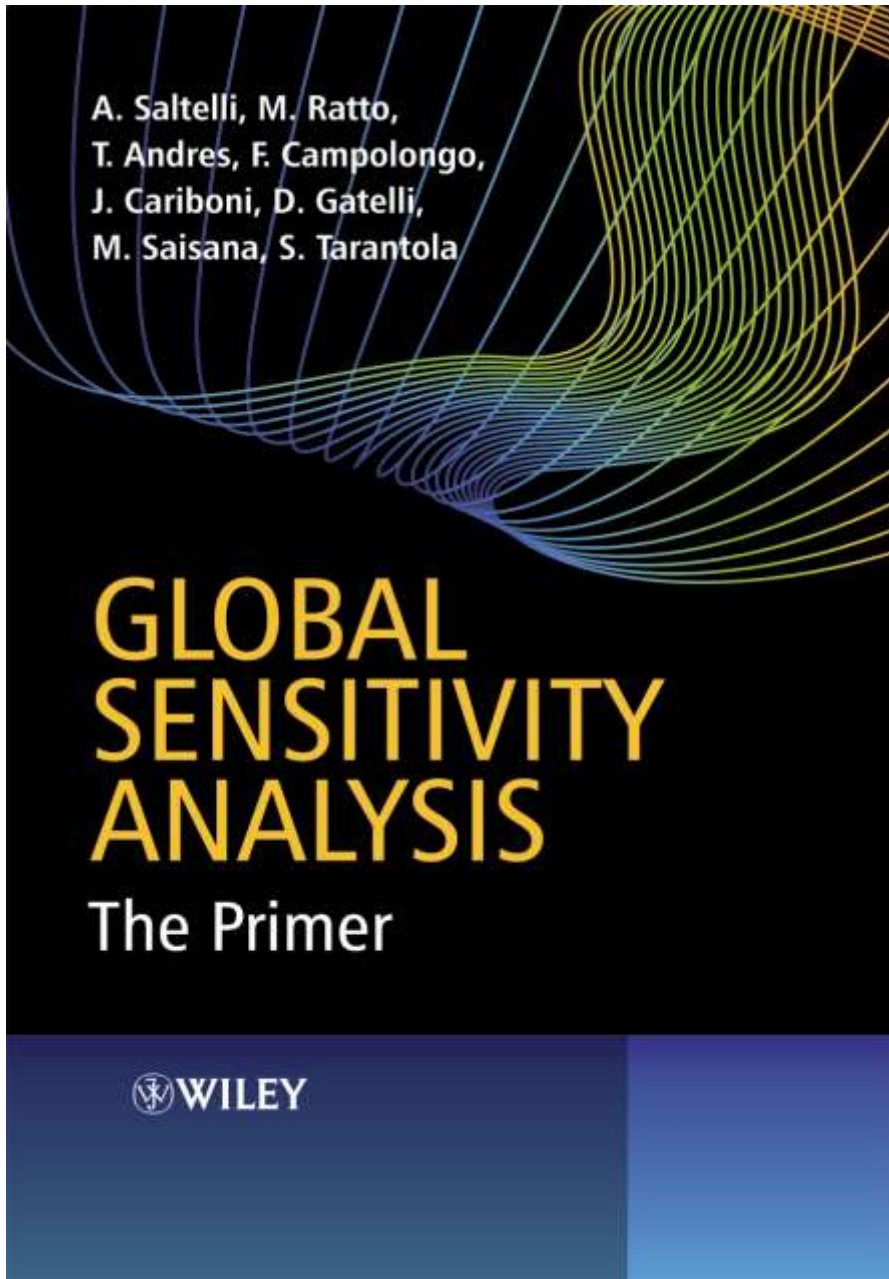


# Sensitivity Analysis

Andrea Saltelli

Presentation at the Barcelona  
Supercomputing Center, September 18, 2023





Partly based on  
Global sensitivity analysis.  
The Primer



# EC impact assessment guidelines: sensitivity analysis & auditing



*Better Regulation*  
**TOOLBOX**

November 2021

European Commission. November 2021. “Better Regulation: Guidelines and Toolbox.”

[https://ec.europa.eu/info/law/law-making-process/planning-and-proposing-law/better-regulation-why-and-how/better-regulation-guidelines-and-toolbox\\_en](https://ec.europa.eu/info/law/law-making-process/planning-and-proposing-law/better-regulation-why-and-how/better-regulation-guidelines-and-toolbox_en)

## Better regulation: guidelines and toolbox

### General principles

The better regulation guidelines set out the principles that the European Commission follows when preparing new initiatives and proposals and when managing and evaluating existing legislation.

The guidelines apply to each phase of the law-making cycle.



3 NOVEMBER 2021

**Better regulation guidelines**  
English (863.75 KB - PDF)

Download



20 JULY 2023

**Better regulation toolbox**  
English (6.46 KB - PDF)

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[Better regulation toolbox by chapters](#)

# EC impact assessment guidelines: sensitivity analysis & auditing

## TOOL #65. UNCERTAINTY AND SENSITIVITY ANALYSIS

### 1. MAIN FEATURES

What	<b>Uncertainty analysis</b> aims at quantifying uncertainties in model results provided to the decision-makers due to uncertain assumptions/inputs. <b>Sensitivity analysis</b> allows identifying the uncertain assumptions mostly responsible for uncertainty in model results.
Why	A <b>transparent</b> and <b>high-quality impact assessment</b> should acknowledge and, to the extent relevant or possible, attempt to quantify the <b>uncertainty in results</b> as it could change the ranking and conclusions about the policy options.
How	<b>Assessing</b> the uncertainties in model results by propagating model input uncertainties through the model and <b>inferring</b> a posteriori the relevant uncertain inputs by subsequent statistical analysis.



Who do these have in common?

