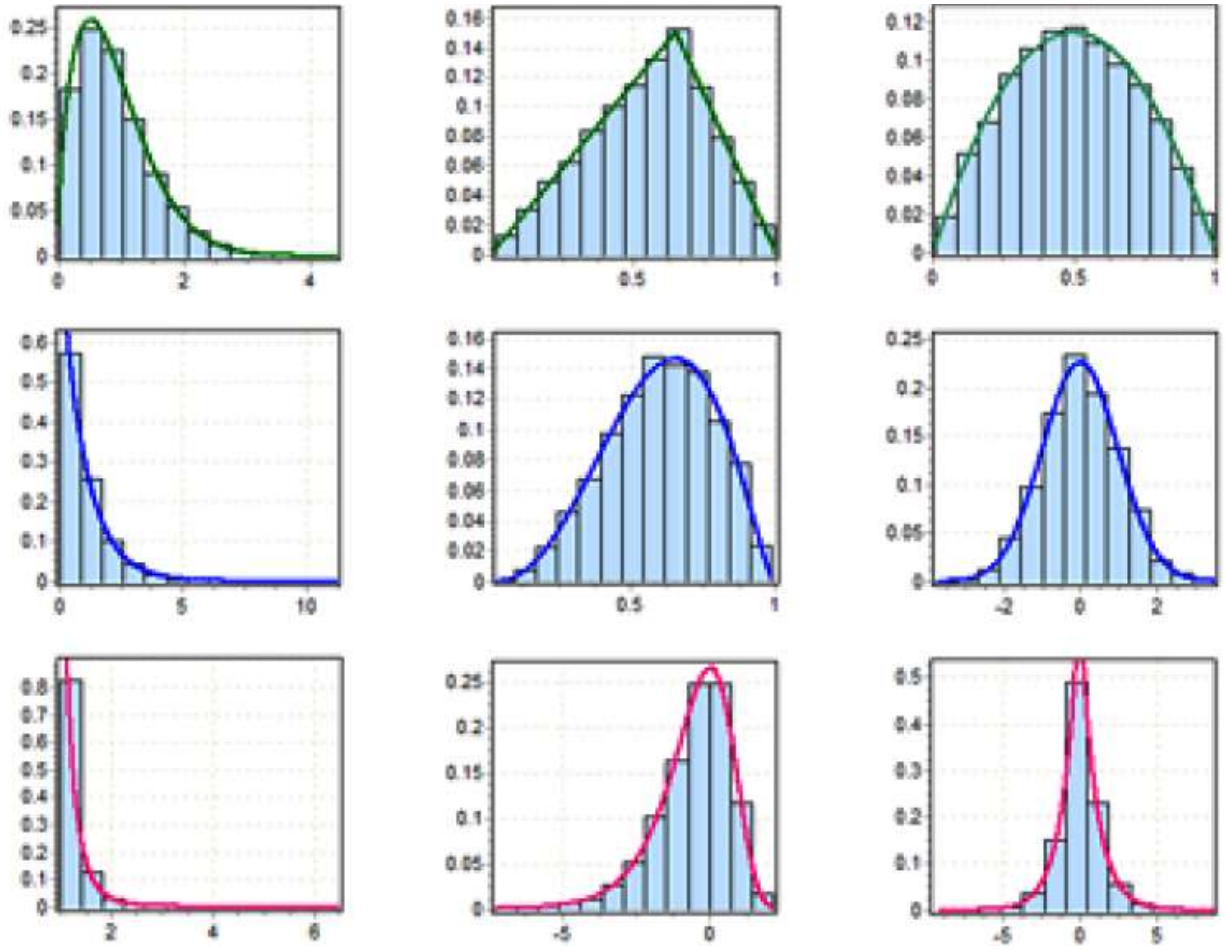
Space of alternatives





$$\begin{matrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{Nk} \end{matrix}$$

Each column is a sample from the distribution of a factor
Each row is a sample trial to generate a value of y



Examples of distributions of input factors

Why Sensitivity analysis?

It is in the guidelines!

European Commission, 2015

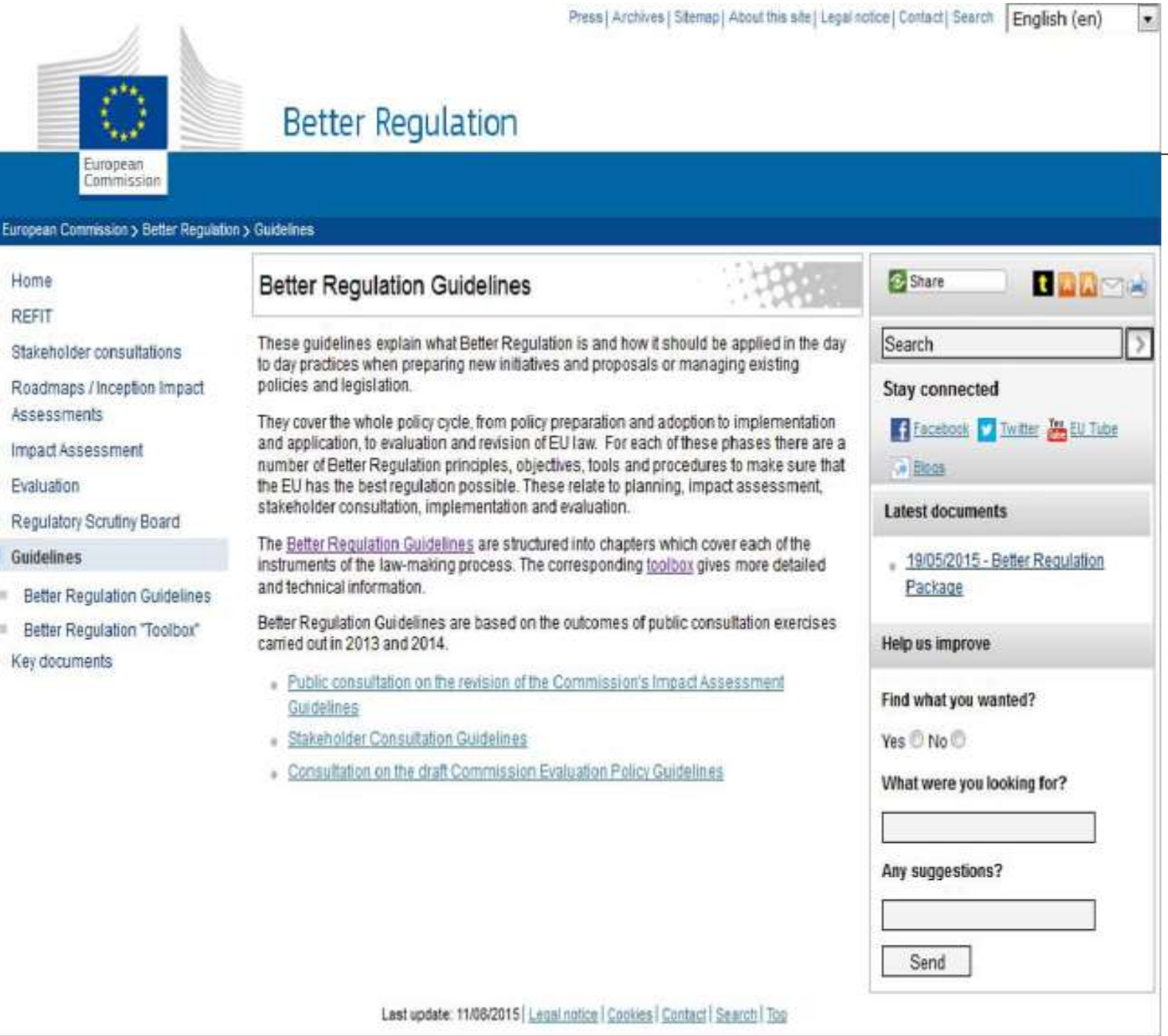
Office for the Management and Budget, 2006

Environmental Protection Agency, 2009

EPA, 2009, March. Guidance on the Development, Evaluation, and Application of Environmental Models. Technical Report EPA/100/K-09/003. Office of the Science Advisor, Council for Regulatory Environmental Modeling, http://nepis.epa.gov/Exe/ZyPDF.cgi?Dockey=P1003E4R.PDF, Last accessed December 2015.

EUROPEAN COMMISSION, Better regulation toolbox, appendix to the Better Regulation Guidelines, Strasbourg, 19.5.2015, SWD(2015) 111 final, COM(2015) 215 final, http://ec.europa.eu/smart-regulation/guidelines/docs/swd_br_guidelines_en.pdf.

OMB, Proposed risk assessment bulletin, Technical report, The Office of Management and Budget's – Office of Information and Regulatory Affairs (OIRA), January 2006, https://www.whitehouse.gov/sites/default/files/omb/assets/omb/inforeg/proposed_risk_assessment_bulletin_010906.pdf, pp. 16–17, accessed December 2015.



http://ec.europa.eu/smart-regulation/

Source: IA Toolbox, p. 391

4. SENSITIVITY AND UNCERTAINTY ANALYSES

Page 391

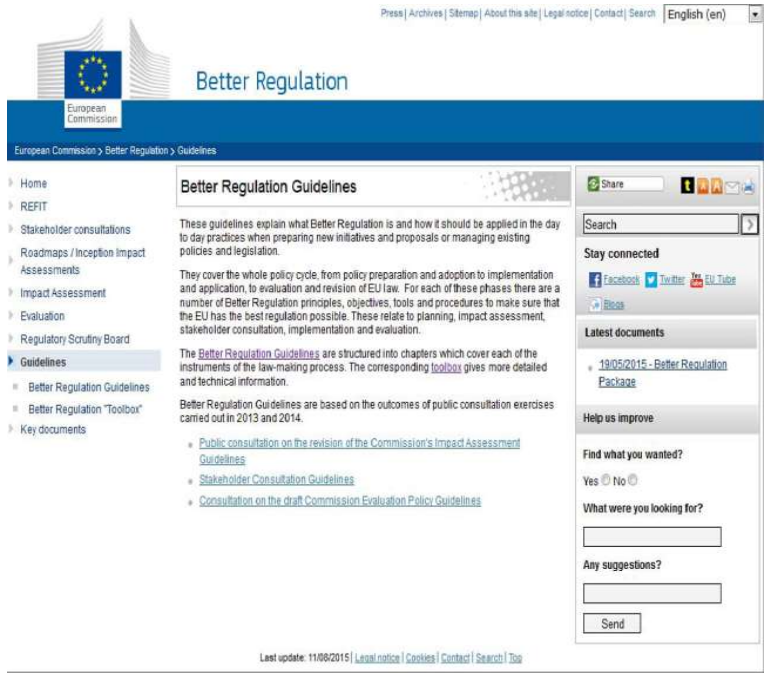
Six steps for a global SA:

1. Select <u>one</u> output of interest;
2. Participatory step: discuss which input may matter;
3. Participatory step (extended peer review): define distributions;
4. Sample from the distributions;
5. Run (=evaluate) the model for the sampled values;
6. Obtain in this way bot the uncertainty of the prediction and the relative importance of variables.

Is something wrong with this statement (p. 384 of EC guidelines)

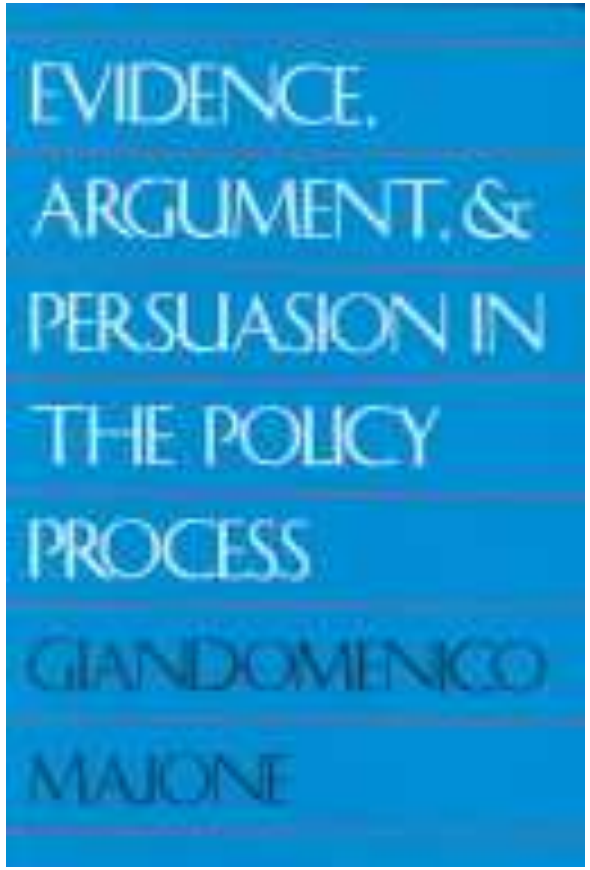
The influence of the key variables

should be investigated by a sensitivity analysis.



Why Sensitivity analysis?

It can answer interesting questions



"Are the results from a particular model more sensitive to changes in the model and the methods used to estimate its parameters, or to changes in the data?"

Why sensitivity analysis?

It can detect garbage in garbage out (GIGO)

Funtowicz & Ravetz's GIGO (Garbage In, Garbage Out) Science "where uncertainties in inputs must be suppressed least outputs become indeterminate"

Leamer's "Conclusions are judged to be sturdy only if the neighborhood of assumptions is wide enough to be credible and the corresponding interval of inferences is narrow enough to be useful"

S. Funtowicz and J. R. Ravetz, *Uncertainty and Quality in Science for Policy*. Dordrecht: Kluwer, 1990; E. E. Leamer, "Sensitivity Analyses Would Help," *Am. Econ. Rev.*, vol. 75, no. 3, pp. 308–313, 1985.

Global Environmental Change 20 (2010) 298–302

Contents lists available at ScienceDirect

Global Environmental Change

journal homepage: www.elsevier.com/locate/gloenvcha

Sensitivity analysis didn't help. A practitioner's critique of the Stern review

Andrea Saltelli *, Beatrice D'Hombres

Joint Research Centre, Institute for the Protection and Security of the Citizen, Ispra, Italy

The case of Stern's Review – Technical Annex to postscript

Nicholas Stern, London School of Economics

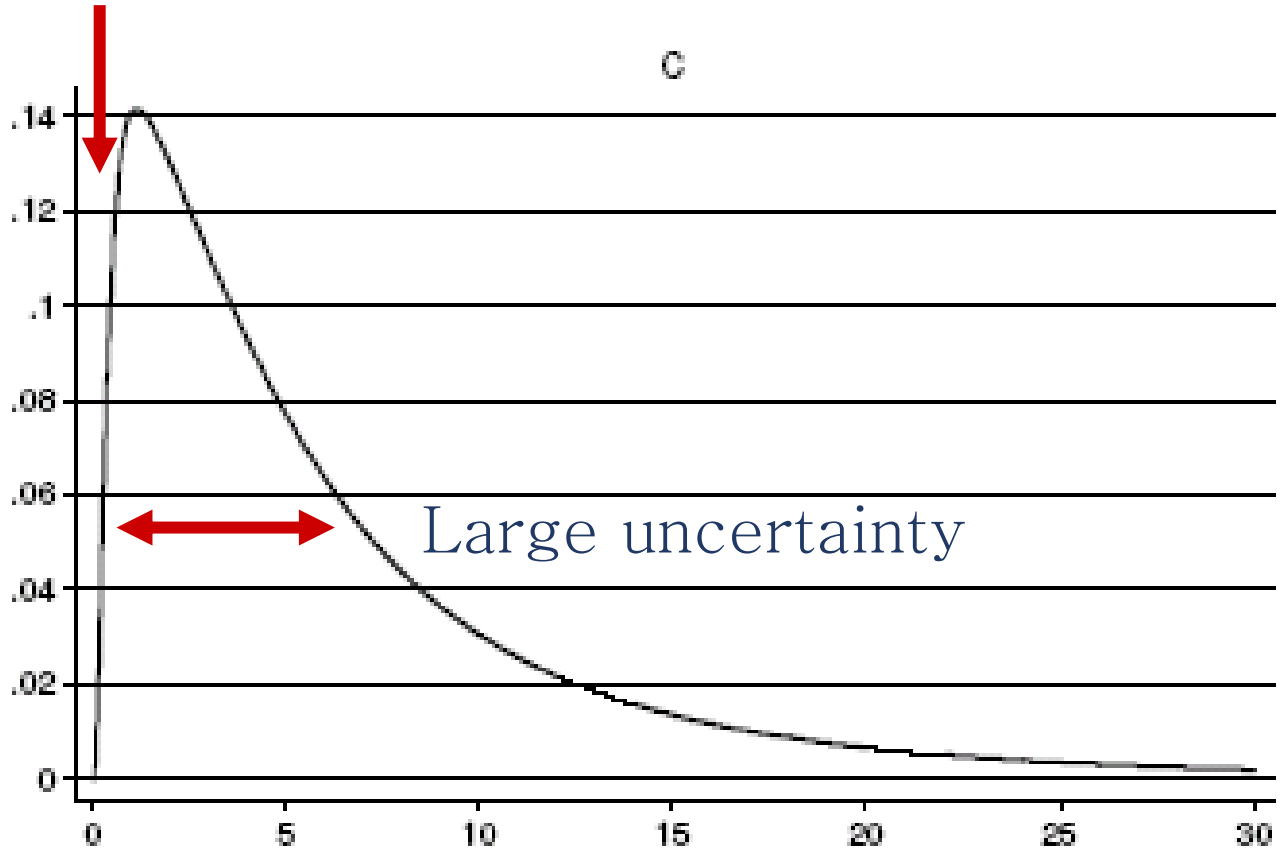
William Nordhaus, University of Yale Nobel 'Economics' 2018

Stern, N., Stern Review on the Economics of Climate Change. UK Government Economic Service, London, www.sternreview.org.uk.

Nordhaus W., Critical Assumptions in the Stern Review on Climate Change, SCIENCE, 317, 201-202, (2007).

