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Comparison of global sensitivity analysis techniques and importance measures in PSA

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Abstract

This paper discusses application and results of global sensitivity analysis techniques to probabilistic safety assessment (PSA) models, and their comparison to importance measures. This comparison allows one to understand whether PSA elements that are important to the risk, as revealed by importance measures, are also important contributors to the model uncertainty, as revealed by global sensitivity analysis. We show that, due to epistemic dependence, uncertainty and global sensitivity analysis of PSA models must be performed at the parameter level. A difficulty arises, since standard codes produce the calculations at the basic event level. We discuss both the indirect comparison through importance measures computed for basic events, and the direct comparison performed using the differential importance measure and the Fussell–Vesely importance at the parameter level. Results are discussed for the large LLOCA sequence of the advanced test reactor PSA.

Keywords: Importance measures; Global sensitivity analysis; Probabilistic safety assessment; Sensitivity analysis; Epistemic uncertainty

1. Introduction

Probabilistic safety assessment (PSA) is a methodology that produces numerical estimates for a number of risk metrics for complex technological systems. The core damage frequency (CDF) and the large early release frequency (LERF) are the common risk metrics of interest in nuclear power plants (NPP).

The generic risk metric can be written as a function of the frequencies of the initiating events, i.e. events that disturb the normal operation of the facility such as a power excursion and the conditional probabilities of the failure modes of structures, systems and components (SSCs)

$$R = h(\underline{f}^{\mathrm{IE}}, \underline{q}) \tag{1}$$

where $\underline{f}^{\mathrm{IE}} = \{f_i^{\mathrm{IE}}\}$, $i = 1, \ldots, Z$, is the set of the frequencies of initiating events with Z the total number of initiating events included in the PSA model and $\underline{q} = \{q_j\}$, $j = 1, \ldots, N$, is the set of the basic event probabilities, with N, the total number of basic events in the PSA. More synthetically, $q_j = p(\mathrm{BE}_j)$, $j = 1, \ldots, N$.

Once the logical expression of the minimal cut sets is expanded and the rare event approximation is considered, R is linear in $\underline{f}^{\mathrm{IE}}$ and $\underline{q}$ [4].

Since Eq. (1) relates the risk metric to the basic events, we refer to Eq. (1) as the basic event representation or basic event level of the PSA model.

A 'point estimate' of the risk metric R can be produced by Eq. (1) using point ('best estimate') values of the inputs ($\underline{f}^{\mathrm{IE}}$ and $\underline{q}$ in this case). We write

$$R_0(\underline{\phi}_0) = h(\underline{q}_0, \underline{f}_0) \tag{2}$$

where we have introduced the symbol $\underline{\phi}$ to denote the generic q_j or f_i ($\underline{\phi} = \{q_j, f_i\}$, $j = 1, 2, \ldots, N$, $i = 1, 2, \ldots, Z$). One refers to R_0 as to the nominal value or the risk metric, or, shortly, the nominal risk.

The risk metric is often expressed as a function of more fundamental parameters. For example, the failure time of a component is usually assumed to follow an exponential distribution with a failure rate λ. In the case the component is renewed every τ units of time, then, its average (over time) unavailability is [2]:

$$q_j = p(\mathrm{BE}_j) = \frac{\lambda_j \tau}{2} \tag{3}$$

However, more rigorously, we acknowledge that these inputs are uncertain and express this uncertainty using

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state-of-knowledge or epistemic probability distributions (Kaplan and Garrick, 1981) [1,12–15,17]. The propagation of these distributions produces the epistemic distribution of R. Epistemic or state of knowledge dependencies and conditional dependencies are not captured by the basic event expression of R. Eq. (1) needs to be replaced by its parametric representation, if we want to take them into account [4]. We denote the expression of the risk metric as a function of the PSA model parameters as:

$$R(\underline{x}) = g(x_1, x_2, \dots, x_n) \tag{4}$$

The importance of a PSA element with respect to the risk is found applying PSA importance measures. Importance measures traditionally used are the Fussell–Vesely (FV), risk achievement worth (RAW) [8,26] These measures show shortcomings when applied to set of basic events (Eq. (1)). Furthermore, RAW cannot be used to compute the importance of parameters (Eq. (4)) [4]. The differential importance measure (DIM) proposed recently by Borgonovo and Apostolakis [4] remedies this situation. In addition, DIM is defined for both Eqs. (1) and (4), providing measures of the risk-significance of both basic events and parameters (Section 2).

PSA importance measures (FV, RAW and DIM) are local measures, i.e. they deal with a point value of R and of the parameters. However, to assess the relevance of a parameter with respect to the model uncertainty, the entire epistemic uncertainty in R and in the parameters should be taken into account. Global sensitivity analysis (GSA) techniques are the appropriate techniques for this task [21]. We have investigated several GSA techniques in this work. In this paper we focus on the results and performance of global sensitivity indices computed via extended fourier amplitude sensitivity test (FAST) [12,22,24].

We show that, due to epistemic dependencies, the appropriate level to perform GSA is the parameter level of the PSA model. Thus, the comparison of importance measures and GSA technique results is not direct, since importance measures are produced at the basic event level by most standard PSA software tools, while GSA techniques are computed at the parameter level. We propose both an indirect approach for the comparison of FV and RAW results at the basic event level to GSA results, and a direct comparison that makes use of DIM and FV at the parameter level as measures of risk. We provide quantitative results through the use of the large loss of coolant accident (LLOCA) PSA model of the advanced test reactor (ATR) [10].

In Section 2, we present DIM, FV, and RAW and discuss their properties. In Section 3, we introduce variance-based techniques and the definition of model coefficient of determination. In Section 4, we discuss dependencies caused by epistemic uncertainty. In Section 5, we present the application and results of GSA and importance measures, and their comparison for the large LLOCA sequence of the ATR PSA model. In Section 6 a number of conclusions is offered.

2. PSA importance measures

In this section, we discuss the definitions and properties at both the parameter and basic event level of DIM, FV and RAW.