J. Campbell, *et al.*, *Science* **322**, 1085 (2008).

R. Bailis, M. Ezzati, D. Kammen, *Science* **308**, 98 (2005).

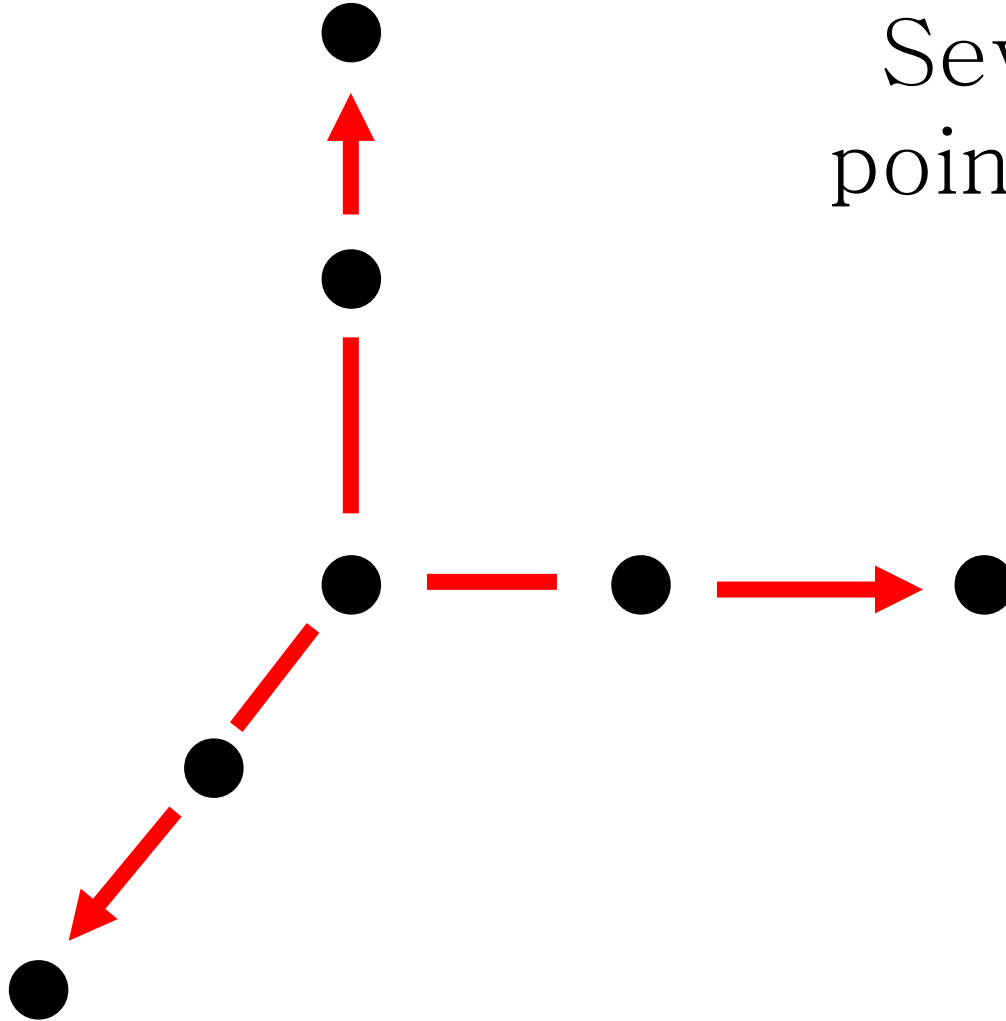
E. Stites, P. Tramont, Z. Ma, K. Ravichandran, *Science* **318**, 463 (2007).

J. Murphy, *et al.*, *Nature* **430**, 768–772 (2004).

J. Coggan, *et al.*, *Science* **309**, 446 (2005).

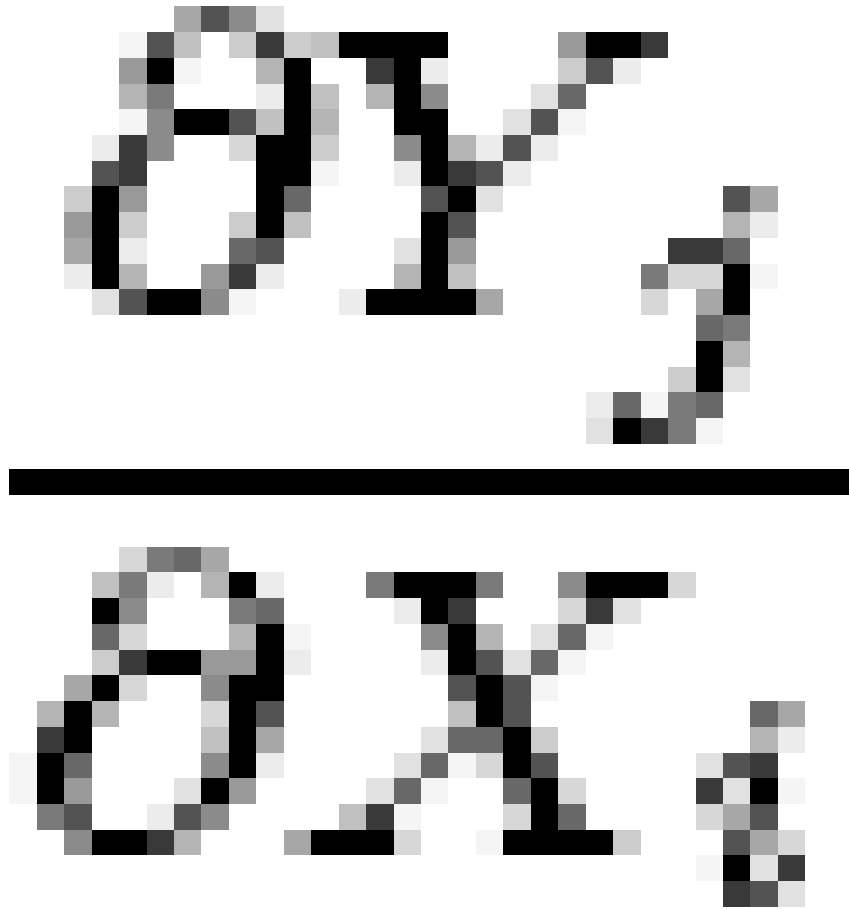
OAT

Seven OAT  
points in a 3D  
space



Before we go on to discuss OAT a premise:

We don't know if a model is linear  
before we do the analysis!



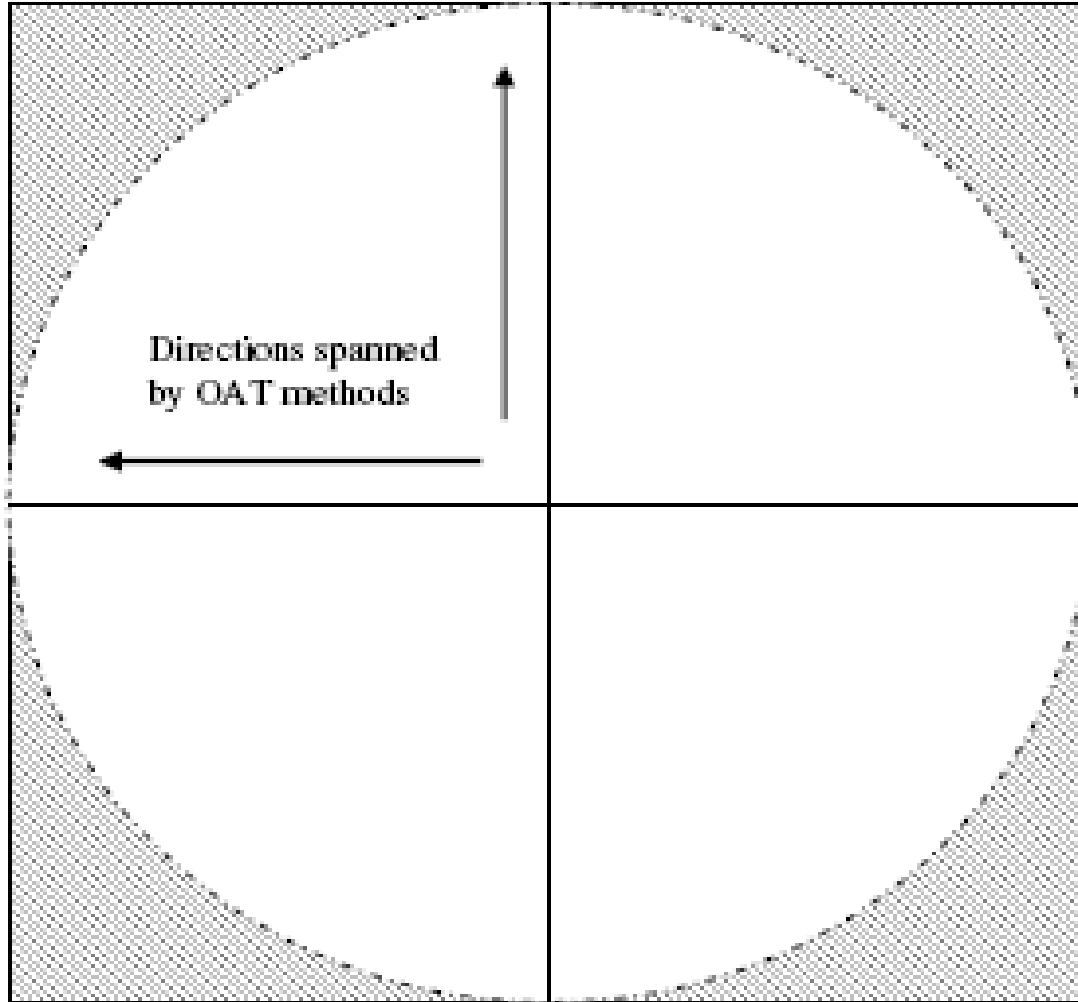
Otherwise the model could be declared linear or additive (or otherwise well behaved) and one could make it do with derivatives at a single baseline point.



