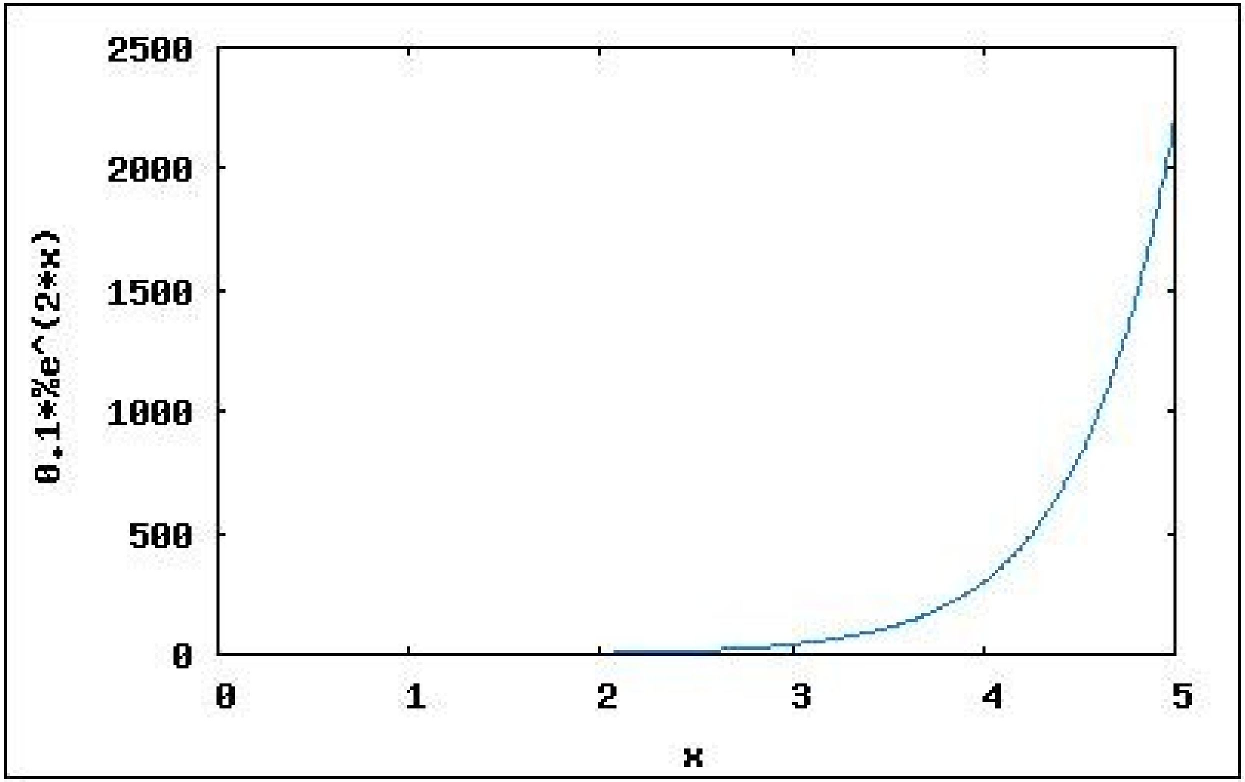
Chaos and complexity

$$dN = rN(t)\,dt, r = const\ .$$

$$N(t) = rN(0)^{rt}$$



Answer

We assume that the growth rate decreases as the population increases:

$r' = r\,(1 - N/K)$

r and K are constants. K is the “carrying capacity of the system.

$$dN = r\left(1 - N/K\right)N\,dt$$

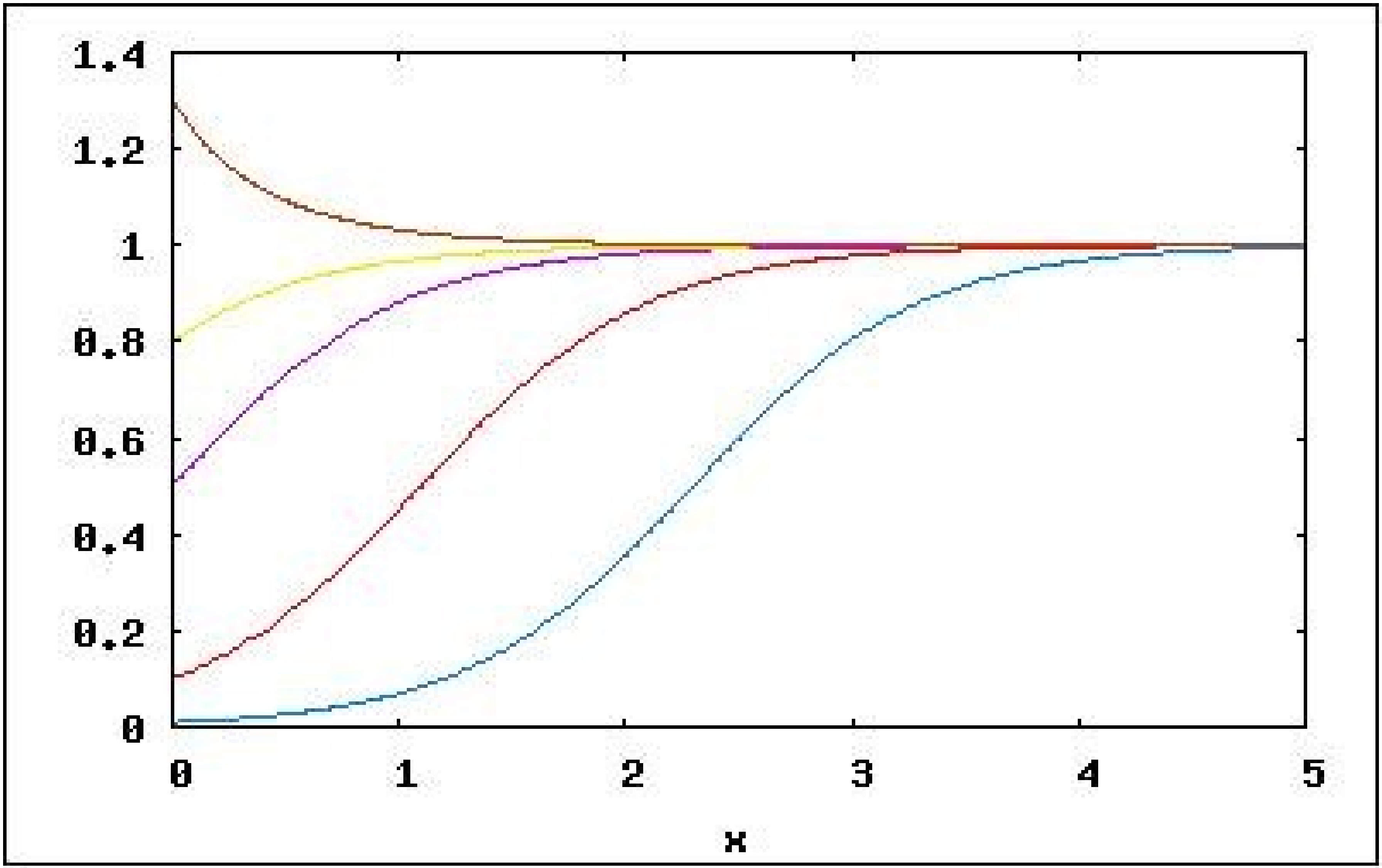
Solution of the continuous logistic equation

$$dy/dx = ry(1-y)$$

$$y(x) = \frac{1}{1-\left(1-\frac{1}{y_0}\right)e^{-rt}}$$

r=2

Initial values 0.1, 0.5, 0.8, 1.3



Donelle Meadows, Dennis Meadows and Jørgen Randers: *Limits to Groth* (1972)