DIM is defined for both PSA model parameters and basic events. The definition of DIM for parameters is as follows [4]:

$$\mathrm{DIM}_{x_i}(\underline{x}_0, \mathrm{d}\underline{x}) = \left.\frac{\mathrm{d}R_{x_i}}{\mathrm{d}R}\right|_{x_0} = \frac{\left.\dfrac{\partial R}{\partial x_i}\right|_{x_0} \mathrm{d}x_i}{\displaystyle\sum_j \left.\frac{\partial R}{\partial x_j}\right|_{x_0} \mathrm{d}x_j} \tag{5}$$

where $\underline{x}_0 = \{x_{1_0}, x_{2_0}, \dots, x_{n_0}\}$ is the set of the parameters in Eq. (4) fixed at a reference point value, $\mathrm{d}\underline{x} = \{\mathrm{d}x_1, \mathrm{d}x_2, \dots, \mathrm{d}x_n\}$ is the change vector,

$$\mathrm{d}R_{x_i} = \left.\frac{\partial R}{\partial x_i}\right|_{\underline{x}_0} \mathrm{d}x_i$$

is the differential of R with respect to x_i,

$$\mathrm{d}R = \left.\frac{\partial R}{\partial x_1}\right|_{\underline{x}_0} \mathrm{d}x_1 + \left.\frac{\partial R}{\partial x_2}\right|_{\underline{x}_0} \mathrm{d}x_2 + \dots + \left.\frac{\partial R}{\partial x_n}\right|_{\underline{x}_0} \mathrm{d}x_n$$

is the total differential of R.

DIM (Eq. (5)) is the fraction of the local change in R that is due to a change in parameter x_i.

The definition of DIM at the basic event level is

$$\mathrm{DIM}_{E_j}(\underline{\phi}_0, \mathrm{d}\underline{\phi}) = \frac{\mathrm{d}R_{E_j}}{\mathrm{d}R} = \frac{\left.\dfrac{\partial R}{\partial \phi_j}\right|_{\underline{\phi}_0} \mathrm{d}\phi_j}{\displaystyle\sum_k \left.\frac{\partial R}{\partial \phi_k}\right|_{\underline{\phi}_0} \mathrm{d}\phi_k} \tag{6}$$

where E_j denotes the generic basic event or initiating event, ϕ_j denotes the corresponding probability (if E_j is a basic event) or frequency (if E_j is an initiating event), $\mathrm{d}R_{E_j}$ denotes the differential of R in ϕ_j, $\mathrm{d}R$ is the total differential of in R. Eq. (6) states that basic events that cause the greater change in the risk metric have the highest DIM. We note that Eq. (6) is based on the expression of R as function of the basic events (Eq. (1)), while the definition in Eq. (5) applies to the expression of the risk metric as a function of the parameters (Eq. (4)).

As it appears from Eqs. (5) and (6), DIM depends on both the parameter reference values and the vector of changes in the parameters. DIM can be computed under different assumptions regarding the way parameters or basic events are affected by the changes [4]. The following assumptions are considered in this paper:

$$\text{Uniform changes}: \quad \delta y_i = \delta y_j \quad \forall i,j \tag{H1}$$

$$\text{Proportional changes}: \quad \frac{\delta y_i}{y_i} = \frac{\delta y_j}{y_j} = \omega \quad \forall i,j \tag{H2}$$

where y_i stands for the generic parameter x_i or basic event probability q_j depending on whether we are dealing with the basic event or parameter levels. We note that, under H1, DIM measures the parameter/event importance with respect to a small change that is the same for all the parameters. Under H2, DIM ranks the parameters according to the effect they produce on R when they are changed by the same fraction of their nominal values. Clearly, the two situations are different and the results for the measure are different, depending on the chosen assumption.

DIM possesses the additivity property, i.e. the DIM of a group of parameters/events is the sum of the individual DIMs of the parameters/events in the group [4]. This property is useful when the analyst is interested in the evaluation of changes that affect multiple parameters/events, and remedies one of the shortcomings in the use of FV and RAW for these applications.

FV and RAW are traditionally used to identify basic events/initiating events that contribute to risk the most. Hence, they are defined at the basic event level (Eq. (1)). The definition of FV for a basic event is [8,28].

$$\mathrm{FV}_{\mathrm{BE}_j}(\underline{\phi}_0) = \frac{\mathrm{Fr}\left(\bigcup_{t=1}^{m} \mathrm{MCS}_j^t\right)}{\mathrm{Fr}\left(\bigcup_{k=1}^{n} \mathrm{MCS}_m\right)} = \frac{\mathrm{Fr}\left(\bigcup_{t=1}^{m} \mathrm{MCS}_j^t\right)}{R_0} \tag{7}$$

where E_j stands for initiating/basic event j, $\mathrm{Fr}(\bigcup_{t=1}^{m} \mathrm{MCS}_j^t)$ is the frequency of the union of all the minimal cut-sets (MCS) containing event j, and $\mathrm{Fr}(\bigcup_k \mathrm{MCS}_k) = R_0$ is the nominal risk (Eq. (2)).

The definition of FV can be extended at the parameter level [4] as follows

$$\mathrm{FV}_{x_j}(\underline{x}_0) = \frac{\sum_i T_{x_j}^i(\underline{x}_0)}{R_0} \tag{8}$$

where $T_{x_j}^i(\underline{x}_0)$ denotes the generic term in the risk metric expression as a function of the parameters (Eq. (4)) that contains parameter x_j; the numerator is the sum over all the terms in the expression of $R(\underline{x})$ that contain parameter x_i and the denominator is the base case value of the risk. $\mathrm{FV}_{x_j}(\underline{x}_0)$ is the fraction of the risk that is associated with parameter x_j.

RAW is defined for basic events as [8,27]:

$$\mathrm{RAW}_{E_j}(\underline{\phi}_0) = \frac{R_j^+}{R_0} \tag{9}$$

where R_0 is the nominal risk and R_j^+ is the new risk that is produced when the Boolean variable of basic event j is set equal to 'true'. Setting to true the Boolean variable of an event means to assume that the event has happened. Then, in the rigorous definition of RAW, the PSA model is modified accordingly and a new estimate of the risk is obtained from the model. For example, if E_j represents a particular failure mode, then this failure is assumed to exist when R_j^+ is calculated.

The relations among the three importance measures presented above are detailed in Ref. [4]. We note that DIM, FV, and RAW are computed with all the basic event probabilities/parameters fixed at their nominal values. Hence, they are local methods and their results provide information about the importance of PSA elements fixed at one point of the uncertainty space only.

3. Epistemic uncertainty

We note that, in the presence of epistemic uncertainty, uncertainty and global sensitivity analyses must be performed on Eq. (3), the parameter level, and not on Eq. (1), the basic event level. Thus uncertainty importance measures [19] are properly defined for parameters and not for basic events.

For example, let us consider the simple case of a one-out-of-two parallel system. The risk metric is the unavailability, Q, of the system. Since the two components are in parallel, this unavailability is $Q = P(1 \text{ and } 2)$, that is the probability that components fail. This is written as:

$$Q = P(1|2)P(2) = P(2|1)P(1)$$

If the two failures are assumed independent, we get:

$$Q = P(1)P(2) \quad \text{or} \quad Q = q_1 q_2$$

Then, Eq. (1) becomes

$$Q = q_1 q_2 \tag{10}$$

where q_1 and q_2 are the unavailabilities of the two components. Eq. (10) is the basic event representation of the risk metric. We note that Eq. (10) is linear in q_1 and q_2.

