

Sensitivity analysis

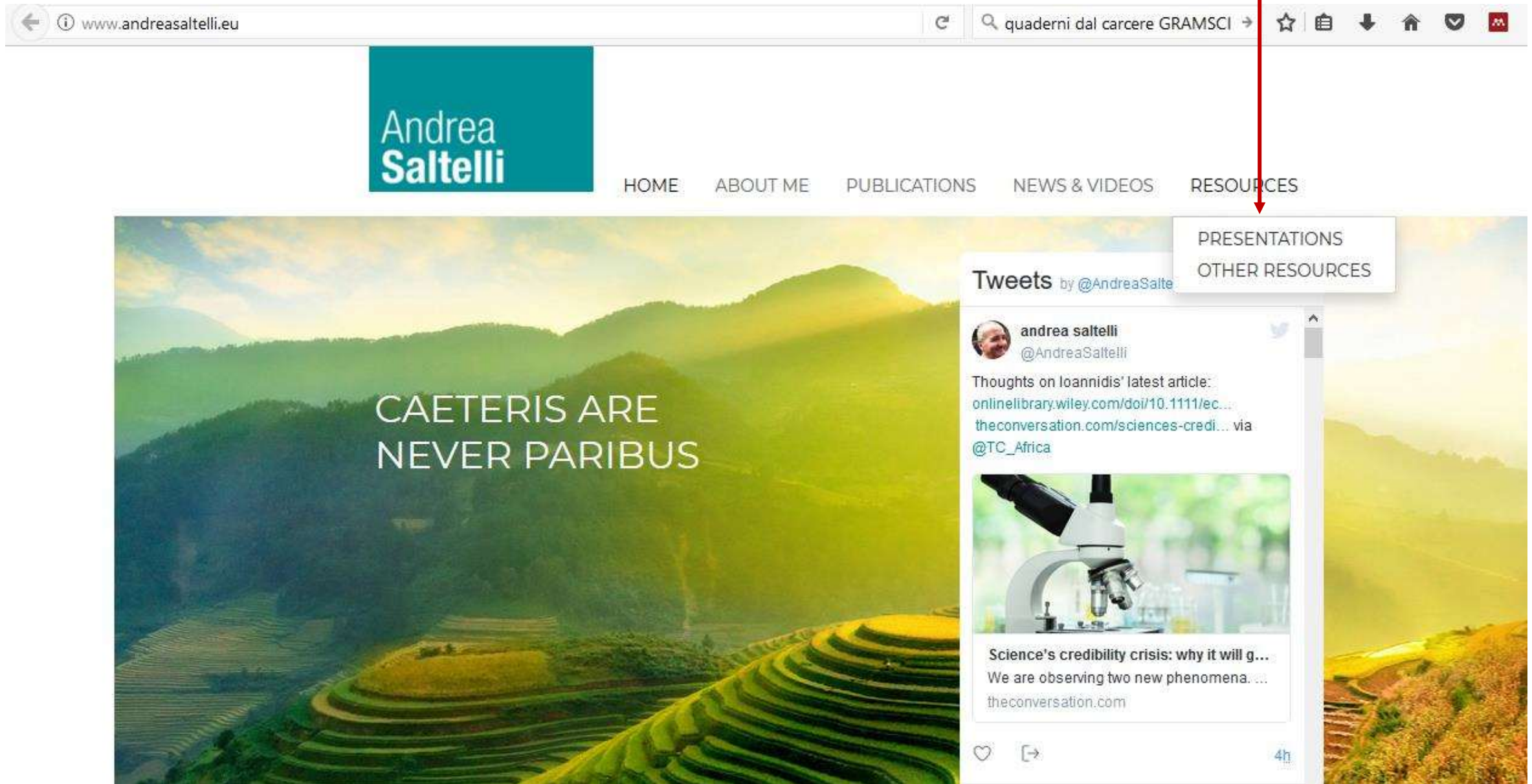
Andrea Saltelli

Centre for the Study of the Sciences and the Humanities, University of Bergen, and Open Evidence Research, Open University of Catalonia

Berkeley, September 24-26, 2018



Where to find the talks: www.andreasaltelli.eu



The image shows a screenshot of the website www.andreasaltelli.eu. The browser address bar shows the URL. The website header features the name "Andrea Saltelli" in a teal box. Below the header is a navigation menu with the following items: HOME, ABOUT ME, PUBLICATIONS, NEWS & VIDEOS, and RESOURCES. A red arrow points from the "RESOURCES" link to a dropdown menu that contains "PRESENTATIONS" and "OTHER RESOURCES". The main content area features a large background image of terraced rice fields with the text "CAETERIS ARE NEVER PARIBUS" overlaid. On the right side, there is a "Tweets" section by @AndreaSalte, featuring a tweet from andrea saltelli (@AndreaSaltelli) discussing a science credibility crisis, accompanied by a photo of a microscope.