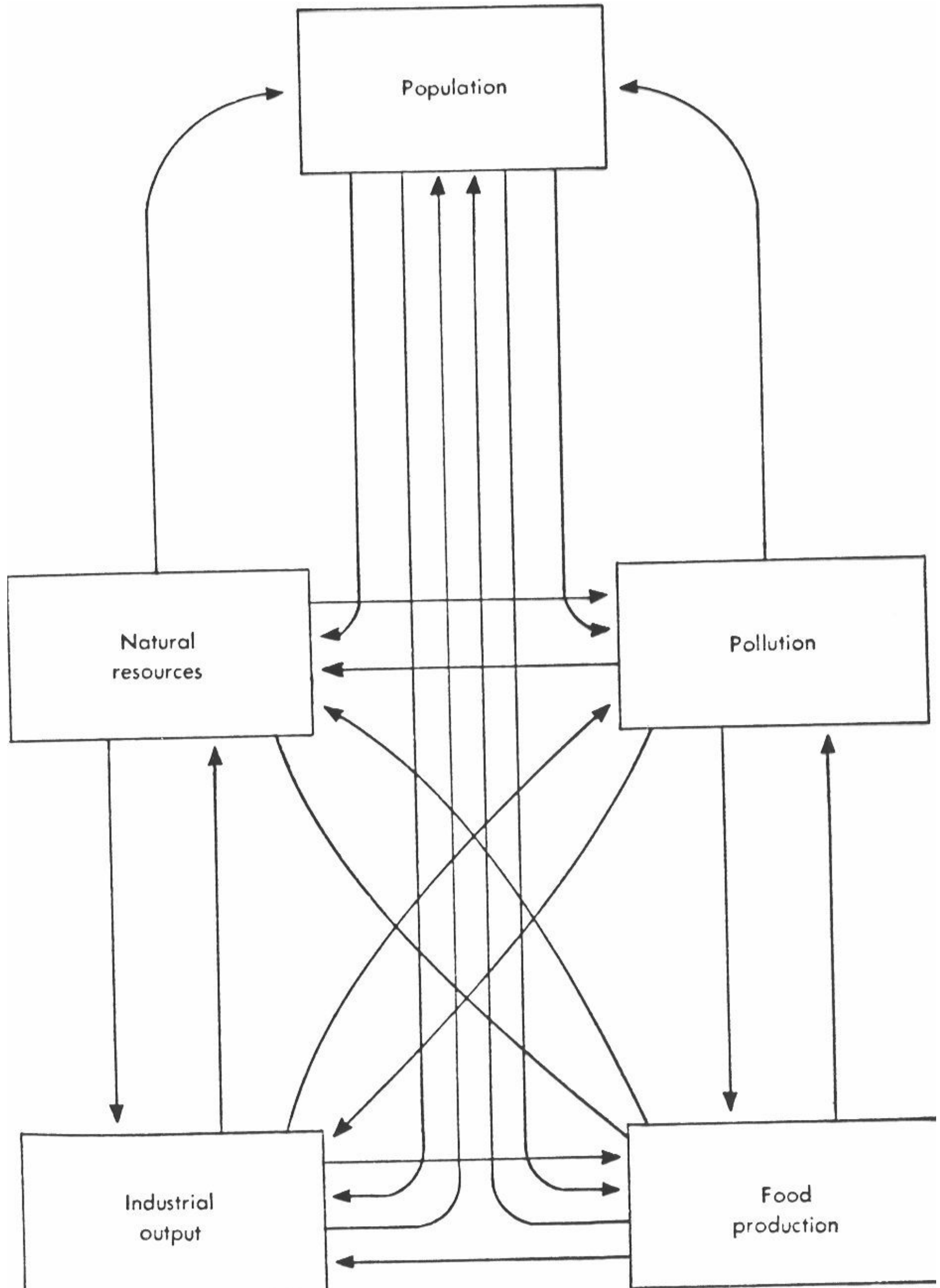
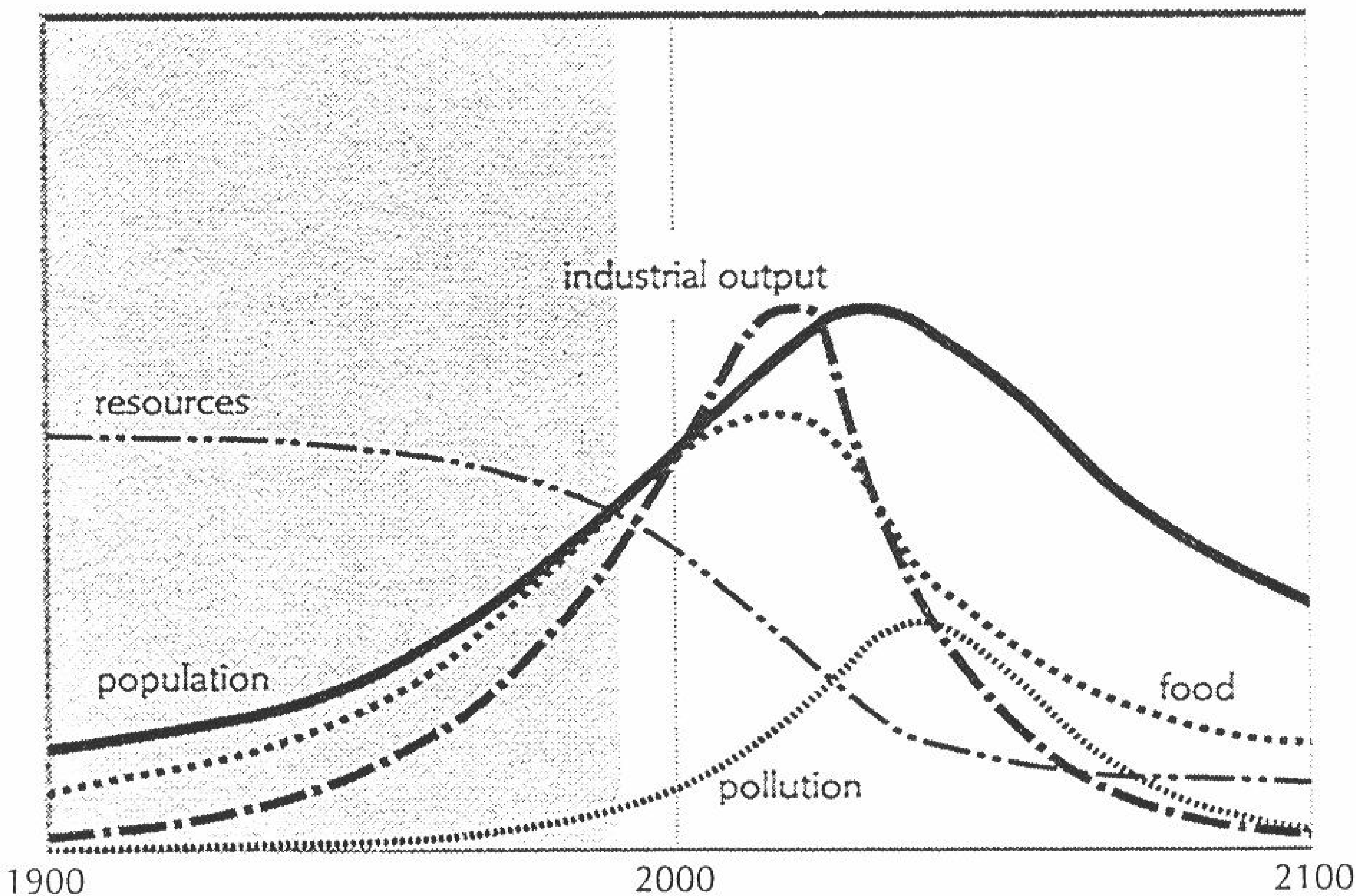
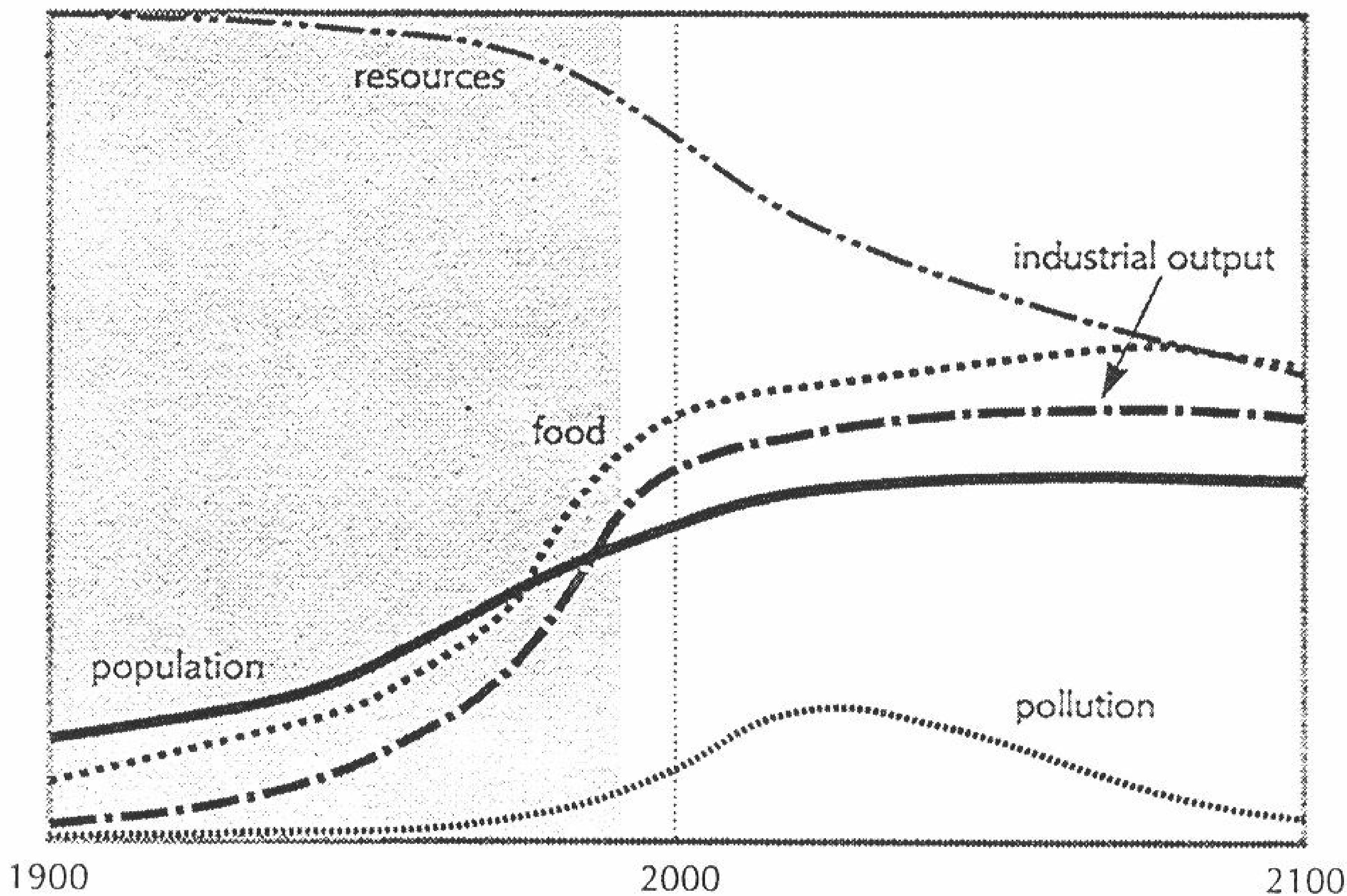


FIGURE 7.4 Interactions in the World II model


industrial output
resources
population
food
pollution
1900
2000
2100



resources
industrial output
food
population
pollution
1900
2000
2100

Discrete case

N is a natural number

$$N_{i+1} = N_i + rN_i$$

$$r' = r\left(1 - N_i / K\right)$$

$$N_{i+1} = N_i + r\left(1 - N_i / K\right) N_i$$

$$n_{i+1} = n_i + r\left(1 - n_i\right)n_i$$

Standard form of the logistic equation:

$$x_{i+1} = ax_i\left(1 - x_i\right)$$

$$x_{i+1} = a x_i \left(1 - x_i\right)$$

a = 2

0.1
0.18
0.2952
0.41611392
0.4859262511644672
0.4996038591874287
0.4999996861449132
0.4999999999999803
0.5
0.5
0.5

a = 3

0.1
0.2700000000000001
0.5913000000000002
0.7249929299999999
0 .5981345443500454
0 .7211088336156269
0.6033326651091411
0.7179670896552621
0.6074710434816448
0.7153499244388992
0.6108732301324813

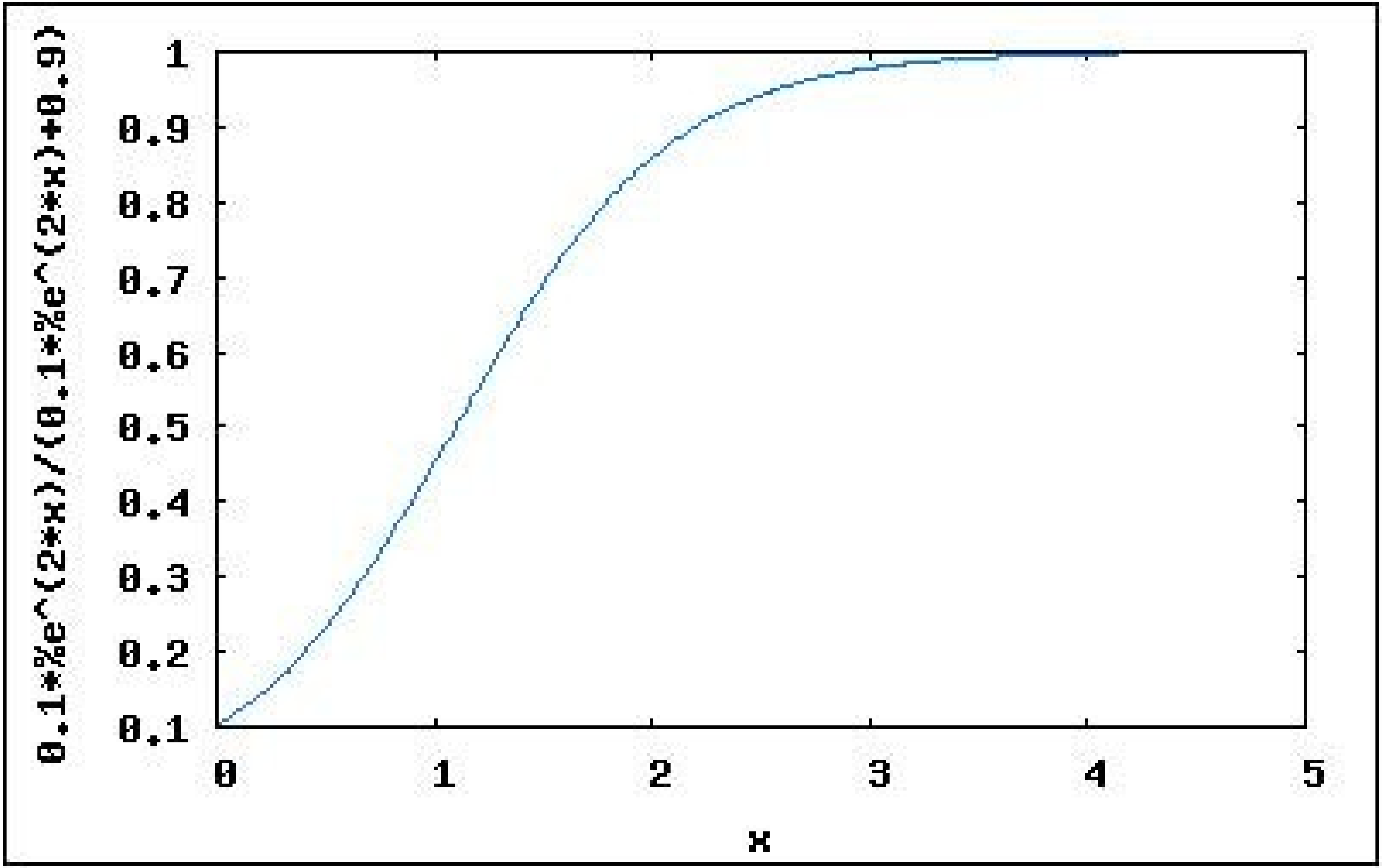
a = 3.5

0.1
0.31500000000000001
0.75521250000000001
0.6470330294531249
0.7993345088744278
0.5613959812891678
0.8618068671853906
0.416835268001226
0.8507926957305024
0.444305696177445
0.864143505826023

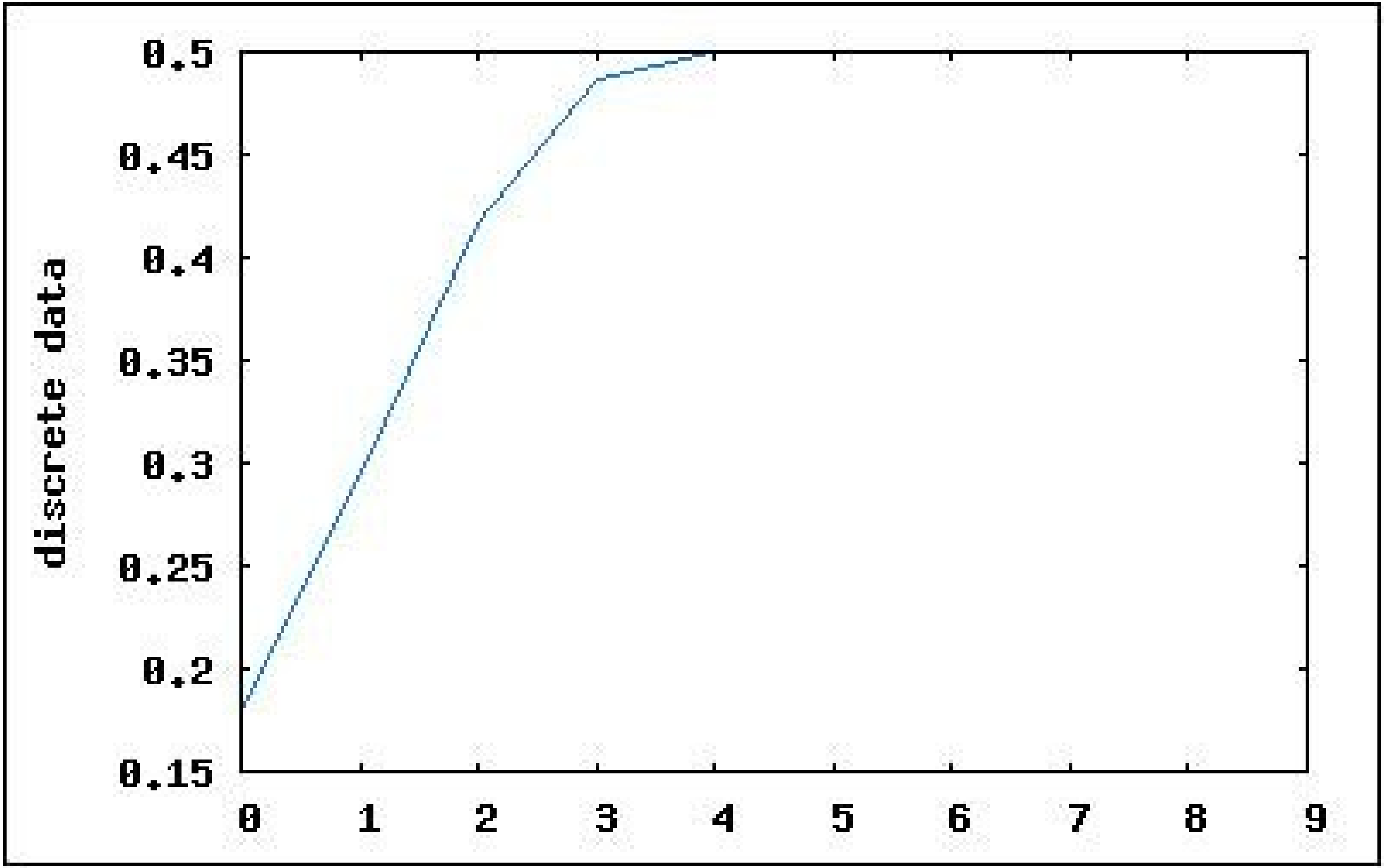
a = 4

0.1
0.36
0.9216
0.28901376000000001
0.8219392261226498
0.5854205387341974
0.970813326249438
0.1133392473037612
0.4019738492975123
0.9615634951138128
0.1478365599132853