We now consider the fact that both q_1 and q_2 are uncertain. Then Q becomes a function of random variables. Its variance is given by:

$$V'[Q] = V[q_1 q_2] = E[q_1^2]E[q_2^2] - E[q_1]^2 E[q_2]^2 \tag{11}$$

Suppose now that the two components are nominally identical. Then, q_1 and q_2 are described by the same distribution because of epistemic (state-of-knowledge) uncertainty [1]. In addition, when one samples the distribution of Q from the two distributions of q_1 and q_2, the same value for the two unavailabilities must be used in each Monte Carlo trial [3]. Thus, q_1 and q_2 are perfectly correlated, or, equivalently $q_1 = q_2 = q$.

Therefore, Q is described by just one parameter (q) and the epistemic representation of Q as a function of the parameter (Eq. (4)) is:

$$Q = q^2 \tag{12}$$

We note that Q in no longer linear. Eq. (12) is the expression that allows the correct computation of the uncertainty in Q:

$$V[Q] = V[q^2] = E[q^4] - E[q^2]^2 \tag{13}$$

Thus, Eq. (1) needs to be replaced by Eq. (4), in performing uncertainty analysis when epistemic uncertainty is taken into account.

4. Global sensitivity analysis techniques

To determine the influence of individual parameters and parameter groups on the uncertainty in R, GSA techniques must be used. These methods consider the parameter uncertainty distributions and reflect their contribution to the epistemic uncertainty in R. As we mentioned, epistemic uncertainty is properly dealt with at the parameter level. Therefore, we will set the expression of R as a function of the parameters (Eq. (3)).

Variance based techniques (VBTs) explain V_R, i.e. the variance of R, in terms of the individual parameters or parameter groups. They identify the parameters that contribute to the overall uncertainty in R the most, as follows. V_R is generated by the epistemic uncertainty in the parameter values. V_R can be written in terms of individual parameter and parameter group contribution as [16]:

$$V_R = \sum_i V_i + \sum_{i<j} V_{ij} + \sum_{i<j<m} V_{ijm} + \cdots + V_{12\ldots k} \tag{14}$$

where k is the number of the uncertain parameters R denotes the risk metric, X_i denotes the ith parameter, $E(R|X_i = x_i^*)$ denotes the expectation of R conditional on X_i having a fixed value x_i^*, $V_i = V(E(R|X_i = x_i^*))$ stands for the variance over all possible values of x_i, and analogous definitions hold for the higher order terms.

First order global sensitivity indexes can be introduced using Eq. (14) as [19]:

$$S_1(x_i) = \frac{V_i}{V_R} \tag{15}$$

Parameters that have a higher contribution to the variance will have higher conditional variances V_i, and therefore will have higher $S_1(x_i)$. $S_1(x_i)$ is then taken as the uncertainty importance measure of the individual parameter x_i. Estimation procedures for $S_1(X_i)$ are the classical FAST [9], the method of Sobol [25], and others [19,20].

We note that, if R is can be expressed as sum of terms containing only one parameter at a time

$$R(\underline{x}) = \sum_{i=1}^{k} f_k(x_k) \tag{16}$$

then, in Eq. (14), all the terms involving more than one parameter in the summation would be zero and V_R would be exactly the sum of first order terms ($V_R = \sum_i V_i$). In this case, the importance of the parameters with respect to the model uncertainty is fully contained in $S_1(X)$.

In general, we expect the risk metric not to be additive with respect to the parameters [11]. Therefore, there will be some interactions between the parameters. The $S_T(X_i)$, defined as the sum of all effects (first and higher order) involving parameter X_i, however, are capable of giving to the analyst information on the importance of terms involving more than one parameter. In this paper we use the extended FAST method proposed by [24], that allows the simultaneous computation of the first and total effect indices.

Finally, we remark that, through VBTs, we are able to identify the parameters that individually or as groups contribute most to the uncertainty in R in a quantitative fashion and without stating any assumption on the type of the dependence of R on the individual parameters. This gain in information may lead to an increase in computational cost [7,16]. However, the use of extended FAST allows us to compute first and total order sensitivity at a cost of the order N, where now N is the sample size required for the computation of the $S_1(X_i)$ and $S_T(X_i)$.

5. Application to a PSA accident sequence

5.1. The model

The reference model is the LLOCA sequence of the ATR [10]. Two major safety systems are involved in the large LLOCA accident, namely, the scram system (SCRAM) and the firewater injection system (FIS). Failure of SCRAM leads directly to core damage. The SCRAM system failure is dominated by the common cause failure event ‘failure to insert four safety rods’, since a system failure due to a series of independent events is very unlikely. If the SCRAM is successful, then FIS must also be successful to assure that no core damage results. The FIS cools the core after the LLOCA. Water is injected into the core by 4 injection lines. During a large LLOCA, one of these lines is assumed failed. Failure of the other three lines is necessary to fail the system. If FIS fails, core damage results.

The number of basic events for the model used in this exercise is 45, for a total of 289 MCSs. Of these, 10 MCSs contain only one basic event, 32 are formed by two basic events, and the remaining 247 are given by three basic events.

The risk metric used in this case is the CDF that is associated with the LLOCA initiating event (CDF^{LLOCA}). CDF^{LLOCA} for the reference example, is written using these data and the MCS above using the rare event approximation as a function of the event probabilities as

$$R(\underline{\phi}) = \mathrm{CDF}^{\mathrm{LLOCA}}(\underline{\phi}) = f_{\mathrm{LLOCA}}\left(\sum_j p(\mathrm{MCS}_j^i)\right)$$
$$= f_{\mathrm{LLOCA}}\left[\sum_{j=1}^{10} p(\mathrm{BE}_1^j) + \sum_{j=1}^{32}\prod_{i=1}^{2} p(\mathrm{BE}_i^j) + \sum_{j=1}^{247}\prod_{i=1}^{3} p(\mathrm{BE}_i^j)\right] \tag{17}$$

where f_{LLOCA} is the frequency of the LLOCA initiating event; $\sum_{j=1}^{10} p(\mathrm{BE}^j)$ is the sum of the probabilities of the MCS that contain one basic event; j is the index in the sum that goes from 1 to the number of minimal cut sets; $\sum_{j=1}^{32}\prod_{i=1}^{2} p(\mathrm{BE}_i^j)$ is the sum of the probabilities of the MCS that contain two basic events; i is the index in the product that runs from 1 to the number of basic events that are contained in the MCS; $\sum_{j=1}^{247}\prod_{i=1}^{3} p(\mathrm{BE}_i^j)$ is the sum of the probabilities of the 247 MCS that contain three basic events ($i=1,\ldots,3$).