www.andreasaltelli.eu

Andrea Saltelli

HOME ABOUT ME PUBLICATIONS NEWS & VIDEOS RESOURCES

PRESENTATIONS OTHER RESOURCES

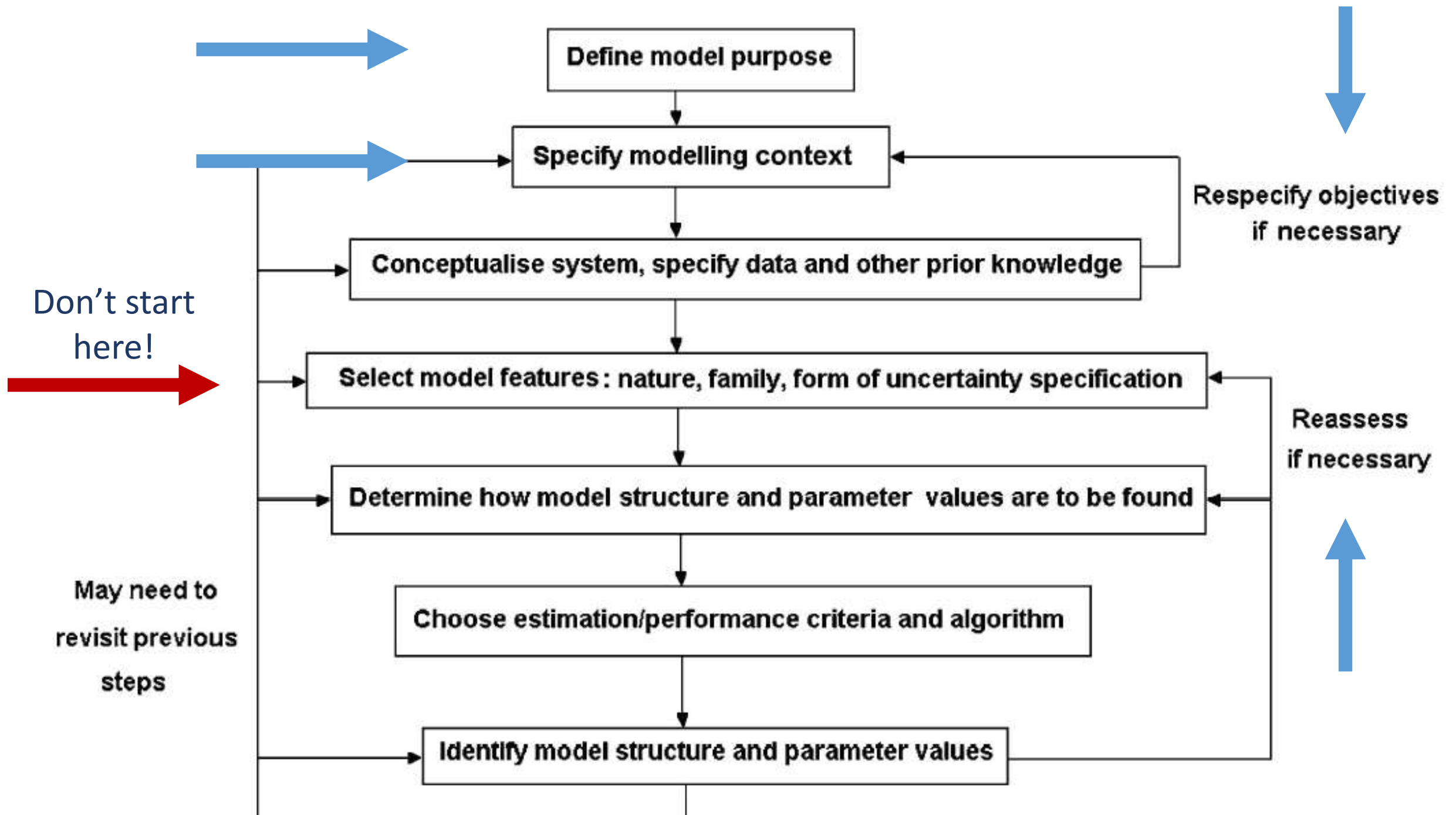
Tweets by @AndreaSalte

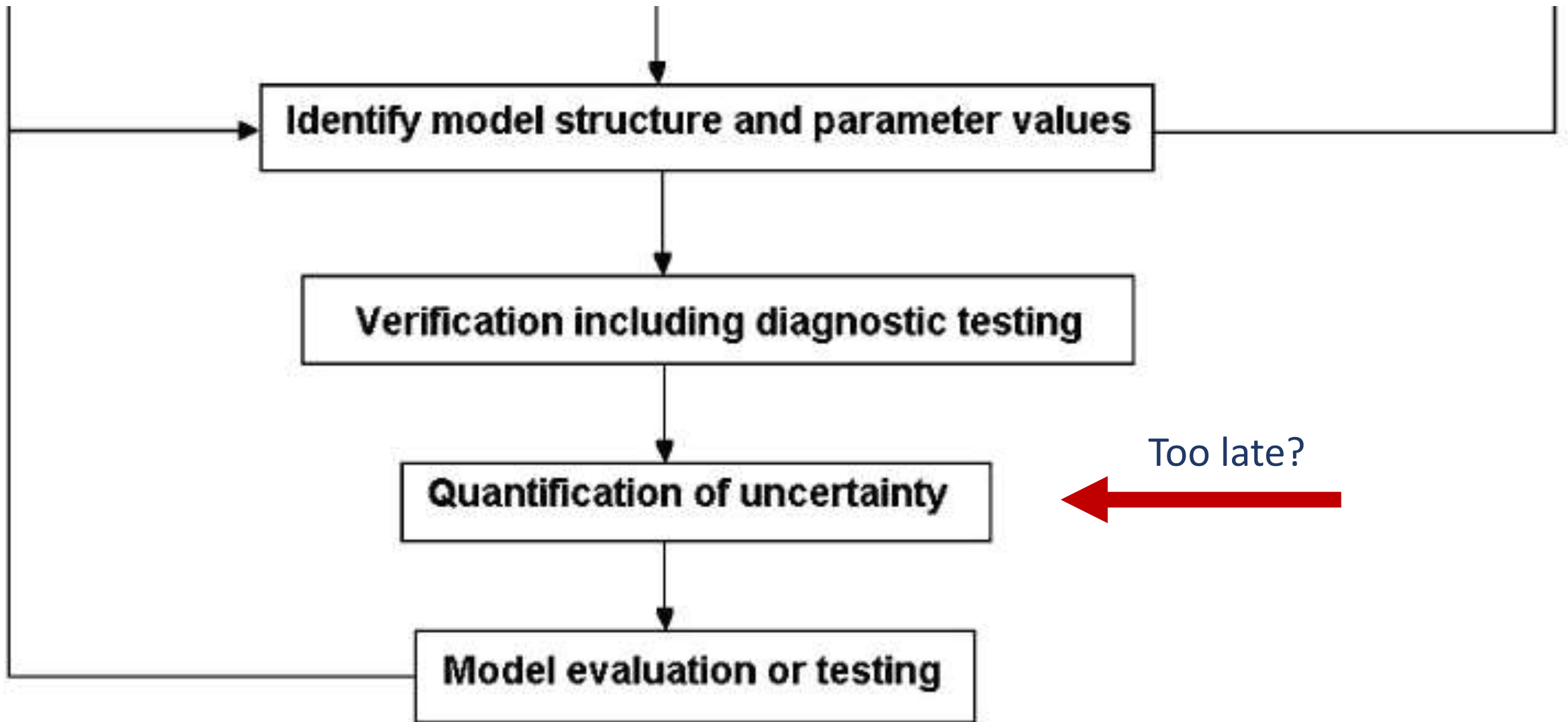
andrea saltelli @AndreaSaltelli

Thoughts on Ioannidis' latest article: onlinelibrary.wiley.com/doi/10.1111/ec... theconversation.com/sciences-credi... via @TC_Africa

Science's credibility crisis: why it will g... We are observing two new phenomena... theconversation.com

4h





Not a discipline

Unlike statistics, mathematical modelling is not a discipline, hence the lack of universally accepted quality standards, disciplinary fora and journals and recognized leaders

Making sensitivity analysis part of the syllabus of statistics?

Peder Sather project with Berkeley, Bergen, Trondheim universities

Saltelli, A., Does Modelling need a reformation? Ideas for a new grammar of modelling, available at <https://arxiv.org/abs/1712.06457>

Uncertainty and sensitivity analysis

Definitions

Uncertainty analysis: Focuses on just quantifying the uncertainty in model output

Sensitivity analysis: The study of the relative importance of different input factors on the model output

4. SENSITIVITY AND UNCERTAINTY ANALYSES

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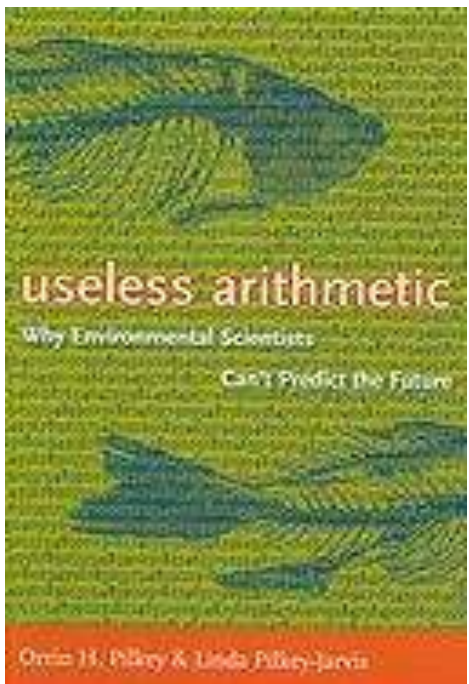
Six steps for a global SA:

1. Select one output of interest;
2. Participatory step: discuss which input may matter;
3. Participatory step (extended peer review): define distributions;
4. Sample from the distributions;
5. Run (=evaluate) the model for the sampled values;
6. Obtain in this way both the uncertainty of the prediction and the relative importance of variables.

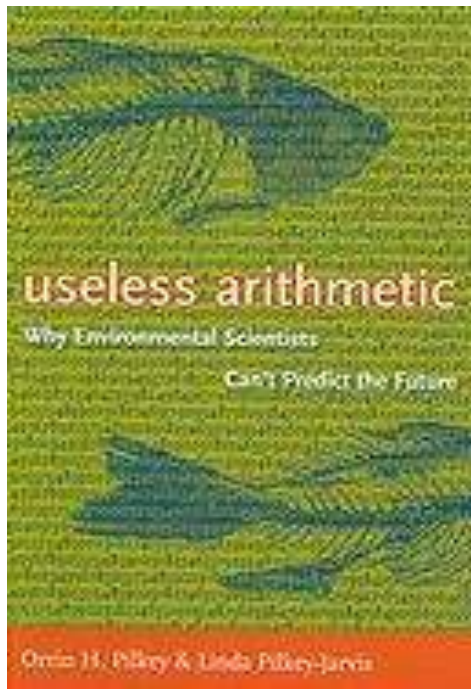
Limits of sensitivity analysis



Orrin H.
Pilkey



Useless Arithmetic: Why
Environmental Scientists Can't
Predict the Future
by Orrin H. Pilkey and Linda
Pilkey-Jarvis

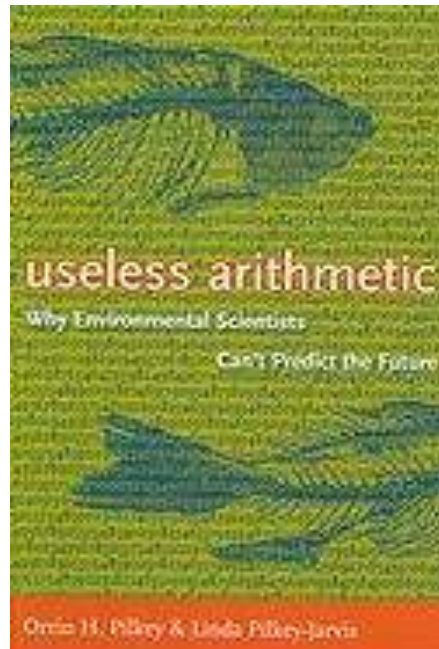


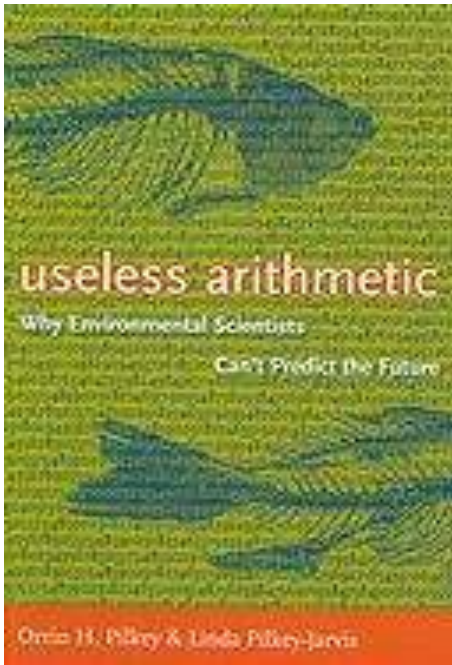
<<It is important, however, to recognize that the sensitivity of the parameter in the equation is what is being determined, not the sensitivity of the parameter in nature.

[...] If the model is wrong or if it is a poor representation of reality, determining the sensitivity of an individual parameter in the model is a meaningless pursuit.>>

One of the examples discussed concerns the **Yucca Mountain** repository for radioactive waste. TSPA model (for total system performance assessment) for safety analysis.

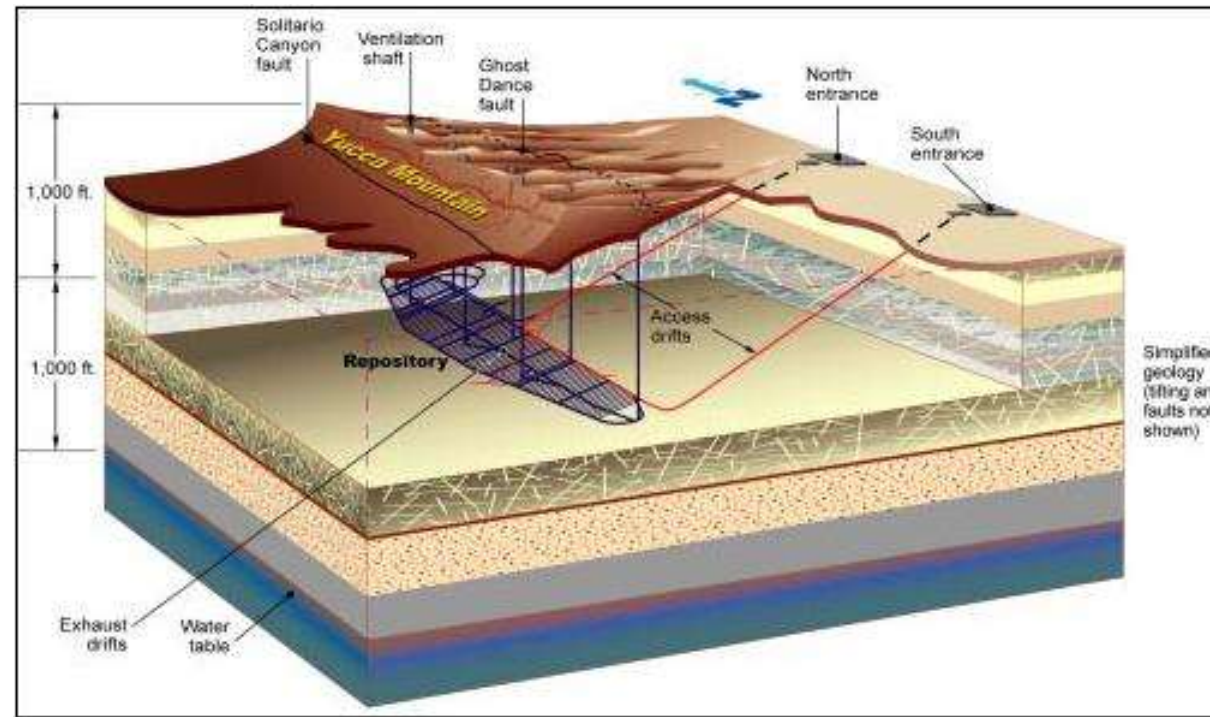
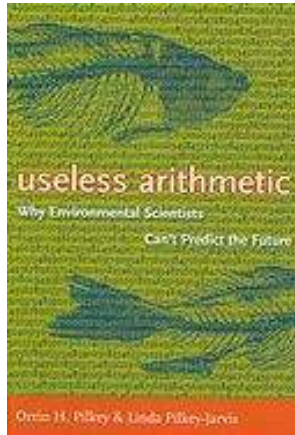
TSPA is Composed of 286 sub-models.