Thus derivatives are **out**, but is OAT OK?

Or how bad is it?

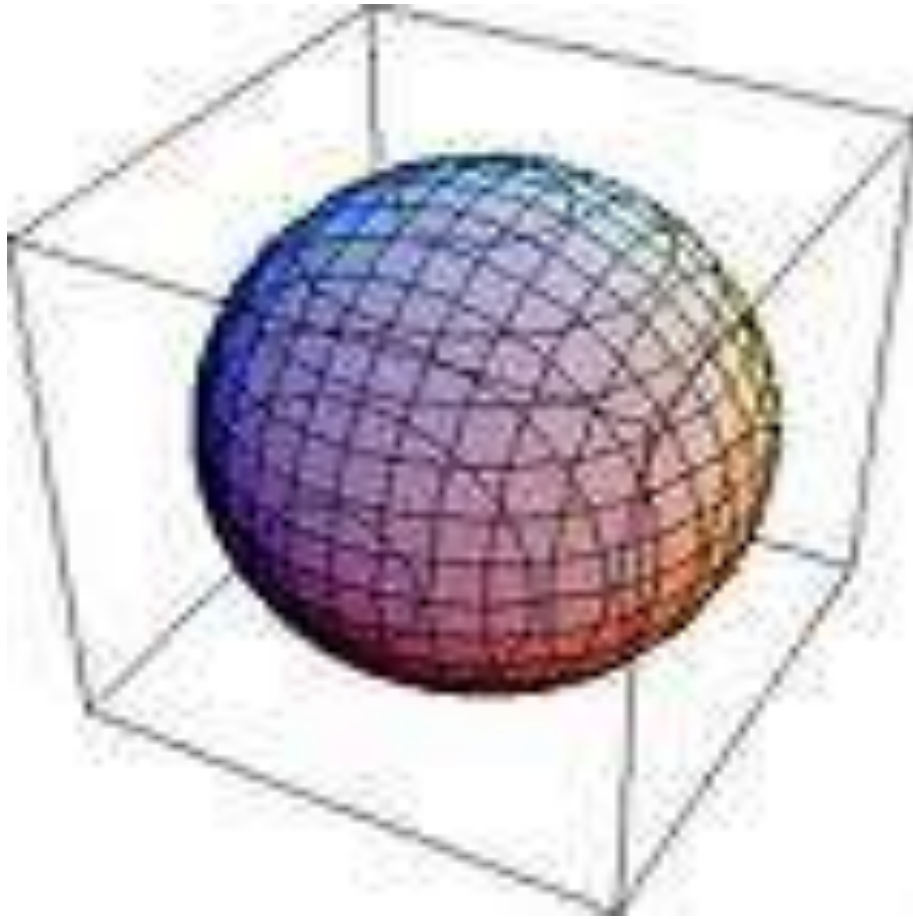
# OAT in 2 dimensions



Area circle / area  
square = ?

$\sim 3/4$

# OAT in 3 dimensions



Volume sphere /  
volume cube =?

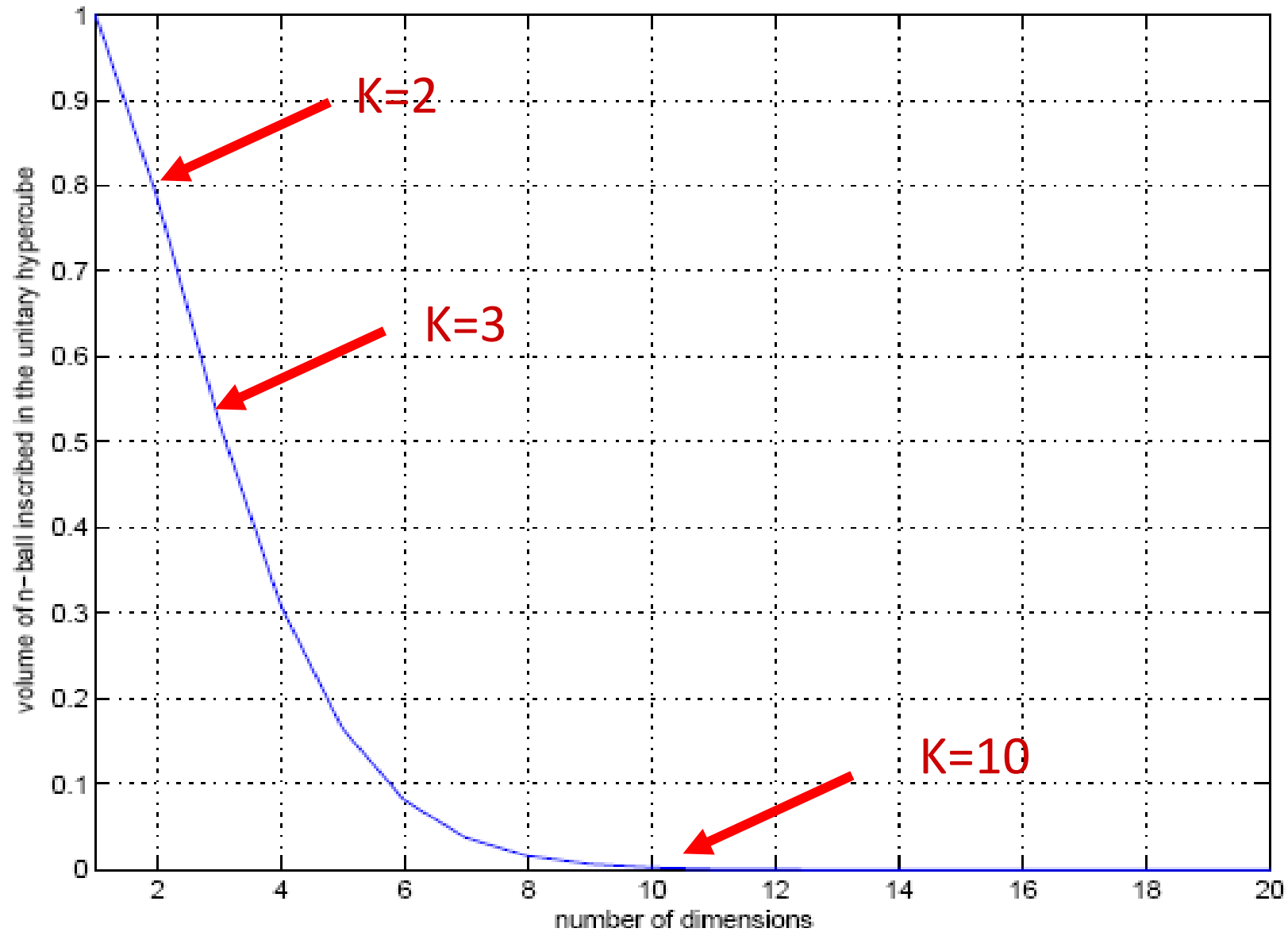
$\sim 1/2$

# OAT in 10 dimensions

Volume hypersphere / volume ten dimensional hypercube =? ~ 0.0025



# OAT in k dimensions



Thus OAT is very poor in exploring the space of the factors – it is also non conservative.