The 45 basic events used and their point estimate failure probabilities are listed in Table 1, where the presence of a failure rate in the fourth column, indicates that an exponential failure model has been chosen for the corresponding event. The point values are the means obtained averaging the failure probabilities over the corresponding epistemic distributions. The total number of parameters is 31. The number of parameters is lower then the number of basic events due epistemic dependence (Section 3). For example, the three pumps are considered identical, and so are two valves LCV-7A and LCV-7B. Thus, the same parameters characterize their failure modes, as, for example, x_{14} is used for the three pump failure-to-start probability and x_6 for the failure-to-actuate probability of both valves LCV-7A and LCV-7B. As in the original PSA model for the ATR [10], an error factor (EF) of 10 and no correlation on the parameters have been assumed. With these data we obtain an uncertainty distribution of the $\mathrm{CDF}^{\mathrm{LLOCA}}$ that can be approximated by a lognormal, with mean of 10^{-9} (1/y) and EF of 12.

A second effect of the presence of epistemic uncertainty is as follows. We note from Eq. (17) that the $\mathrm{CDF}^{\mathrm{LLOCA}}$ is linear in the individual basic event probabilities. In the presence of epistemic uncertainty $\mathrm{CDF}^{\mathrm{LLOCA}}$ becomes polynomial function of the parameters of the PRA model, since the same parameters can characterize several basic event probabilities that are in the same MCS. From the size of the MCS, the analyst can determine what the maximum order of the polynomial is. For the reference PSA model and from Eq. (17), we see that the maximum size of the MCS is three. Therefore, a parameter may appear at most three times in an the MCS and $\mathrm{CDF}^{\mathrm{LLOCA}}$ the parameters will appear the most in the third power. This is the case when a parameter is shared by all three basic events of the same MCS. This is the case, in our example, for the rate of independent failures while running of the three deep-well pumps. As noted by Apostolakis and Kaplan [3], the epistemic dependence of the failure rates of the pumps will result in a term of the form $(\lambda t)^3$. Furthermore, in general, MCS that involve independent events are characterized by lower probability than MCS involving the corresponding single basic events. One could then hypothesize that only the terms involving one basic event are relevant in R and in its uncertainty. Using the model coefficient of determination, we show that this is not the case. The analyst can have a quantitative indication of the linear regression of $\mathrm{CDF}^{\mathrm{LLOCA}}$ by computing the model coefficient of determination, $\Gamma^2_{\mathrm{CDF}^{\mathrm{LLOCA}}}$. This coefficient $\Gamma^2_{\mathrm{CDF}^{\mathrm{LLOCA}}}$ is defined as [23]

$$\Gamma^2_{\mathrm{CDF}^{\mathrm{LLOCA}}} = \frac{\sum_{i=1}^{m}(\mathrm{CDF}_i^{\mathrm{LLOCA}} - \overline{\mathrm{CDF}^{\mathrm{LLOCA}}})^2}{\sum_{i=1}^{m}(\mathrm{CDF}_i^{\mathrm{LLOCA}} - \overline{\mathrm{CDF}^{\mathrm{LLOCA}}})^2} \tag{18}$$

where R_i^* denotes the estimate of R_i from the regression model, obtained regressing on the epistemic distributions of the input parameters. Γ_R^2 represents the fraction of the variance of the risk metric explained by the linear regression. The closer Γ_R^2 is to unity, the greater the linearity of the model. For the reference model, we found $\Gamma_R^2 = 0.12$. Thus, this tells us that the model is non-linear at the parameter level. From the one hand this result confirms our expectations in consideration of the presence of epistemic uncertainty, and, on the other hand, it has consequences on both the results of the importance measures and GSA techniques, as discussed in the following sections.

5.2. Importance measure results at the basic event and the parameter level

The FV and RAW of the basic events as produced from a standard PRA software code and the corresponding rankings are given in Table 2. Table 2 also gives the results of the computation of DIM for the basic events, and the rankings obtained with this measure. We note that DIM and FV produce the same rankings. This is due to the facts that (a) we are considering the importance of both initiating and basic events. Thus, the variables in this case are basic event probabilities (p(BE)) and initiating event frequencies (f_{LLOCA}). They are characterized by different units: p(BE), as anticipated in Section 3. The most important event according to the three measures is LLOCA, the initiating event.

The results of the importance measure analysis at the parameter level are reported in Table 3. From this table, we note that the most relevant parameter is the frequency of the initiating event, F_{LLOCA}. This parameter is also