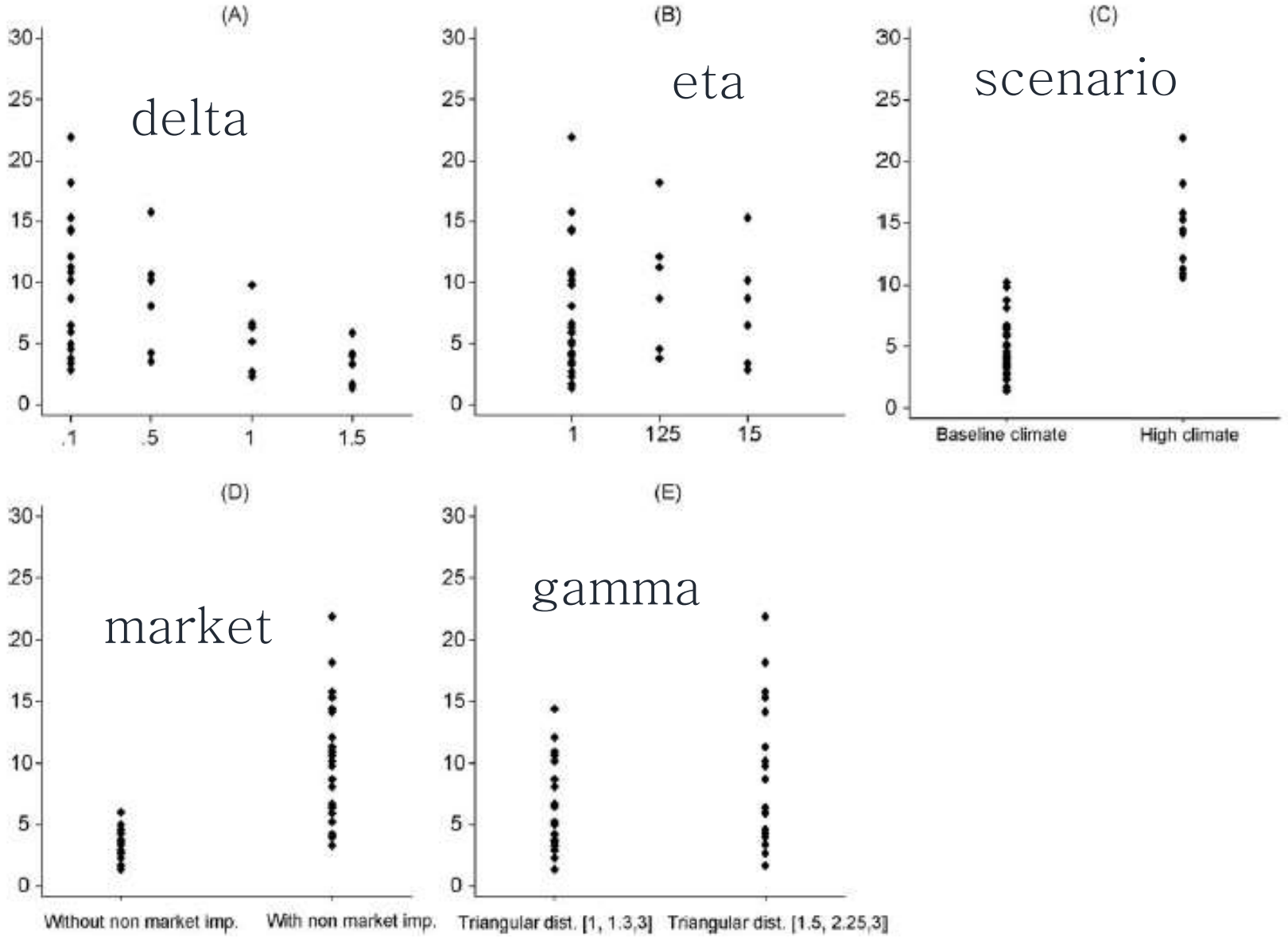
How was it done? A reverse engineering of the analysis

Missing points



% loss in GDP per capita

Sensitivity analysis here (by reverse engineering)



Why sensitivity analysis?

It allows interesting discoveries

Journal of the Royal Statistical Society
SERIES A Statistics in Society

J. R. Statist. Soc. A (2013)
176, *Part* 3, *pp.* 609–634

Ratings and rankings: voodoo or science?

Paolo Paruolo

University of Insubria, Varese, Italy

and Michaela Saisana and Andrea Saltelli

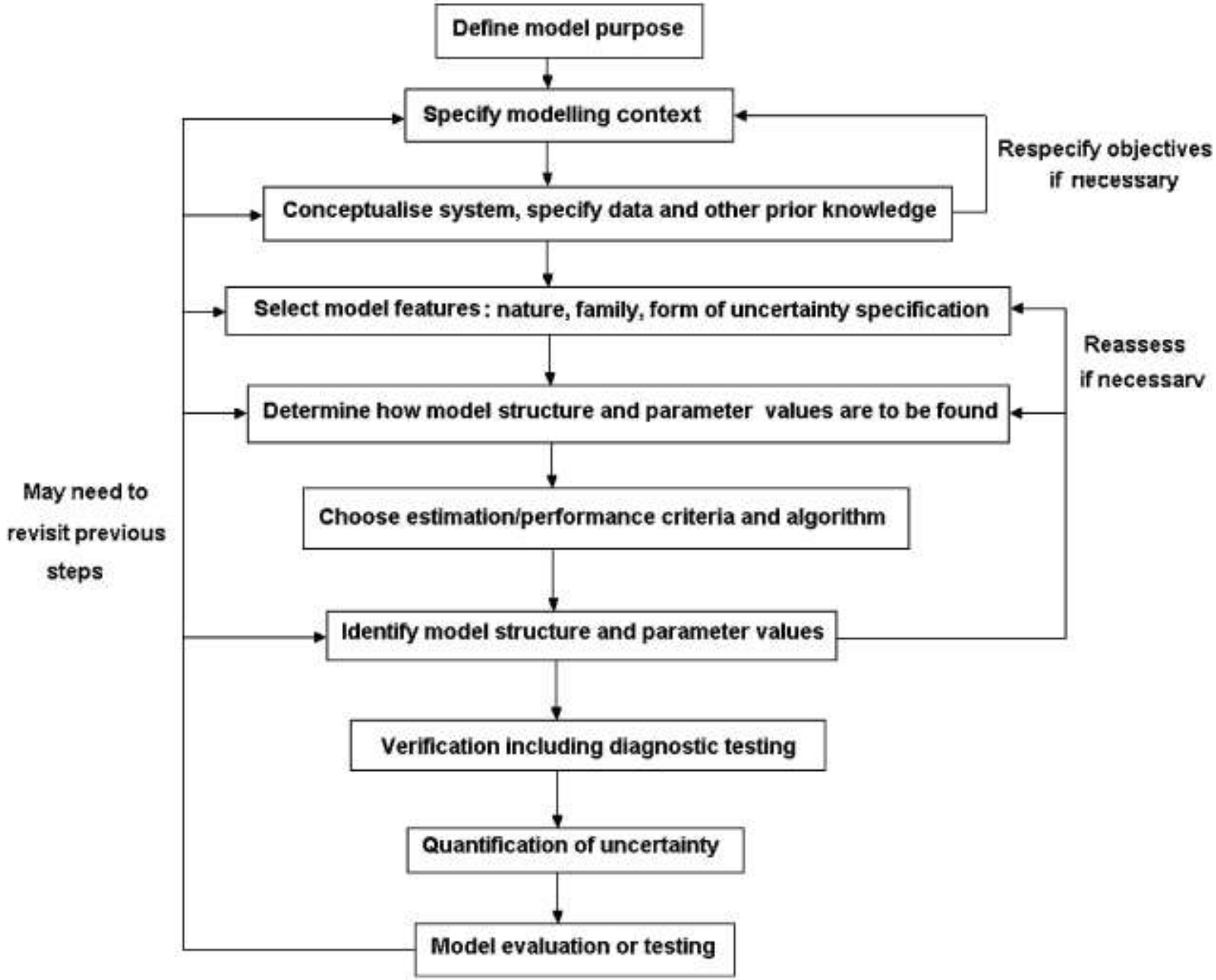
European Commission, Ispra, Italy

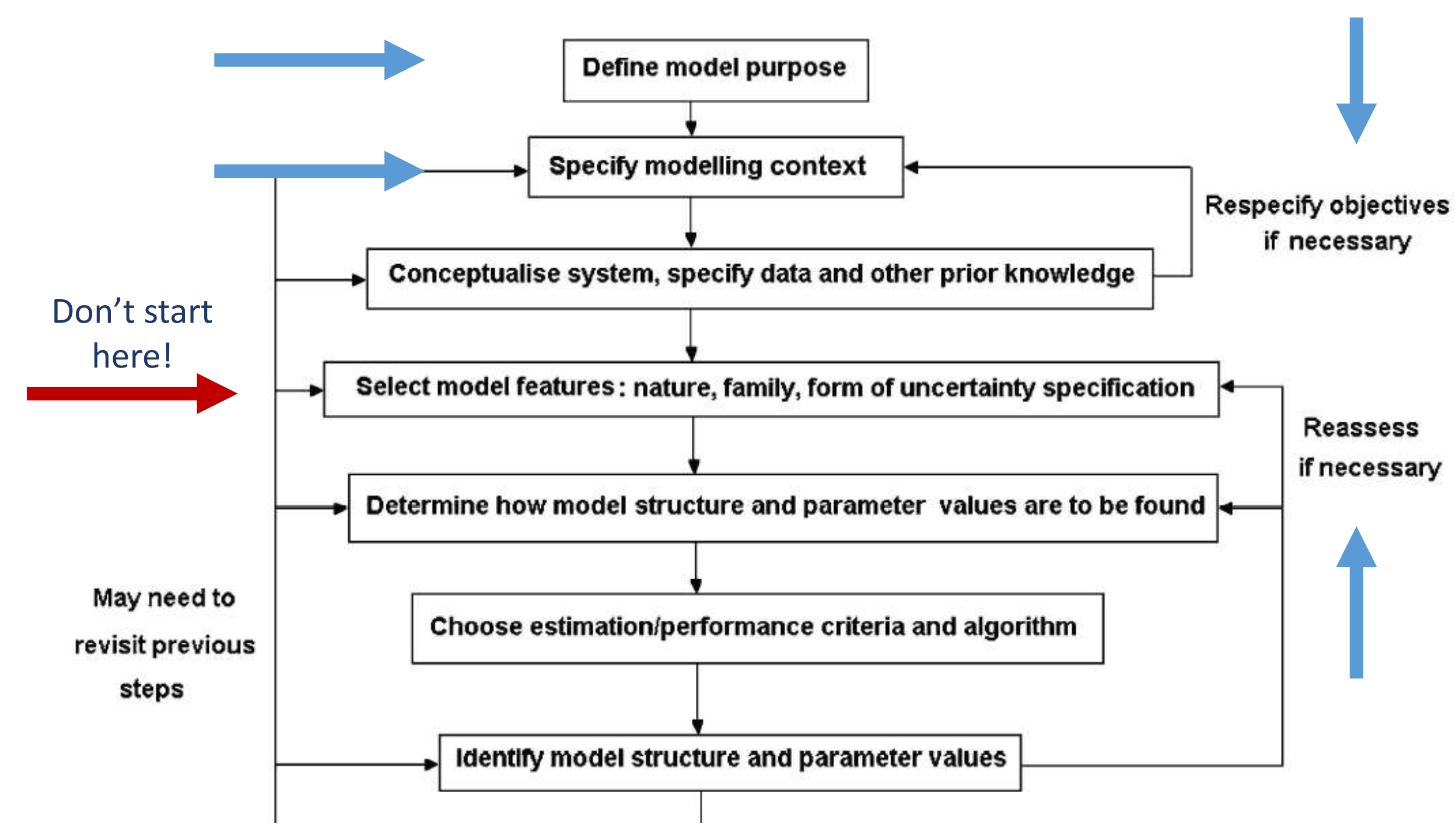
Why sensitivity analysis

A 10 points participatory checklist (Jakeman et al. 2006)

Padilla, J. J., Diallo, S. Y., Lynch, C. J., & Gore, R. (2018). Observations on the practice and profession of modeling and simulation: A survey approach. SIMULATION, 94(6), 493–506.

Jakeman, A. J., Letcher, R. A., & Norton, J. P. (2006). Ten iterative steps in development and evaluation of environmental models,. Environmental Modelling & Software, 21(5), 602–614.


Define model purpose
Specify modelling context
Conceptualise system, specify data and other prior knowledge
Respecify objectives if necessary
Select model features : nature, family, form of uncertainty specification
Reassess if necessarv
Determine how model structure and parameter values are to be found
May need to revisit previous steps
Choose estimation/performance criteria and algorithm
Identify model structure and parameter values
Verification including diagnostic testing
Quantification of uncertainty
Model evaluation or testing


Define model purpose
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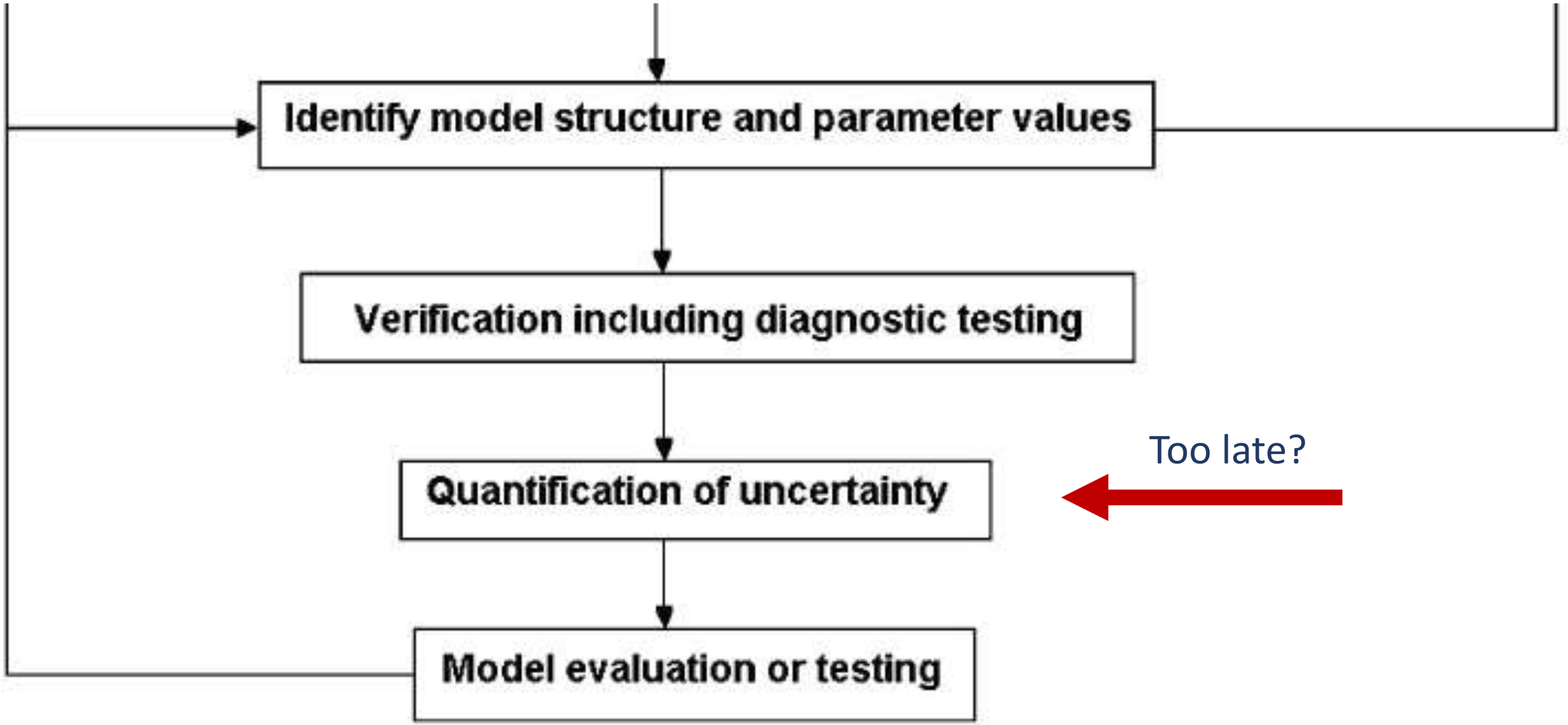
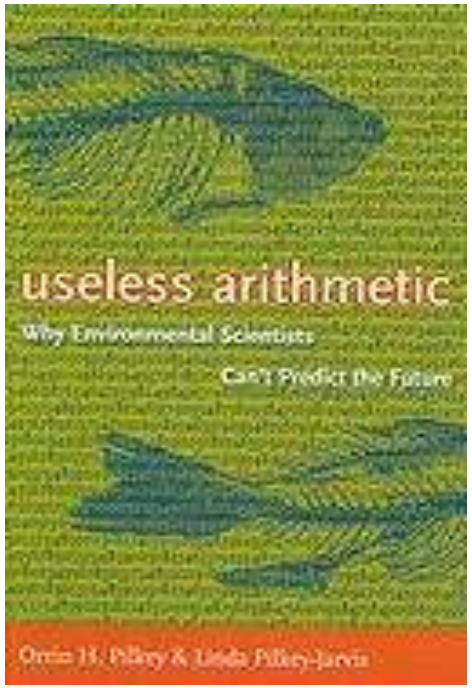
Select model features : nature, family, form of uncertainty specification
Determine how model structure and parameter values are to be found
Choose estimation/performance criteria and algorithm
Identify model structure and parameter values
Respecify objectives if necessary
Reassess if necessary
Don't start here!
May need to revisit previous steps


Identify model structure and parameter values
Verification including diagnostic testing
Quantification of uncertainty
Model evaluation or testing
Too late?

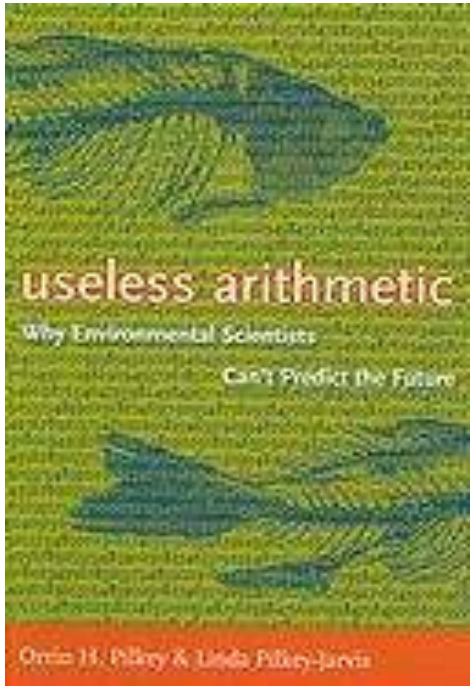
Limits of sensitivity analysis

Orrin H.
Pilkey



Useless Arithmetic: Why Environmental Scientists Can't Predict the Future by Orrin H. Pilkey and Linda Pilkey-Jarvis, Columbia University Press, 2009.