Why?

OAT is not roughly  
right ... it is precisely  
wrong!



# Reading about dubious or absent sensitivity analysis





Environmental Modelling & Software

Volume 114, April 2019, Pages 29-39



## Why so many published sensitivity analyses are false: A systematic review of sensitivity analysis practices

Andrea Saltelli<sup>a, b</sup>  , Ksenia Aleksankina<sup>c</sup>, William Becker<sup>d</sup>, Pamela Fennell<sup>e</sup>,  
Federico Ferretti<sup>d</sup>, Niels Holst<sup>f</sup>, Sushan Li<sup>g</sup>, Qiongli Wu<sup>h</sup>

Show more 



For the papers using OAT points a better (statistical theory based) alternative is available, be it:

- A two level factorial design,
- A trajectory analysis (a-la-Morris) or
- A linear regression based on a Monte Carlo Sample

Using perhaps the same low number of points.

## Another story of SA



William Nordhaus,  
University of Yale



Nicholas Stern, London  
School of Economics

Stern's Review –  
Technical Annex to  
postscript

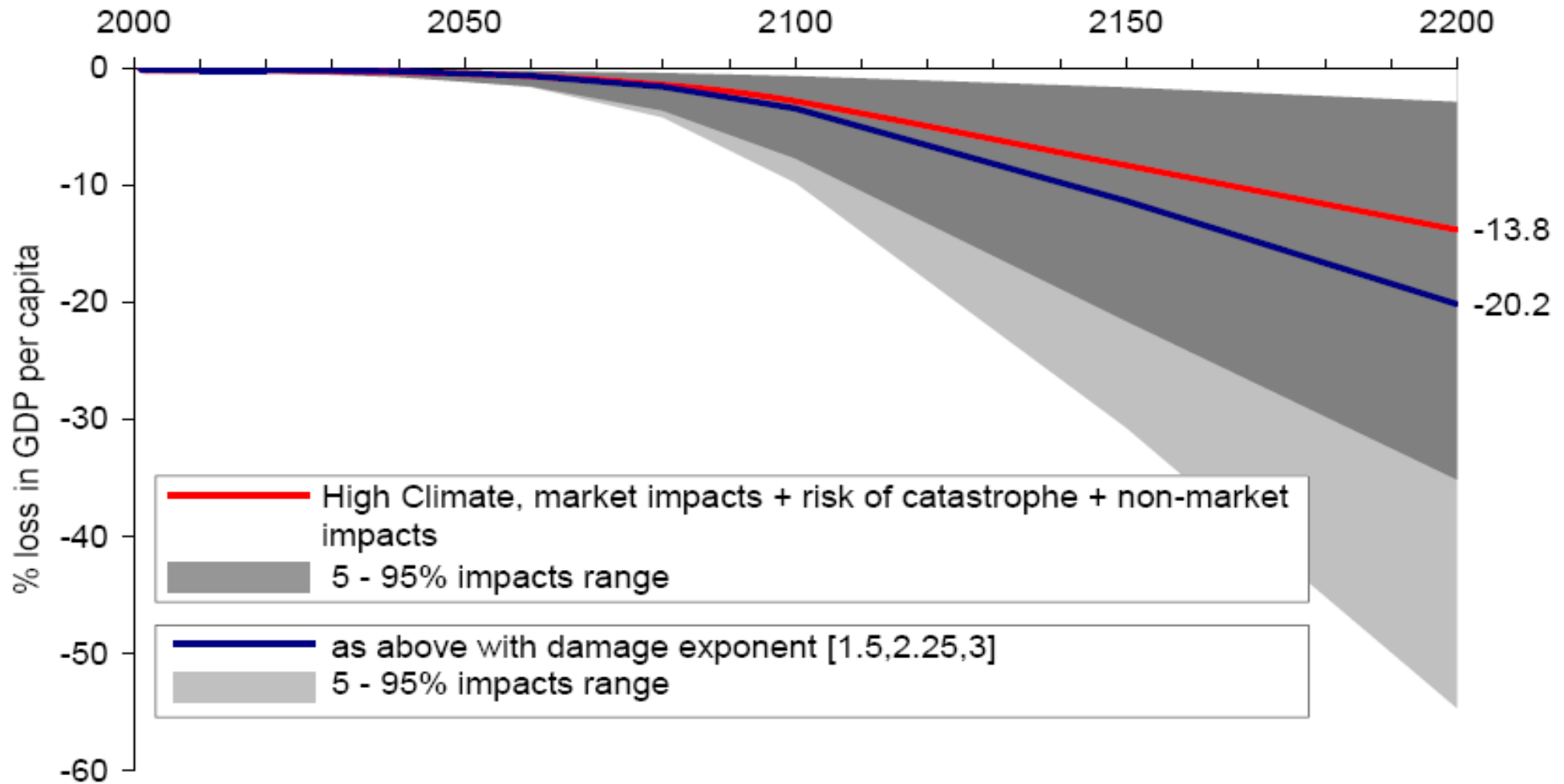
Stern's Review – Technical Annex To postscript  
(a sensitivity analysis of a cost benefit analysis)