Table 1
Model basic events list, meaning and failure probability

Basic event	Meaning	Probability	Parameter
BE_1	Operator failure to isolate FIS path after excavation error	8.000×10^{-2}	x_1
BE_2	Firewater injection system disabled by excavation error	1.250×10^{-4}	x_2
BE_3	Insufficient flow through bottom head injection	1.503×10^{-6}	x_3
BE_4	Lower FIS manual valve GT-T-84 failure to restore after TM	2.700×10^{-5}	x_4
BE_5	No flow from firewater injection system	3.484×10^{-5}	x_5
BE_6	Failure to actuate valve lcv-7b	5.001×10^{-4}	x_6
BE_7	Failure to actuate valve lcv-7a	5.001×10^{-4}	x_6
BE_8	Lower FWIS injection valve LCV-7B spuriously closes	3.000×10^{-4}	$x_7=\lambda_v=3.000\times10^{-6}$ (1/h)
BE_9	Valve LCV-7B ICC fails to operate	1.000×10^{-3}	x_8
BE_{10}	Lower FIS injection valve LCV-7B fails to open	7.000×10^{-4}	x_9
BE_{11}	Common cause failure of valve paths LCV-7A and LCV-7B to open	7.000×10^{-5}	x_{10}
BE_{12}	Common cause loss of both FIS paths due to failure of AOVs	4.300×10^{-5}	x_{11}
BE_{13}	Lower FIS injection valve LCV-7A fails to open	7.000×10^{-4}	x_9
BE_{14}	Valve LCV-7A ICC fails to operate	1.000×10^{-3}	$x_8=\lambda_v=3.000\times10^{-6}$
BE_{15}	Lower FWIS injection valve LCV-7A spuriously closes	3.000×10^{-4}	x_7
BE_{16}	Deepwell pump 1 heating and ventilation fails	1.40×10^{-2}	x_{12}
BE_{17}	Deepwell pump 1 is in TM (plant-specific)	1.940×10^{-2}	x_{13}
BE_{18}	Deepwell pump 1 fails to start	3.000×10^{-3}	x_{14}
BE_{19}	Deepwell pump 1 fails to run	2.996×10^{-3}	$x_{15}=\lambda_p=3.000\times10^{-5}$
BE_{20}	Deepwell Pump 1 instrumentation and control (ICC) fails	1.000×10^{-3}	x_{16}
BE_{21}	Deepwell Pump #1 Breaker Spuriously Opens	3.000×10^{-5}	$x_{17}=\lambda_b=3.000\times10^{-7}$
BE_{22}	Level control faults	8.383×10^{-5}	x_{19}
BE_{23}	Power failure at 4160 vac etr commercial bus 'd'	5.601×10^{4}	x_{20}
BE_{24}	Deepwell pump 3 is in TM (plant-specific)	7.050×10^{-3}	x_{18}
BE_{25}	Deepwell pump 3 fails to start	3.000×10^{-3}	x_{14}
BE_{26}	Deepwell pump 3 fails to run	2.996×10^{-3}	$x_{15}=\lambda_p=3.000\times10^{-5}$
BE_{27}	Deepwell Pump 3 instrumentation and control (ICC) fails	1.000×10^{-3}	x_{16}
BE_{28}	Deepwell Pump #3 Breaker Spuriously Opens	3.000×10^{-5}	$x_{17}=\lambda_b=3.000\times10^{-7}$
BE_{29}	Deepwell pump 3 heating and ventilation fails	1.40×10^{-2}	x_{12}
BE_{30}	Deepwell pump 4 TM (plant-specific)	2.620×10^{-2}	x_{21}
BE_{31}	Deepwell pump 4 fails to start	3.000×10^{-3}	x_{14}
BE_{32}	Deepwell pump 4 fails to run	2.996×10^{-3}	$x_{15}=\lambda_p=3.000\times10^{-5}$
BE_{33}	Deepwell Pump 4 instrumentation and control (ICC) fails	1.000×10^{-3}	x_{16}
BE_{34}	Deepwell pump #4 breaker spuriously opens	3.000×10^{-5}	$x_{17}=\lambda_b=3.000\times10^{-7}$
BE_{35}	Heating and ventilation fails for pump 4	1.40×10^{-2}	x_{12}
BE_{36}	Power failure at 4160 V atr bus 670-e-1	1.080×10^{-3}	x_{22}
BE_{37}	Common cause loss of scram system	1.500×10^{-5}	x_{23}
BE_{38}	Common cause failure of low outlet pressure sensor trains (C)	7.200×10^{-6}	x_{24}
BE_{39}	Common cause failures of low outlet pressure 2:3 logics	3.000×10^{-5}	x_{25}
BE_{40}	Failure of rod clutch coil controllers (rcccs)	2.601×10^{6}	x_{26}
BE_{41}	2/3 Sensor trains fail to signal lop sublogic u	4.776×10^{7}	x_{27}
BE_{42}	Failure to insert at least three safety rods in	6.500×10^{-7}	x_{28}
BE_{43}	Common cause failure of RCCCs to release	5.001×10^{-4}	x_{29}
BE_{44}	Failure of sufficient rcccs to release	5.001×10^{-4}	x_{30}
LL0CA	Initiating event frequency	4.56×10^{-6} (1/y)	f_{lloca}

associated with the most relevant basic event. We note that there is now a discrepancy between the rankings of $\text{DIM}(x_i)$ and $\text{FV}(x_i)$. In particular, 19 parameters are ranked in a different position by the two measures. This discrepancy shows that the model is non-linear at the parameter level, as we anticipated in the previous section.

5.3. Global sensitivity analysis results

We discuss the results of the extended FAST, reported in Table 4. For the reference model, a sample size of $N=27{,}000$ was required for the computation of the first and total indices, with a corresponding total CPU time of 110 s.

We note that the parameters ranked in the first three positions by $S_1(x_i)$, F_{LLOCA}, x_{12}, and $x_{19,}$ are responsible for about 10% of V_R. All the other parameters have values $0.0017 < S(x_i) < 0.0045$, the only exception being x_{20} that has by far the lowest $S_1(x_i)$. Their total contribution amounts to about 13% of V_R. We note that the sum of the $S_1(x_i)$ over all the parameters is equal to 0.23. This means that the portion of the variance explainable in terms of individual parameters is about 23%. Parameter groups and interactions among parameters account for 77% of V_R, as confirmed by the high values of the $S_T(x_i)$ in Table 4.

Table 2
FV and RAW and DIM of the basic events of the ATR large LLOCA sequence

Basic event	FV	FV rankings	RAW	RAW rankings	DIM	DIM rankings
BE_1	2.42×10^{-2}	16	1.28	42	1.02×10^{-2}	16
BE_2	2.42×10^{-2}	16	1.95×10^{2}	13	1.02×10^{-2}	16
BE_3	3.64×10^{-3}	31	2.42×10^{3}	2	1.53×10^{-3}	31
BE_4	6.54×10^{-2}	10	2.42×10^{3}	2	2.75×10^{-2}	10
BE_5	8.44×10^{-2}	5	2.42×10^{3}	2	3.55×10^{-2}	5
BE_6	3.03×10^{-3}	29	7.05	15	1.28×10^{-3}	29
BE_7	3.03×10^{-3}	29	7.05	15	1.28×10^{-3}	29
BE_8	1.82×10^{-3}	37	7.05	15	7.66×10^{-4}	37
BE_9	6.05×10^{-3}	23	7.04	15	2.55×10^{-3}	23
BE_{10}	4.24×10^{-3}	28	7.05	15	1.78×10^{-3}	28
BE_{11}	1.70×10^{-1}	3	2.42×10^{3}	2	7.16×10^{-2}	3
BE_{12}	1.04×10^{-1}	4	2.42×10^{3}	2	4.38×10^{-2}	4
BE_{13}	4.24×10^{-3}	27	7.05	15	1.78×10^{-3}	27
BE_{14}	6.05×10^{-3}	22	7.04	15	2.55×10^{-3}	22
BE_{15}	1.82×10^{-3}	36	7.05	15	7.66×10^{-4}	36
BE_{16}	4.70×10^{-2}	11	4.29	36	1.98×10^{-2}	11
BE_{17}	6.47×10^{-2}	13	4.27	36	2.72×10^{-2}	13
BE_{18}	1.00×10^{-2}	25	4.32	36	4.21×10^{-3}	25
BE_{19}	9.99×10^{-3}	26	4.32	36	4.21×10^{-3}	26
BE_{20}	3.34×10^{-3}	35	4.33	36	1.41×10^{-3}	35
BE_{21}	1.00×10^{-4}	42	4.33	36	4.21×10^{-5}	42
BE_{22}	2.03×10^{-1}	2	2.42×10^{3}	2	8.55×10^{-2}	2
BE_{23}	6.61×10^{-2}	12	1.17×10^{2}	14	2.78×10^{-2}	12
BE_{24}	3.37×10^{-2}	15	5.74	23	1.42×10^{-2}	15
BE_{25}	1.43×10^{-2}	24	5.76	23	6.02×10^{-3}	24
BE_{26}	1.43×10^{-2}	19	5.76	23	6.02×10^{-3}	19
BE_{27}	4.78×10^{-3}	34	5.77	23	2.01×10^{-3}	34
BE_{28}	1.43×10^{-4}	41	5.78	23	6.02×10^{-5}	41
BE_{29}	6.79×10^{-2}	6	5.71	23	2.86×10^{-2}	6
BE_{30}	1.08×10^{-1}	8	5.02	35	4.55×10^{-2}	8
BE_{31}	1.24×10^{-2}	20	5.12	29	5.22×10^{-3}	20
BE_{32}	1.24×10^{-2}	21	5.12	29	5.22×10^{-3}	21
BE_{33}	4.13×10^{-3}	33	5.12	29	1.74×10^{-3}	33
BE_{34}	1.24×10^{-4}	40	5.13	29	5.22×10^{-5}	40
BE_{35}	5.95×10^{-2}	9	5.07	34	2.50×10^{-2}	9
BE_{36}	4.46×10^{-3}	32	5.12	29	1.88×10^{-3}	32
BE_{37}	3.63×10^{-2}	14	2.42×10^{3}	2	1.53×10^{-2}	14
BE_{38}	1.74×10^{-2}	18	2.42×10^{3}	2	7.32×10^{-3}	18
BE_{39}	7.27×10^{-2}	7	2.42×10^{3}	2	3.06×10^{-2}	7
BE_{40}	1.64×10^{-8}	43	1.01	43	6.90×10^{-9}	43
BE_{41}	1.16×10^{-3}	39	2.42×10^{3}	2	4.88×10^{-4}	39
BE_{42}	1.58×10^{-3}	38	2.42×10^{3}	2	6.65×10^{-4}	38
BE_{43}	1.64×10^{-8}	44	1.01	43	6.90×10^{-9}	44
BE_{44}	7.53×10^{-12}	45	1.01	43	3.17×10^{-12}	45
LLOCA	1.00	1	2.22×10^{5}	1	4.21×10^{-1}	1