The map is not the territory

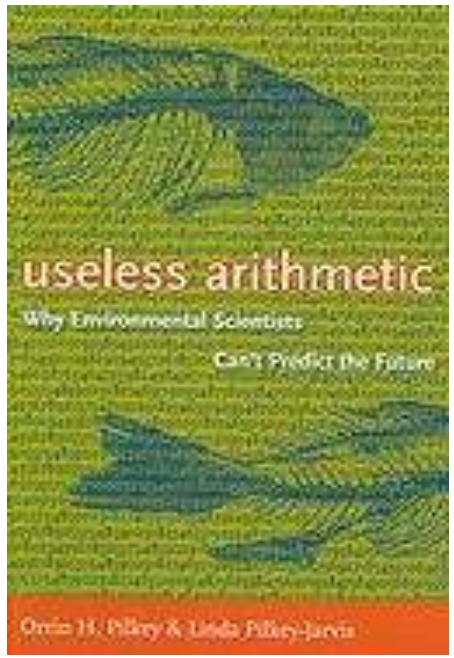


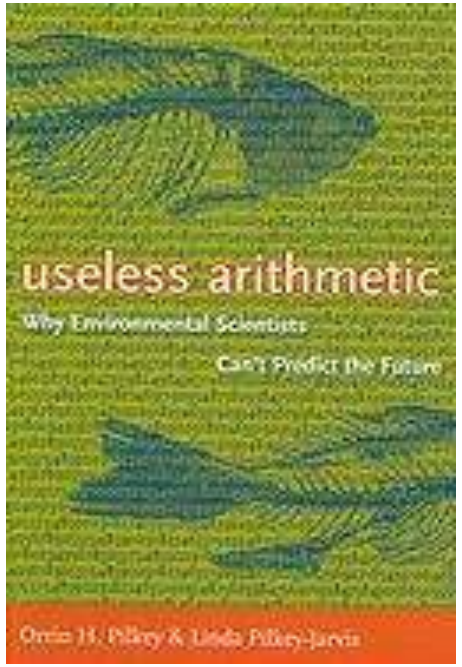
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

[…] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

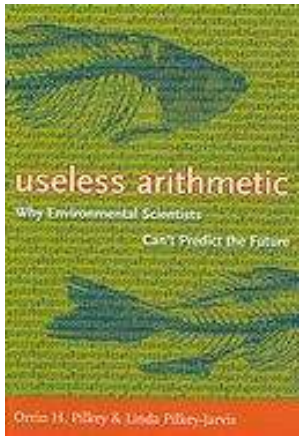
One of the examples discussed concerns the Yucca Mountain repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

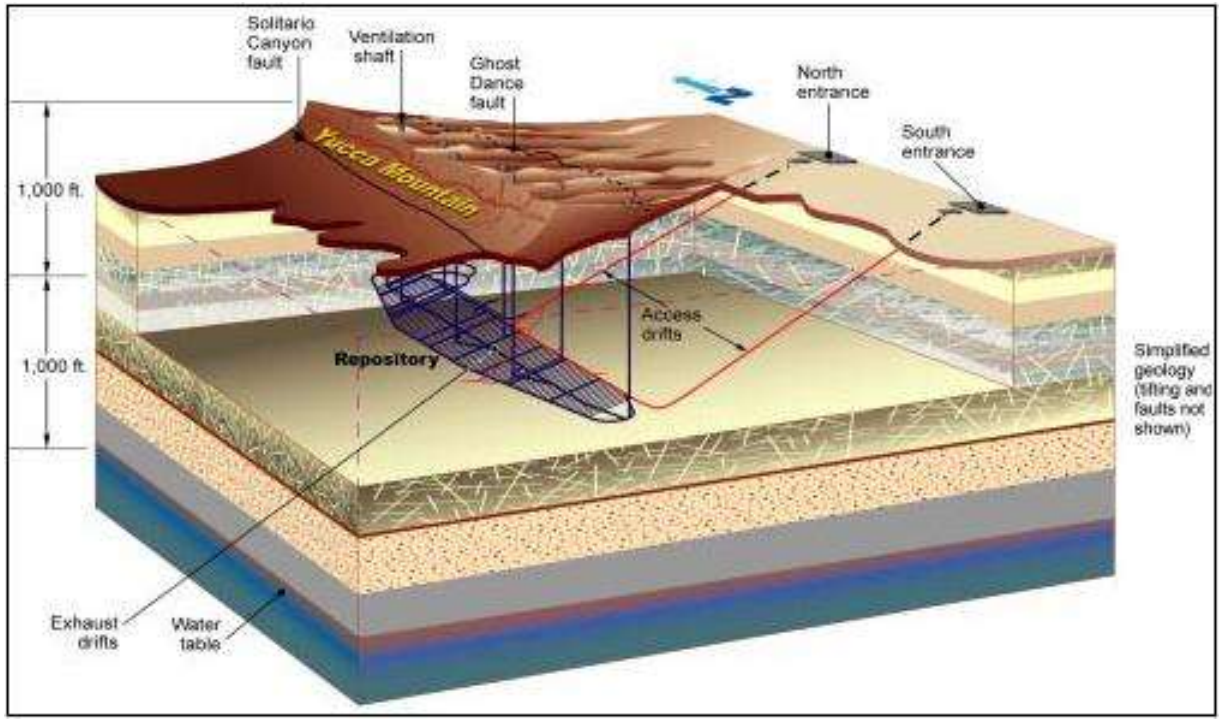
TSPA is Composed of 286 sub-models.





TSPA (like any other model) relies on assumptions → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.





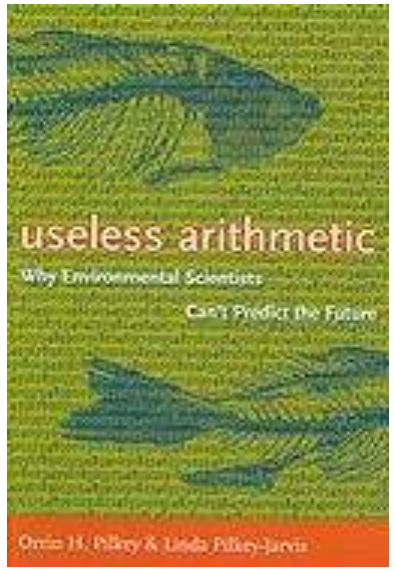
The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter

(the ^{36}Cl story)

Type III error in sensitivity: Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

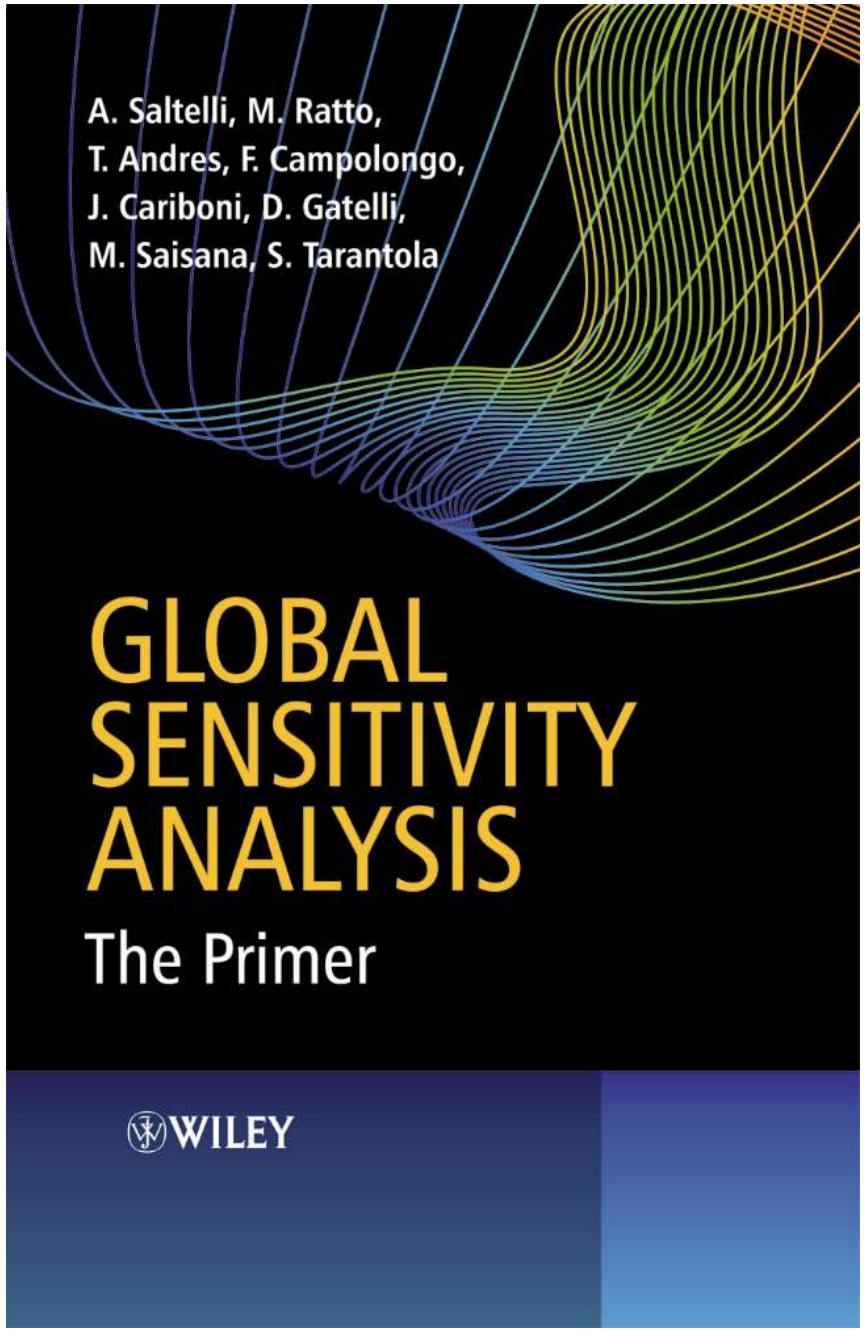
→… SA useless if it is instead ~ 3,000 millimetres per year.

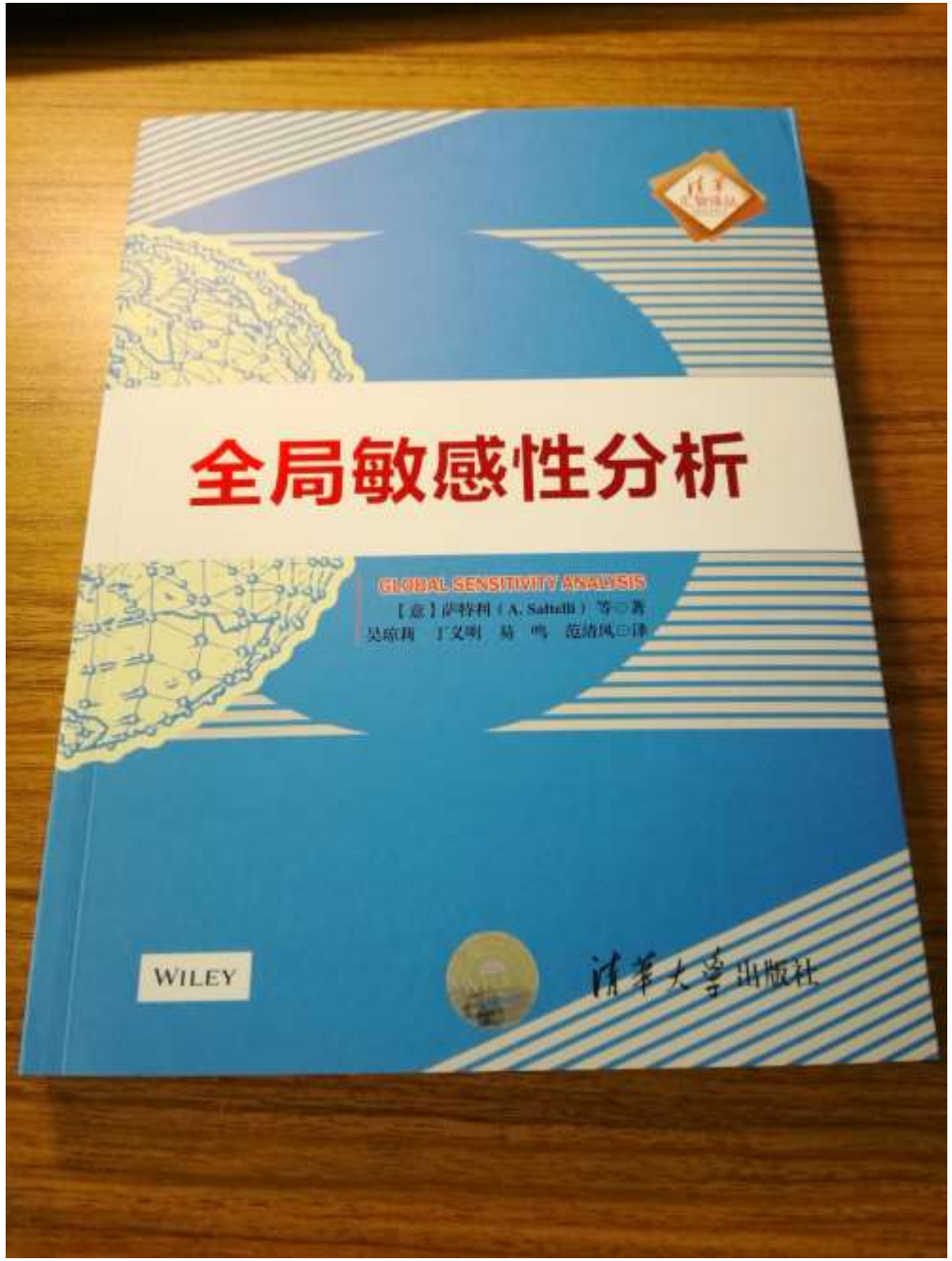


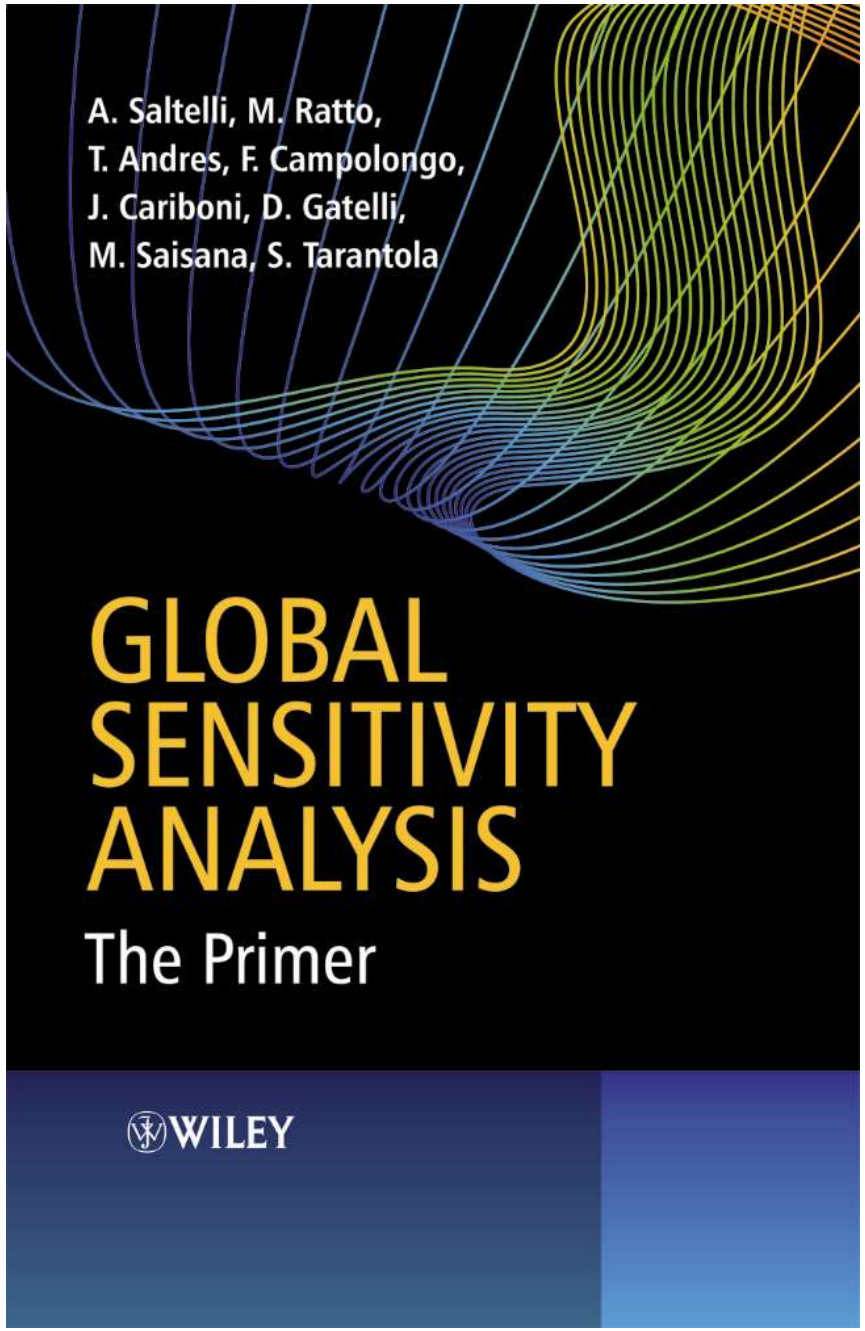
"Scientific mathematical modelling should involve constant efforts to falsify the model"

➔ Organized skepticism (as per CUDOS)

Where to study sensitivity analysis?


A. Saltelli, M. Ratto,
T. Andres, F. Campolongo,
J. Cariboni, D. Gatelli,
M. Saisana, S. Tarantola
GLOBAL
SENSITIVITY
ANALYSIS
The Primer
WILEY

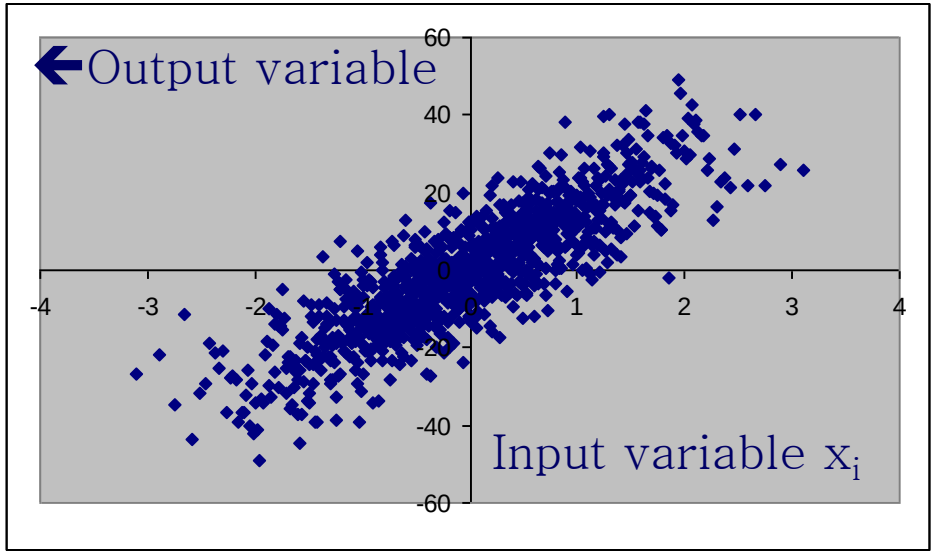

全局敏感性分析
GLOBAL SENSITIVITY ANALYSIS
WILEY

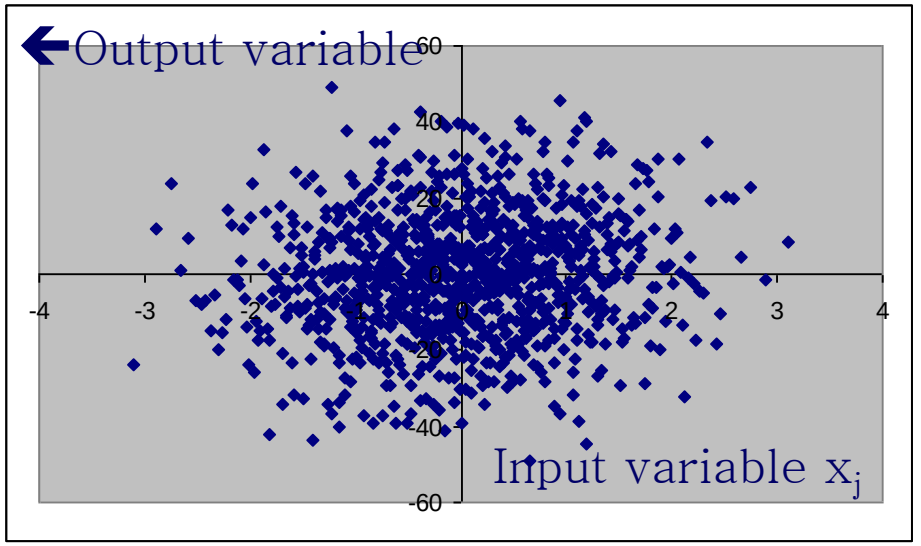


Available for free at

http://www.andreasaltelli.eu

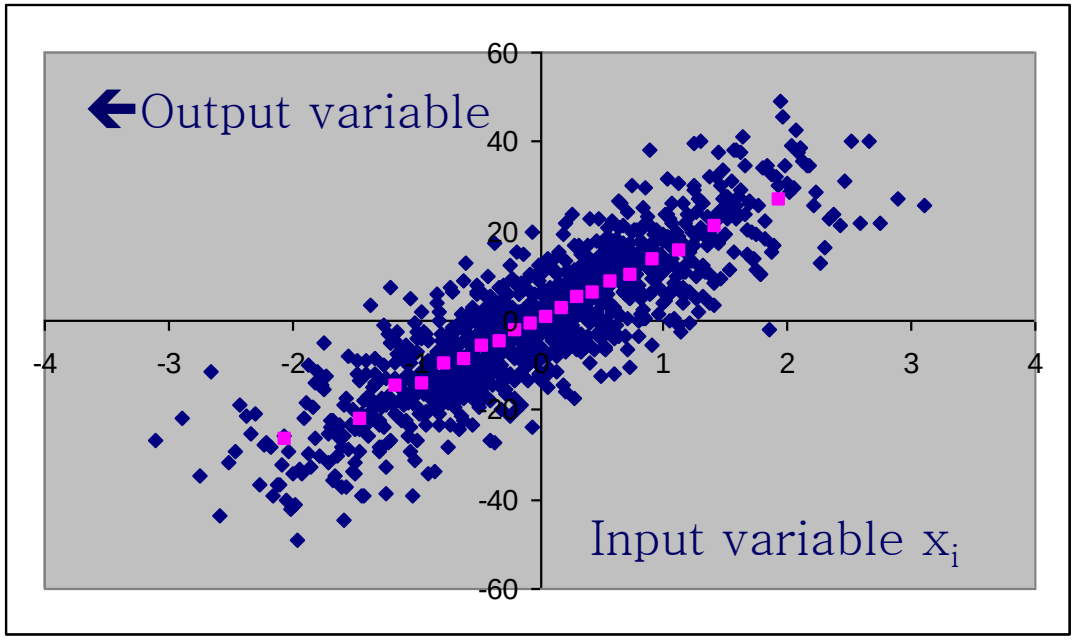
How is it done in practice?