The Stern – Nordhaus exchange on *SCIENCE*

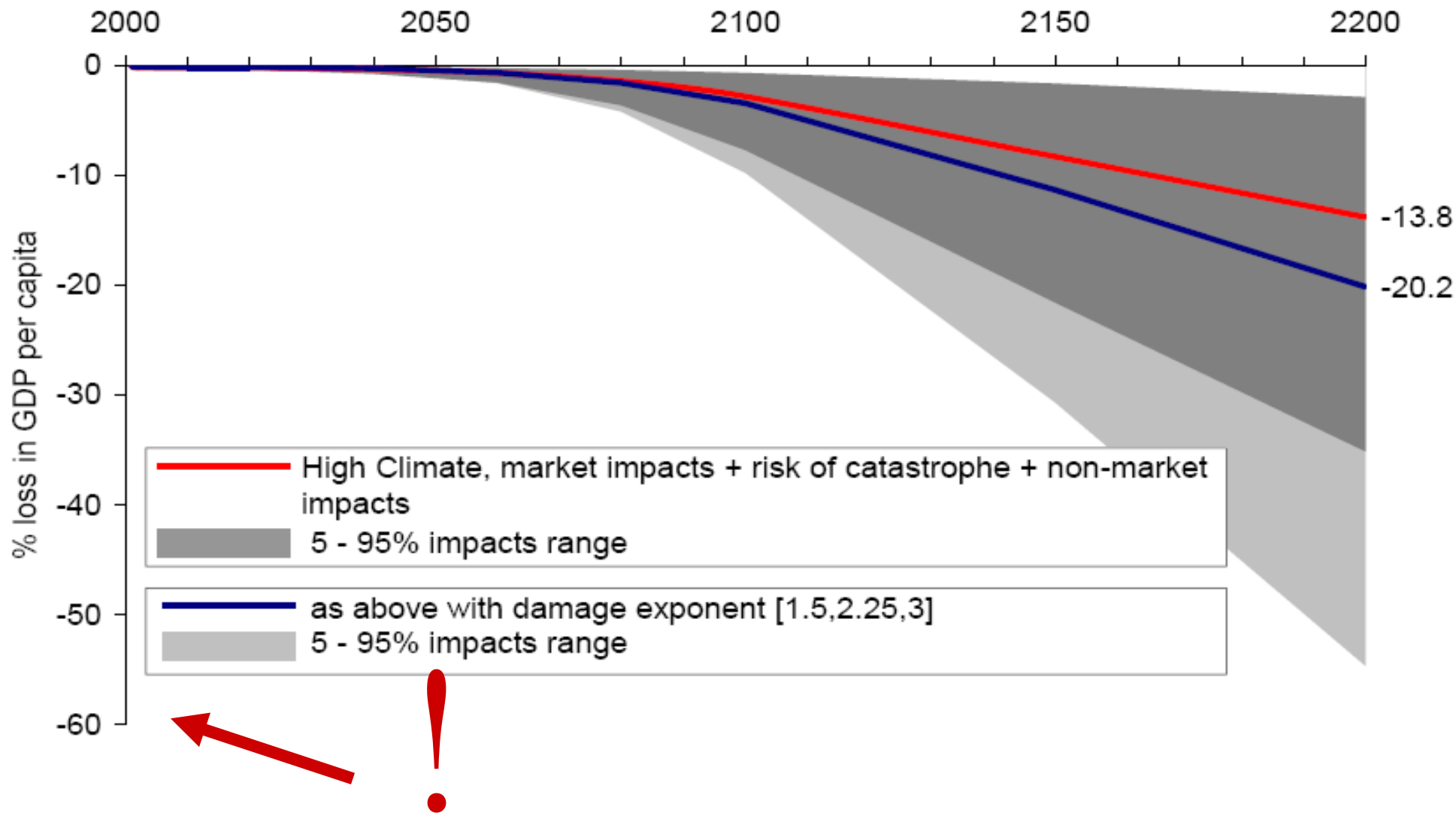
Nordhaus → falsifies Stern based on 'wrong'  
range of discount rate (~ you GIGOing)

Stern → 'My analysis shows robustness'

# From Stern's Review SA



# My problems with it:

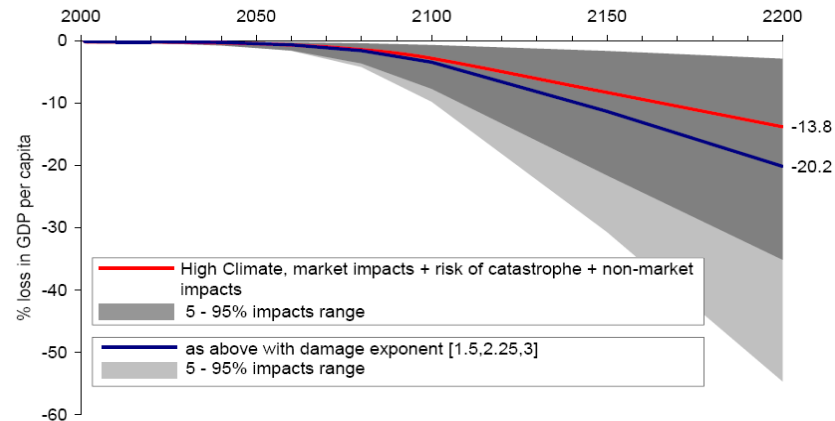


... but foremost he says:

changing assumptions → important effect

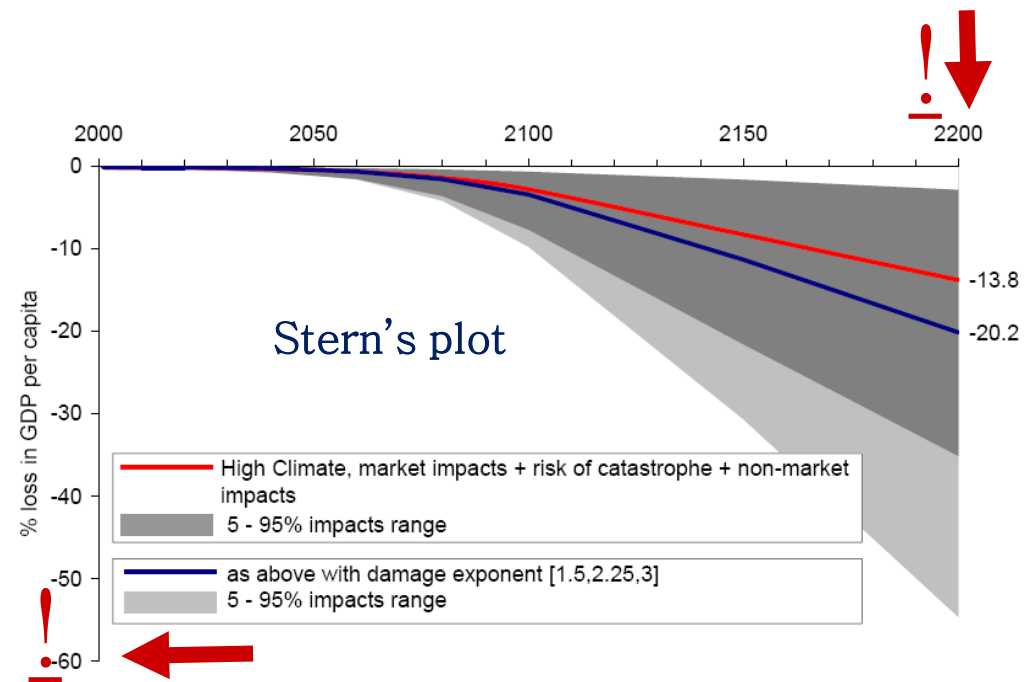
when instead he should admit that:

changing assumptions → all changes a lot



The Stern–Nordhaus controversy;  
 a reverse engineering the model:

→ uncertainty is too large to take decisions → both Stern and Nordhaus are wrong

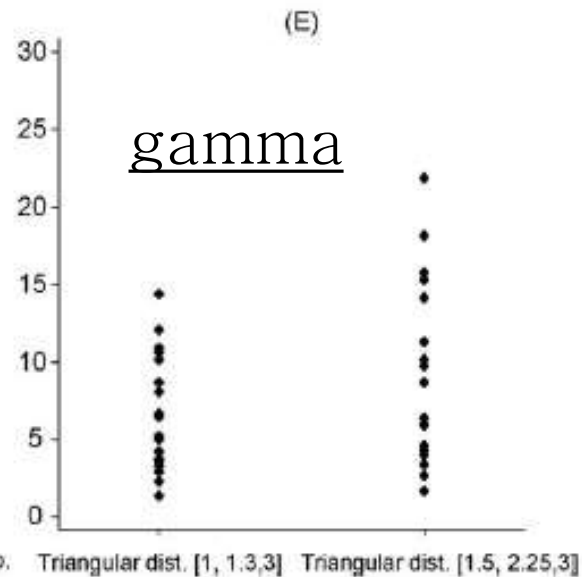
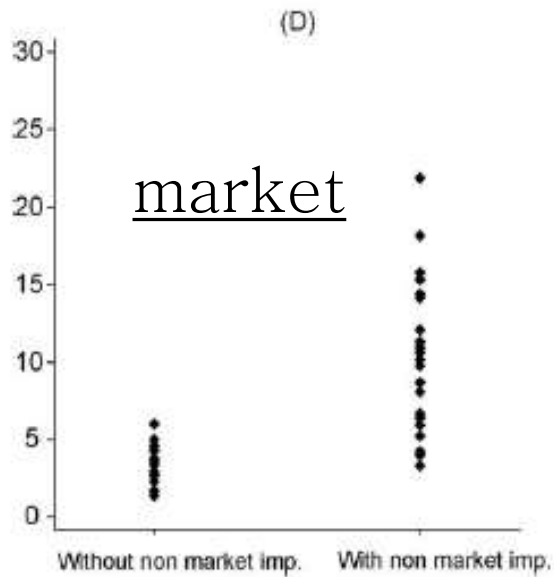
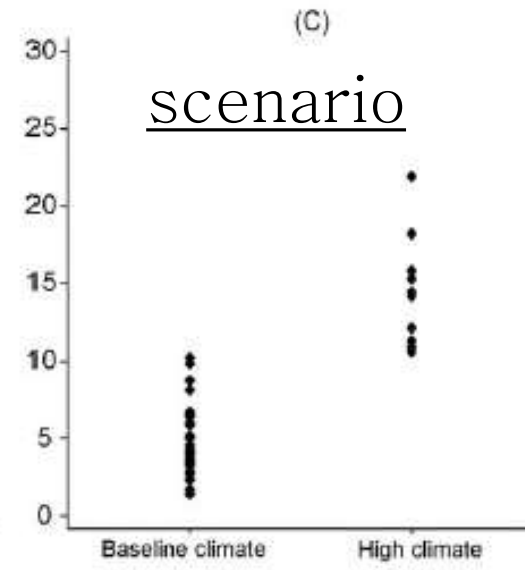
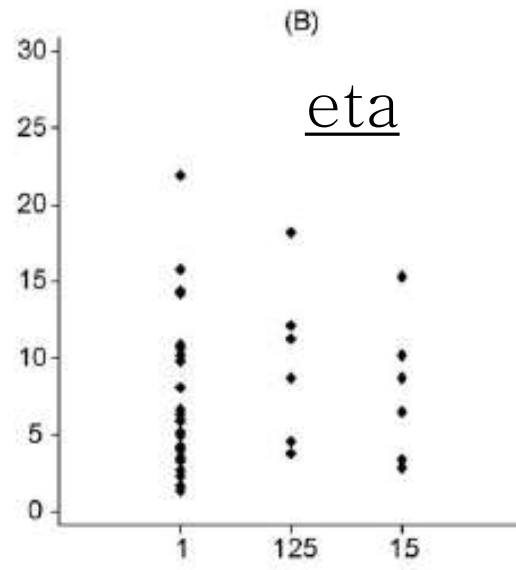
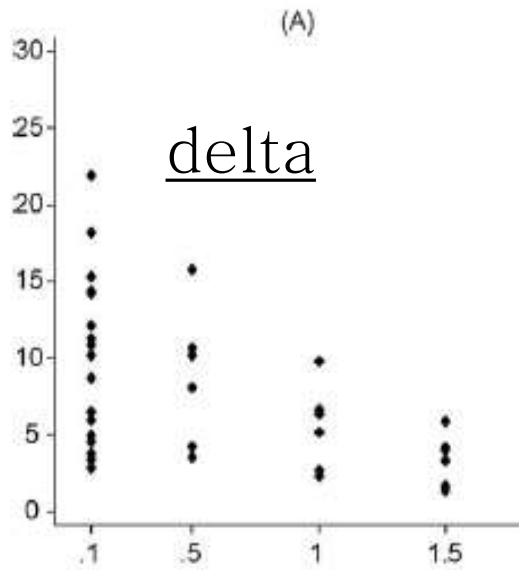


**Sensitivity analysis didn't help. A practitioner's critique of the Stern review**

Andrea Saltelli\*, Beatrice D'Hombres

Joint Research Centre, Institute for the Protection and Security of the Citizen, Ispra, Italy

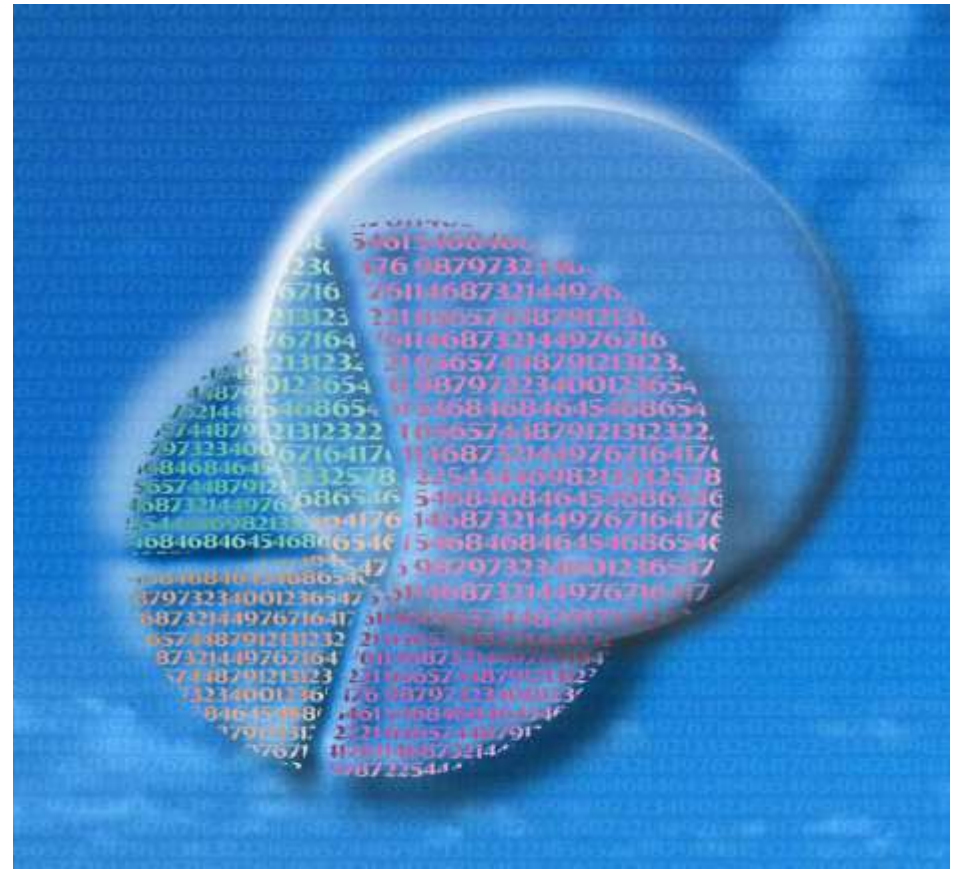




Sensitivity  
analysis,  
also by  
reverse  
engineering

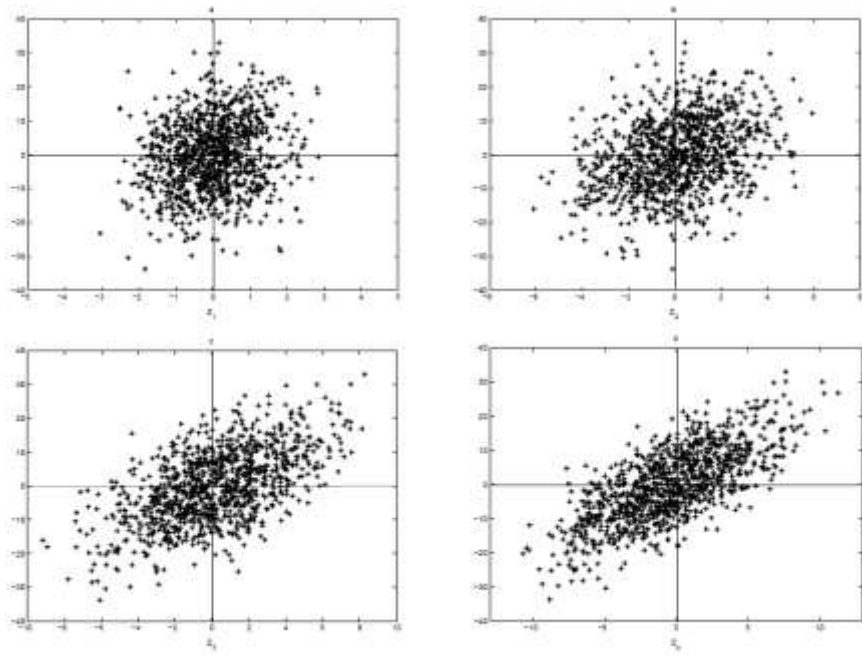


Variance based methods; a best practice?