GSA results also allow us to understand how uncertainty is partitioned between the two safety systems. Table 4 tells us that out of the first 10 parameters, seven pertain to basic events of the FIS, and two pertain to the reactor SCRAM. The first ranked parameters of the SCRAM fault tree are x_{23} and x_{30}, ranked 8th and 10th, respectively. Furthermore, the sum of the $S_1(x_i)$ of the parameters associated with FIS is 0.1104, while their sum over the parameters of the SCRAM fault tree is 0.0232 (the remaining is attributed to F_{LLOCA}). These results qualitatively tell us that uncertainty in R is unequally distributed between the two safety systems, with FIS being responsible for most of it.

Suppose we have collected further data for some of the top ranked parameters with $S_1(x_i)$ and that we can, in turn, fix them to their true value. The interpretation of the $S_1(x_i)$ and $S_{\mathrm{T}}(x_i)$ indices computed through extended FAST is that the percentage reduction of V_{CDFLLOCA} will be between $S_1(x_i)$ and $S_{\mathrm{T}}(x_i)$, when x_i is fixed. The actual reduction in the output variance V_R that is obtained by fixing x_i to its true value is obtained by re-computing the whole uncertainty analysis, with assumed x_i known. We fixed the values of the first 10 parameters of Table 4, to assess the reduction in uncertainty that the knowledge of their value would bring. We obtained a reduction in V_{CDFLLOCA} of almost two orders

Table 3
Point estimate results and rankings for FV and DIM at the parameter level

Parameter	$FV(x_i)$	$DIM(x_i)$	FV rankings	FV Savage scores	DIM rankings	DIM Savage scores
x_1	0.024	0.010	16	0.7757	16	0.7090
x_2	0.024	0.010	16	0.7757	16	0.7090
x_3	0.0036	0.001537	23	0.3364	24	0.2930
x_4	0.066	0.02761	9	1.1983	10	1.1983
x_5	0.085	0.03562	7	1.7439	7	1.5772
x_6	0.0055	0.00255	21	0.4295	22	0.3819
x_7	0.0034	0.00153	24	0.3819	25	0.2513
x_8	0.0097	0.005114	20	0.5321	20	0.4795
x_9	0.0000030	0.00357	31	0.0323	21	0.4295
x_{10}	0.17	0.07157	3	2.5272	4	2.1939
x_{11}	0.10	0.04396	5	1.4344	6	1.7439
x_{12}	0.11	0.07244	4	2.1939	3	2.5272
x_{13}	0.064	0.02703	11	1.0074	11	1.0983
x_{14}	0.033	0.01541	15	0.6465	12	1.0074
x_{15}	0.034	0.01536	13	0.9240	13	0.9240
x_{16}	0.012	0.00513	19	0.4795	19	0.5321
x_{17}	0.00034	0.0001541	27	0.1728	28	0.1358
x_{18}	0.034	0.01414	13	0.9240	15	0.7757
x_{19}	0.20	0.0857	2	3.0272	2	3.0272
x_{20}	0.066	0.02771	9	1.1983	9	1.3094
x_{21}	0.10	0.04557	5	1.4344	5	1.9439
x_{22}	0.0044	0.00188	22	0.2930	23	0.3364
x_{23}	0.036	0.01533	12	1.0983	14	0.8471
x_{24}	0.017	0.007362	18	0.5877	18	0.5877
x_{25}	0.073	0.030675	8	1.5772	8	1.4344
x_{26}	0.0000064	2.71166×10^{-6}	28	0.1358	29	0.1001
x_{27}	0.0012	0.000488	26	0.2113	27	0.1728
x_{28}	0.0016	0.000664	25	0.2513	26	0.2113
x_{29}	0.0000032	1.35583×10^{-6}	30	0.0656	30	0.0656
x_{30}	0.0000032	1.35583×10^{-6}	30	0.0656	30	0.0656
F_{LLOCA}	1.00	0.421351315	1	4.0272	1	4.0272

of magnitude (i.e. from 10^{-15} to 10^{-17}) with the EF falling from 12 to 9.94.