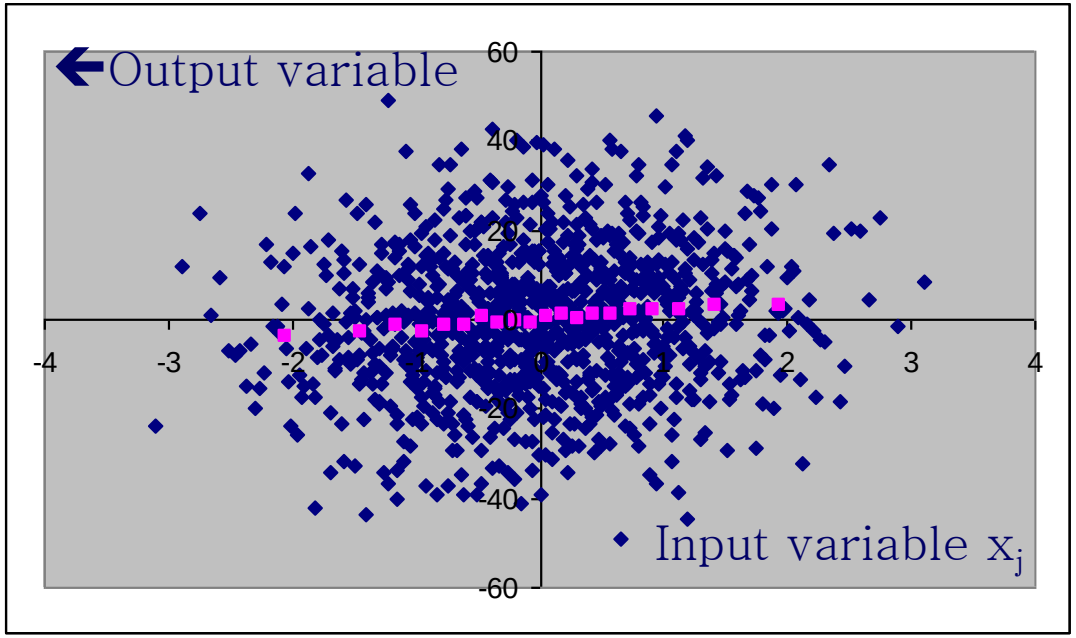




Plotting the output as a function of two different input factors

Which factor is more important?

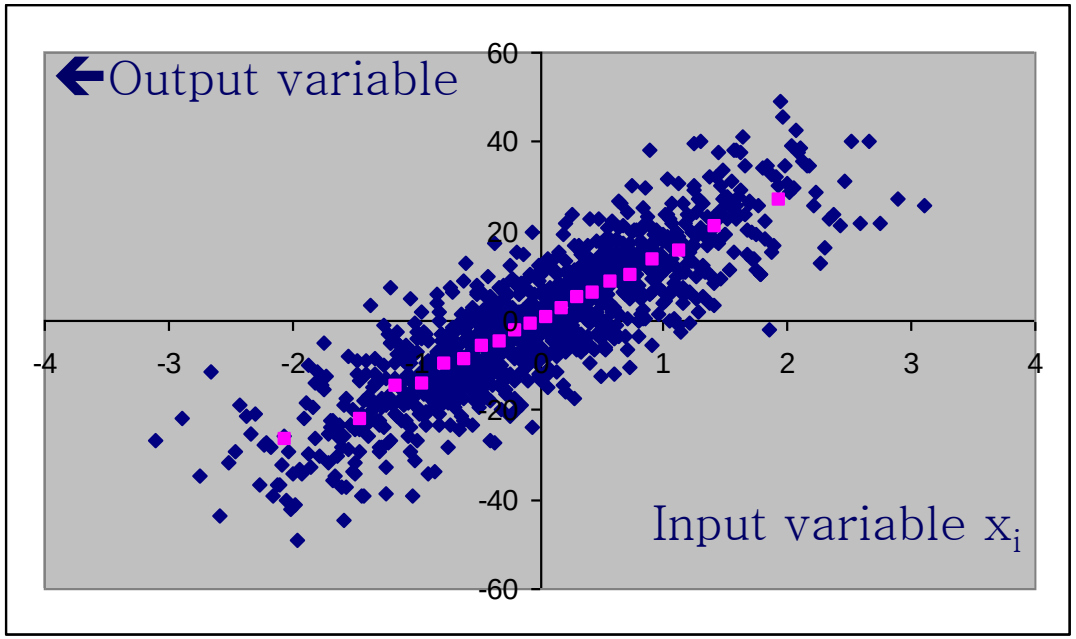




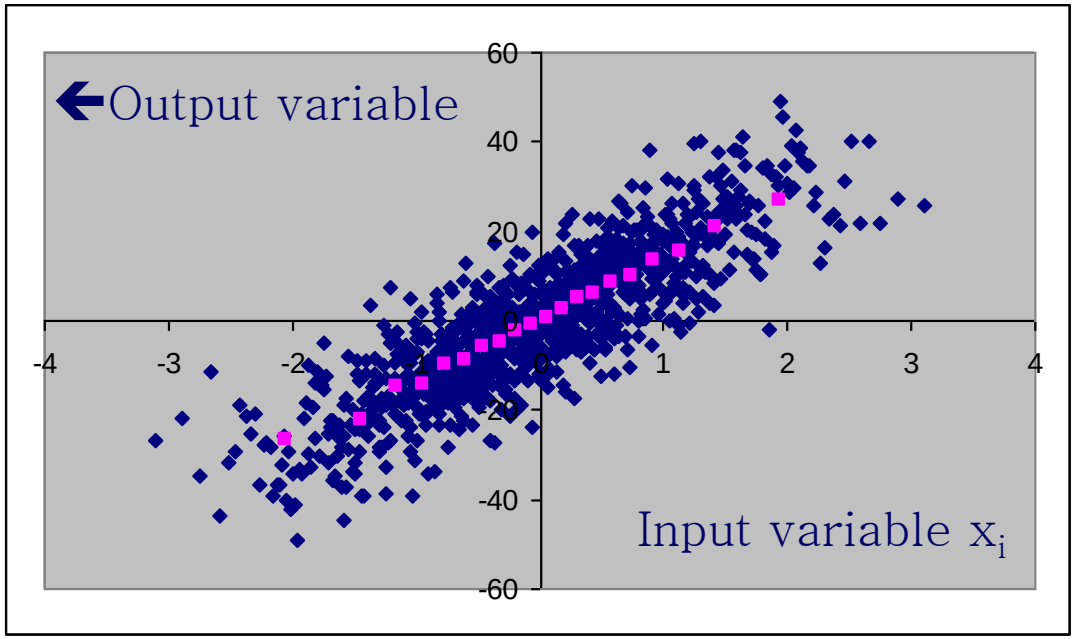
~1,000 blue points

Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)

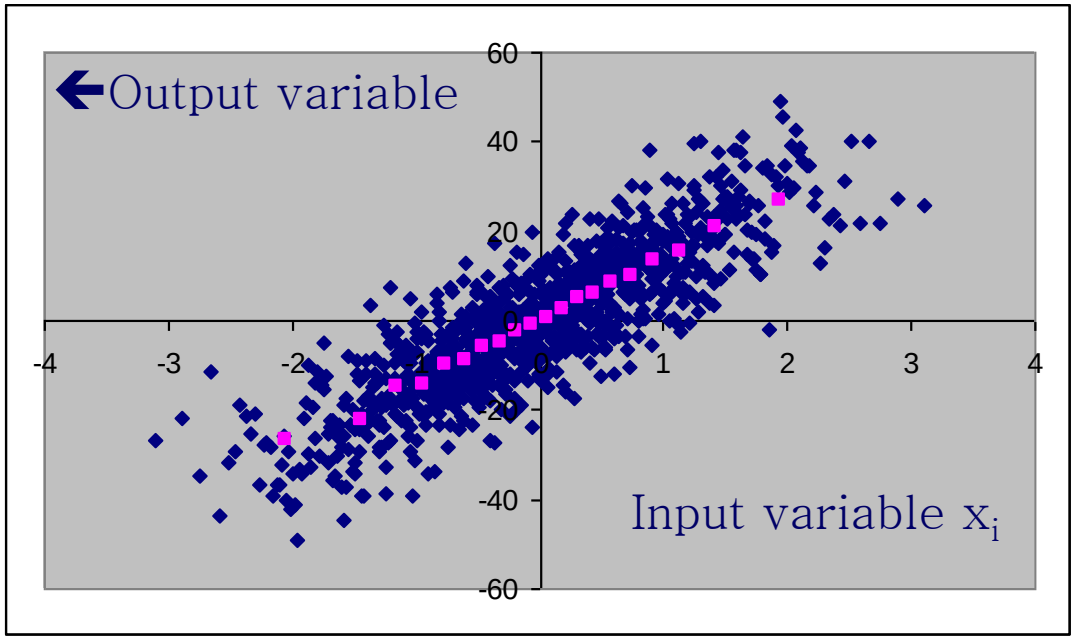


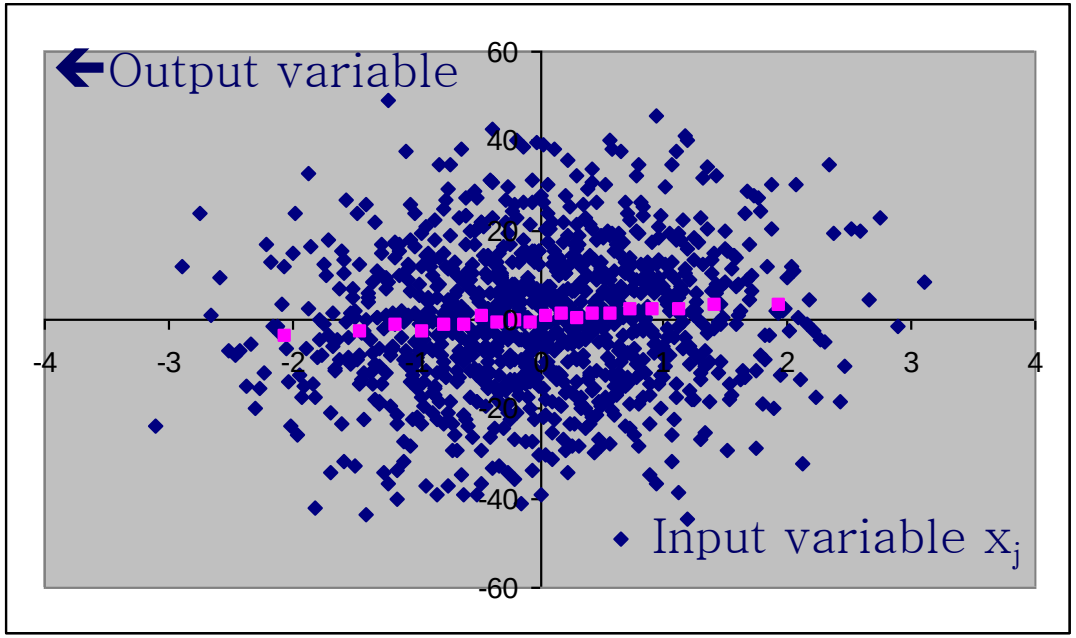
Each pink point is ~ $E_{\mathbf{X}_{\sim i}}\left(Y \middle| X_i\right)$



Take the variance of the pink points one obtains a sensitivity measure

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)$$





Which factor has the highest

$$V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right)?$$

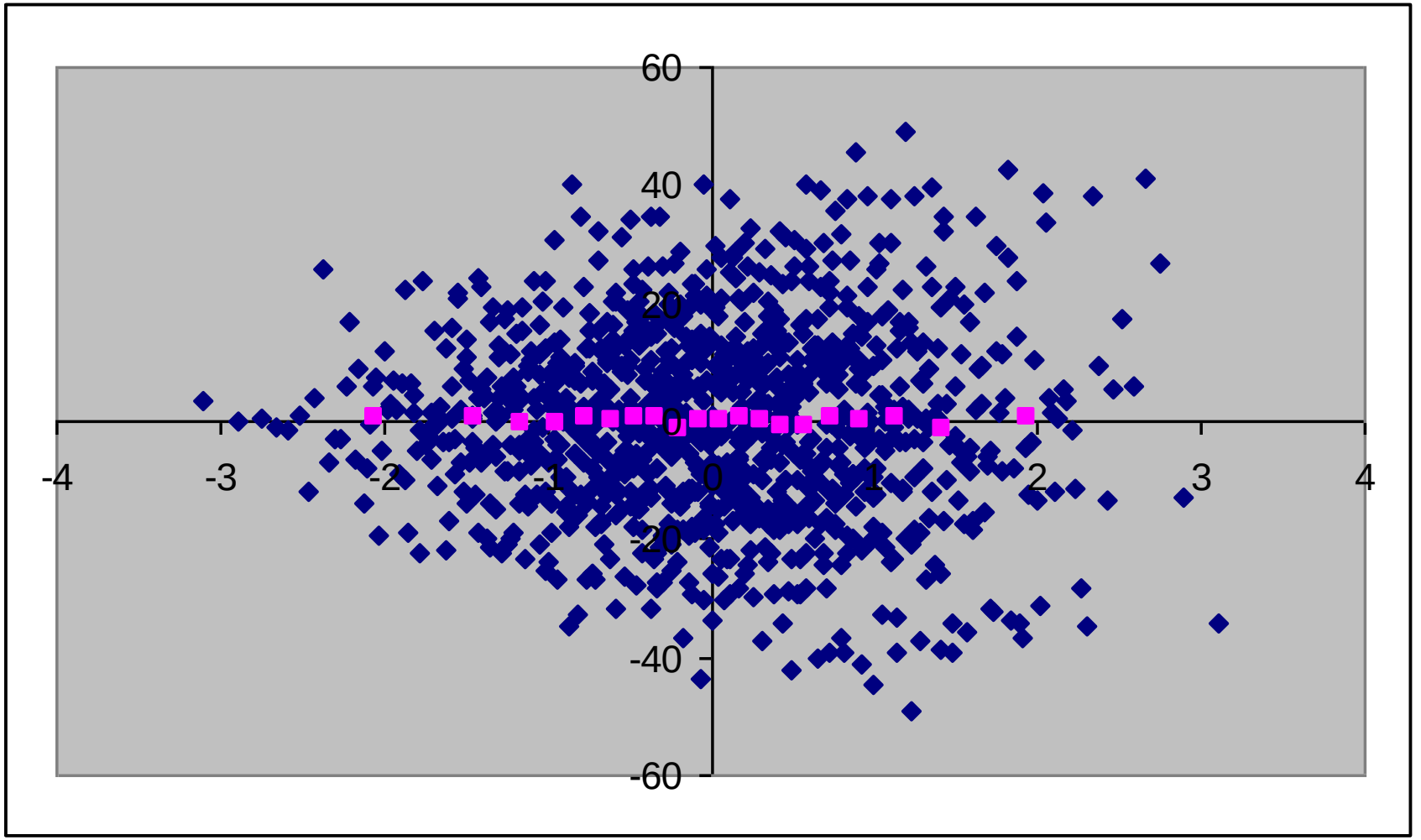
For additive models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i}\left(E_{\mathbf{X}_{\sim i}}\left(Y|X_i\right)\right) \approx V(Y)$$

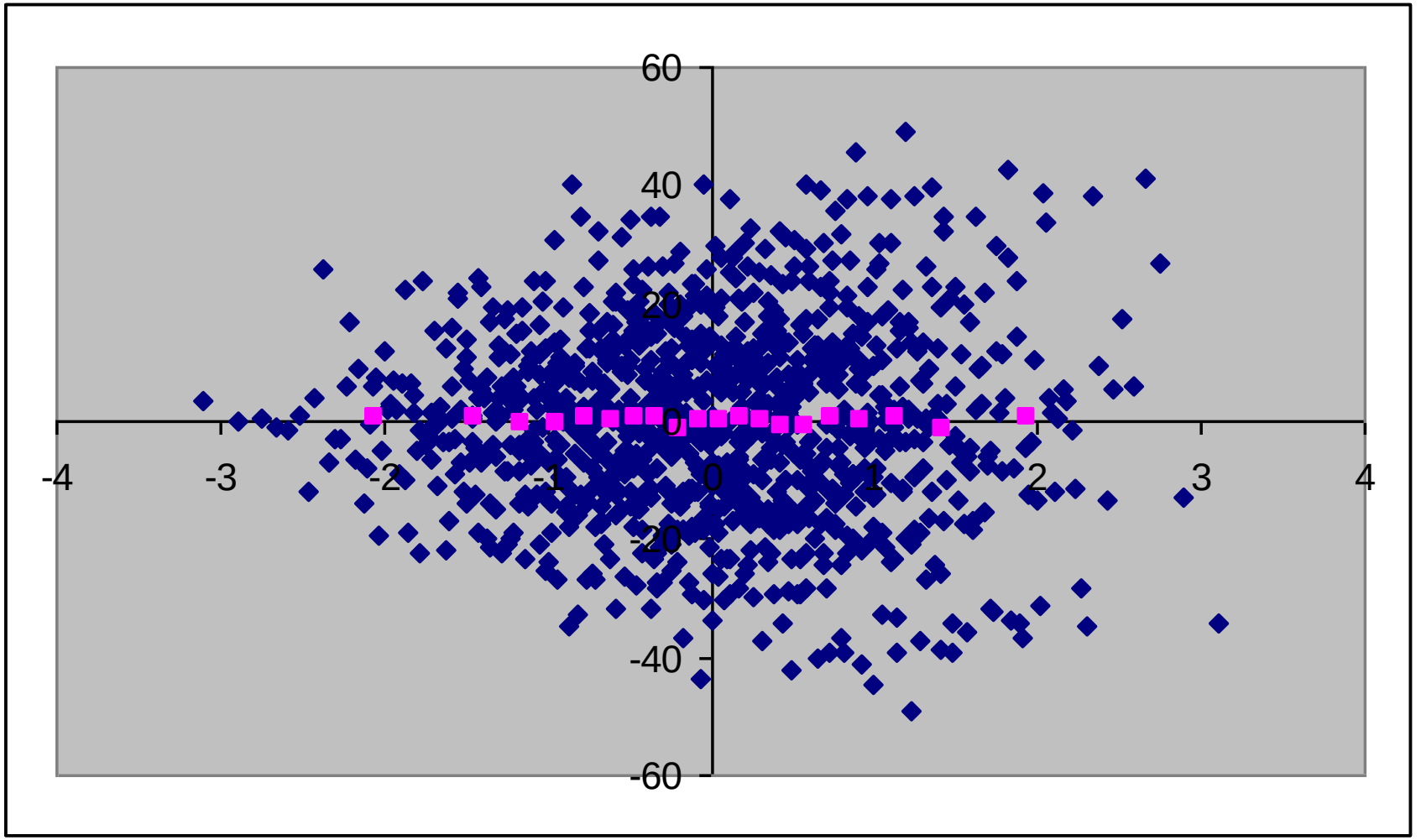
… which is also how additive models are defined

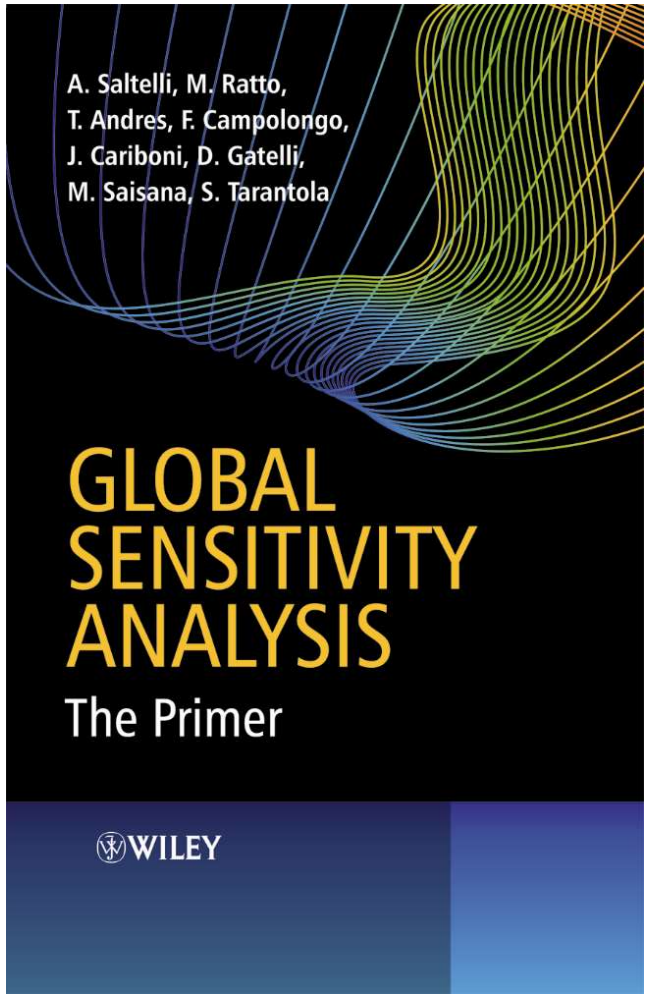
Non additive models

Is $S_i = 0$?