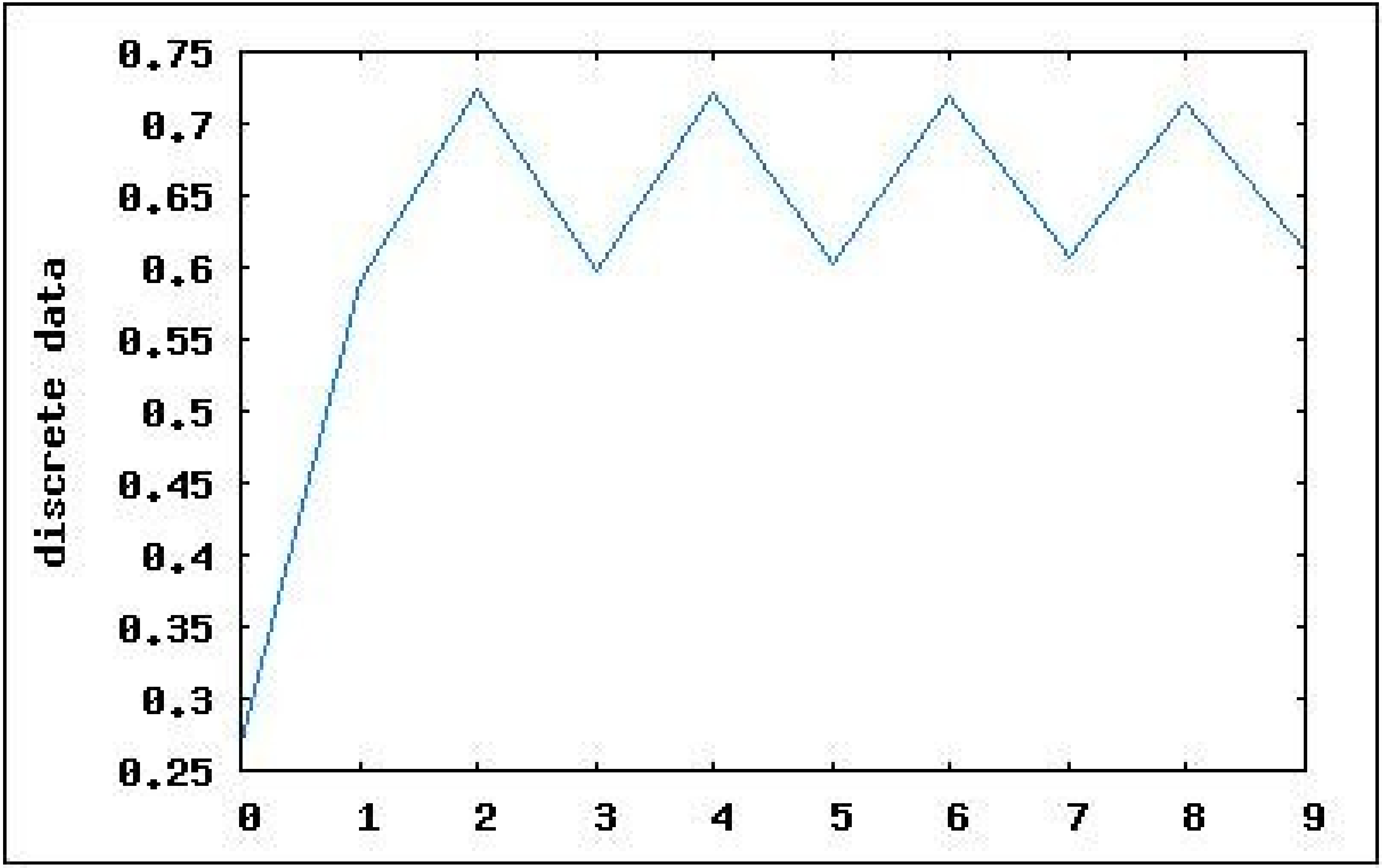
r=2



a=2 (r=1)

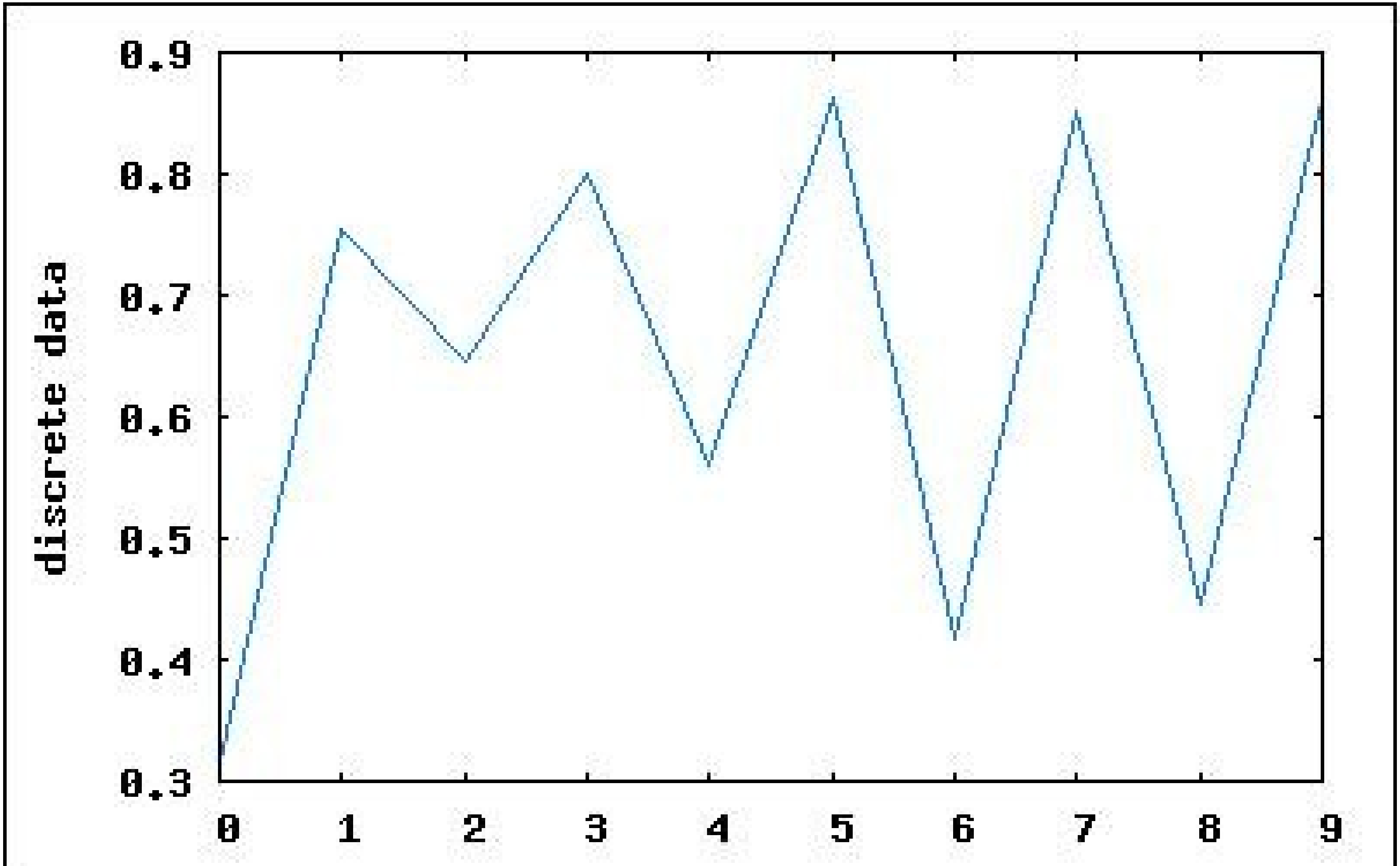


a=3 (r=2)

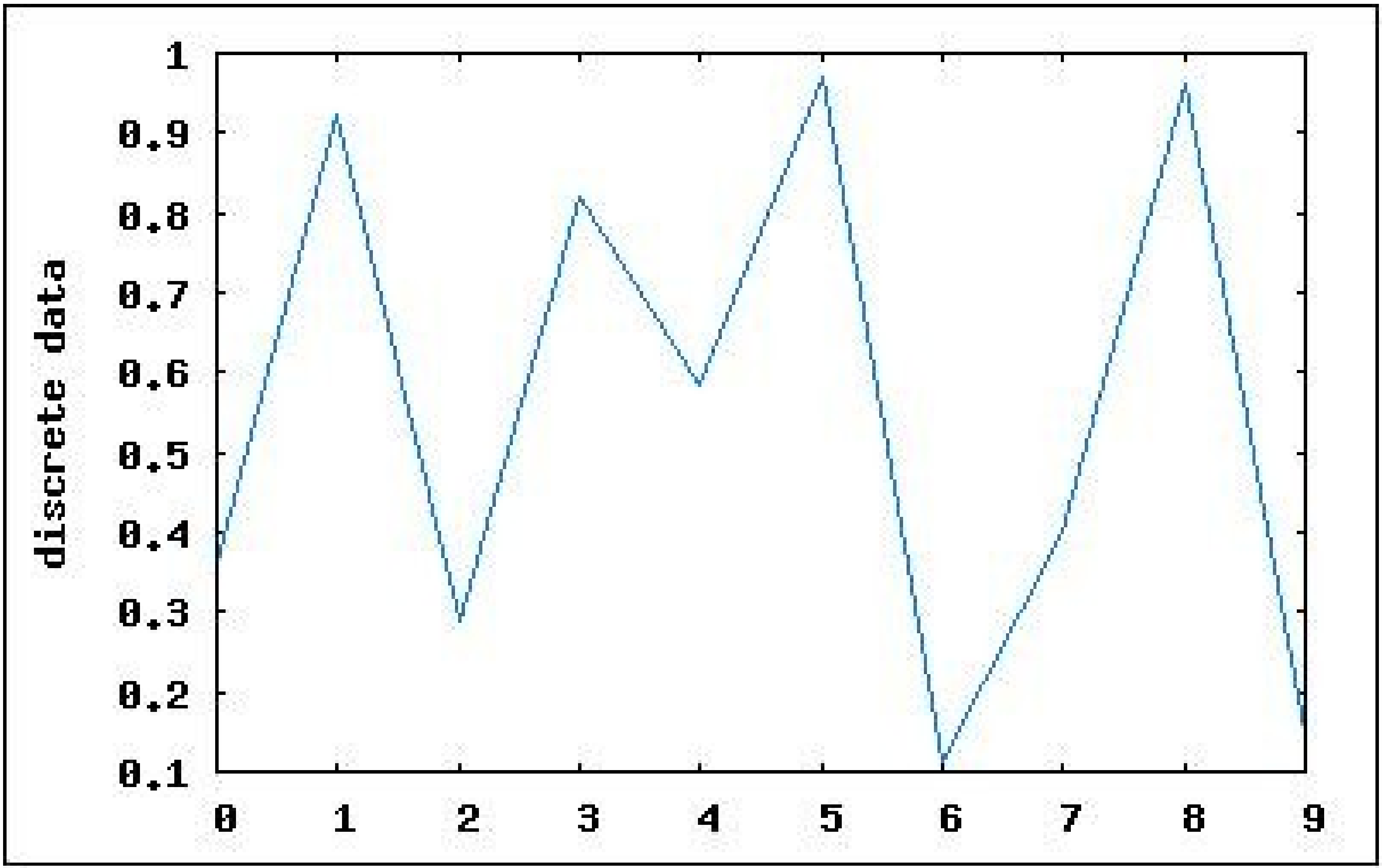


discrete data
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0
1
2
3
4
5
6
7
8
9