TSPA (like any other model) **relies on assumptions** → one is the low permeability of the geological formation → long time for the water to percolate from surface to disposal.



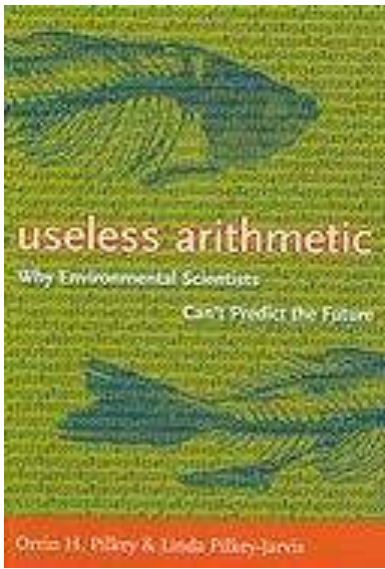


The confidence of the stakeholders in TSPA was not helped when evidence was produced which could lead to an upward revision of 4 orders of magnitude of this parameter
(the ^{36}Cl story)

Type III error in sensitivity:
Examples:

In the case of TSPA (Yucca mountain) a range of 0.02 to 1 millimetre per year was used for percolation of flux rate.

→... SA useless if it is instead ~ 3,000 millimetres per year.



“Scientific mathematical modelling should involve constant efforts to falsify the model”

Ref. → Robert K. Merton’s ‘Organized skepticism’



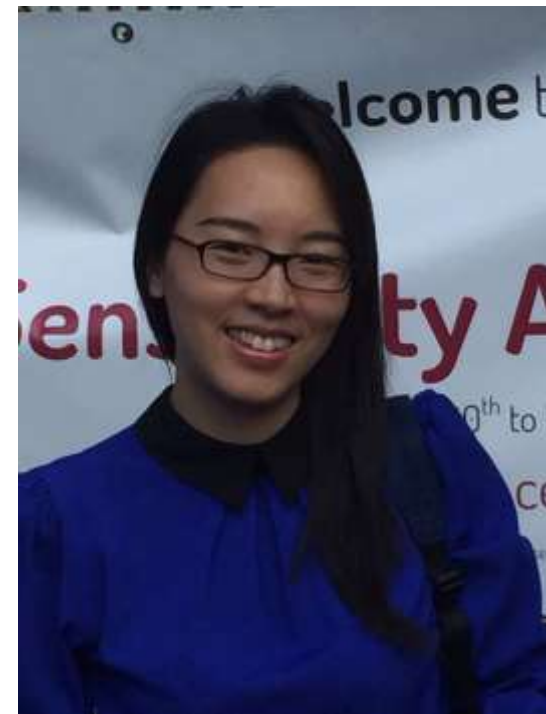
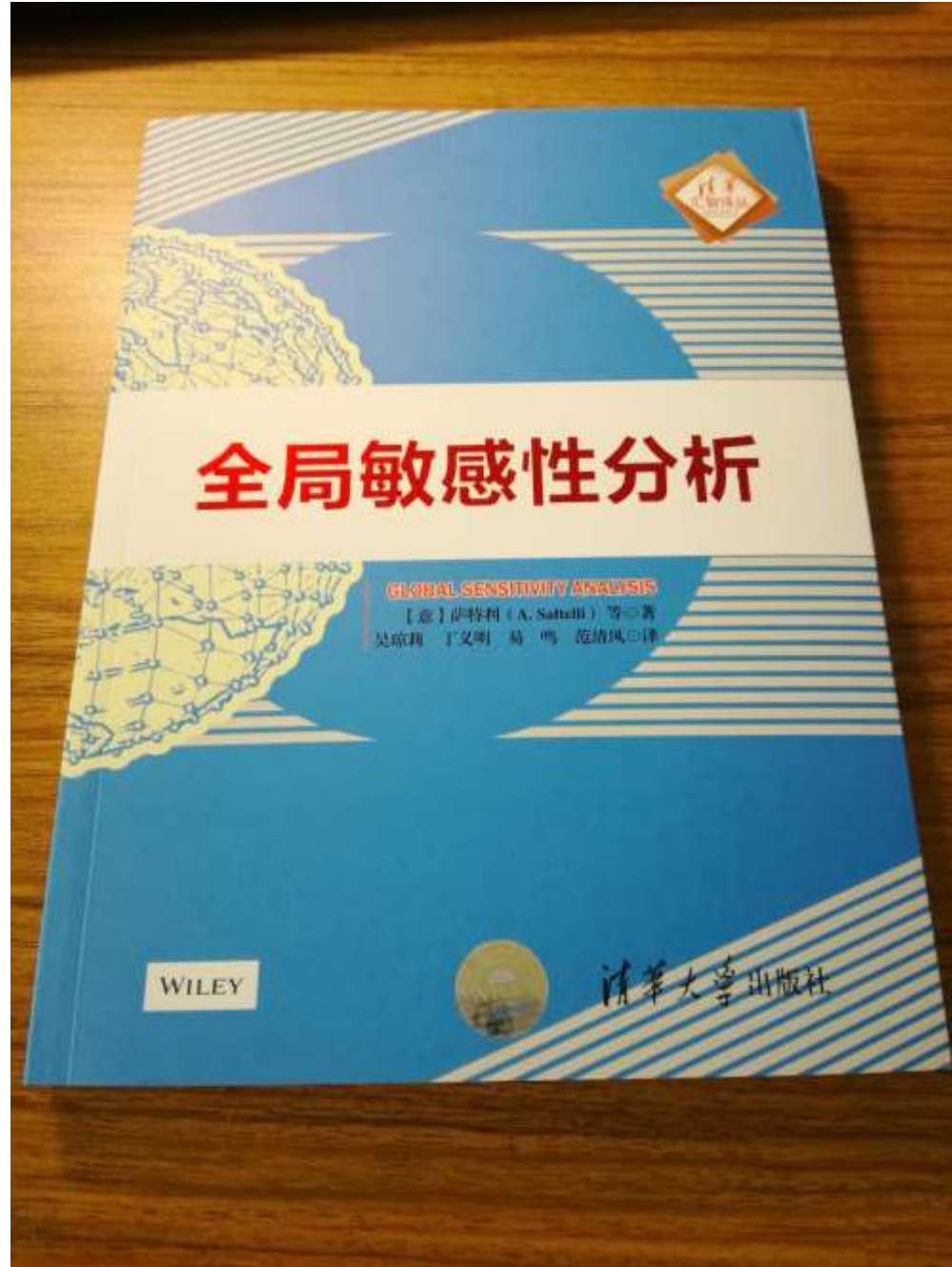
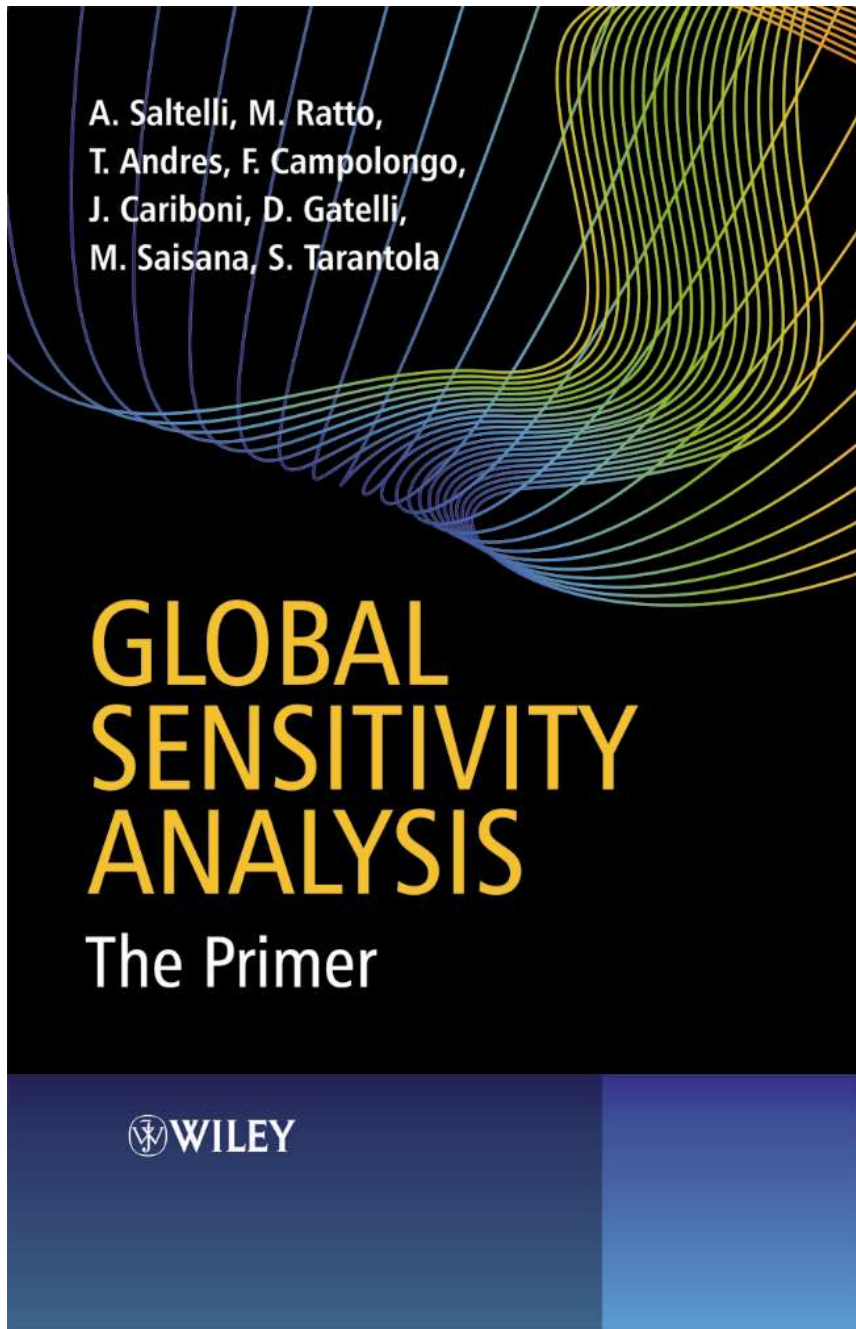
Robert K. Merton

Communalism – the common ownership of scientific discoveries, according to which scientists give up intellectual property rights in exchange for recognition and esteem (Merton actually used the term Communism, but had this notion of communalism in mind, not Marxism);

Universalism – according to which claims to truth are evaluated in terms of universal or impersonal criteria, and not on the basis of race, class, gender, religion, or nationality;