Is this factor non-important?





The theory of variance based sensitivity measures can cope … with another measure we do not describe today …

Plenty of code available in MATLAB, R and Phyton

Why using variance-based sensitivity analysis methods

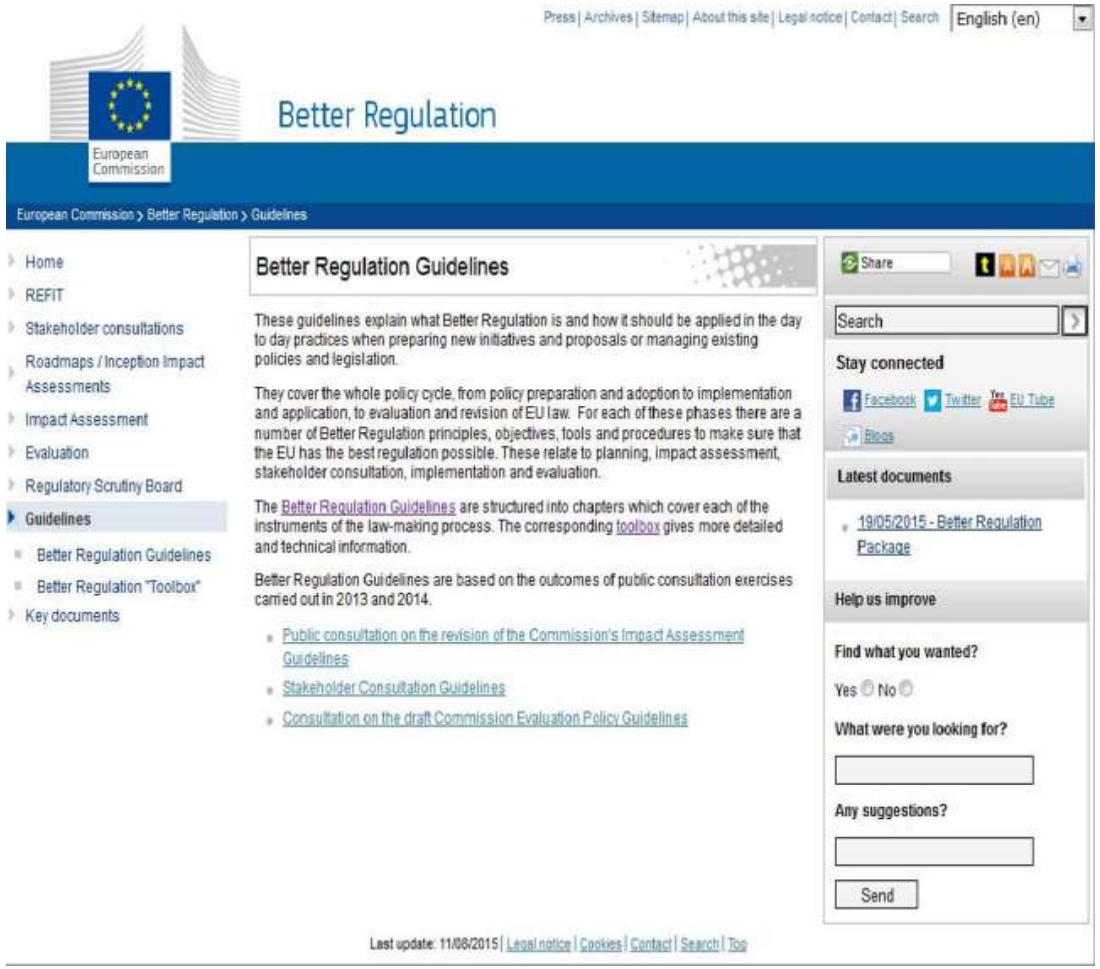
Advantages with variance based methods:

- graphic interpretation scatterplots
- statistical interpretation
- expressed plain English
- working with sets
- relation to settings such as factor fixing and factor prioritization

Secrets of sensitivity analysis

Why should one ever run a model just once?

EC impact assessment guidelines: sensitivity analysis & auditing



http://ec.europa.eu/smart-regulation/guidelines/docs/br_toolbox_en.pdf

First secret: The most important question is the question.

Or: sensitivity analysis is not “run” on a model but on a model once applied to a question

Second secret: Sensitivity analysis should not be used to hide assumptions [it often is]

Third secret: If sensitivity analysis shows that a question cannot be answered by the model one should find another question or model

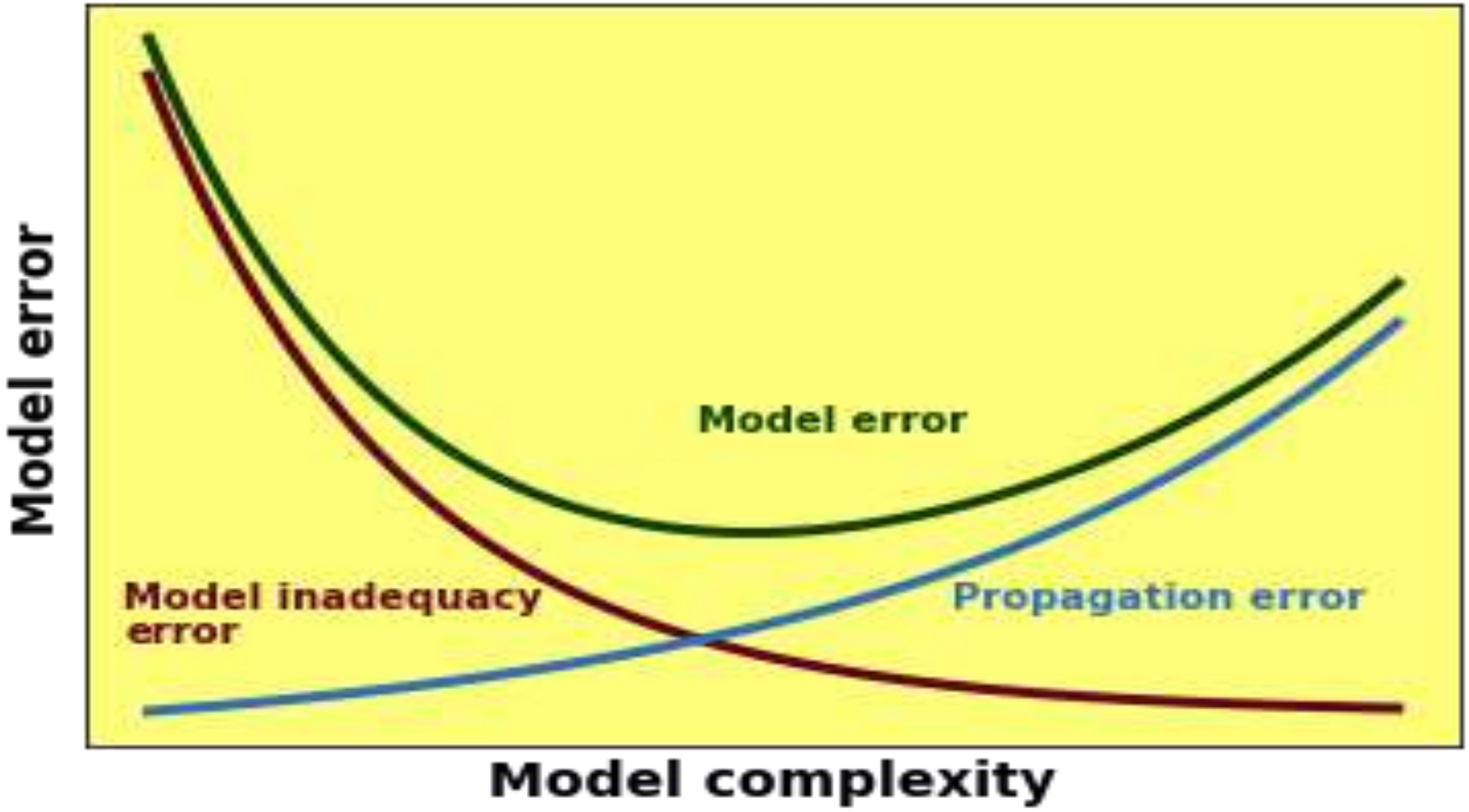
[Often the love for one's own model prevails]

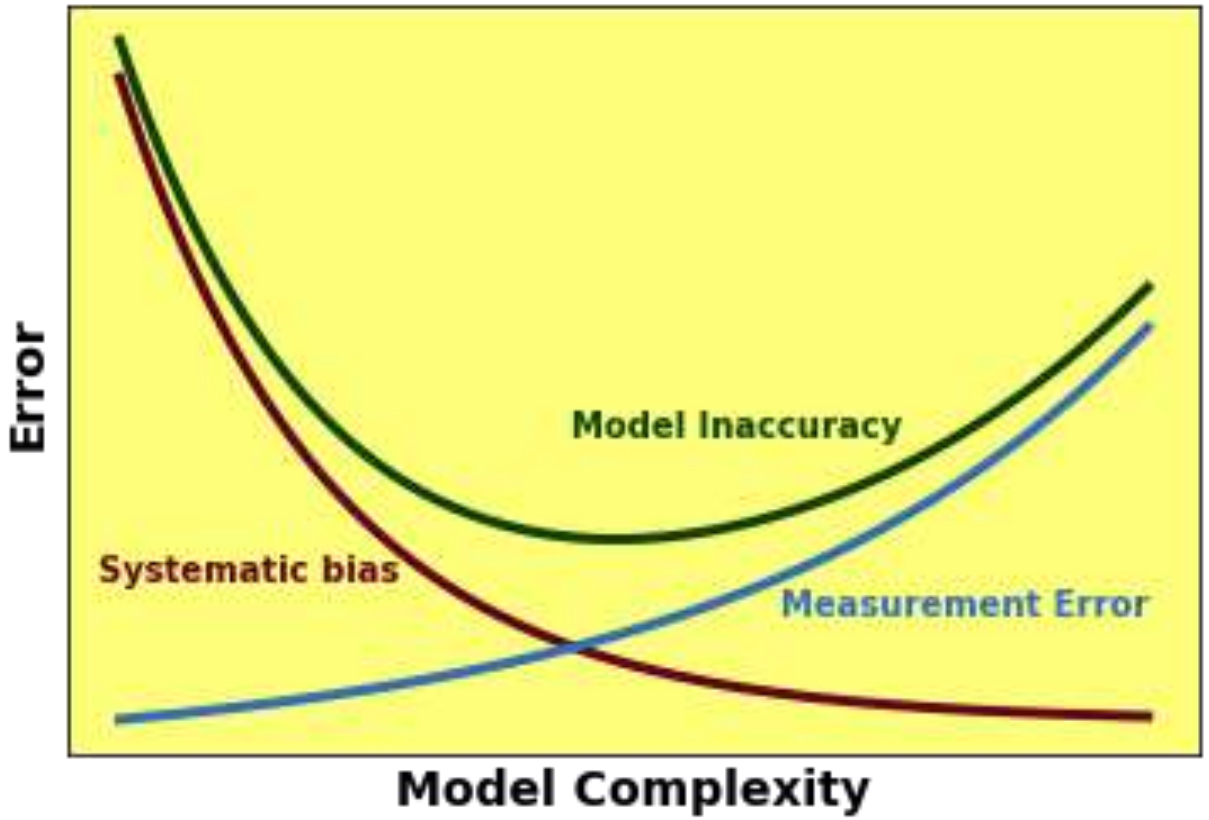
Fourth (badly kept) secret:

There is always one more bug!

=Lubarsky's Law of Cybernetic Entomology

Fifth secret: use SA to calibrate complexity





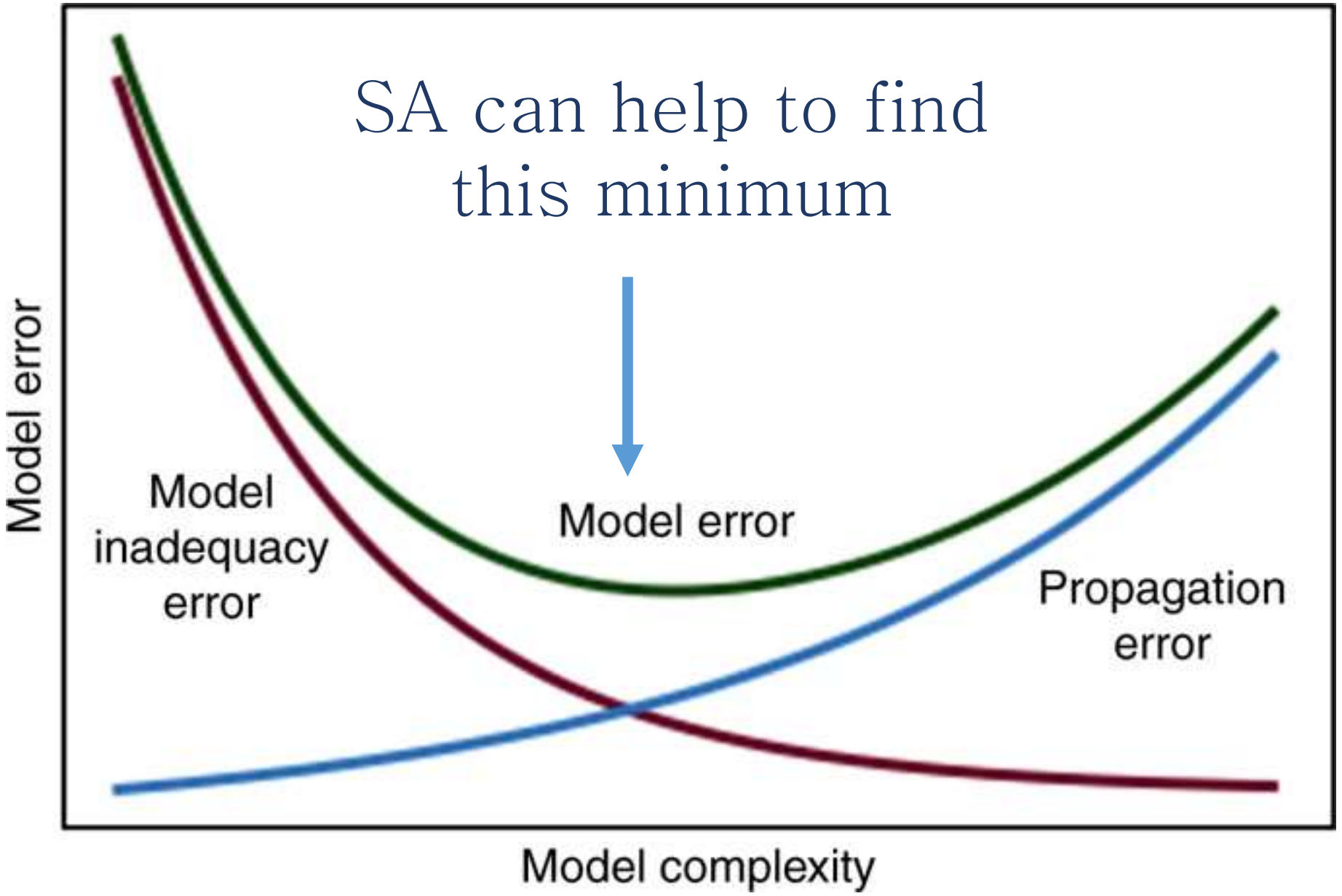
Presented as ‘Conjecture by O’Neill’

In M. G. Turner and R. H. Gardner, “Introduction to Models” in Landscape Ecology in Theory and Practice, New York, NY: Springer New York, 2015, pp. 63–95.

Lofti Aliasker Zadeh

Also known as Zadeh's principle of incompatibility, whereby as complexity increases "precision and significance (or relevance) become almost mutually exclusive characteristics"

L. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," IEEE Trans. Syst. Man. Cybern., vol. 3, no. 1, pp. 28–44, 1973.


SA can help to find
this minimum
Model error
Model
inadequacy
error
Model error
Propagation
error
Model complexity

nature COMMUNICATIONS

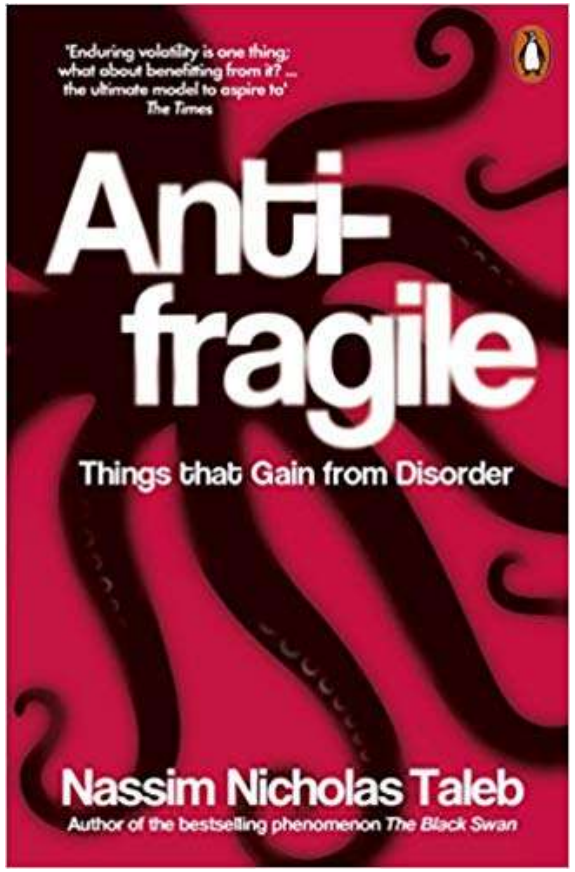
Comment | Open Access | Published: 27 August 2019

A short comment on statistical versus mathematical modelling

Andrea Saltelli

Sixth secret:

With SA it is easier to disprove than to prove; use SA ‘via negativa’:



Doing the right thing

or

Avoiding something wrong?