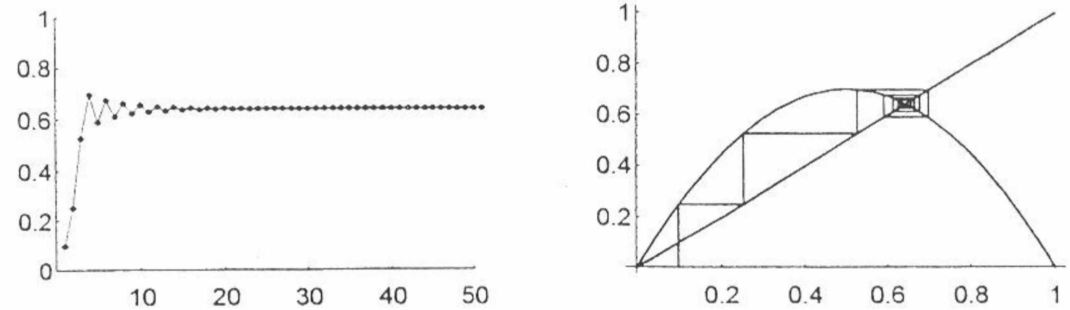
a=3.5 (r=2.5)



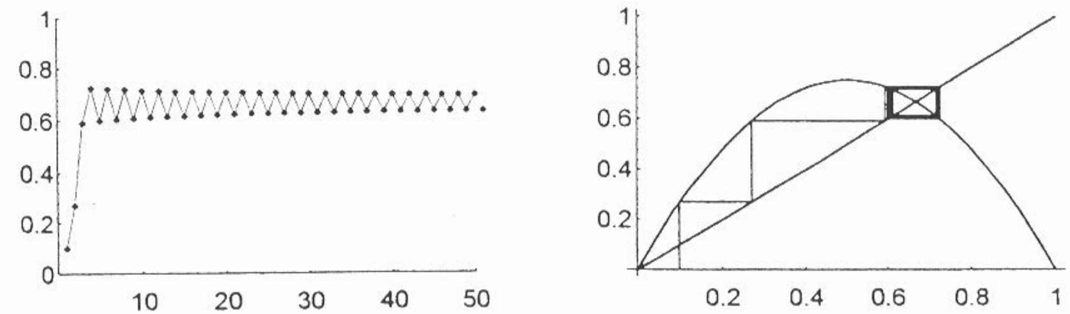
a=4 (r=3)



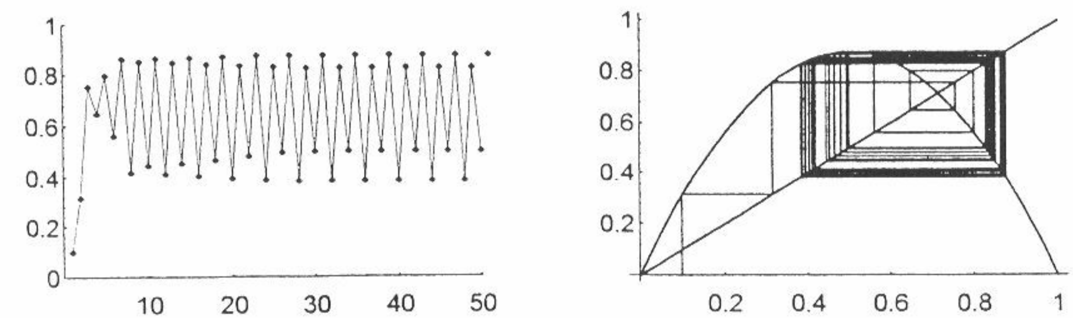
a = 2,8



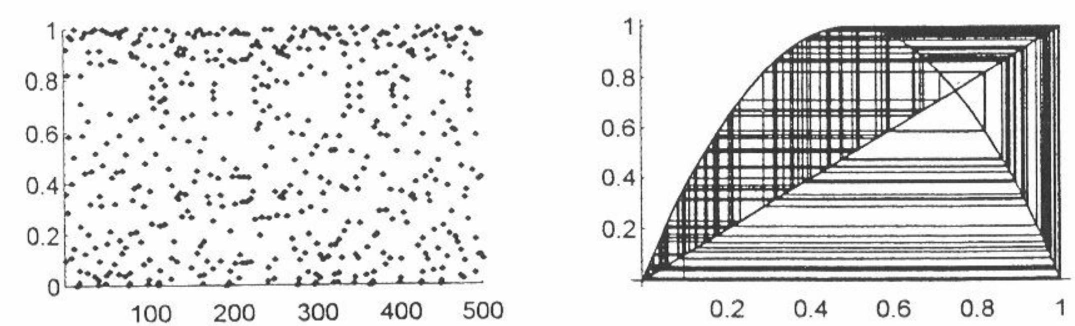
a = 3,0



a = 3,5



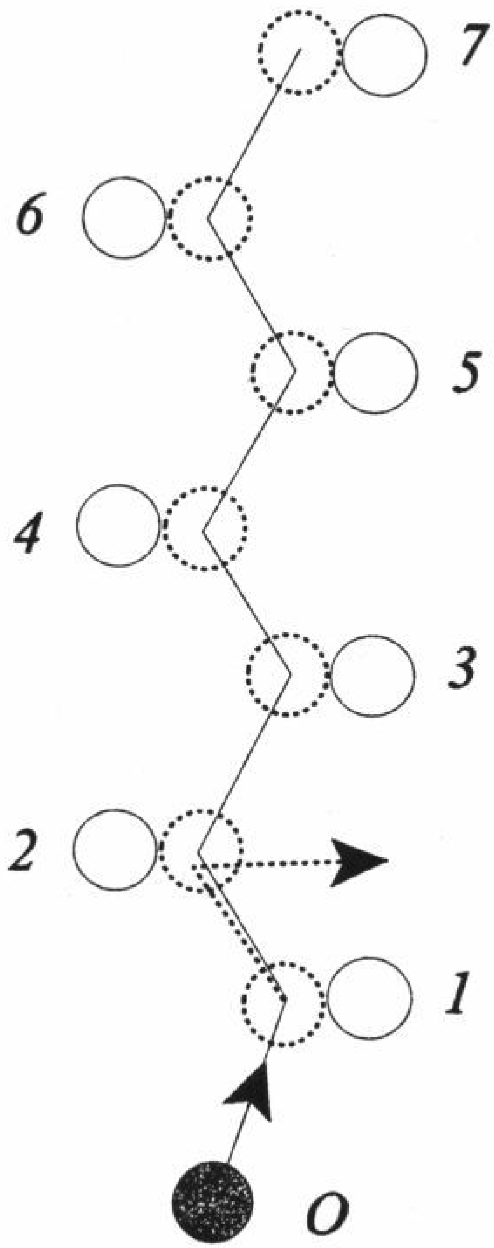
a = 4,0



Chaos theory

Pierre Simon de Laplace envisaged a superhuman intelligence capable of knowing the position at any time of every particle in the universe and all the forces acting on them. For this intelligence "...nothing would be uncertain and the future, as the past, would be present to its eyes. The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence." (1814)

This is sometimes called the “billiard ball universe”, or the “clockwork universe”.



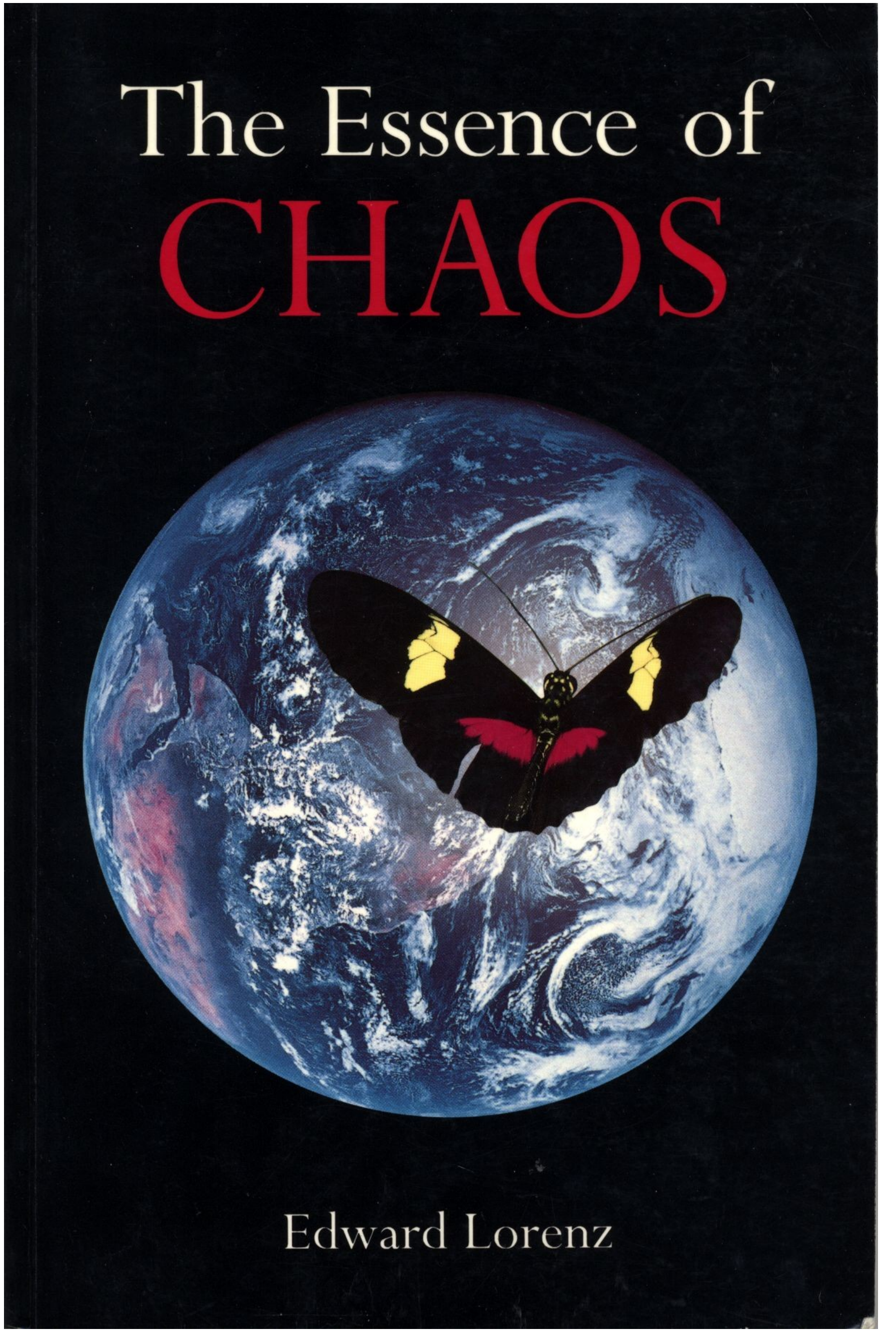
- We place a billiard ball in 0
- 1 – 7 are cylinders
- How many cylinders will we be able to hit?

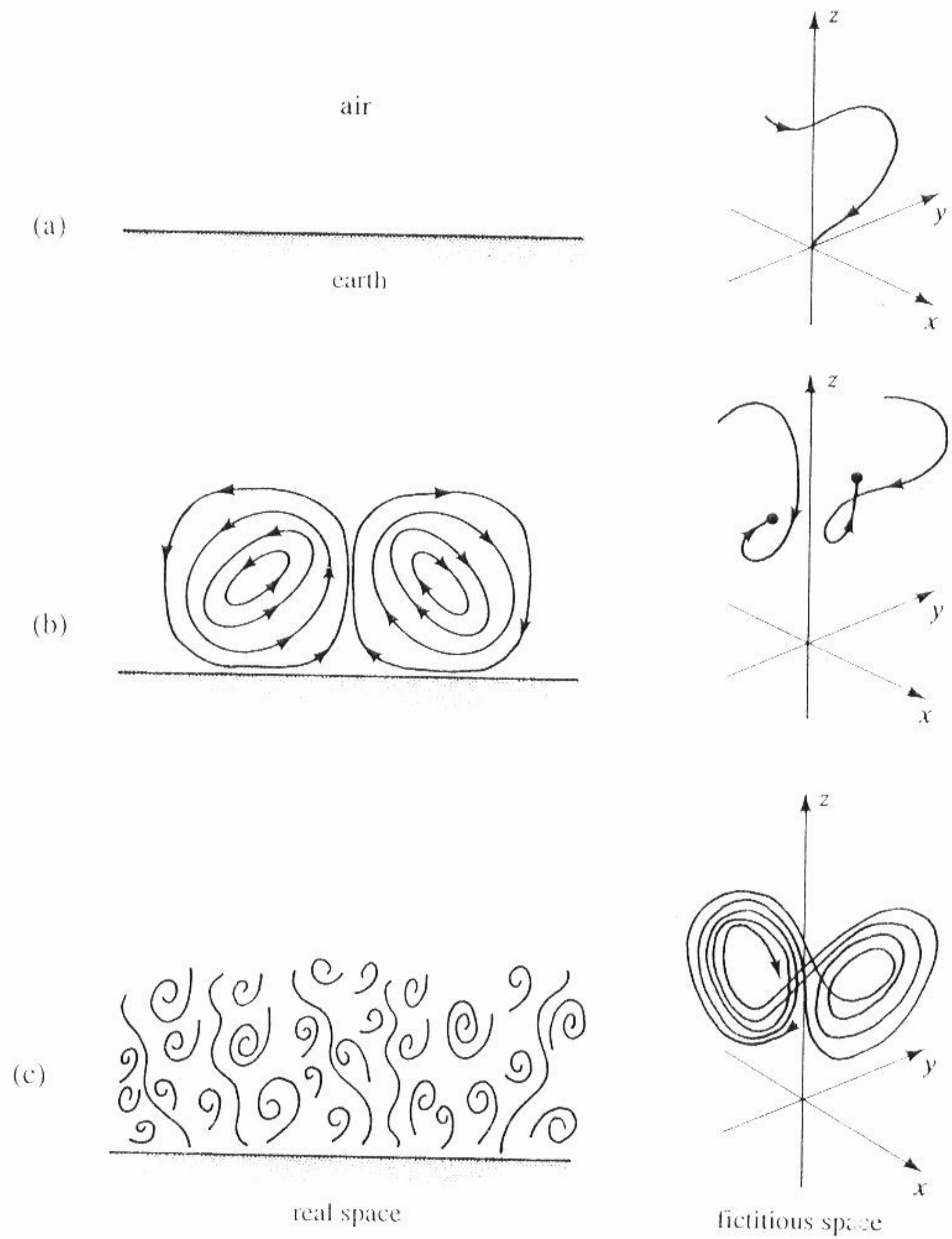
A fundamental tacit assumption

- What scientists actually believed was this: from approximately the same causes follow approximately the same effects – in nature as well as in any good experiment. And this is indeed often the case, especially over short time spans. If this were not so, we could not be able to ascertain any natural laws, nor could we build any functioning machines.