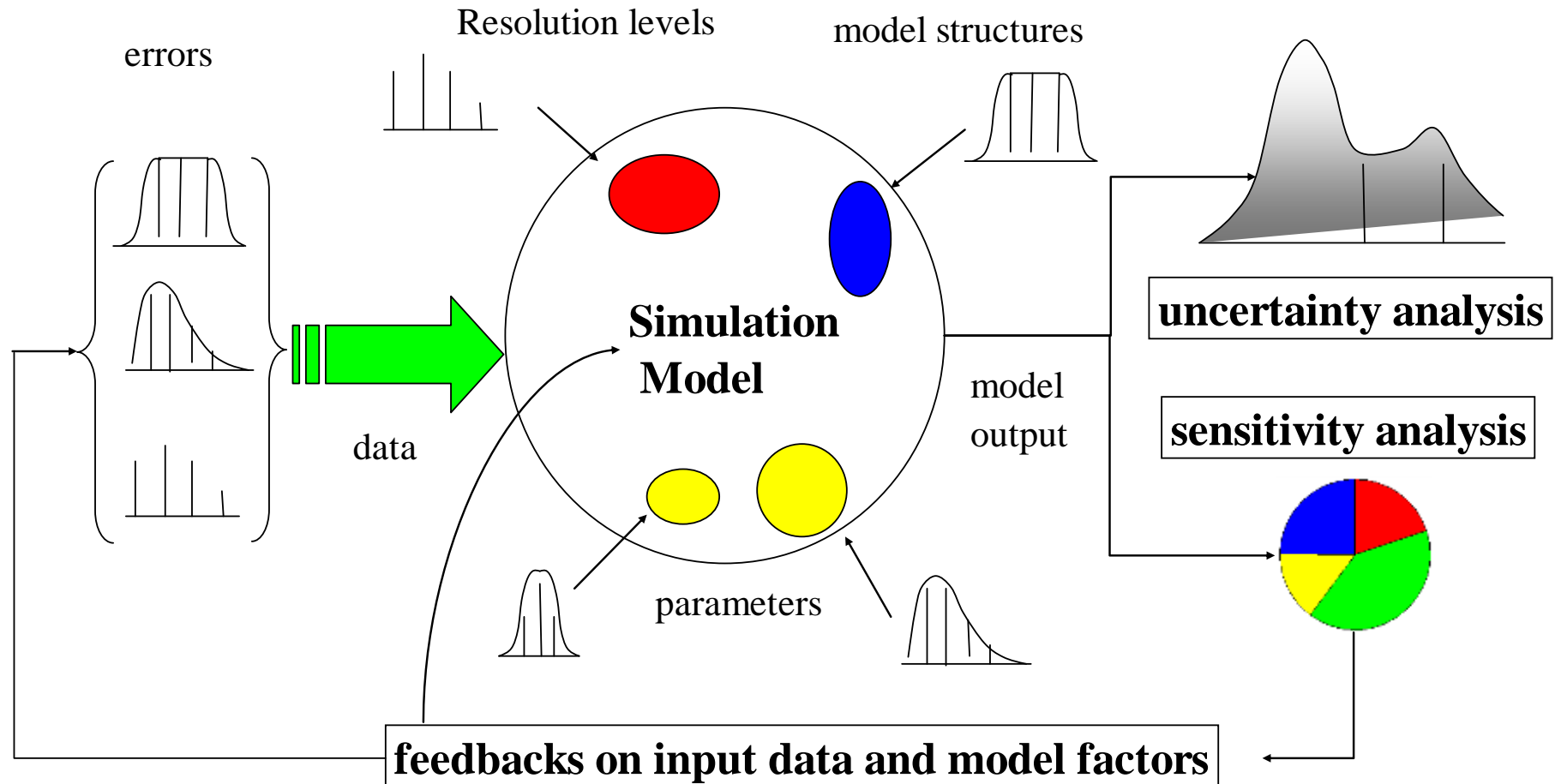
Disinterestedness – according to which scientists are rewarded for acting in ways that outwardly appear to be selfless;

Organized Skepticism – all ideas must be tested and are subject to rigorous, structured community scrutiny.



Wu
Qiongli

An engineer's vision of UA, SA

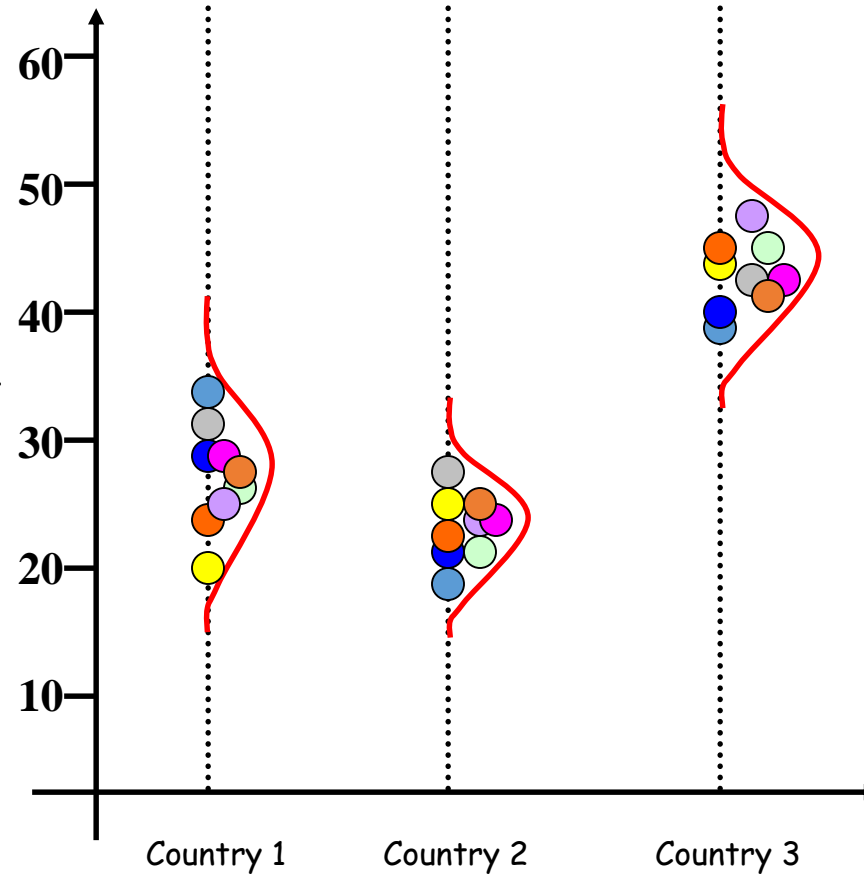
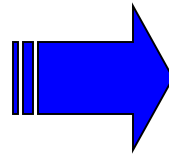
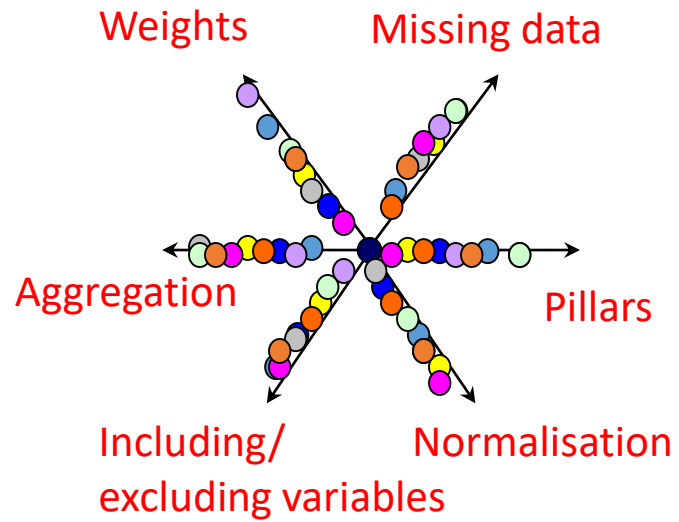


One can sample more than just factors

One can sample modelling assumptions,
alternative data sets, resolution levels,
scenarios ...

Assumption	Alternatives
Number of indicators	<ul style="list-style-type: none">▪ all six indicators included or one-at-time excluded (6 options)
Weighting method	<ul style="list-style-type: none">▪ original set of weights,▪ factor analysis,▪ equal weighting,▪ data envelopment analysis
Aggregation rule	<ul style="list-style-type: none">▪ additive,▪ multiplicative,▪ Borda multi-criterion