And of course please don’t run a sensitivity analysis where each factors has a 5% uncertainty

Why?

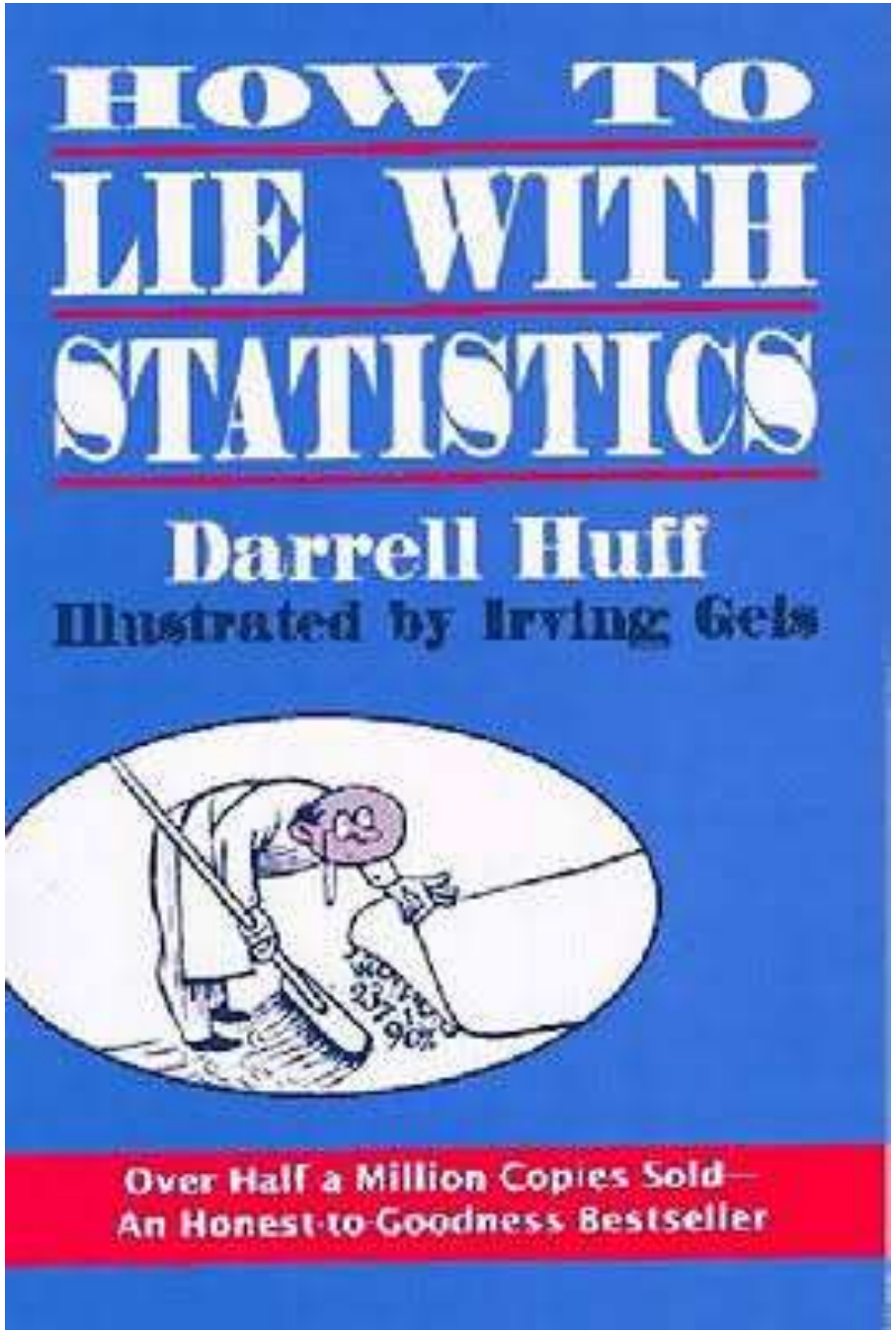
Environmental Modelling & Software

Volume 114, April 2019, Pages 29-39

Why so many published sensitivity analyses are false: A systematic review of sensitivity analysis practices

Andrea Saltelli [a, b], Ksenia Aleksankina [c], William Becker [d], Pamela Fennell [e], Federico Ferretti [d], Niels Holst [f], Sushan Li [g], Qiongli Wu [h]

Can we say that one lies with sensitivity analysis as one can lie with statistics?



Limit of SA: Often no SA (SA conflated with UA e.g. in economics) or one-factor-at-a-time SA

Why is OAT (one-factor-at-a-time) SA so bad?

Environmental Modelling & Software 25 (2010) 1508–1517

Contents lists available at ScienceDirect

Environmental Modelling & Software

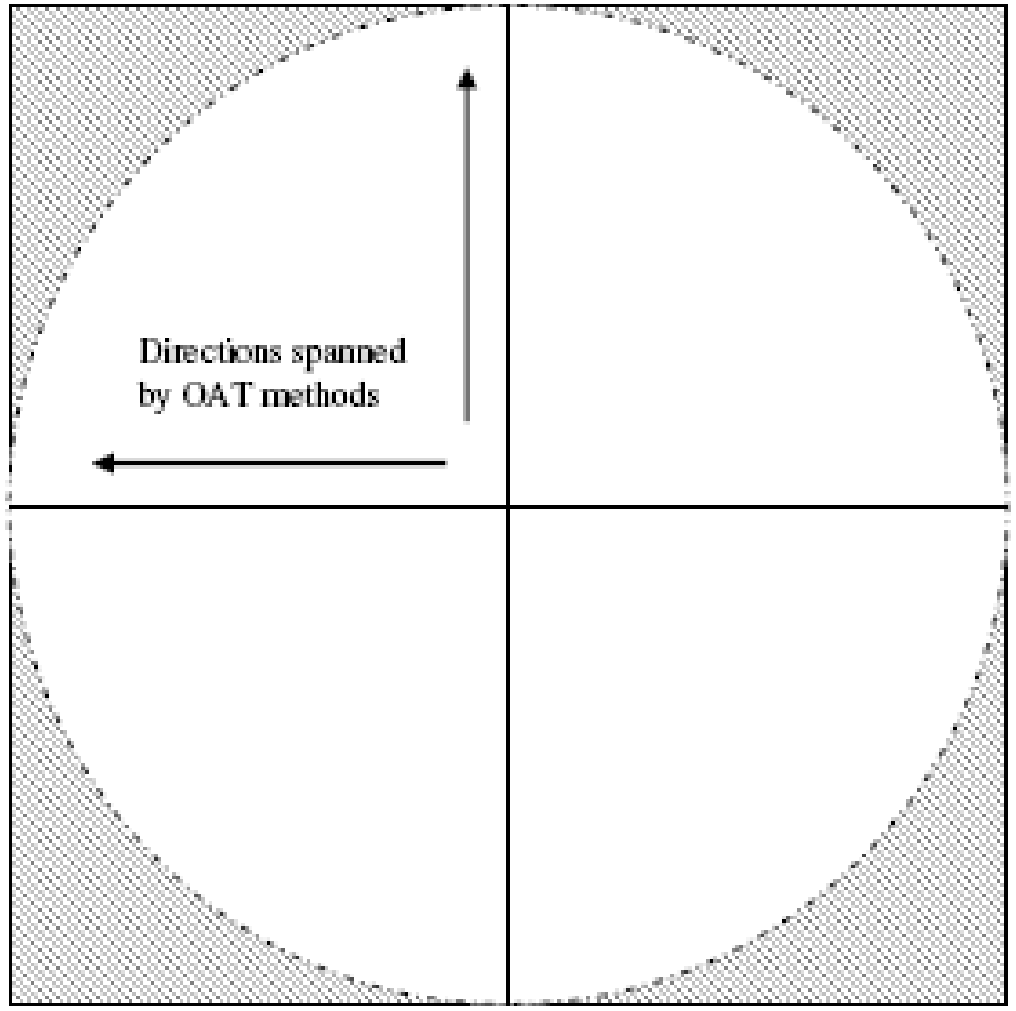
journal homepage: www.elsevier.com/locate/envsoft

How to avoid a perfunctory sensitivity analysis

Andrea Saltelli*, Paola Annoni

Joint Research Center, Institute for the Protection and Security of the Citizen, via E.Fermi, 2749, Ispra VA 21027, Italy

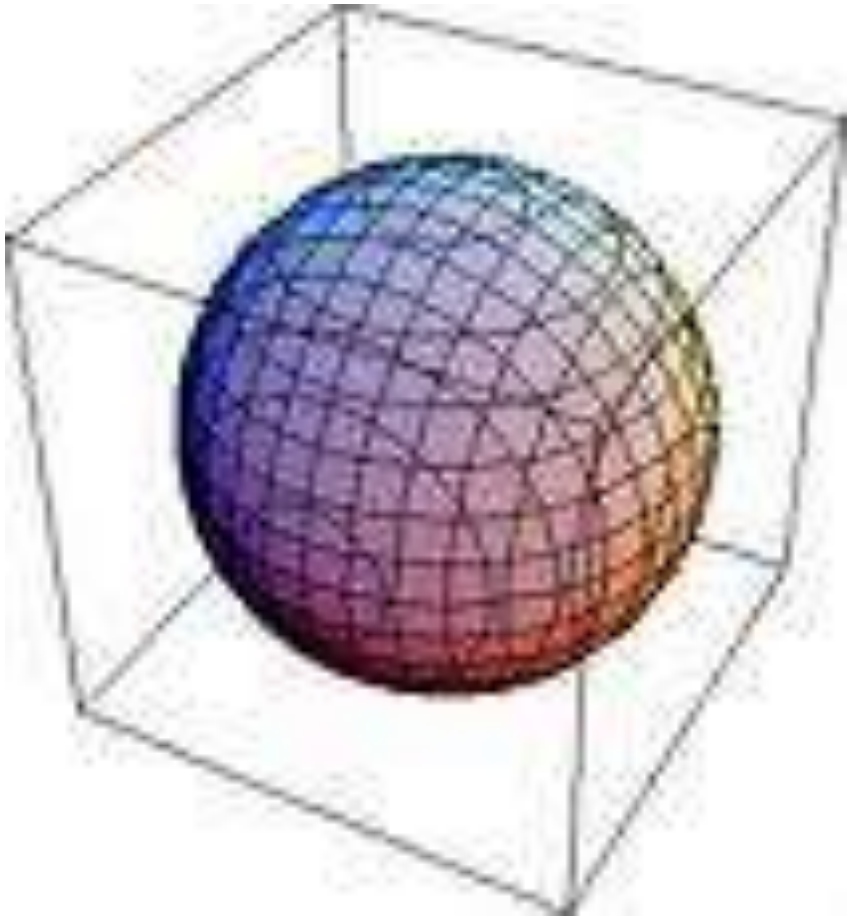
OAT in 2 dimensions



Area circle / area square =?

~ 3/4

OAT in 3 dimensions

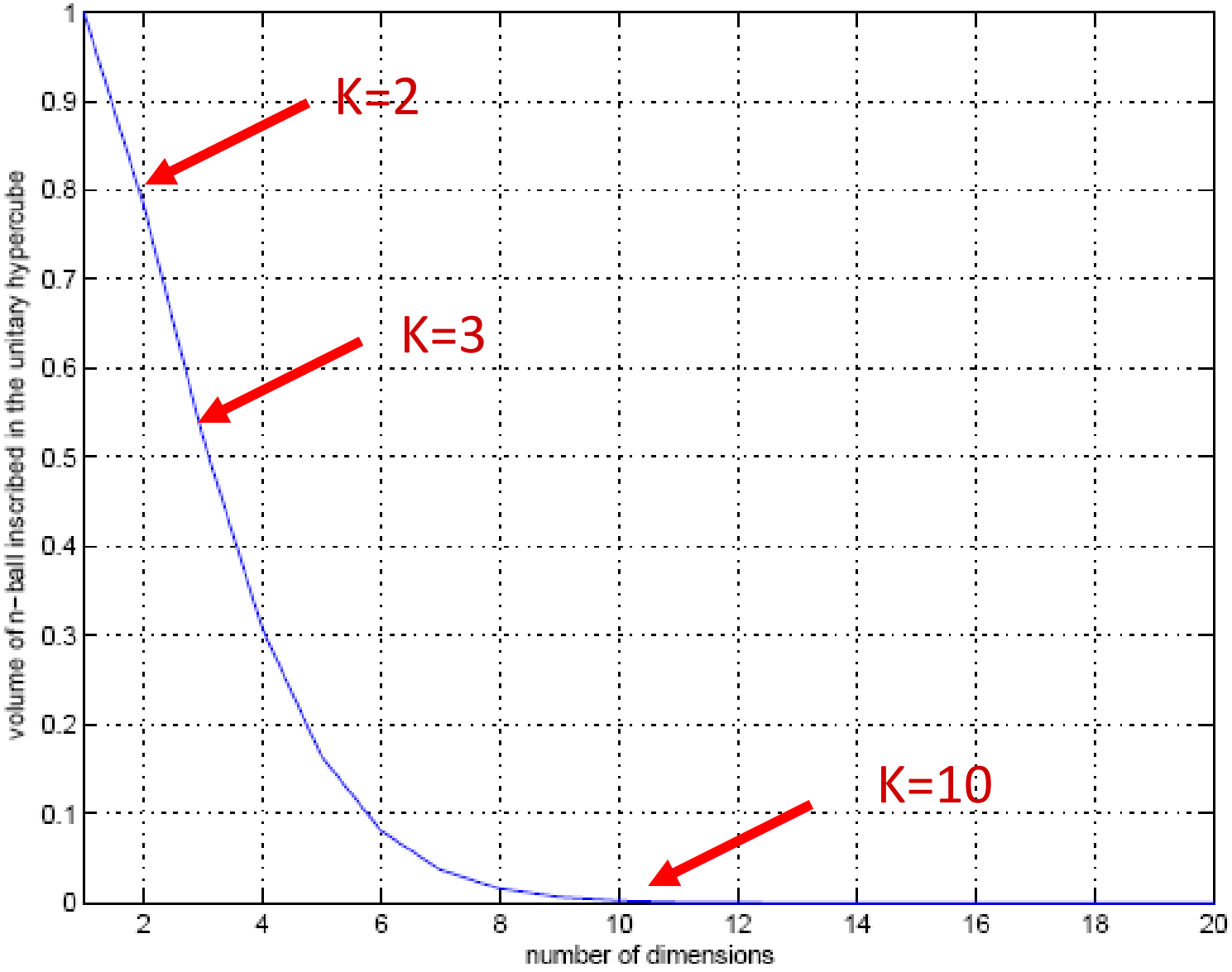


Volume sphere / volume cube =?

~ 1/2

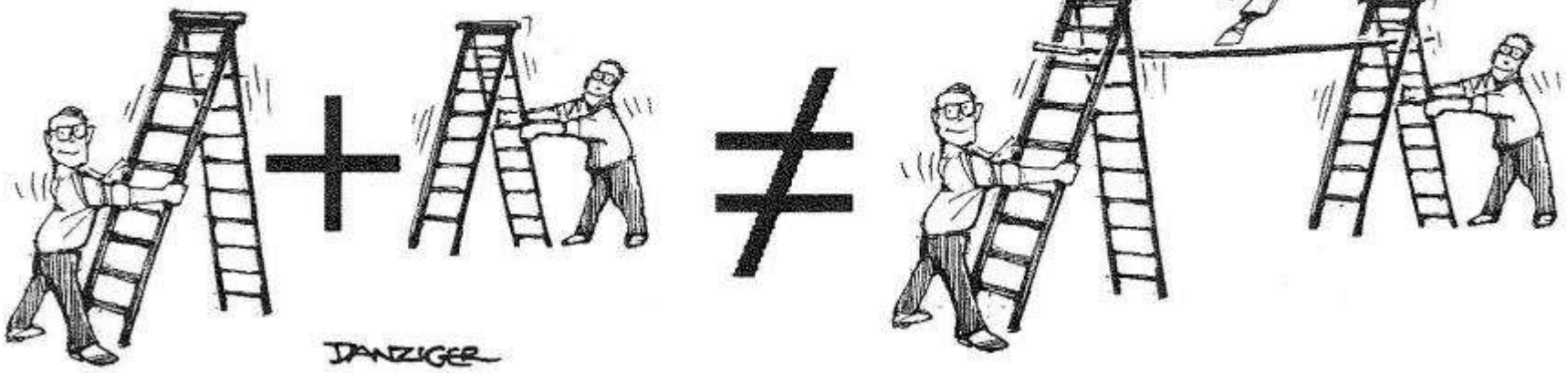
OAT in 10 dimensions; Volume hypersphere / volume ten dimensional hypercube =? ~ 0.0025

OAT in k dimensions

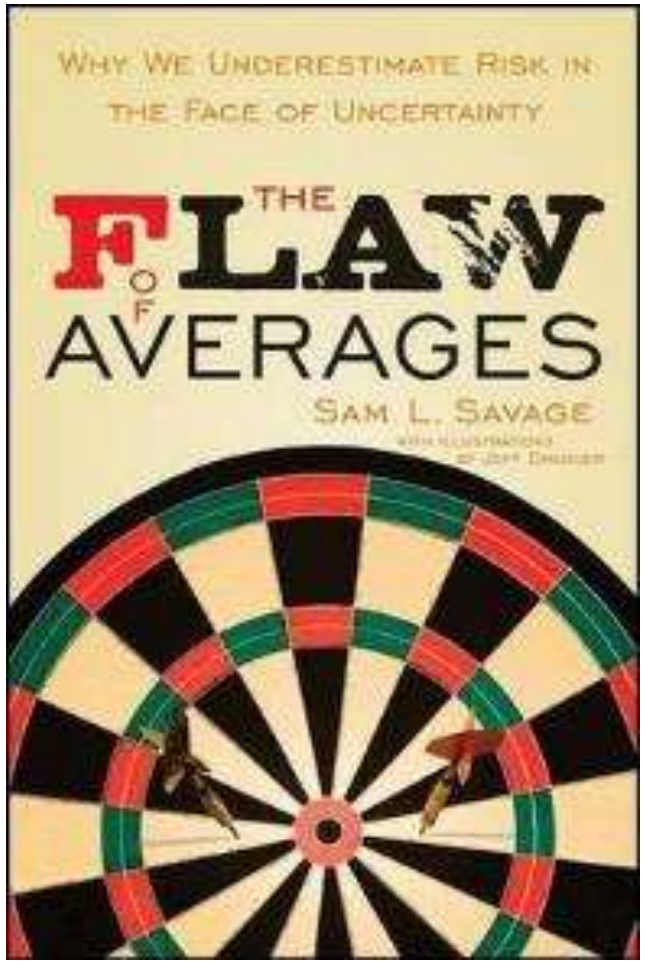


How would you test the scaffolding?