(Peitgen, Jürgens, Saupe: Fractals for the Classroom)

- “Predictability: Does the Flap of a Butterfly's Wings in Brazil Set up a Tornado in Texas” (1972)



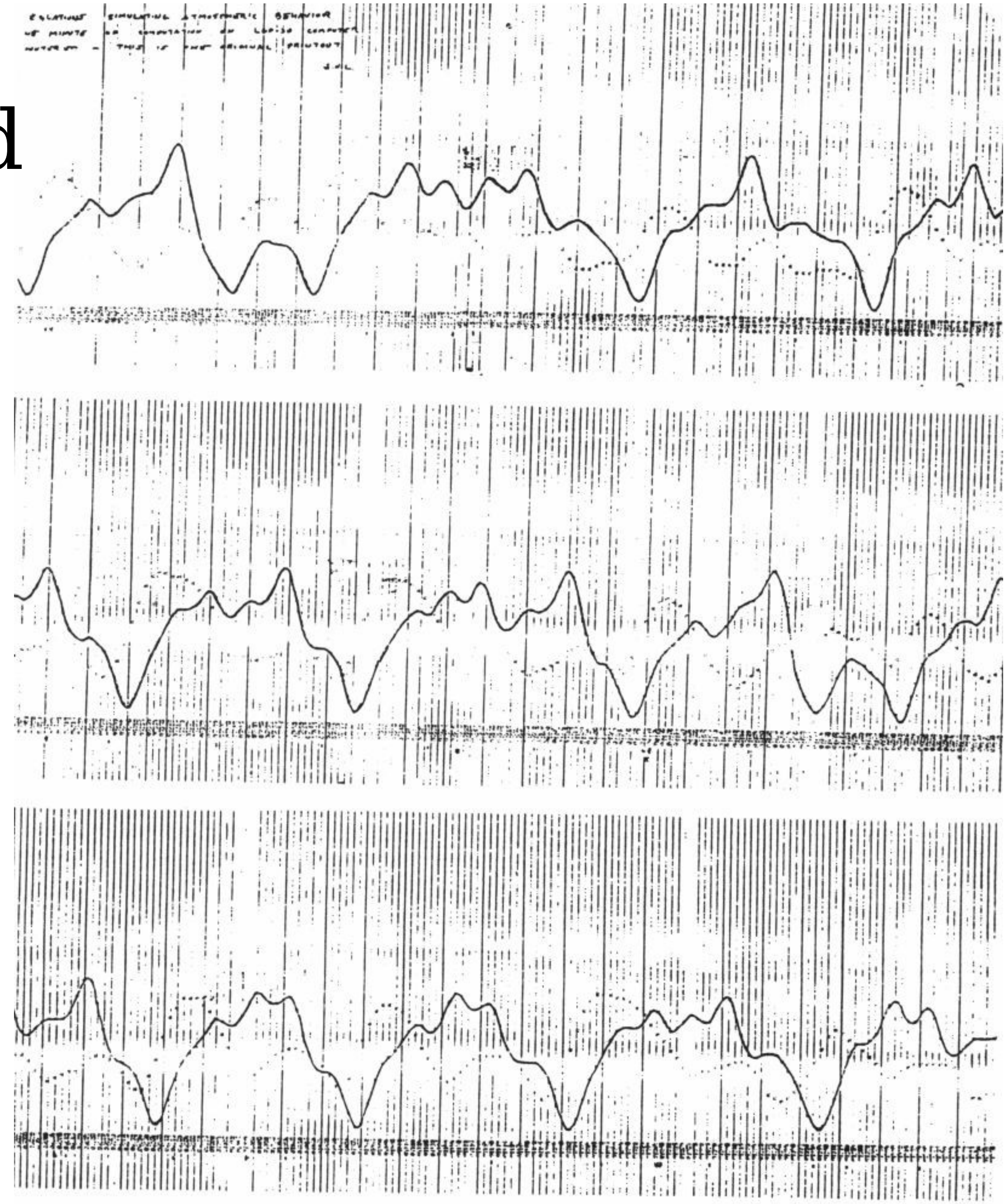


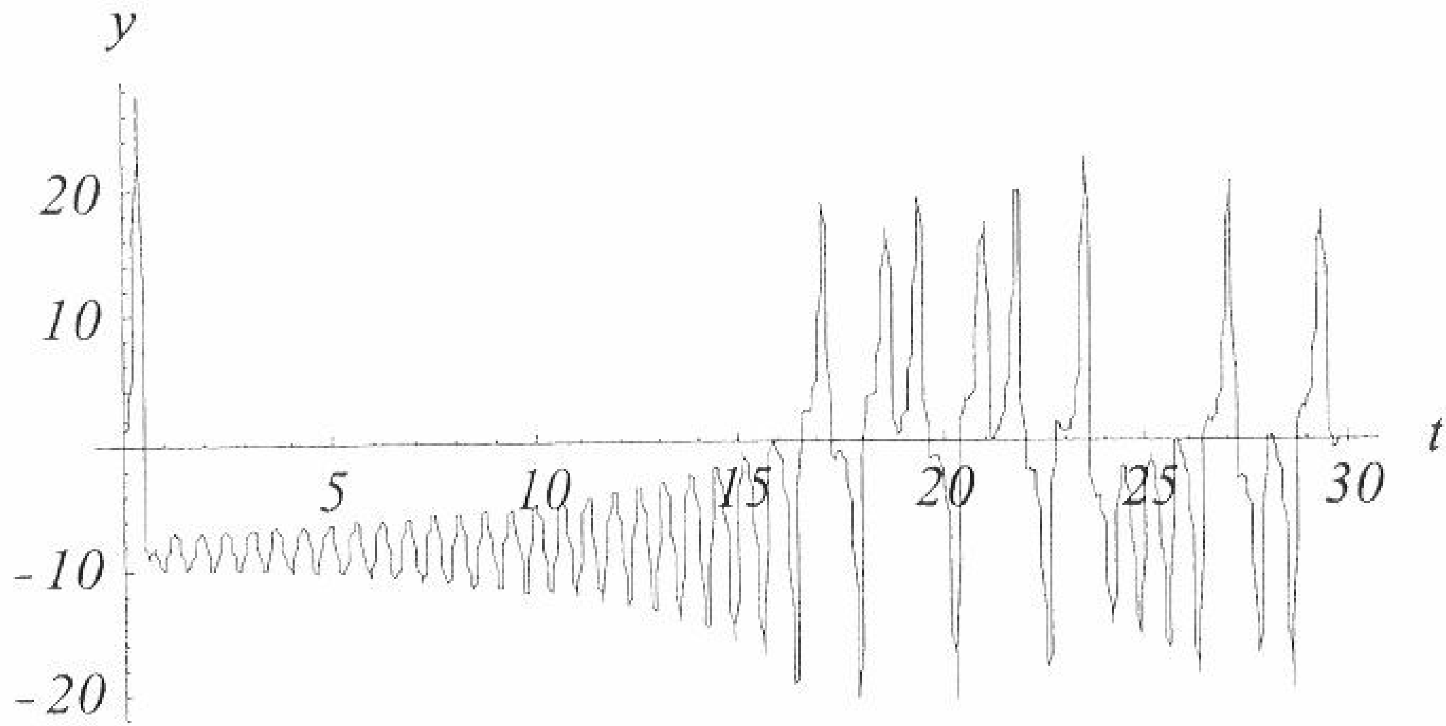
air
(a)
earth
(b)
(c)
real space
fictitious space
z
y
x

Lorenz' equations

$$dx/dt = -\rho x + \rho y$$
$$dy/dt = rx - y - xz$$
$$dz/dt = -bz - xy$$

- Edward Lorenz and the discovery of sensitive dependence of initial conditons (“the Butterfly effect”)