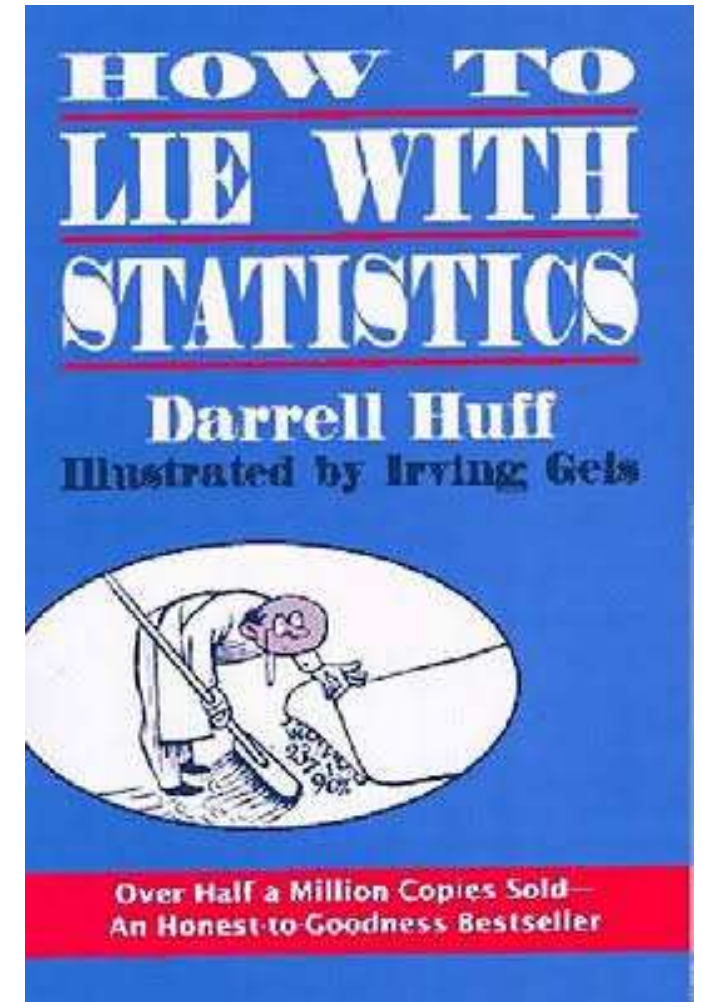
Space of alternatives



EVIDENCE,
ARGUMENT, &
PERSUASION IN
THE POLICY
PROCESS
GIANDOMENICO
MAJONE

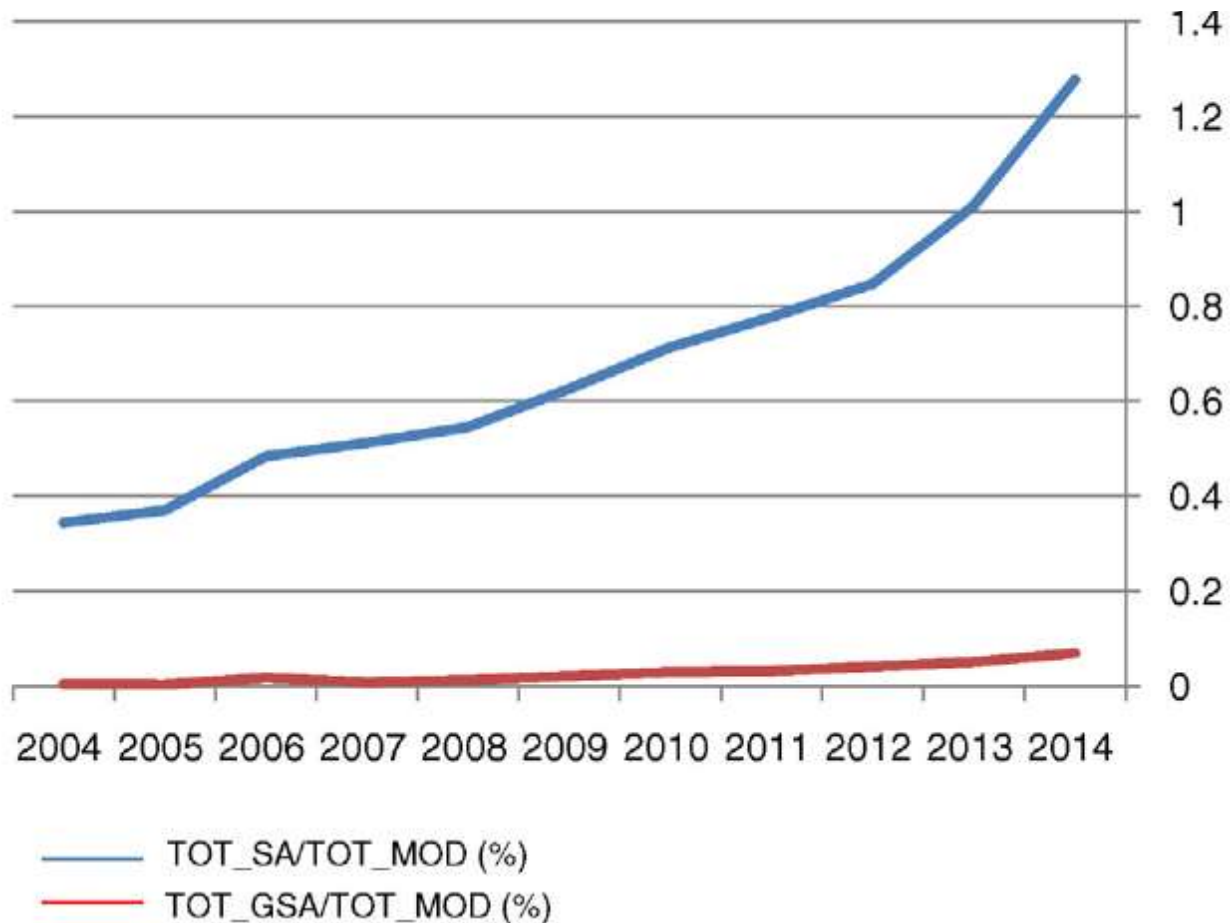
"Are the results from a particular model more sensitive to changes in the model and the methods used to estimate its parameters, or to changes in the data?"

Can one lie with sensitivity analysis as one can lie with statistics?



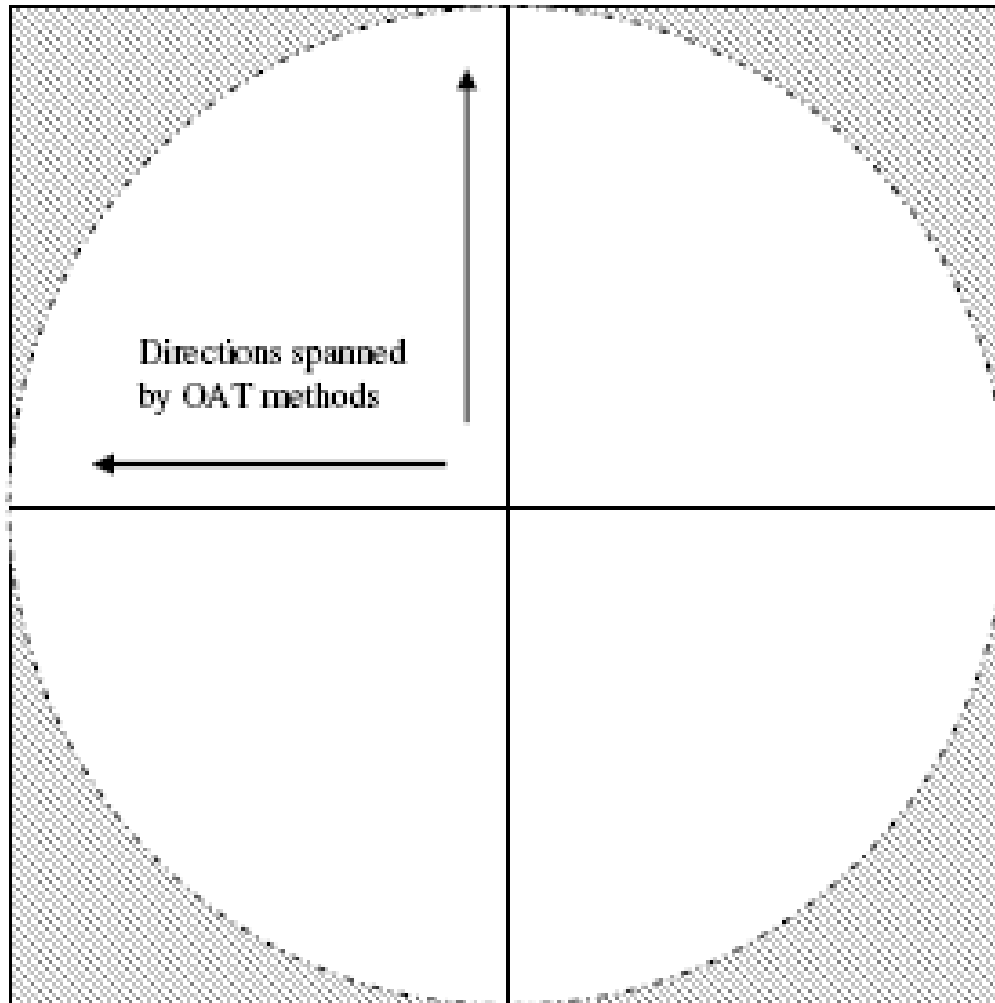
Saltelli, A., Annoni P., 2010, How to avoid a perfunctory sensitivity analysis, Environmental Modeling and Software, 25, 1508–1517.

In 2014 out of 1000 papers in modelling 12 have a sensitivity analysis and < 1 a global SA; most SA still move one factor at a time



Ferretti, F., Saltelli A., Tarantola, S., 2016, Trends in Sensitivity Analysis practice in the last decade, Science of the Total Environment, <http://dx.doi.org/10.1016/j.scitotenv.2016.02.133>

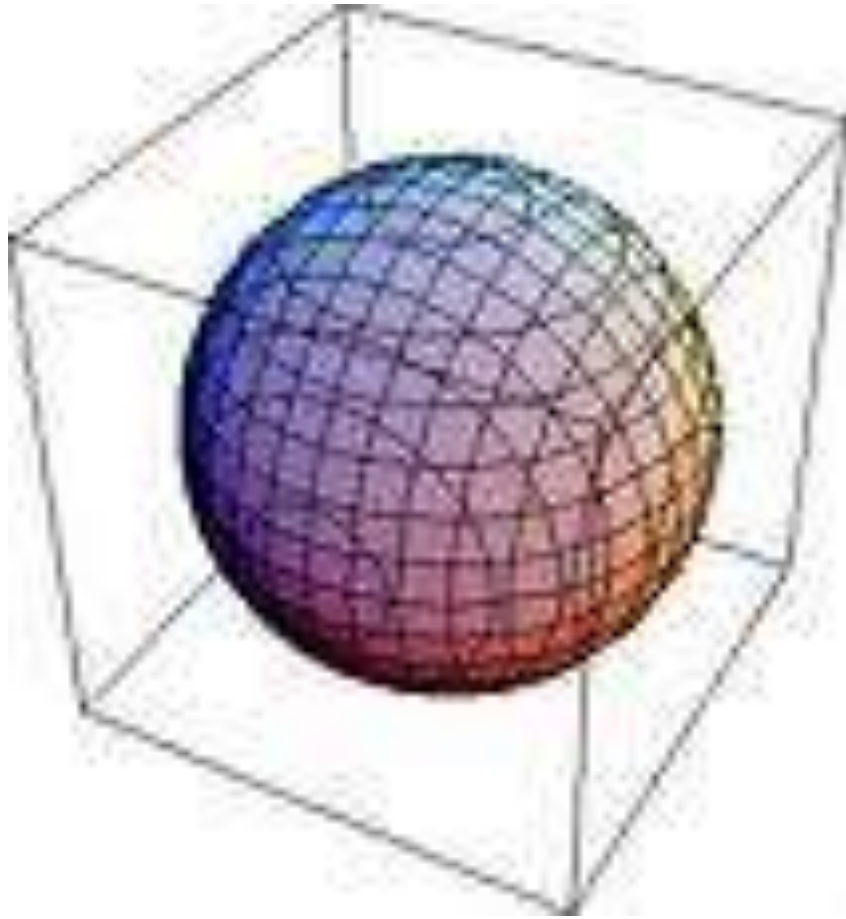
OAT in 2 dimensions



Area circle
/ area
square = ?