Mostly based on the work of Ilya M. Sobol' (1990), who extended the work of R.I. Cukier (1973). Further extensions by T. Homma and myself (1996, onward).





Scatterplots'  
notation:

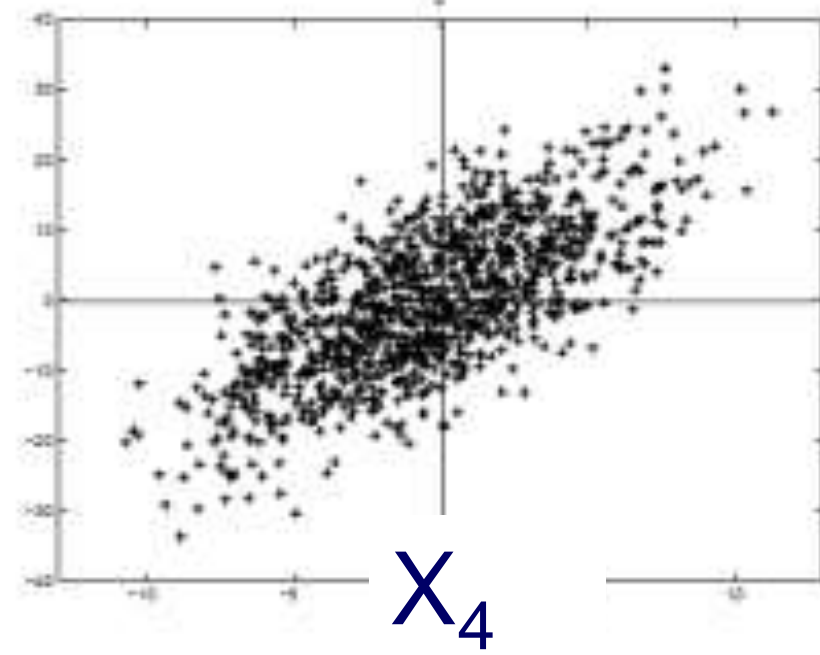
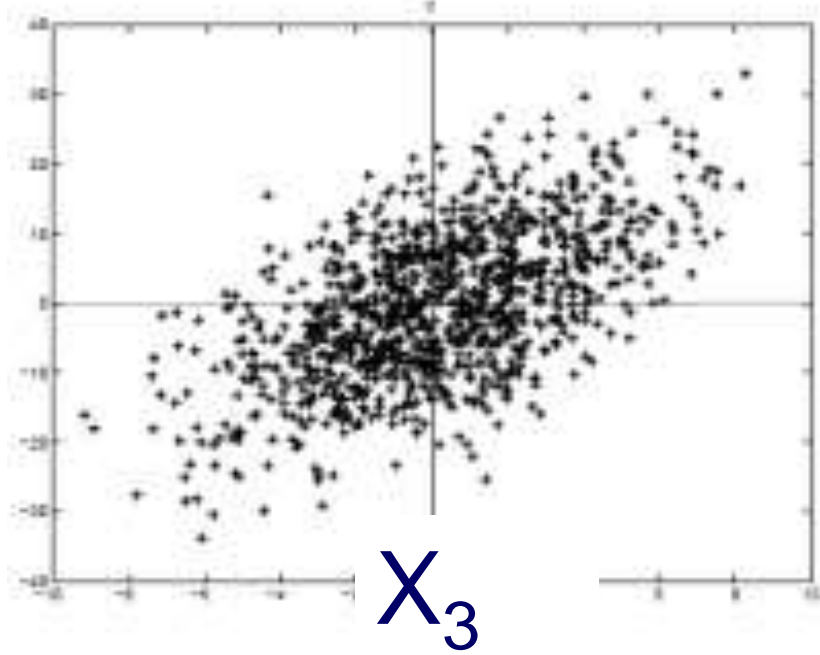
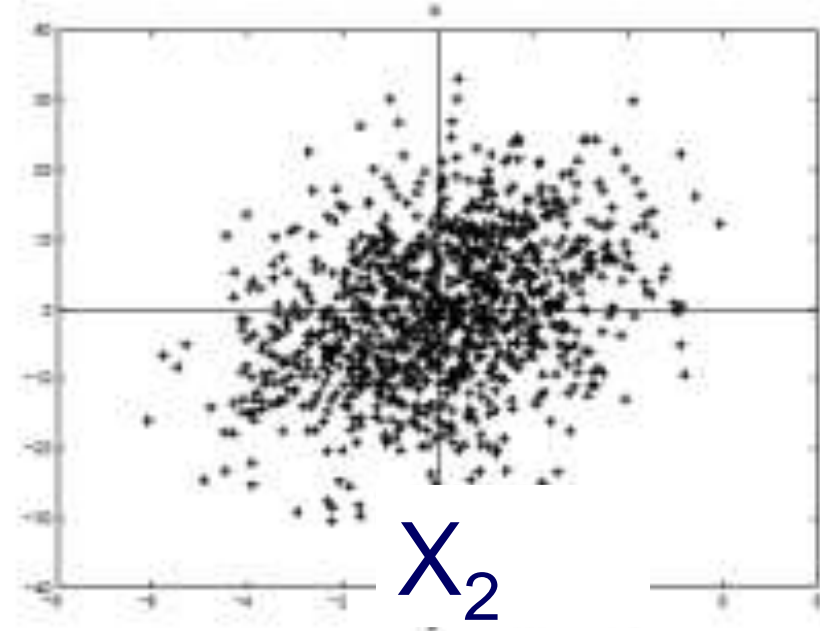
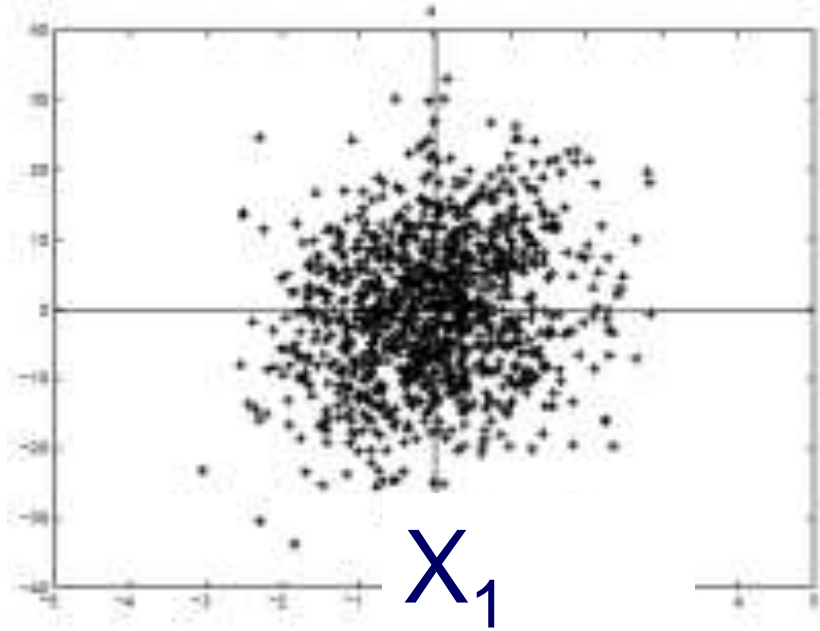
$$Y = f(X_1, X_2, \dots, X_k)$$