$\rho = 10$

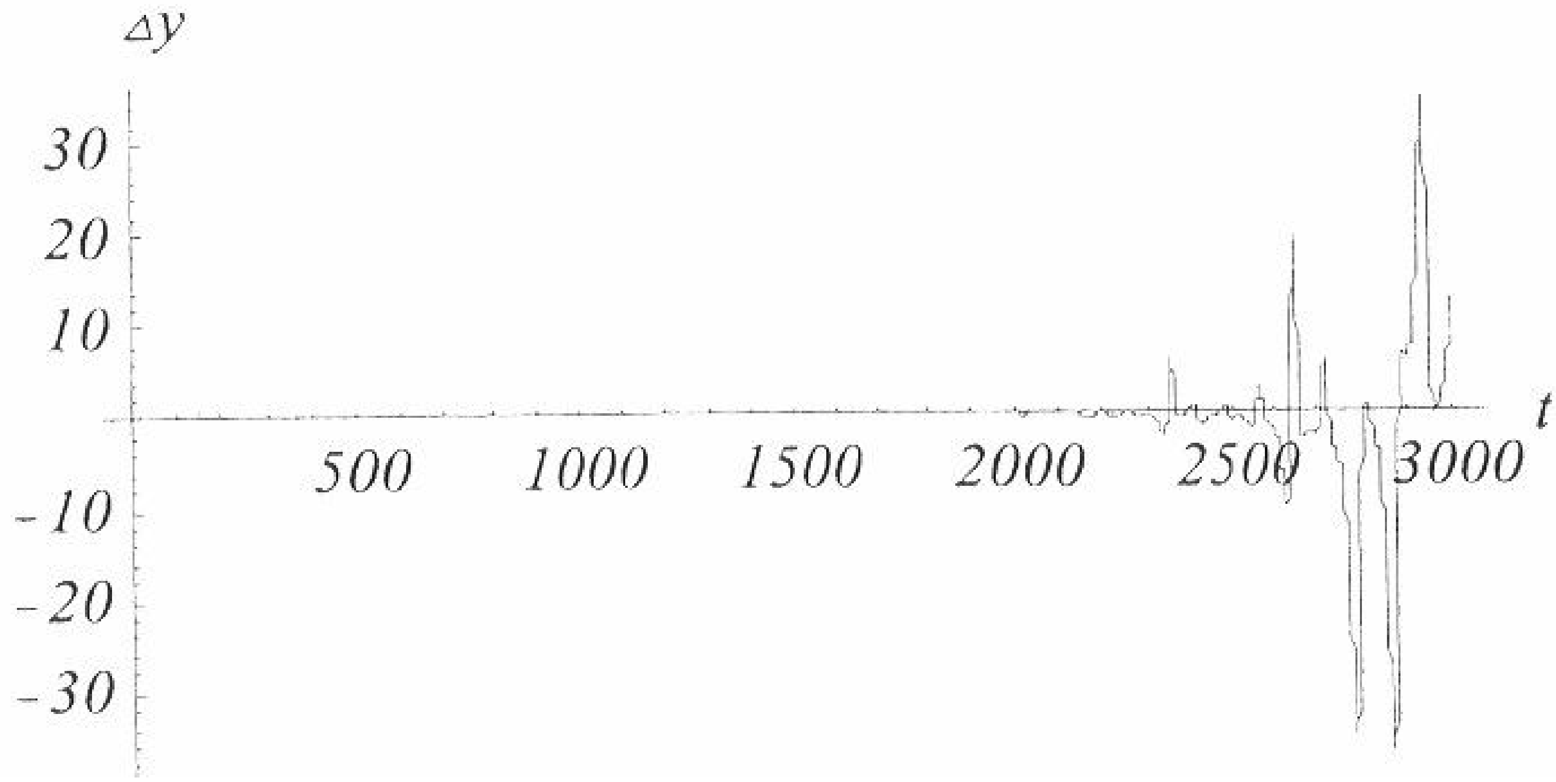
$r = 28$

$b = 8/3$

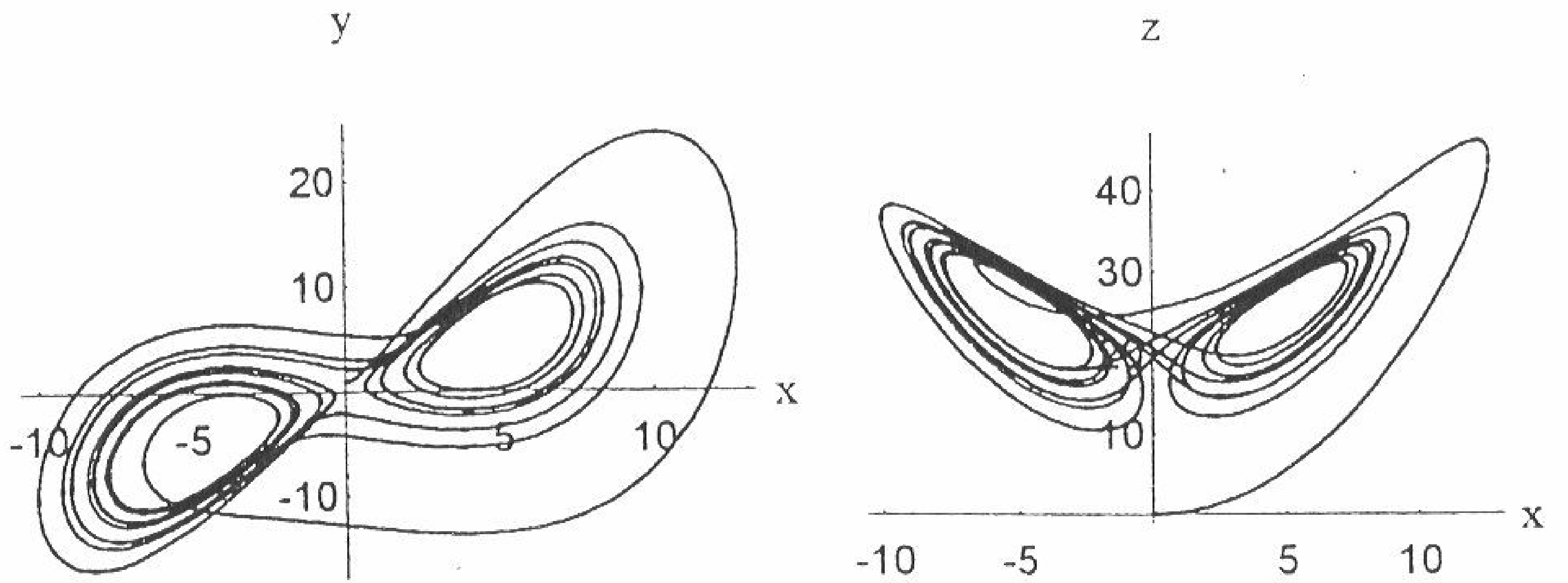
Initialconditions:

$x(0) = 0, y(0) = 1, z(0) = 0$

Example of sensitive dependence on initial conditions ("the butterfly effect")



Only change in initial condition:
y(0) = 0.00000001


y
20
10
-10
-5
5
10
-10
x
z
40
30
10
-10
-5
5
10
x

What can we learn from this? 1

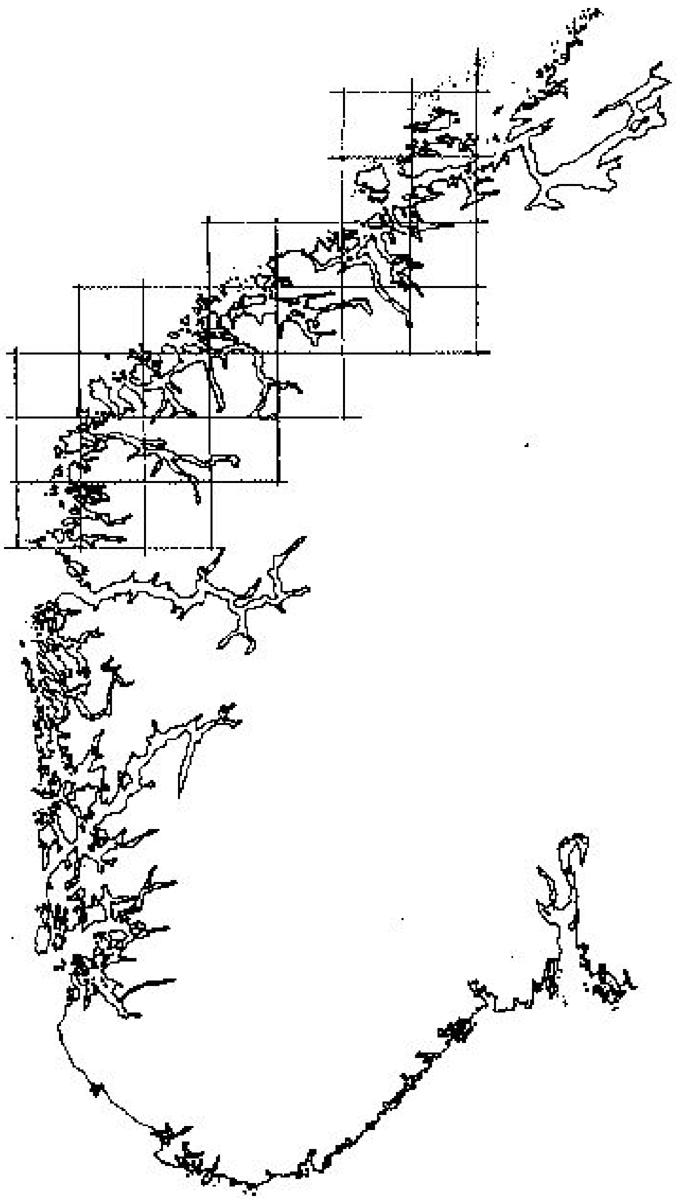
- It is important to know if a system is stable, unstable or chaotic.
- When a system is very unstable or chaotic we cannot make exact predictions, and we normally cannot improve accuracy by more accurate measurements.

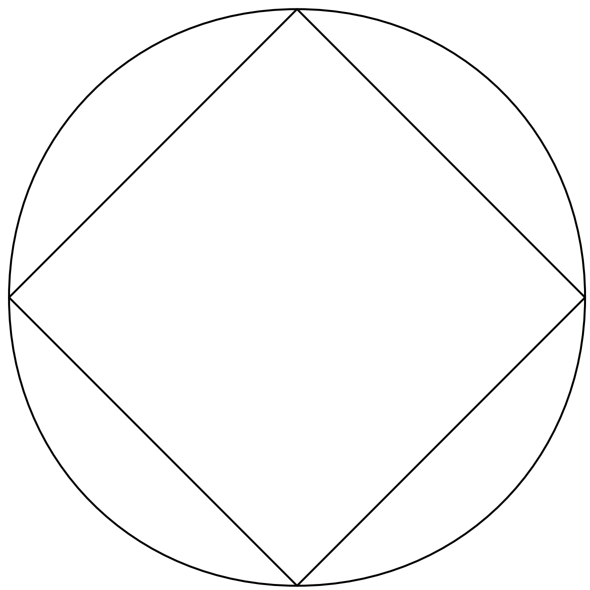
What can we learn from this? 2

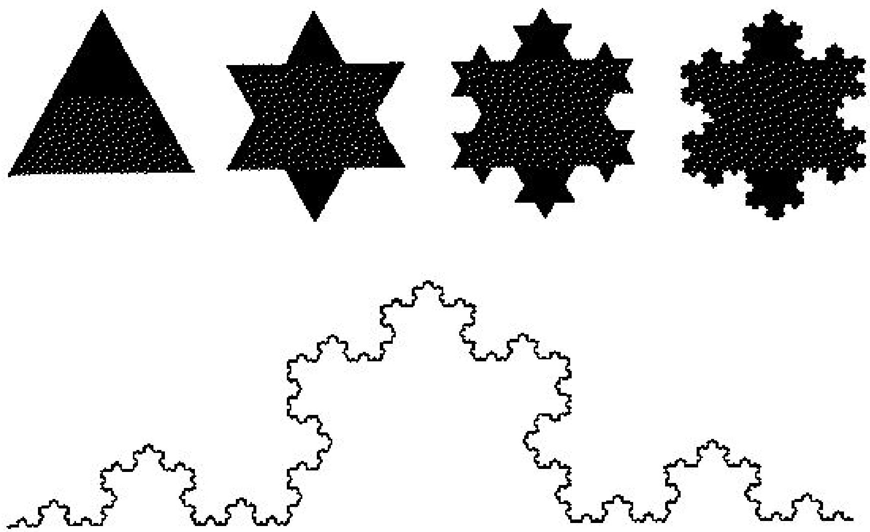
- The world is not as simple and regular as Galileo, Newton etc. assumed.

Fractal geometry and complexity

Benoit Mandelbrot:
“How long is the Coast of Britain?” (1967)