How coupled ladders are shaken in most of available literature



How to shake coupled ladders



Literature search in Scopus

Query: “sensitivity analysis” & “model/modelling” & “uncertainty”; years 2012–2017; journal articles; in English

➔ 6000 articles

- AgrBioSci (Agricultural and Biological Sciences)
- BiochemGenMBio (Biochemistry, Genetics and Molecular Biology)
- BusManAcc (Business, Management and Accounting)
- Chemi (Chemistry)
- ChemEng (Chemical Engineering)
- CompSci (Computer Science)
- DecSci (Decisional Science)
- EarthSci (Earth and Planetary Sciences)
- EconFin (Economy and Finance)
- Energy (Energy)
- Engineering (Engineering)
- EnvSci (Environmental Science)
- ImmunMicrobio (Immunology and Microbiology)
- MatSci (Material Science)
- Math (Math)
- Medicine (Medicine)
- PharTox (Pharmacology and Toxicology)
- PhysAstro (Physics and Astronomy)
- SocSci (Social Science)

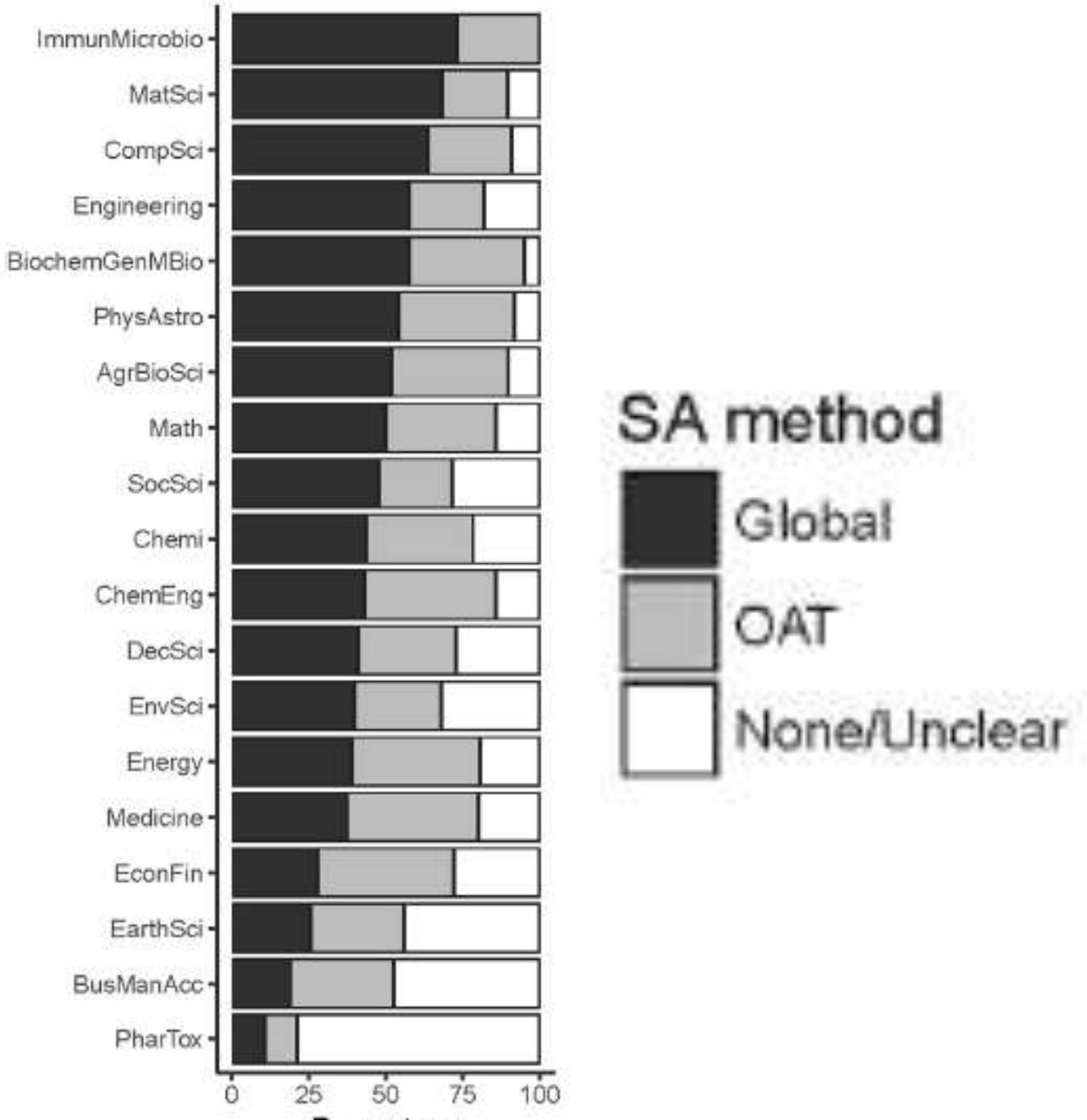
subject areas >100 articles

Taking the top twenty most-cited papers in each subject area:

➔ 324 articles, divided among authors

Cleansing manually irrelevant articles:

➔ 280 articles



Still many papers apply an OAT SA: 65%

What if the model is truly linear?

Linear	7%
Nonlinear	61%
Unclear	32%

Linear	7%
Nonlinear	61%
Unclear	32%

65% highly cited articles are OAT

Taking all unclear = linear ➔ still over 20% of papers wrong

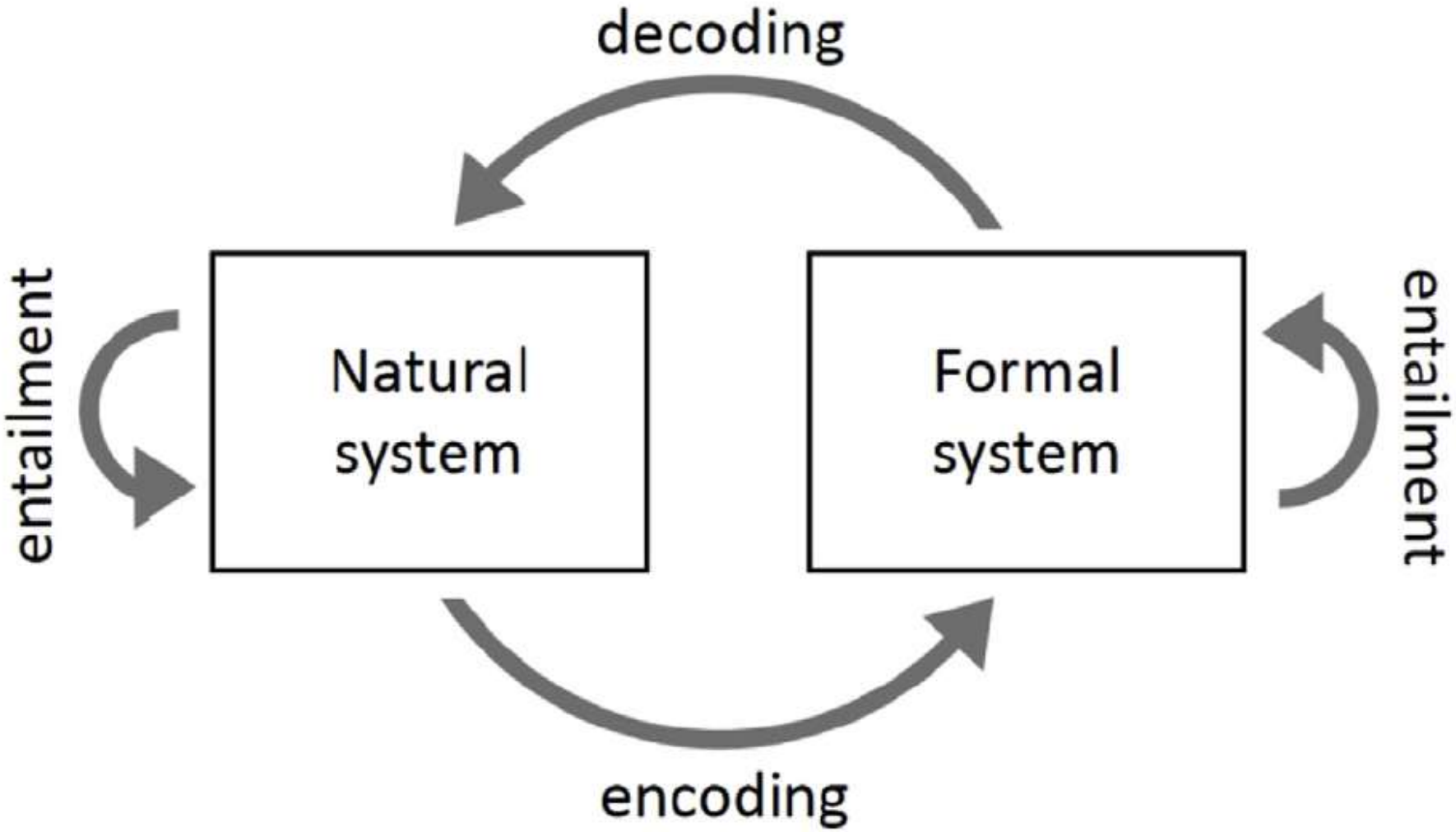
(OAT & non-linear model)

Why?

5. Discussion

5.1. Reasons for bad practice

Why? ➔ 1. Modelling as a craft



Why? ➔ 2. Each discipline going about modelling on its own separate way; pockets of SA practitioners (out of our 280 papers, 35 were methodological, of which 24 suggest global SA)

Why? ➔ 3. Mathematical modelling is not a discipline

… mathematical modelling cannot do this:

Wasserstein, R.L. and Lazar, N.A., 2016. 'The ASA's statement on p-values: context, process, and purpose', The American Statistician, Volume 70, 2016 - Issue 2, Pages 129-133.

Need for a more structured, generalized and standardized approach to verification

Padilla, J. J., Diallo, S. Y., Lynch, C. J., & Gore, R. (2018). Observations on the practice and profession of modeling and simulation: A survey approach. SIMULATION, 94(6), 493–506.

Why? ➔ 4. Good practices require training in statistics

Why? ➔ 5. More time is needed; though mature global sensitivity analysis methods around for more than 25 years researchers tend to emulate methods found in highly cited papers assuming that they are best practice

Why? ➔ 6. Strategical reasons: global SA is bad if one wants to play the uncertainty game, inflating or deflating uncertainties instrumentally

Solutions? 1. Statistics as a discipline takes responsibility for statistical methods for model validation and verification

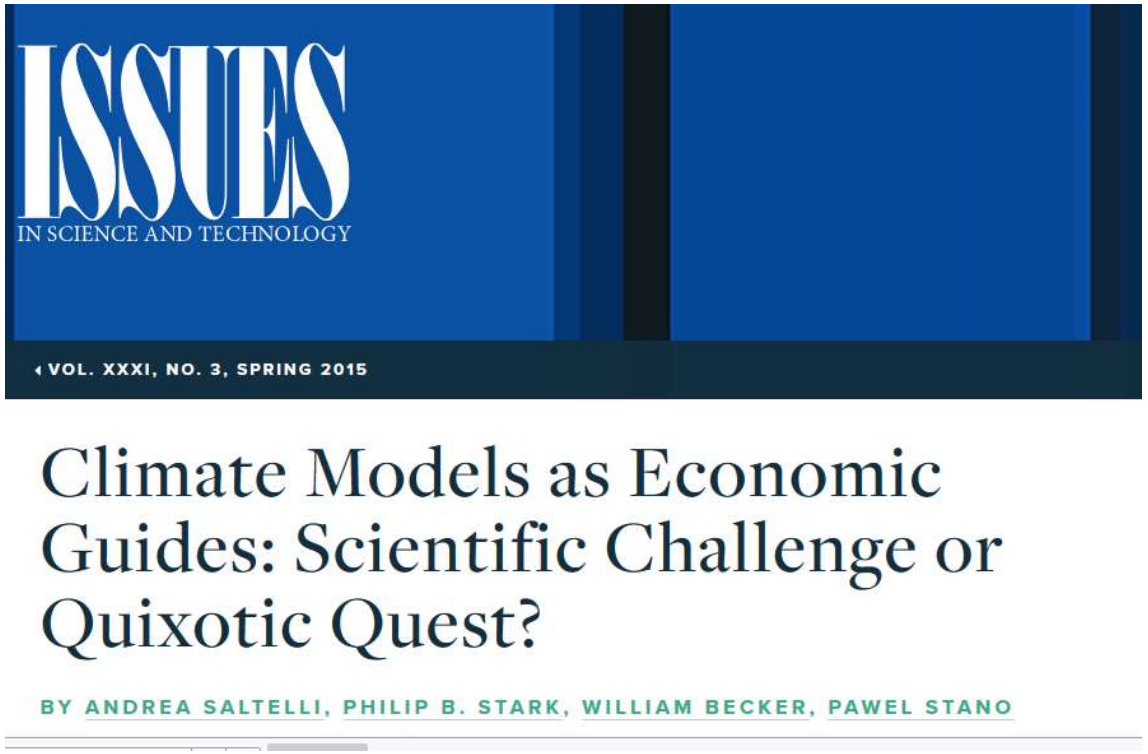
Example: who can authoritatively suggest to modellers not to overinterpret results from multi-model ensembles?

ISSUES
IN SCIENCE AND TECHNOLOGY

VOL. XXXI, NO. 3, SPRING 2015

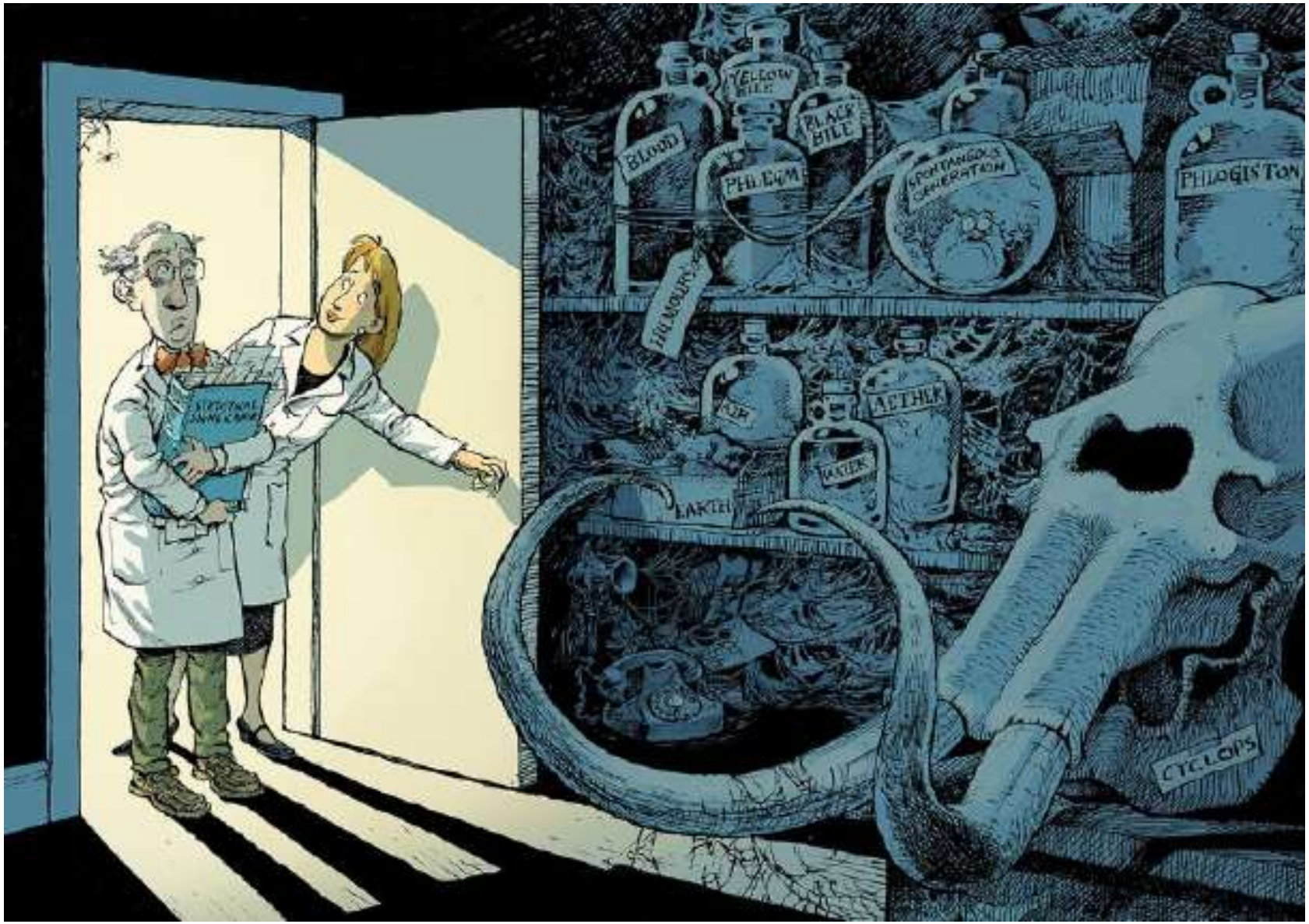
Climate Models as Economic Guides: Scientific Challenge or Quixotic Quest?

BY ANDREA SALTELLI, PHILIP B. STARK, WILLIAM BECKER, PAWEL STANO



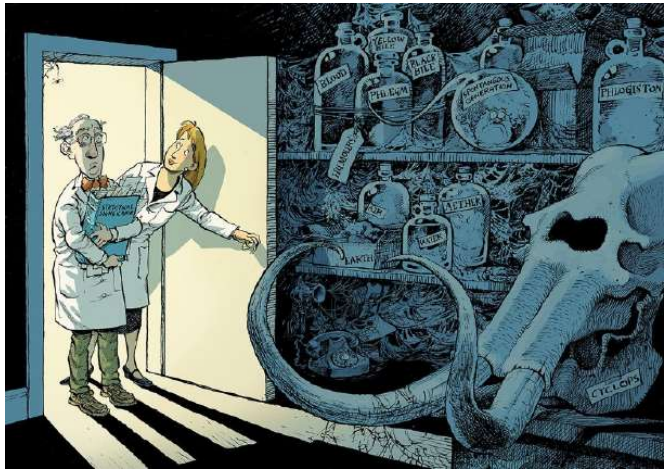
A plea against audacious risk or cost-benefit analysis running over centennial time scales; example: crime rate as modified by climate change at US county level in 2100

Solutions? 2. Learn from what happens in statistics where the p-test crisis is being tackled head on



Throw away the concept of statistical significance?

nature
International journal of science



COMMENT · 20 MARCH 2019

Scientists rise up against statistical significance

Valentin Amrhein, Sander Greenland, Blake McShane and more than 800 signatories call for an end to hyped claims and the dismissal of possibly crucial effects.