$\sim 3/4$

OAT in 3 dimensions



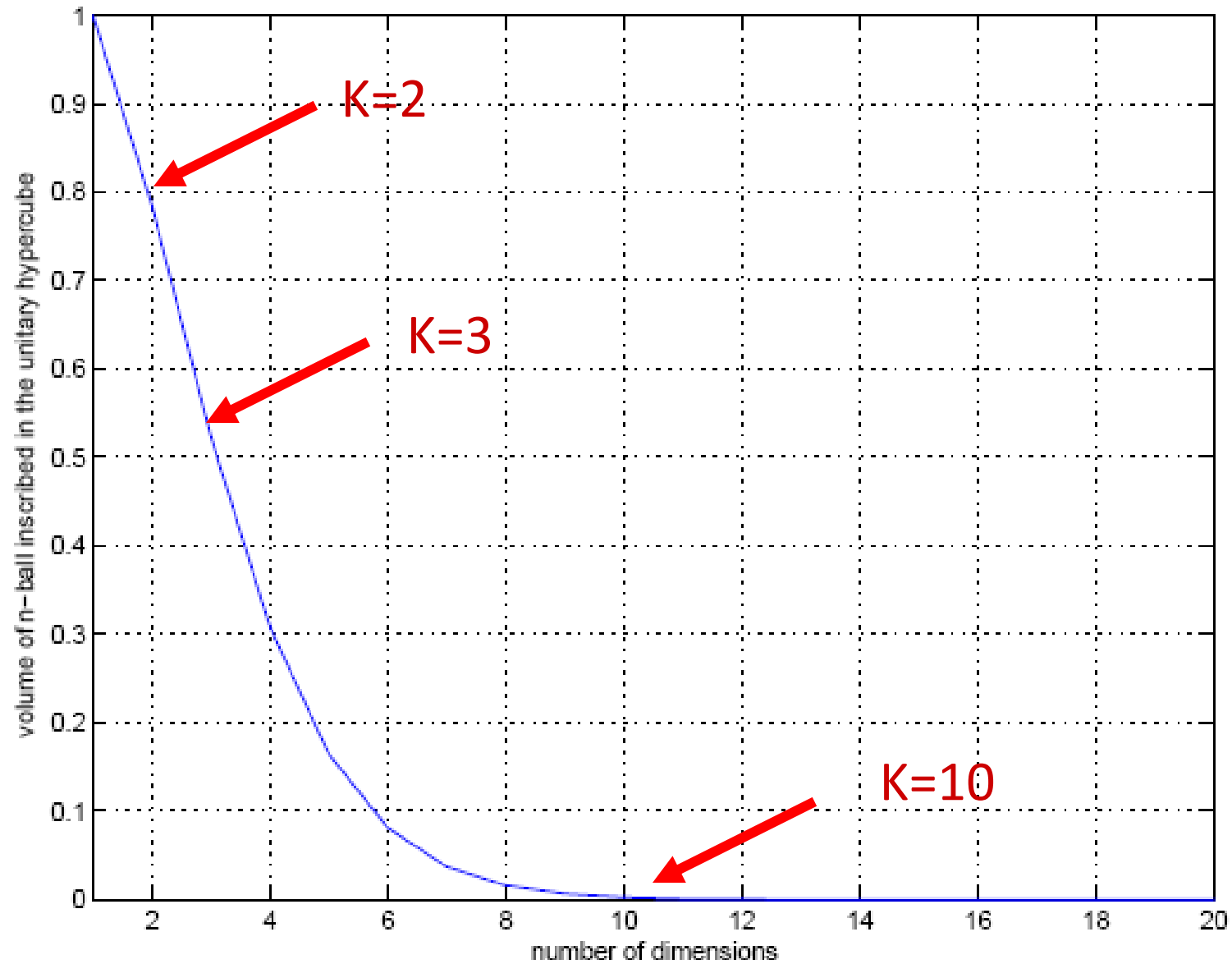
Volume sphere /
volume cube = ?

$\sim 1/2$

OAT in 10 dimensions; Volume
hypersphere / volume ten dimensional
hypercube =? ~ 0.0025



OAT in k dimensions



Once a sensitivity analysis is done via OAT there is no guarantee that either uncertainty analysis (UA) or sensitivity analysis (SA) will be any good:

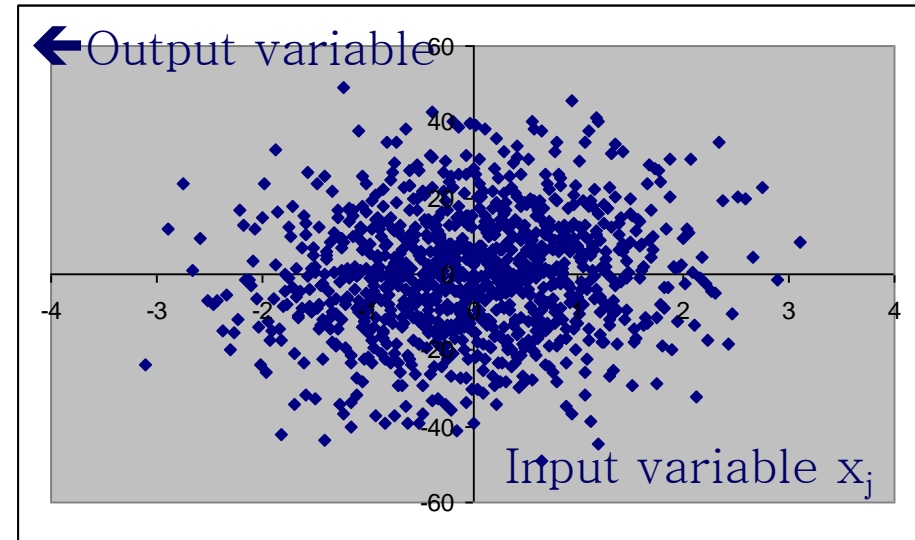
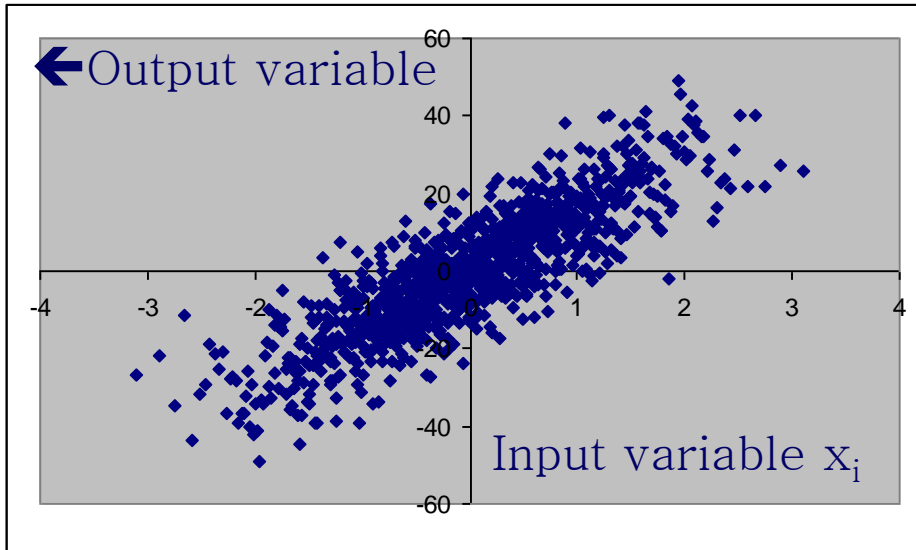
→ UA will be non conservative

→ SA may miss important factors

END

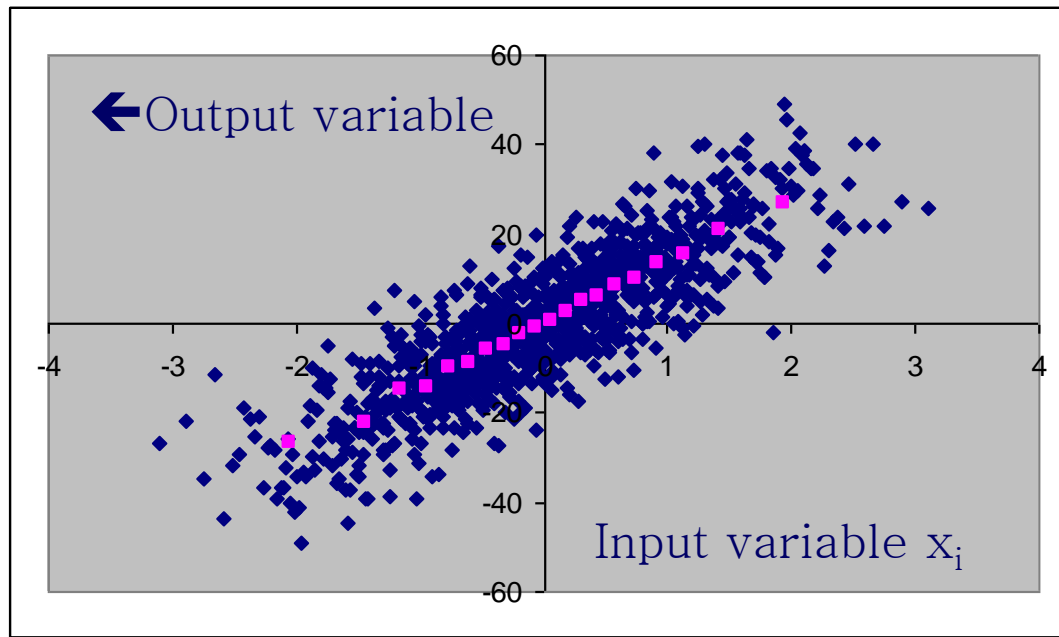


@andreasaltelli



Which factor is more important?

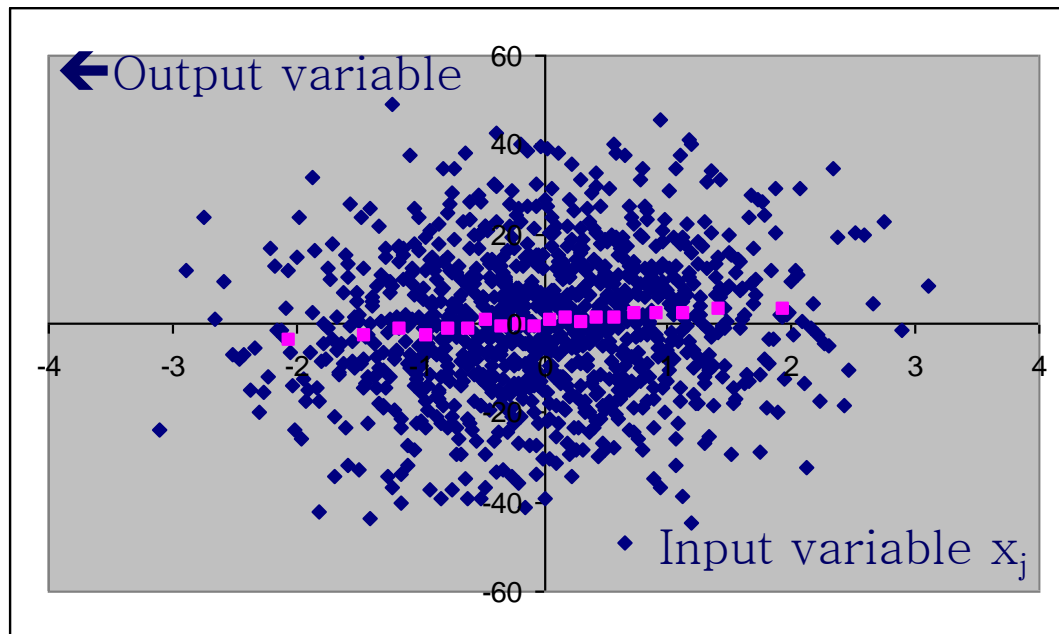
Why?