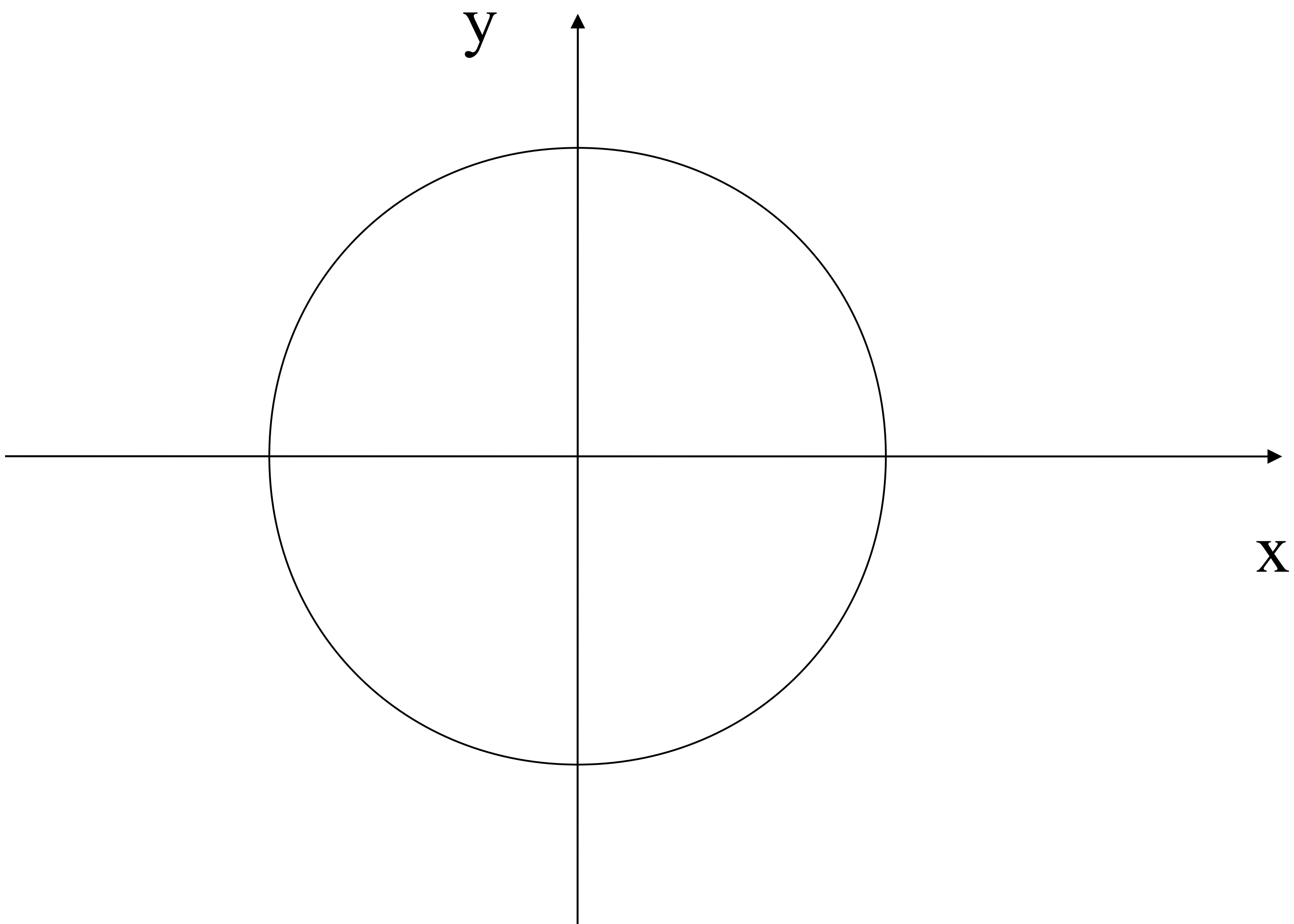


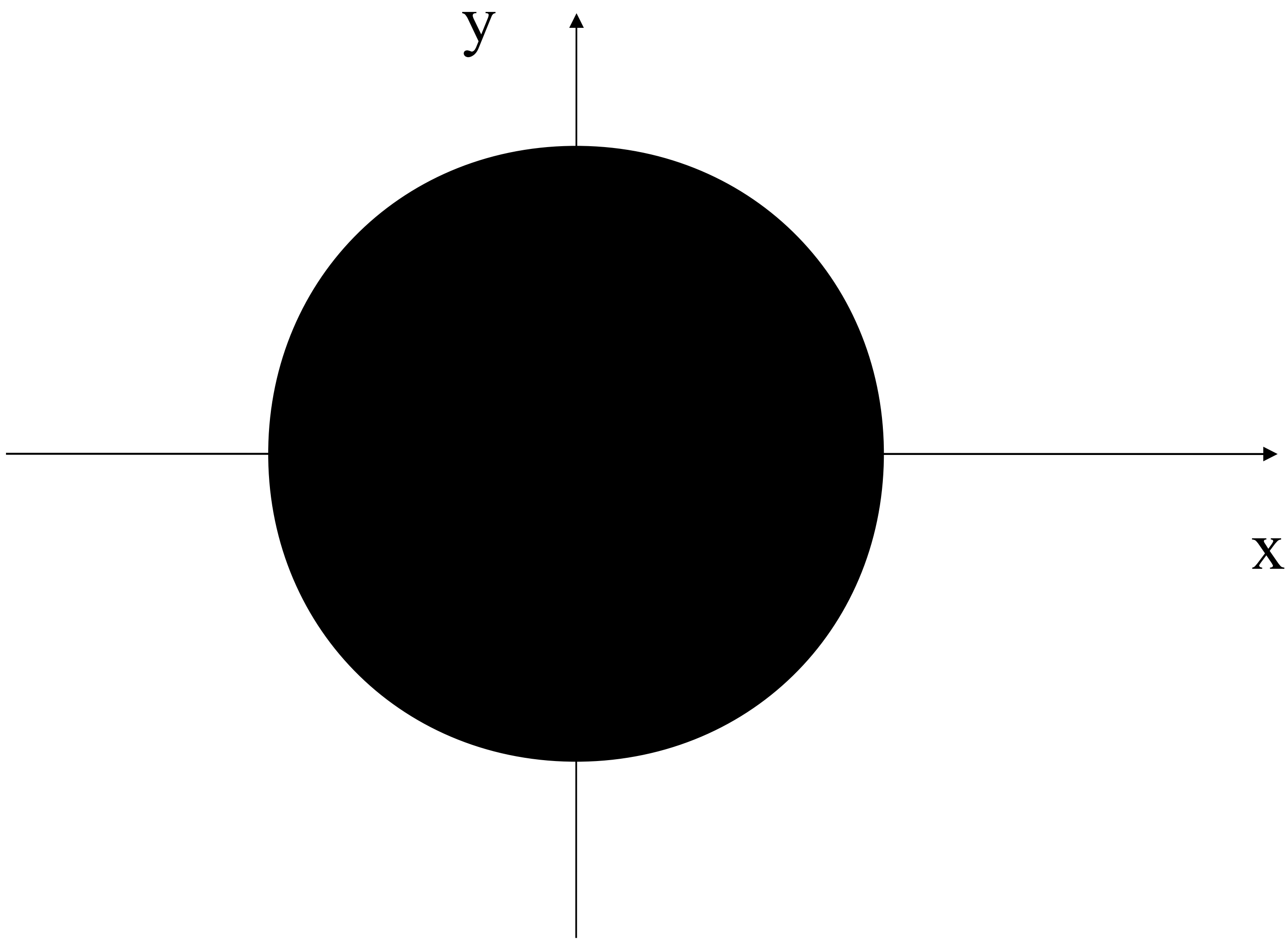
Mandelbrot-mengden

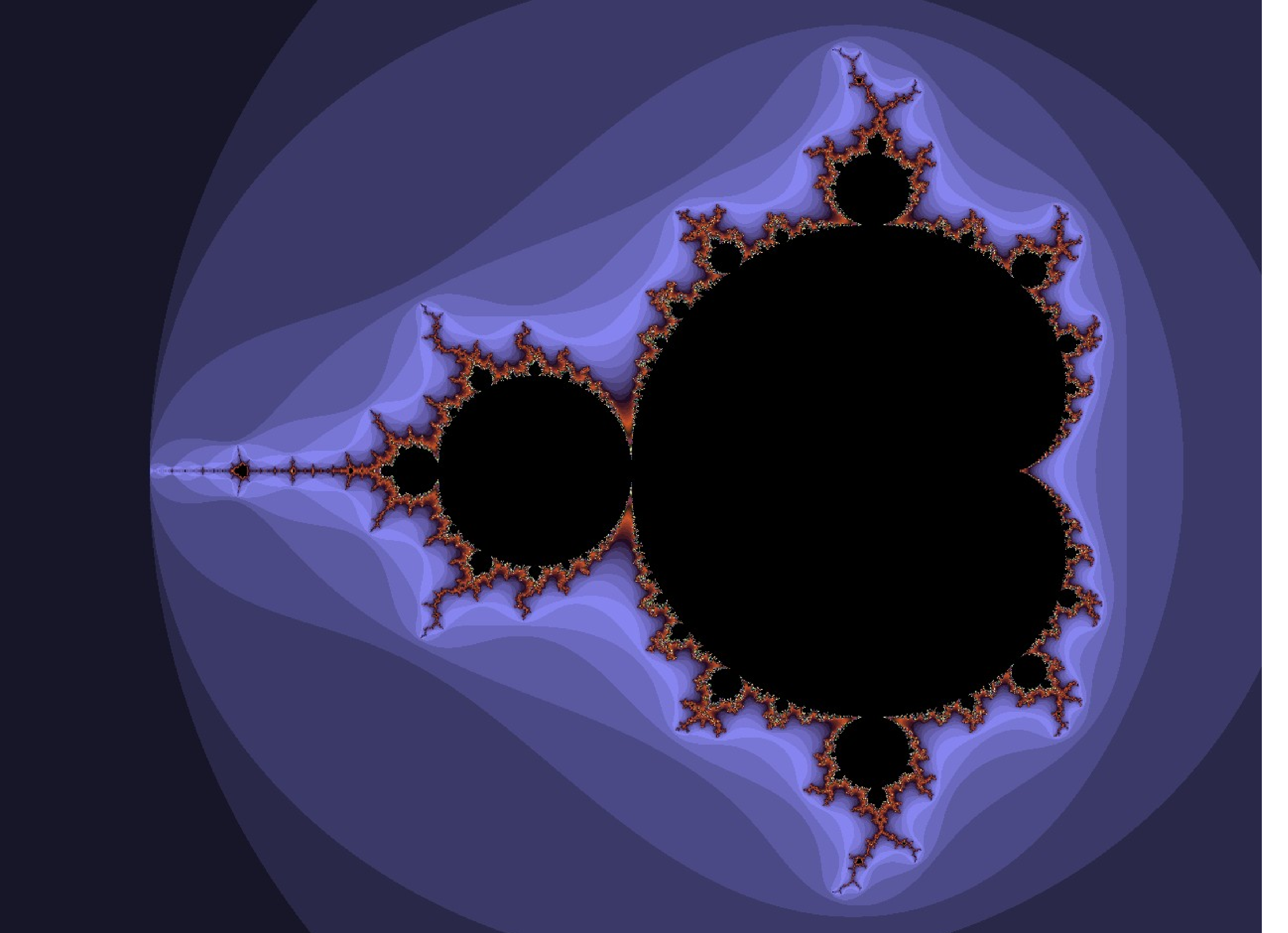
$$z_{n+1} = z_n^2 + c$$

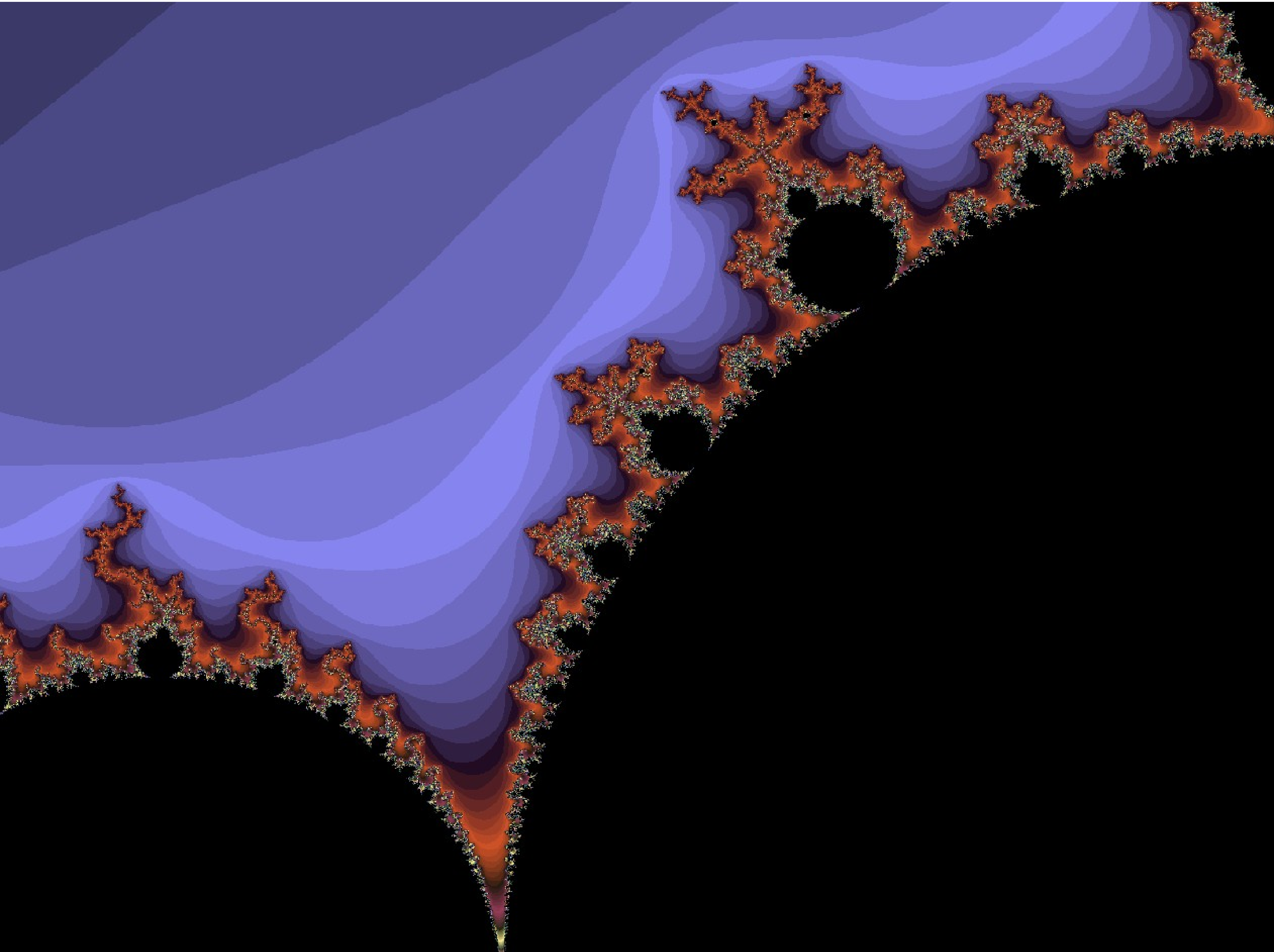
z og c er komplekse tall:

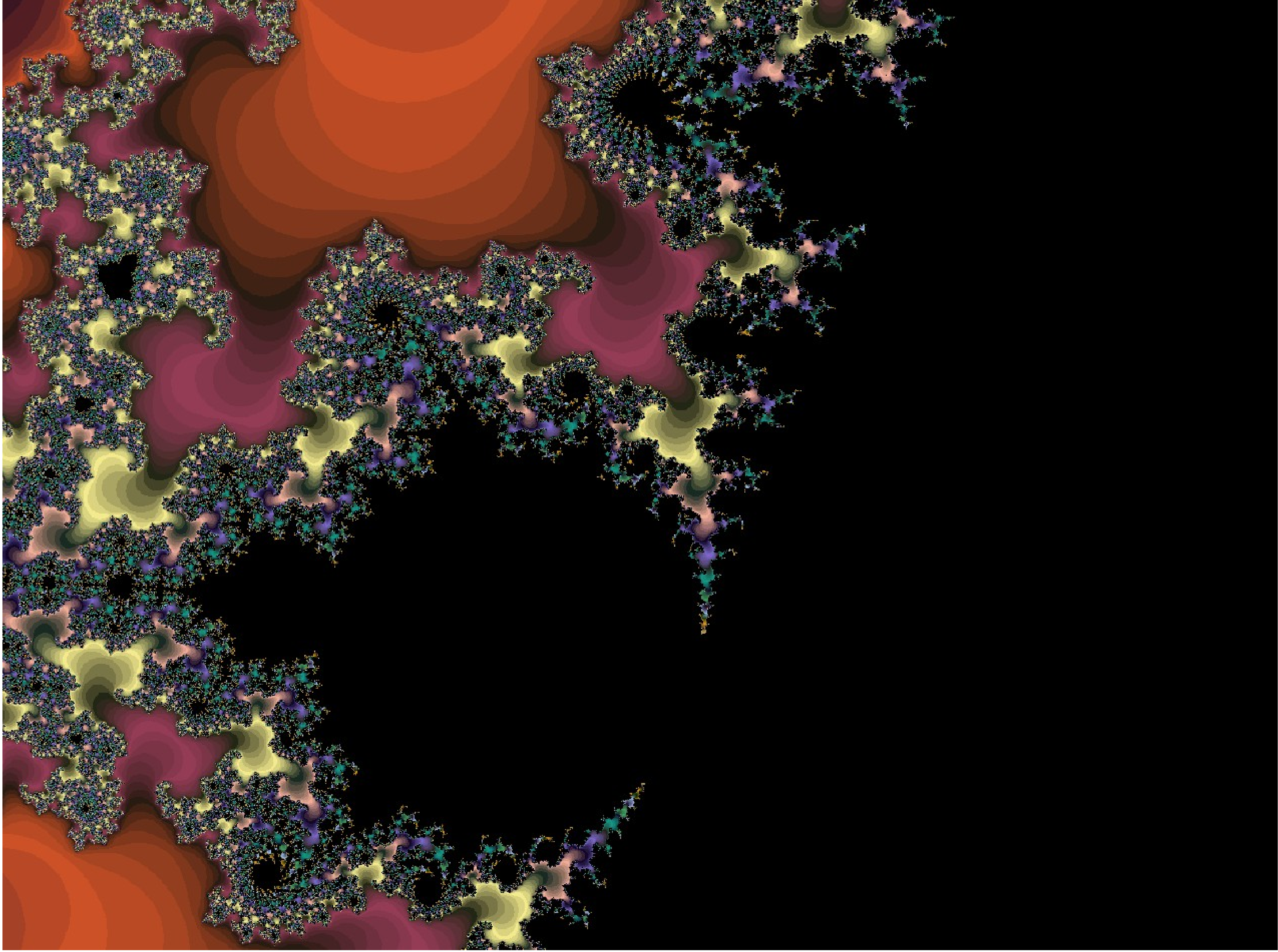
$$z = x + iy$$

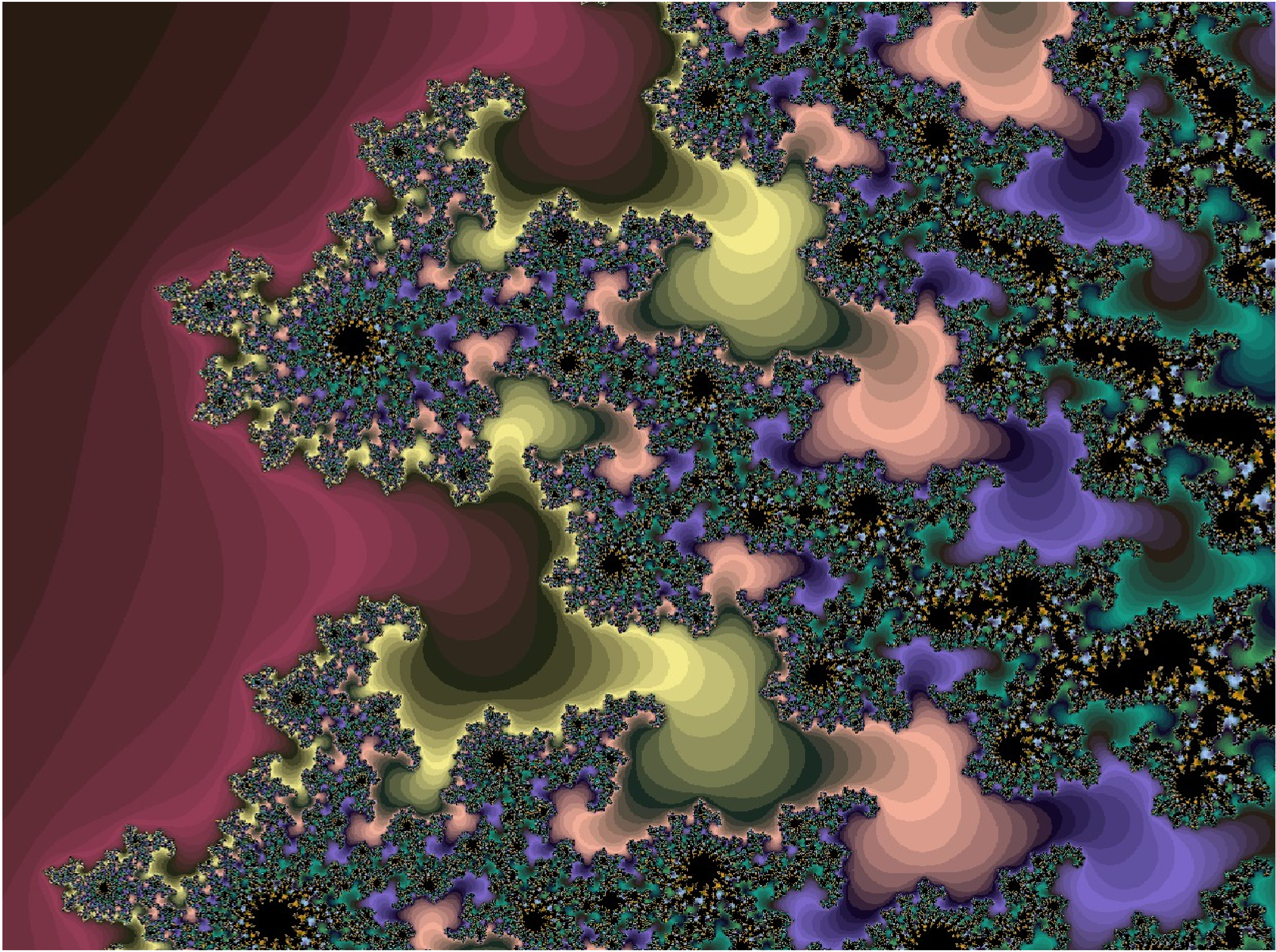


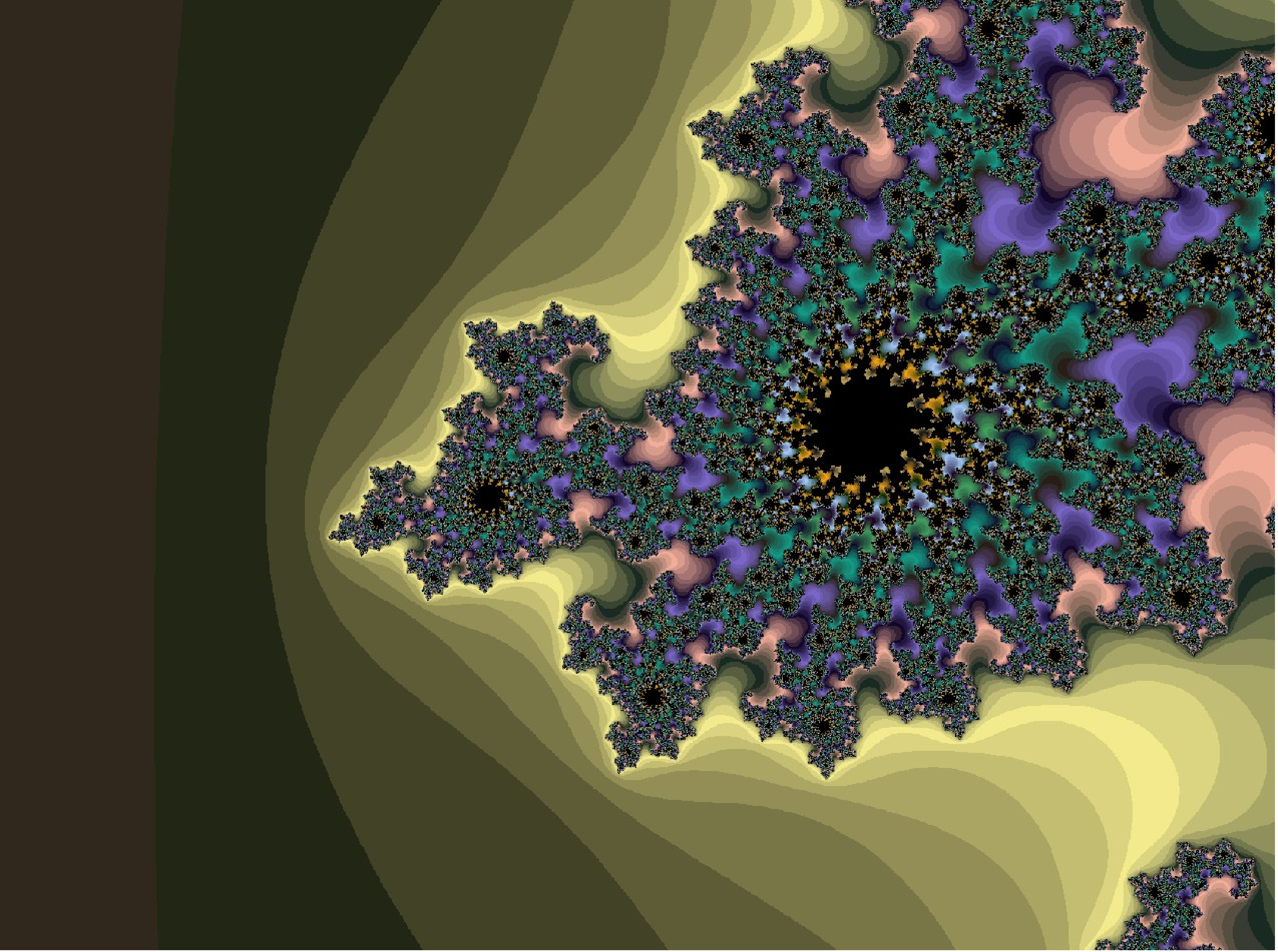

y
x

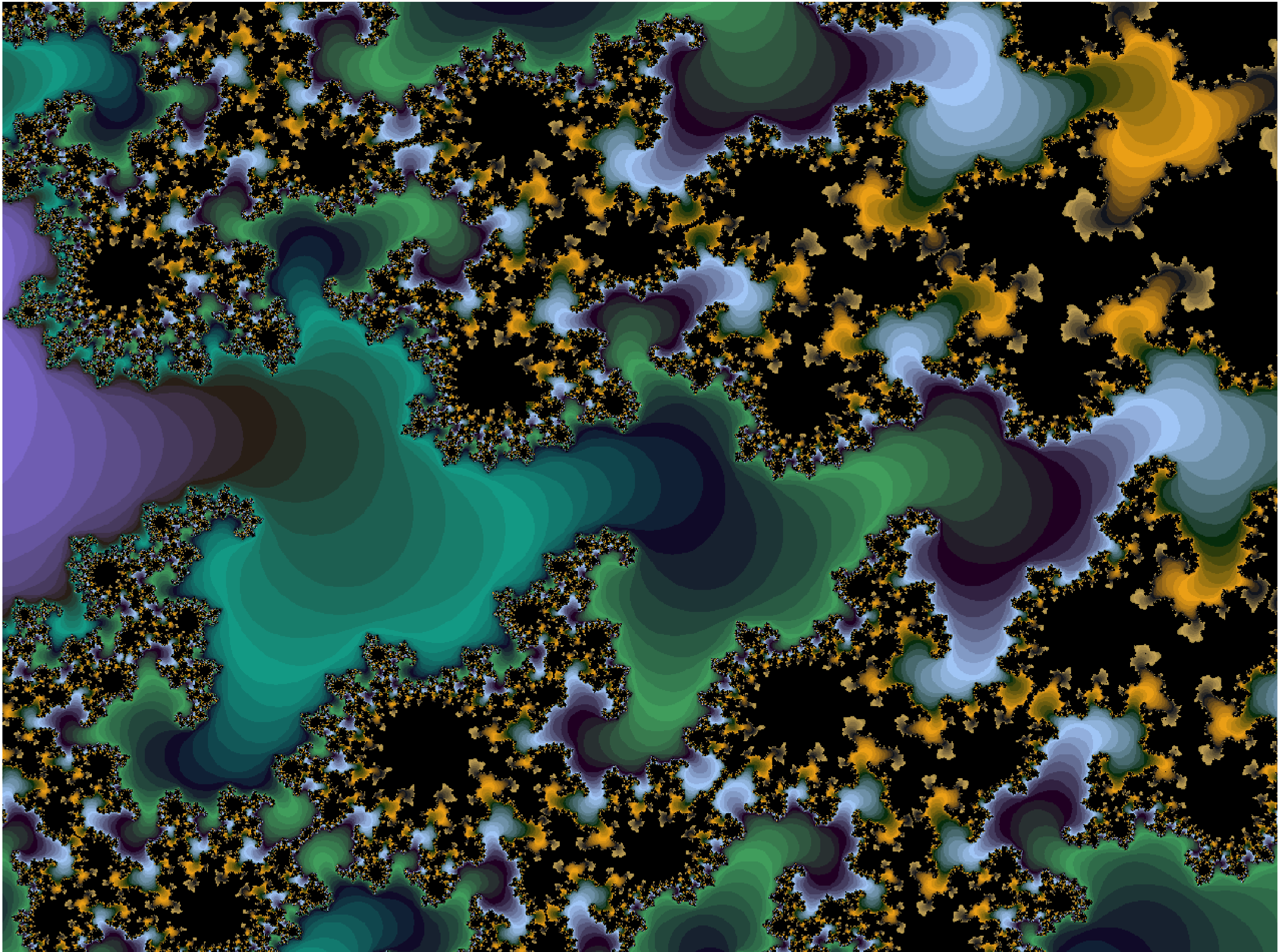












“Why is [Euclidean] geometry often described as “cold” and “dry”? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.”

Benoit Mandelbrot: *The Fractal Geometry of Nature* (1977)