5.4. Uncertainty drivers vs. safety significant contributors

To understand whether uncertainty drivers are risk significant contributors, it is necessary to compare importance measure results to GSA results. Such a comparison is not obvious when dealing with complex PSA models for several reasons. Standard software codes, as for example SAPHIRE, compute R at the basic event level (Eq. (1)) [5, 6]. The model size, often of several hundreds basic events and parameters, requires the PSA model to be solved using computer codes and, it is often not possible to get an analytical expression for R. Thus, deriving the expression for the risk metric as a function of the parameters is not possible. In addition importance measures (FV and RAW) are traditionally defined and computed for basic events and components. We have seen that GSA techniques deal with parameters. This means that a direct comparison of the results of standard PSA codes for importance measures and GSA results is not possible. However, let us for a moment suppose to have the results of importance analysis and GSA techniques at the parameter level. In this case, we could develop the Savage scores from the importance measure rankings and the GSA rankings [7]. Savage scores are utilized in the literature to compare the parameter rankings of different sensitivity analysis techniques [7,17,18]. They are defined as follows

$$\xi(x_i) = \sum_{j=r(x_i)}^{k} 1/j, \tag{19}$$

where $\xi(x_i)$ is the Savage score of parameter x_i, $r(x_i)$ is the rank of parameter x_i and k is the total number of parameters.

The degree of agreement of the rankings of two sensitivity analysis techniques, SA_1 and SA_2, is quantified by the correlation coefficient of the Savage scores of the two techniques. The correlation coefficient computed on Savage scores is defined as

$$\rho_{\xi}^{SA_i/SA_k} = \frac{\sum_{j=1}^{n} (\xi^{SA_i}(x_j) - \overline{\xi^{SA_i}(x_j)})(\xi^{SA_k}(x_j) - \overline{\xi^{SA_k}(x_j)})}{\sqrt{\sum_{j=1}^{n} \xi^{SA_i}(x_j) - \overline{\xi^{SA_i}(x_j)})^2 \sum_{j=1}^{n} (\xi^{SA_k}(x_j) - \overline{\xi^{SA_k}(x_j)})^2}} \tag{20}$$

Table 4
First order indexes (S_i) and total order indexes (S_{Ti}) for the parameters of the Large LLOCA sequence

Parameter	$S_1(x_i)$	$S_T(x_i)$	Rank	Savage scores
x_1	0.0027	0.602582	19	0.5321
x_2	0.0023	0.557544	23	0.3364
x_3	0.0047	0.837081	4	2.1939
x_4	0.0042	0.810726	7	1.5772
x_5	0.0014	0.469973	30	0.0656
x_6	0.0022	0.627429	25	0.2513
x_7	0.0046	0.748013	5	1.9439
x_8	0.0017	0.521022	29	0.1001
x_9	0.0031	0.721116	15	0.7757
x_{10}	0.0043	0.814946	6	1.7439
x_{11}	0.003	0.47721	16	0.7090
x_{12}	0.0437	0.815207	2	3.0272
x_{13}	0.002	0.554586	26	0.2113
x_{14}	0.0036	0.760575	14	0.8471
x_{15}	0.0038	0.7626	11	1.0983
x_{16}	0.0036	0.783453	13	0.9240
x_{17}	0.0027	0.71104	18	0.5877
x_{18}	0.0019	0.623066	27	0.1728
x_{19}	0.0075	0.783292	3	2.5272
x_{20}	0.000873	0.52003	31	0.0323
x_{21}	0.0041	0.65086	9	1.3094
x_{22}	0.0024	0.616027	21	0.4295
x_{23}	0.0041	0.758641	8	1.4344
x_{24}	0.0029	0.557622	17	0.6465
x_{25}	0.0018	0.514783	28	0.1358
x_{26}	0.0036	0.70982	12	1.0074
x_{27}	0.0022	0.579769	24	0.2930
x_{28}	0.0024	0.561865	20	0.4795
x_{29}	0.0023	0.45943	22	0.3819
x_{30}	0.0039	0.788219	10	1.1983
f_{LLOCA}	0.0932	0.838342	1	4.0272

where n is the total number of parameters, $\overline{\xi^{\text{SA}_i}(x_j)}$ is the average Savage score that the sensitivity analysis technique SA_i attributes to parameter x_j and $\overline{\xi^{\text{SA}_i}(x_j)}=1$ independently of the number of parameters. Savage scores are used instead of simple rankings in order to emphasize the agreement of the measures on the top ranked variables [7].

From Eq. (20) we see that, if two measures produce the same parameter rankings, then the parameters will have the same scores. Therefore $\rho_\xi^{\text{SA}_i/\text{SA}_k}$ will be unity. In general, the closer $\rho_\xi^{\text{SA}_i/\text{SA}_k}$ is to unity, the higher the agreement in the rankings obtained with the two measures. A negative value of $\rho_\xi^{\text{SA}_i/\text{SA}_k}$ means that the two measures tend to give opposite rankings, i.e. parameters that are ranked high by one technique tend to be low ranked by the other.

The correlation coefficient on the scores of importance measures and GSA techniques gives quantitative information on whether parameters that are ranked first by importance measures (i.e. are risk significant) are also drivers of the uncertainty of R. In fact, a high value of the correlation coefficient would mean that uncertainty contributors tend to be also risk significant elements.

Table 5
Comparison steps for PSA importance measures and GSA techniques

No.	Step
1	Associate each parameter with the corresponding basic event/events
2	Get the parameter importance measure ranking as average of the basic event rankings
3	Compute the Savage scores on the rankings
4	Compute the correlation coefficient on the scores

To enable the computation of the scores, when dealing with PSA model results produced by PSA standard codes, we have to apply some intermediate steps, since importance measures are computed for basic events [5,6]. We propose to (1) associate to each parameter the corresponding basic event/events. The rankings of the various techniques can be converted into Savage scores, and the correlation coefficients can be computed. These steps are summarized in Table 5.

The application of these steps to the ATR Large LLOCA sequence is summarized in Table 6. The Savage scores are reported in columns 5 and 8. The correlation coefficients of the RAW and extended FAST, $\rho_\xi^{\text{RAW(BE)}/S_i}$, and FV and extended FAST $\rho_\xi^{\text{FV(BE)}/S_i}$, are:

$$\rho_\xi^{\text{RAW(BE)}/S_1}=0.51 \quad \text{and} \quad \rho_\xi^{\text{FV(BE)}/S_1}=0.49 \tag{21}$$

These values indicate that the agreement between RAW and extended FAST, and between FV and extended FAST is, in general, weak. Thus, we can say that uncertainty contributors do not tend to coincide with risk contributors for the model at hand. However, we note that the initiating event, LLOCA is ranked first by both FV and RAW, and its corresponding parameter, F_{LLOCA}, is ranked first by extended FAST. This means that, by improving our knowledge in F_{LLOCA}, we would acquire better knowledge on a risk significant contributor while efficiently reducing the uncertainty in the $\text{CDF}^{\text{LLOCA}}$.