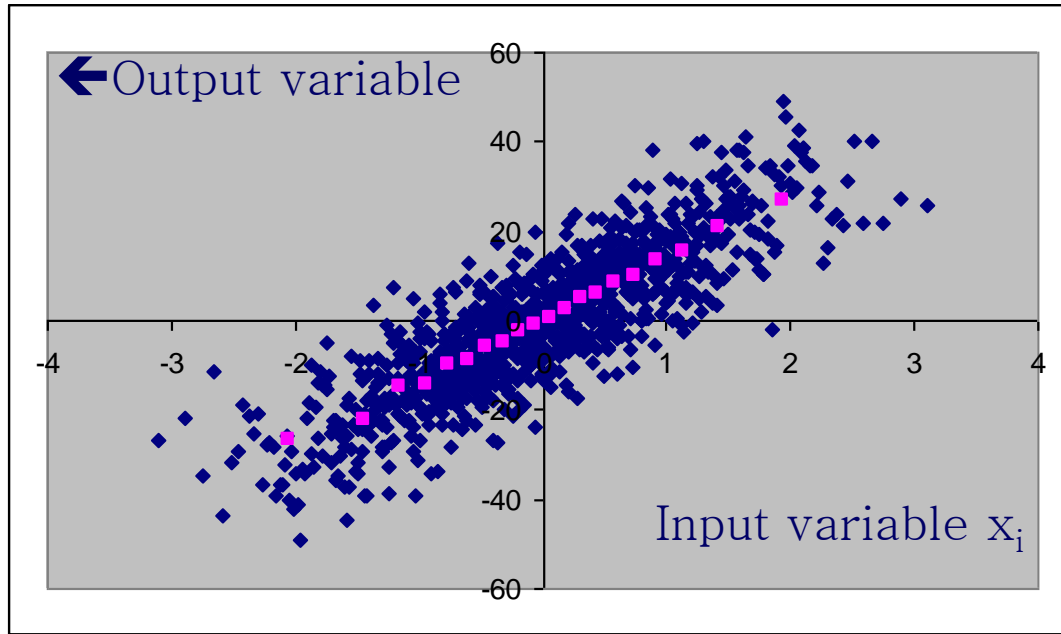


~1,000 blue points

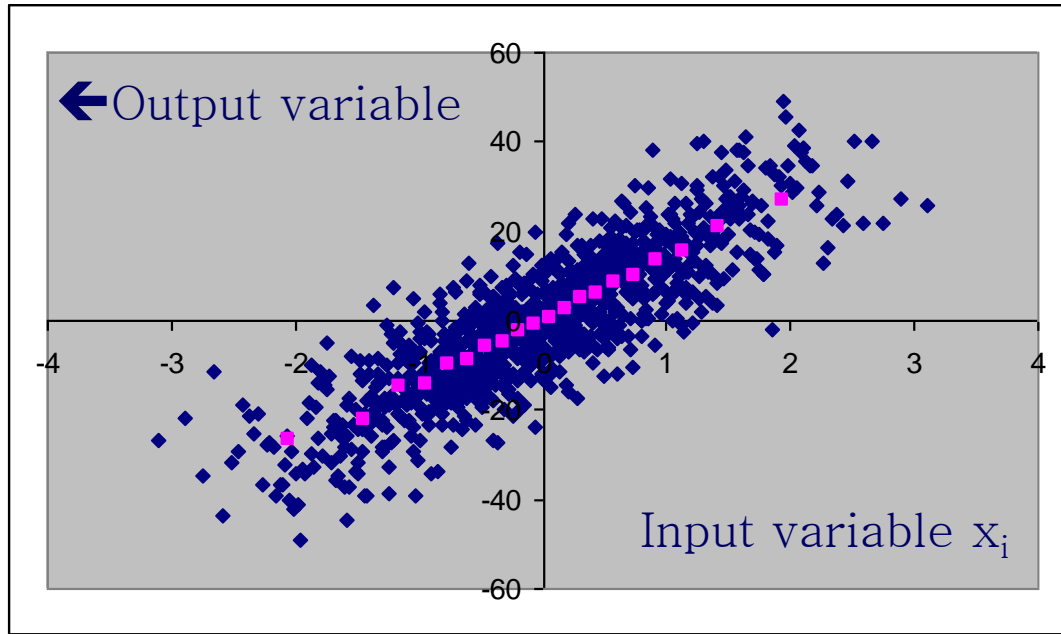
Divide them in 20 bins of ~ 50 points

Compute the bin's average (pink dots)



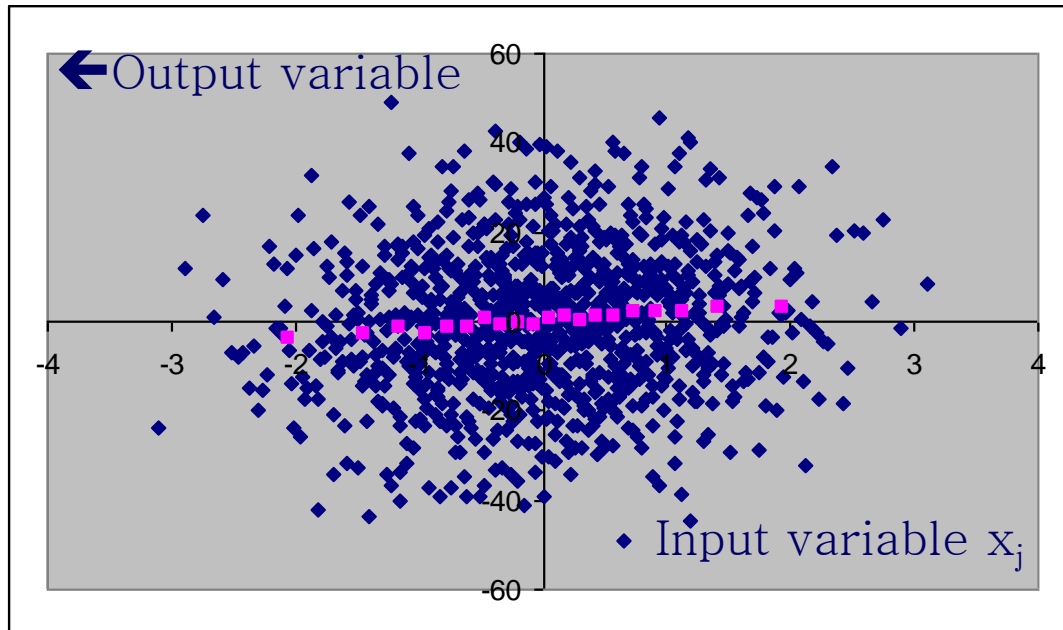
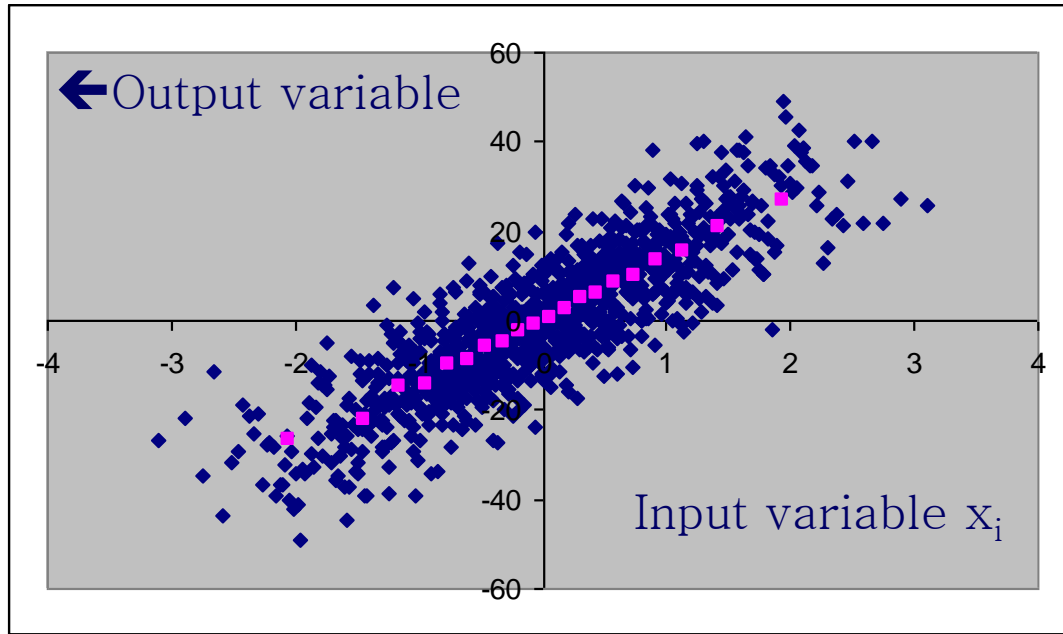


Each pink point is $\sim E_{\mathbf{X}_{\sim i}}(Y|X_i)$



Take the variance of
the pink points and
you have a
sensitivity measure

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$



Which factor
has the highest
 $V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$?

$$S_i \equiv \frac{V(E(Y|X_i))}{V_Y}$$

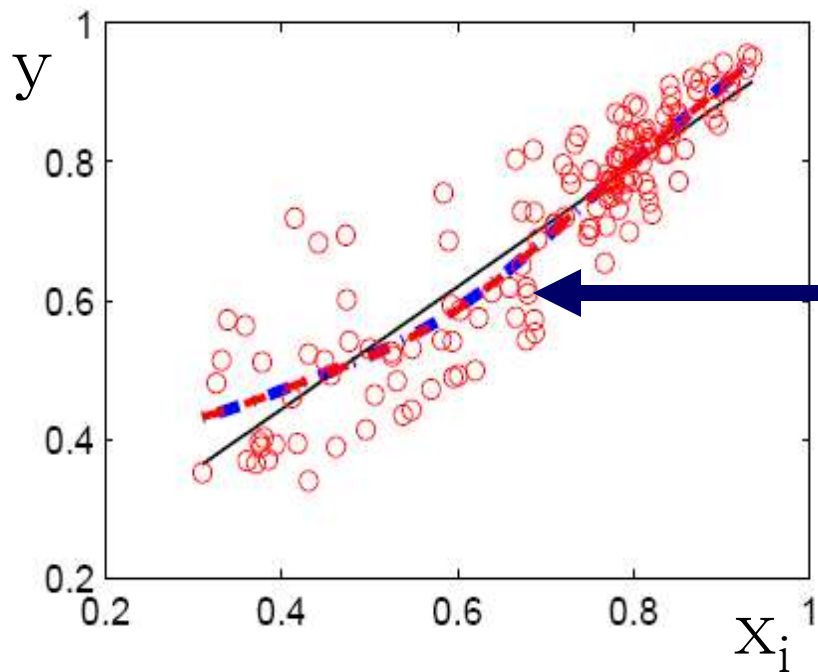
Pearson's correlation
ratio

Smoothed curve

$$S_i \equiv \eta_i^2 := \frac{V_{x_i} (\mathbf{E}_{\mathbf{x}_{\sim i}} (y \mid x_i))}{V(y)}$$

First order sensitivity index

Unconditional
variance



Smoothed curve:

$$\mathbf{E}_{\mathbf{x} \sim i} (y \mid x_i)$$

First order
sensitivity index:

$$\frac{V_{x_i} (\mathbf{E}_{\mathbf{x} \sim i} (y \mid x_i))}{V(y)}$$

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

First order effect, or top marginal variance=

= the expected reduction in variance that would be achieved if factor X_i could be fixed.