Valentin Amrhein ✉, Sander Greenland & Blake McShane

See the discussion on the blog of Andrew Gelman https://statmodeling.stat.columbia.edu/

SIGNIFICANCE

Business Culture Politics

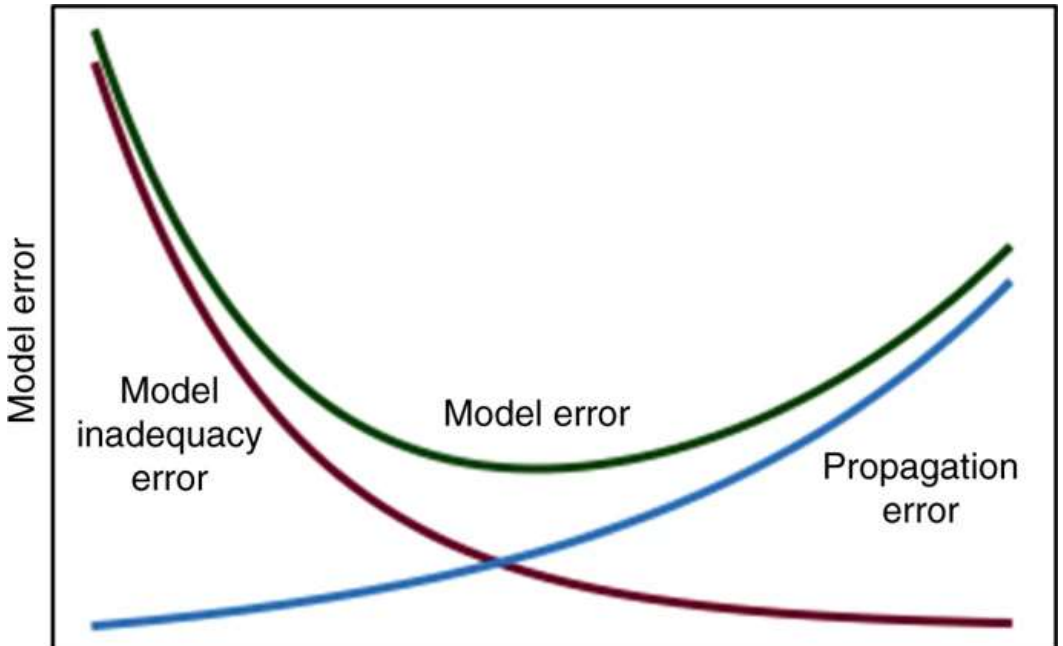
IN PRACTICE

Cargo-cult statistics and scientific crisis

The mechanical, ritualistic application of statistics is contributing to a crisis in science. Education, software and peer review have encouraged poor practice – and it is time for statisticians to fight back. By **Philip B. Stark** and **Andrea Saltelli**

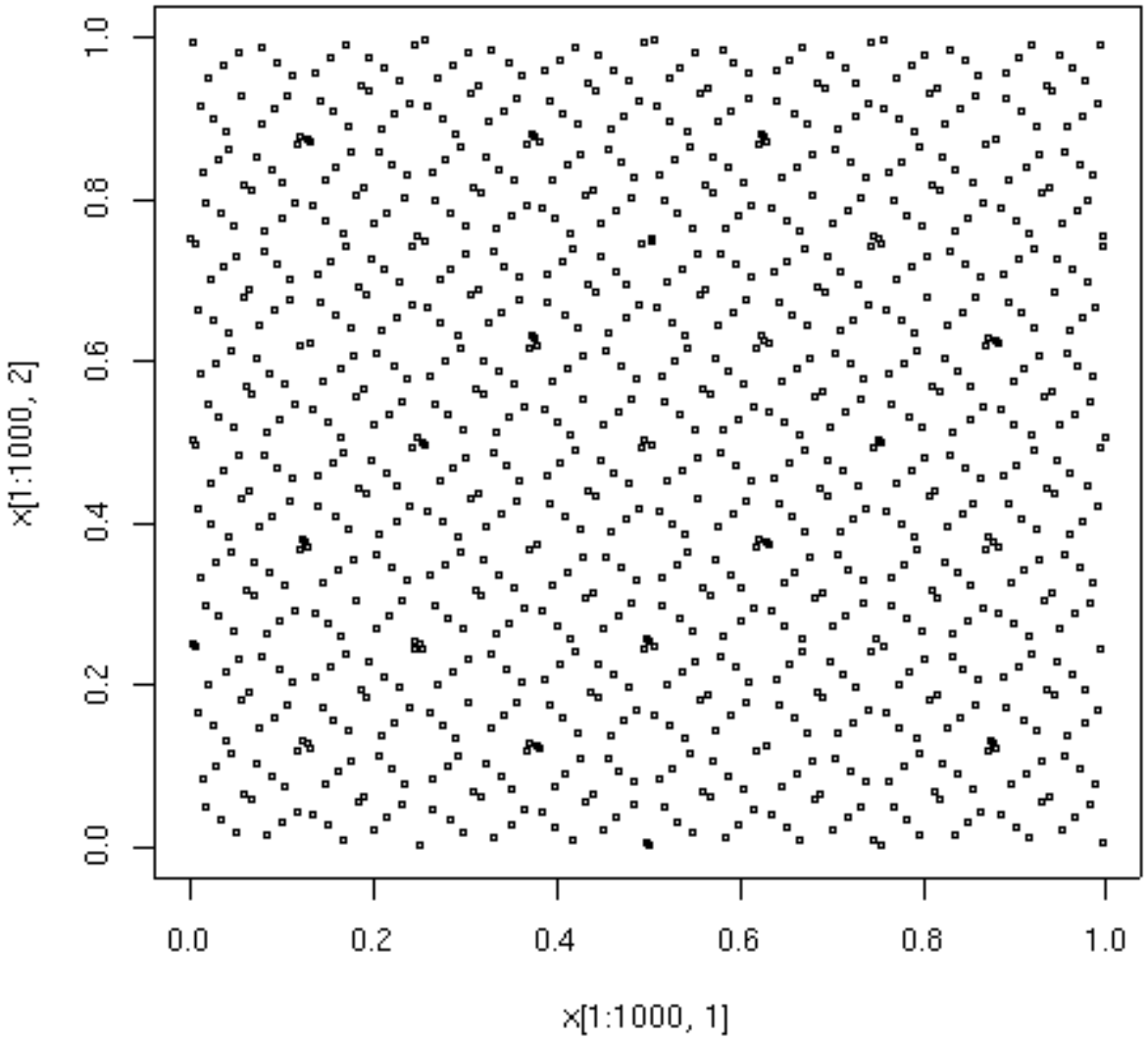
Lessons for sensitivity analysis

- Global SA
- UA and SA coupled
- Purpose- & context-specific
- The map is not the territory
- Memento

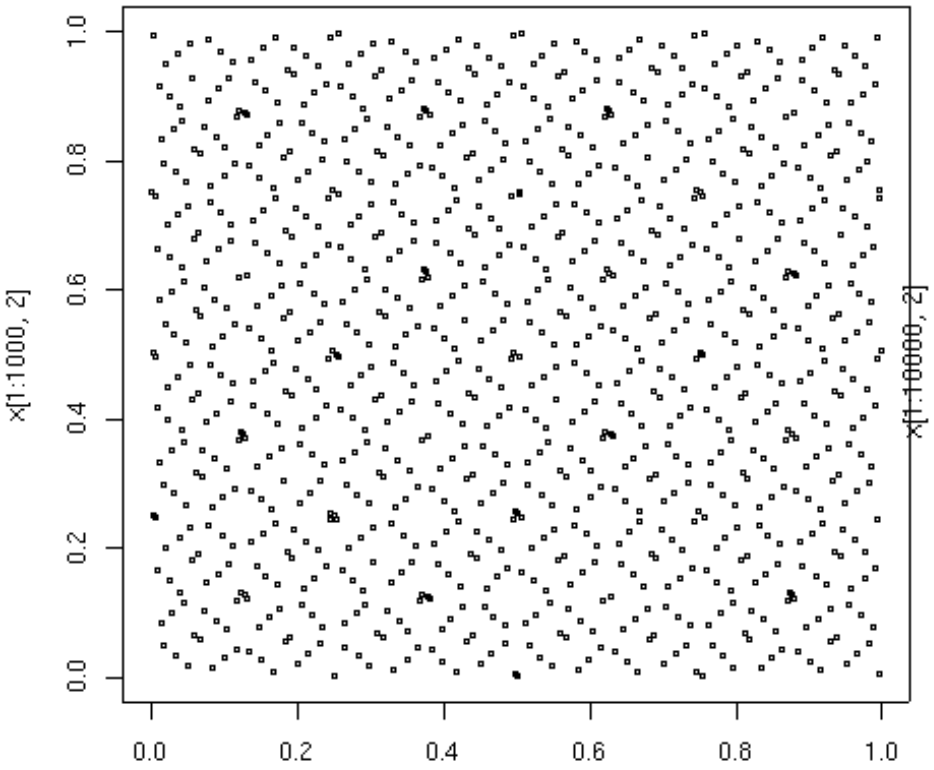


How to generate the random sample?

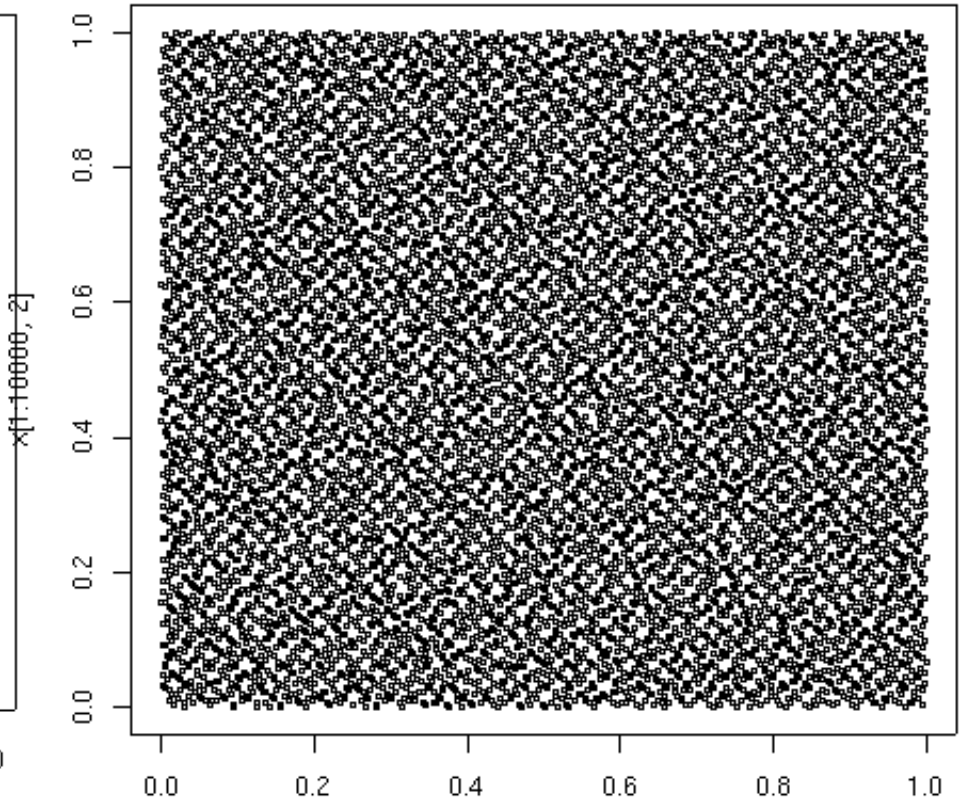
Quasi random sequences developed by I.M. Sobol’



An LP_τ sequence

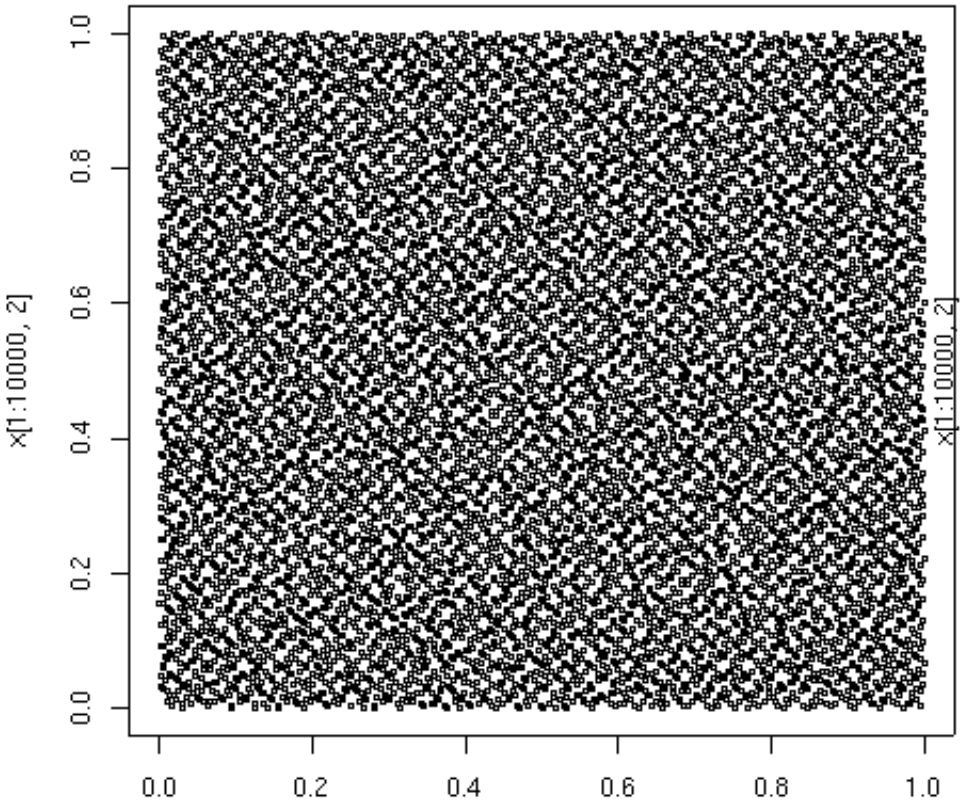


X1,X2 plane, 1000 Sobol' points

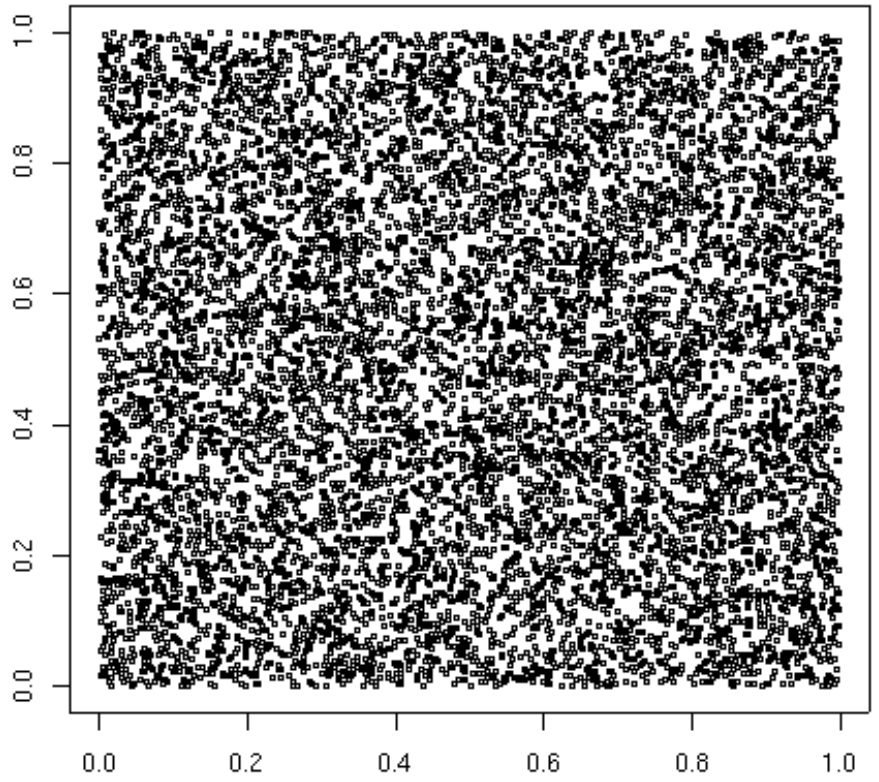


X1,X2 plane, 10000 Sobol' points

Sobol' sequences of quasi-random points



X1,X2 plane, 10000 Sobol' points



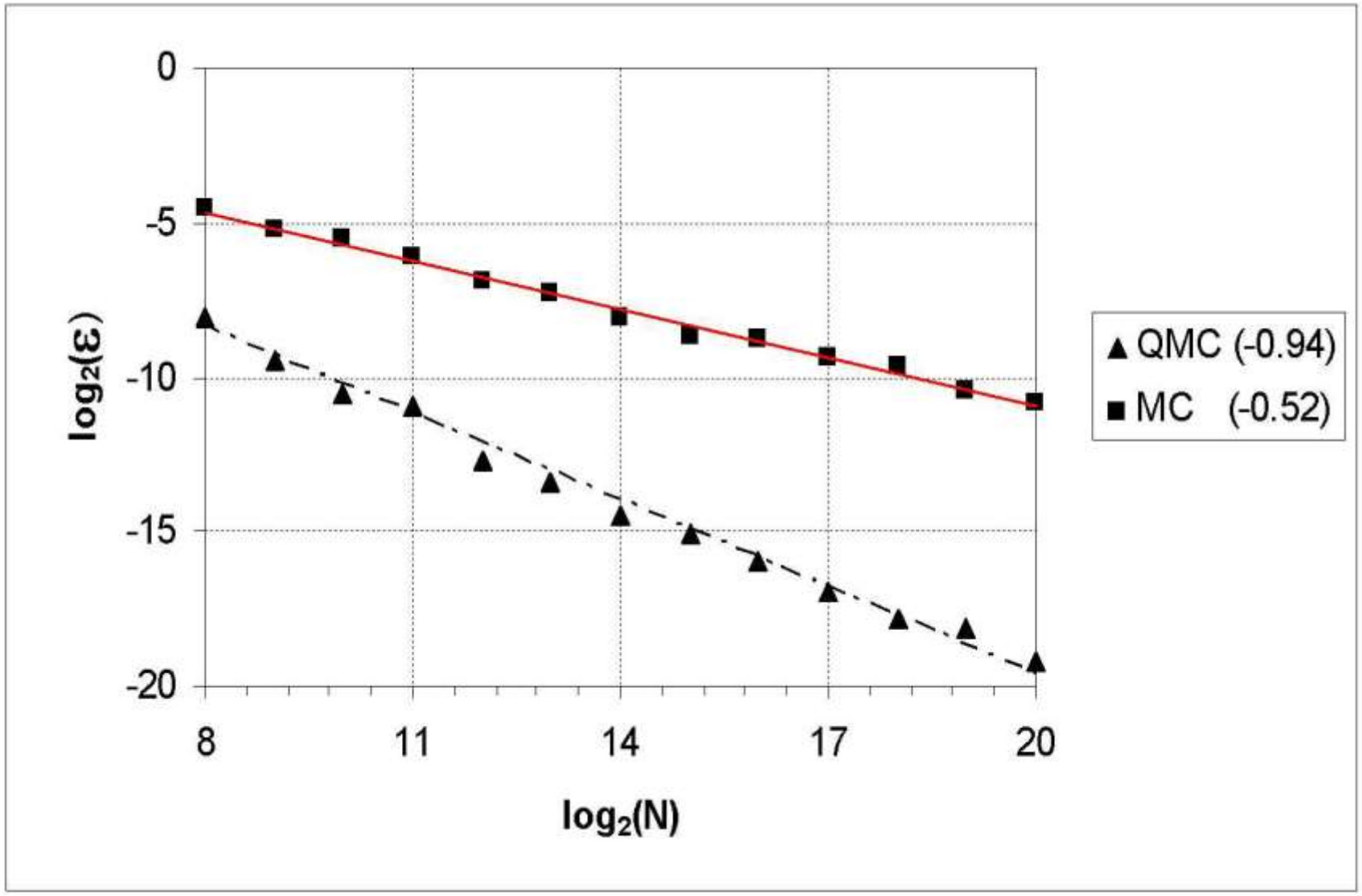
X1,X2 plane, 10000 random points

Sobol' sequences of quasi-random points against random points

Why quasi-random: they have faster convergence

Sergei Kucherenko,
Imperial College London

Kucherenko S., Feil B., Shah N., Mauntz W. The identification of model effective dimensions using global sensitivity analysis Reliability Engineering and System Safety 96 (2011) 440–449.



$$\varepsilon = \left(\frac{1}{K} \sum_{k=1}^{K} (I[f] - I_k[f])^2 \right)^{1/2}$$

$$\sum_{i=1}^{n} (-1)^i \prod_{j=1}^{i} x_j$$

Error=numeric-versus-analytic value the integral of the function (for n=360) over its dominion.

Root mean square error over K=50 different trials.

The End

@andreasaltelli