The comparison of DIM, FV, and extended FAST rankings at the parameter level[1] can be done in a straightforward manner, since all these measures are defined for parameters. We can compute directly the Savage scores obtained from the parameter rankings according to FV(x_i) and DIM(x_i) and extended FAST. These scores are given in Tables 3 and 4.

The Savage scores correlation coefficients turn out to be:

$$\rho_\xi^{\text{DIM}(x)/S_1(x_i)}=0.64 \quad \text{and} \quad \rho_\xi^{\text{FV}(x)/S_1(x_i)}=0.62 \tag{22}$$

The intermediate values of these correlation coefficients show that risk significant parameters do not tend to coincide

[1] As mentioned, DIM and FV for parameters cannot be estimated through a standard code. This calculations were done transposing the cut sets expression for the reference example into a mathematical software.

Table 6
RAW(BE) and FV(BE) Savage scores for comparison with extended FAST results

Parameter	Associated basic event	Raw basic event ranking	Raw average ranking	Raw Savage scores	FV basic event ranking	FV average ranking	FV Savage scores
x_1	BE_1	42	28	0.092	16	14	1.0767
x_2	BE_2	13	13	1.29	16	15	1.0767
x_3	BE_3	2	2	3.39	31	22	3.9996×10^{-1}
x_4	BE_4	2	3	3.39	10	9	1.5660
x_5	BE_5	2	4	3.39	5	5	2.3116
x_6	BE_6, BE_7	15, 15	15	1.14	29, 29	21	4.6778×10^{-1}
x_7	BE_8, BE_{15}	15, 15	15	1.14	37, 36	25	2.2313×10^{-1}
x_8	BE_9, BE_{14}	15, 15	15	1.14	23, 22	18	7.1555×10^{-1}
x_9	BE_{10}, BE_{13}	15, 15	15	1.14	28, 27	20	5.1079×10^{-1}
x_{10}	BE_{11}	2	5	3.39	3	3	2.8949
x_{11}	BE_{12}	2	6	3.39	4	4	2.5616
x_{12}	BE_{16}, BE_{29}, BE_{35}	36, 23, 34	25	0.41	11, 6, 9	8	1.7094
x_{13}	BE_{17}	36	27	0.25	13	11	1.2917
x_{14}	BE_{18}, BE_{25}, BE_{31}	36, 23, 29	20	0.47	25, 24, 20	19	7.0413×10^{-1}
x_{15}	BE_{19}, BE_{26}, BE_{32}	36, 23, 29	21	0.47	26, 19, 21	17	7.4959×10^{-1}
x_{16}	BE_{20}, BE_{27}, BE_{33}	36, 23, 29	22	0.47	35, 34, 33	24	3.0615×10^{-1}
x_{17}	BE_{21}, BE_{28}, BE_{34}	36, 23, 29	23	0.47	42, 41, 40	28	1.1641×10^{-1}
x_{18}	BE_{24}	23	19	0.70	15	13	1.1434
x_{19}	BE_{22}	2	7	3.39	2	2	3.3949
x_{20}	BE_{23}	14	14	1.21	12	10	1.3751
x_{21}	BE_{30}	35	26	0.28	8	7	1.8021
x_{22}	BE_{36}	29	24	0.28	32	23	3.6770×10^{-1}
x_{23}	BE_{37}	2	8	3.39	14	12	1.2148
x_{24}	BE_{38}	2	9	3.39	18	16	9.5540×10^{-1}
x_{25}	BE_{39}	2	10	3.39	7	6	1.9449
x_{26}	BE_{40}	43	29	0.07	43	29	6.8205×10^{-2}
x_{27}	BE_{41}	2	11	3.39	39	27	1.6705×10^{-1}
x_{28}	BE_{42}	2	12	3.39	38	26	1.9336×10^{-1}
x_{29}	BE_{43}	43	30	0.07	44	30	4.4949×10^{-2}
x_{30}	BE_{44}	43	31	0.07	45	31	2.2222×10^{-2}
F_{LLOCA}	Initiating event: LLOCA	1	1	4.39	1	1	4.3949

with uncertainty drivers. This was the case at the basic event level also. We note that the correlation between FV(x_i) and extended FAST is now higher than at the basic event level. This is explained by the definition of FV(x_i) (Eq. (8)). If a parameter appears in several terms, it is likely that it will have a high FV and drive the model uncertainty. We note that again F_{LLOCA} is ranked first by all the techniques. Thus, the analysis at the parameter level confirms that F_{LLOCA} is important both as uncertainty driver and risk contributor.

We finally note that, while the previous analysis required some intermediate steps to compare importance measures to GSA results, the use of DIM(x) and FV(x) at the parameter level enables a direct comparison.

6. Conclusions

We have discussed the application and results of GSA techniques to PSA models, focusing on variance-based techniques, computed through extended FAST. We have seen that the presence of epistemic uncertainty makes it necessary to perform GSA at the parameter level of the model.

Application and results of the extended FAST technique for the computation of the first and total order sensitivity indices for the parameters of the reference model have been discussed. We have computed the reduction in the risk metric variance that we obtain if the parameters ranked in the first 10 positions by first order global sensitivity indices were known with certainty. We have seen that we would be able to reduce V_R by two order of magnitudes. We have analyzed how uncertainty is partitioned between the two safety systems. We have found that parameters of the FIS are responsible for most of the uncertainty.

We have seen that, to understand whether uncertainty drivers are also associated to risk significant element, we must compare GSA results to importance measure results. Since R is computed at the basic event level, and FV and RAW are produced for basic events by standard software codes [4,6], a direct comparison of importance measure and GSA results is not possible. Thus, the results of such a comparison are to be considered qualitative. However, using DIM and FV at the parameter level the comparison is direct and quantitative results can be obtained.

The comparison of risk contributors and uncertainty drivers at the basic event level for model at hand has produced an intermediate value of the correlation coefficient, indicating that uncertainty drivers are not necessarily risk significant contributors. However, LLOCA, the initiating event, is ranked first by all the measures. This means that getting information to reduce the uncertainty in the initiating event frequency (LLOCA), would allow a reduction in the uncertainty of an important risk contributors, while effectively reducing our uncertainty in the CDF^{LLOCA}.

At the parameter level, we have obtained again an intermediate correlation for the rankings obtained with $FV(x_j)$ and DIM (x_j) and those obtained with extended FAST. This means that parameters that are important to risk are not necessarily uncertainty drivers. The most important parameter according to this analysis at the basic event level and confirms that this parameter is the most significant in both the model uncertainty and determination of the risk.

The analyst can then utilize this information to allocate resources and prioritize information and data collection for the model. In this respect, GSA techniques provide useful analytical capabilities that respond to the need of improving uncertainty analysis of PSA models.

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