Why?

Because:

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) + \\ + E_{X_i} \left(V_{\mathbf{X}_{\sim i}} (Y | X_i) \right) = V(Y)$$

Easy to prove using $V(Y) = E(Y^2) - E^2(Y)$

Because:

$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) +$$

$+ E_{X_i} \left(V_{\mathbf{X}_{\sim i}} (Y | X_i) \right) = V(Y)$

This is what variance would be left (on average) if X_i could be fixed...

... then this ...



$$V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) + \\ + E_{X_i} \left(V_{\mathbf{X}_{\sim i}} (Y | X_i) \right) = V(Y)$$

... must be the expected reduction
in variance that would be achieved
if factor X_i could be fixed

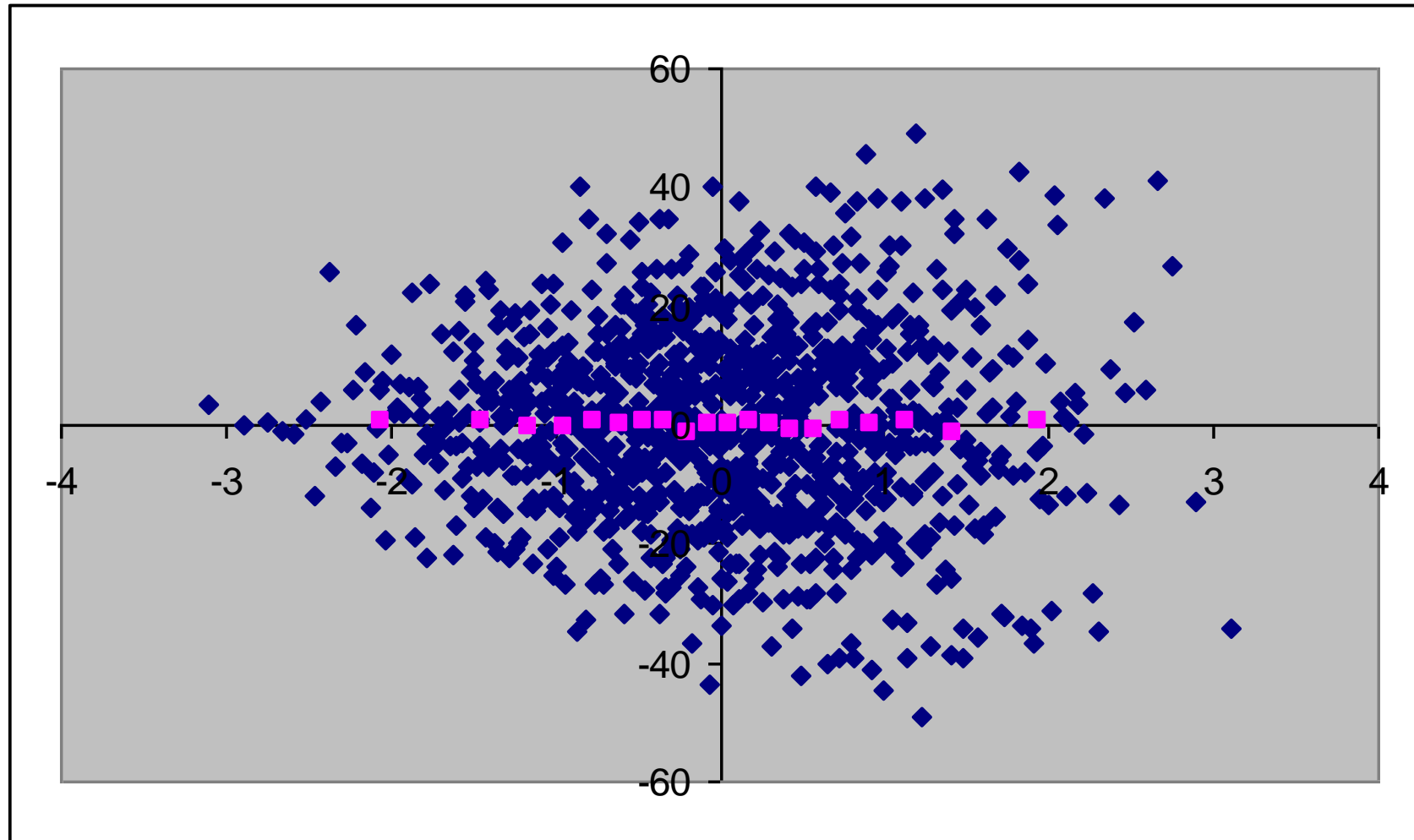
For additive models one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i} \left(E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) \approx V(Y)$$

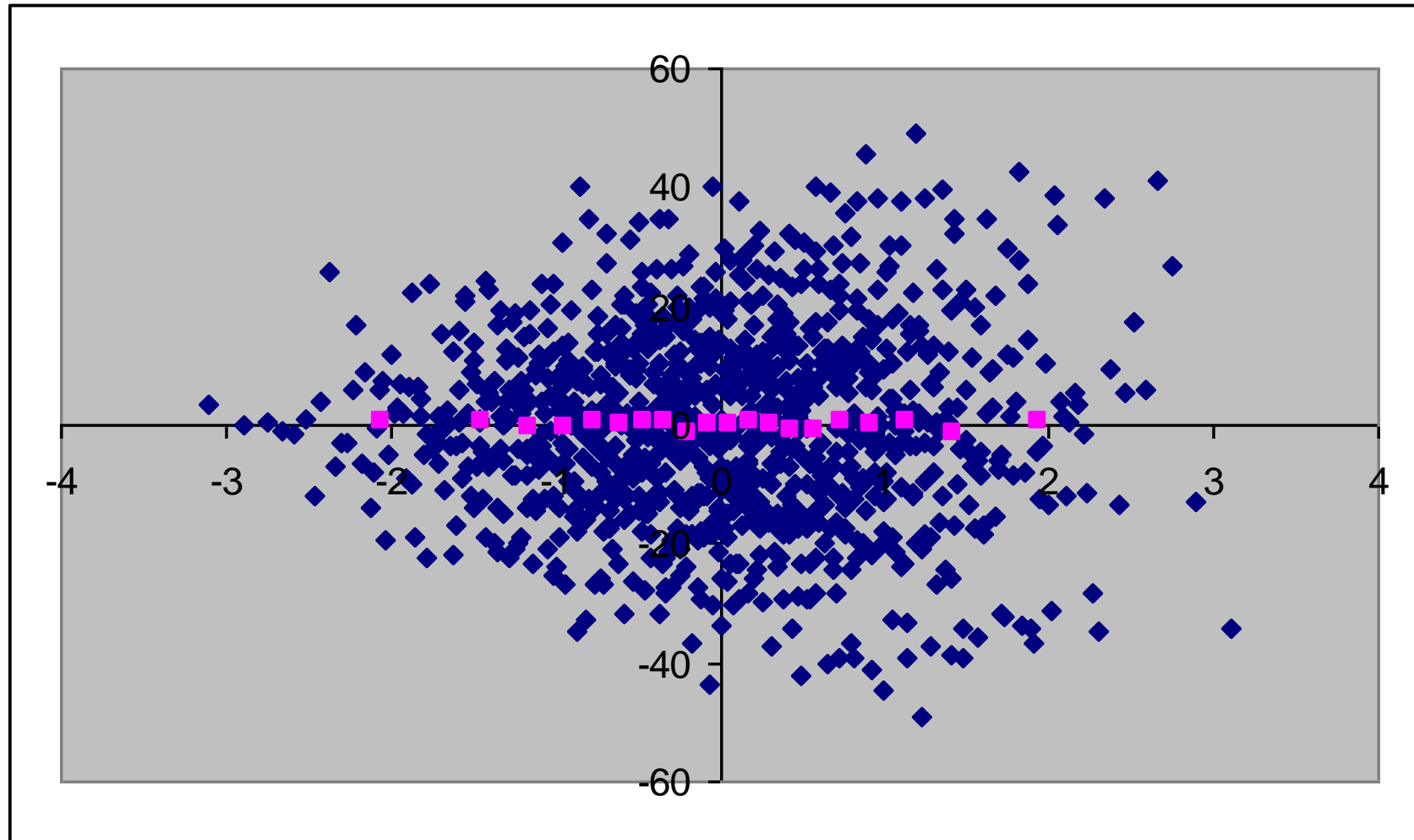
... which is also how additive models are defined

Non additive models

Is $S_i = 0$?



Is this factor non-important?



There are terms which capture two-way, three way, \dots interactions among variables.

All these terms are linked by a formula

Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i,j>i} V_{ij} + \dots + V_{123\dots k}$$

➔ Lesson Stefano Tarantola

EC impact assessment guidelines: sensitivity analysis & auditing



http://ec.europa.eu/smart-regulation/guidelines/docs/br_toolbox_en.pdf

Secrets of sensitivity analysis

Why should one
ever run a model
just once?

First secret: The most important question is the question.

Or: sensitivity analysis is not “run” on a model but on a model once applied to a question

Second secret: Sensitivity analysis should not be used to hide assumptions
[it often is]



Third secret: If sensitivity analysis shows that a question cannot be answered by the model one should find another question or model

[Often the love for one's own model prevails]

Badly kept secret:

There is always one more bug!

(Lubarsky's Law of Cybernetic Entomology)



And of course please don't run a sensitivity analysis where each factors has a 5% uncertainty



More than a technical
uncertainty and sensitivity
analysis?

1. Uncertainty and sensitivity analysis (never execute the model once)

2. Sensitivity auditing and quantitative storytelling (investigate frames and motivations)

Saltelli, A., Guimarães Pereira, Â., Van der Sluijs, J.P. and Funtowicz, S., 2013, 'What do I make of your latinorum? Sensitivity auditing of mathematical modelling', *Int. J. Foresight and Innovation Policy*, (9), 2/3/4, 213–234.

Saltelli, A., Does Modelling need a reformation? Ideas for a new grammar of modelling, available at <https://arxiv.org/abs/1712.06457>

3. Replace ‘model to predict and control the future’ with ‘model to help mapping ignorance about the future’ ...

... in the process exploiting and making explicit the metaphors embedded in the model

J. R. Ravetz, “Models as metaphors,” in Public participation in sustainability science : a handbook, and W. A. B. Kasemir, J. Jäger, C. Jaeger, Gardner Matthew T., Clark William C., Ed. Cambridge University Press, 2003, available at <http://www.nusap.net/download.php?op=getit&lid=11>