$$f_0 = E(Y)$$

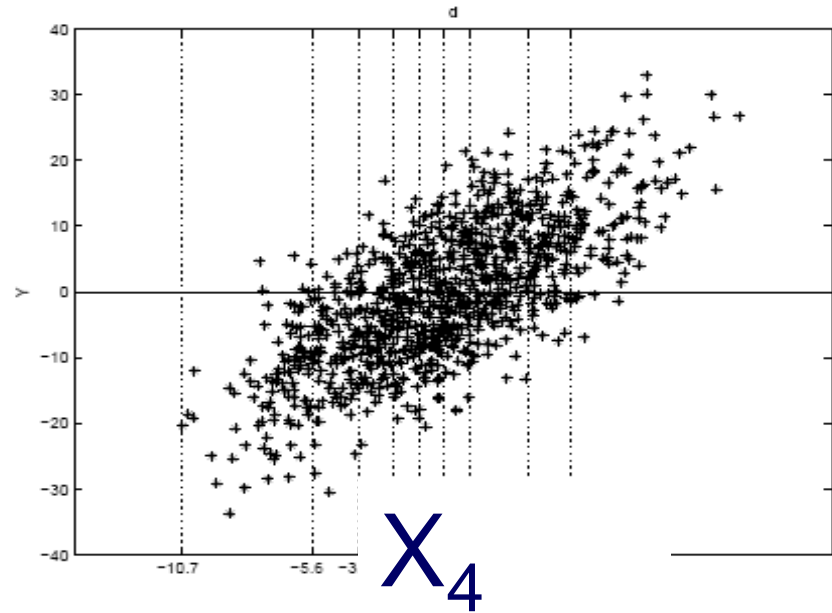
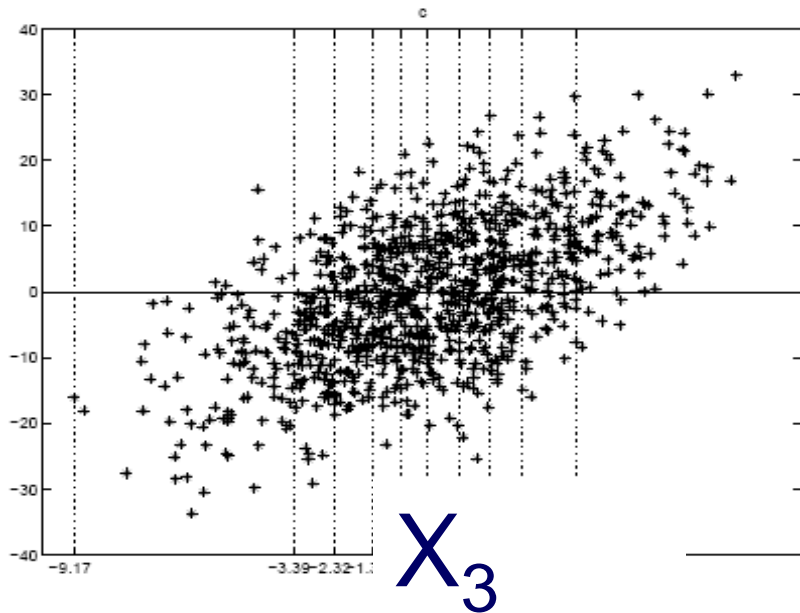
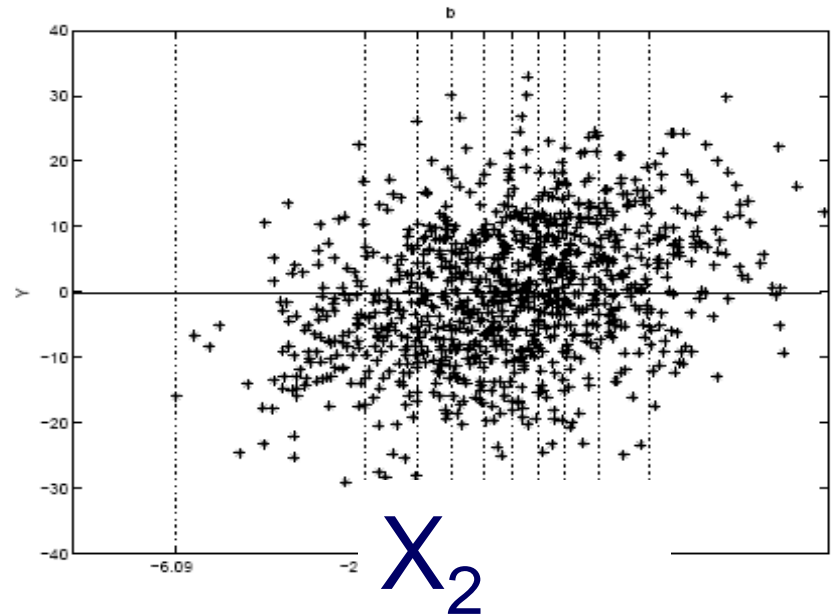
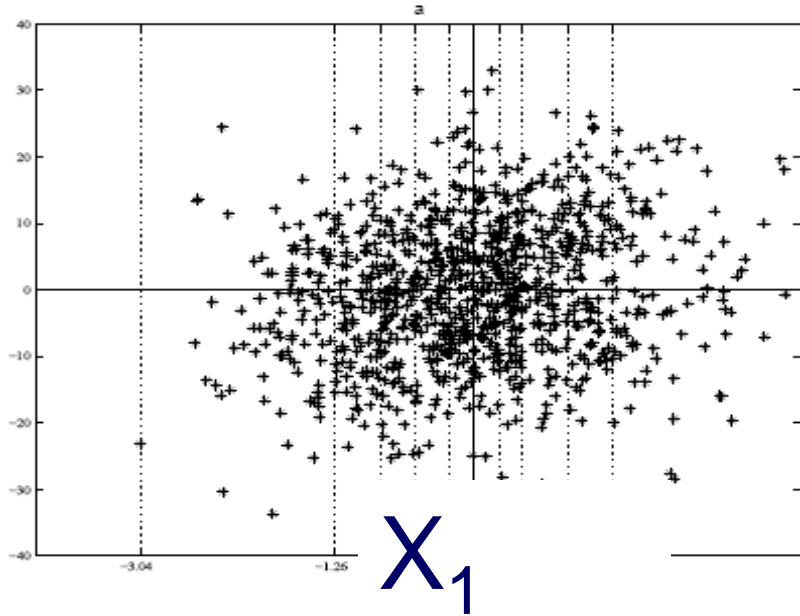
The ordinate axis is always  $Y$

The abscissa are the various  
factors  $X_i$  in turn.

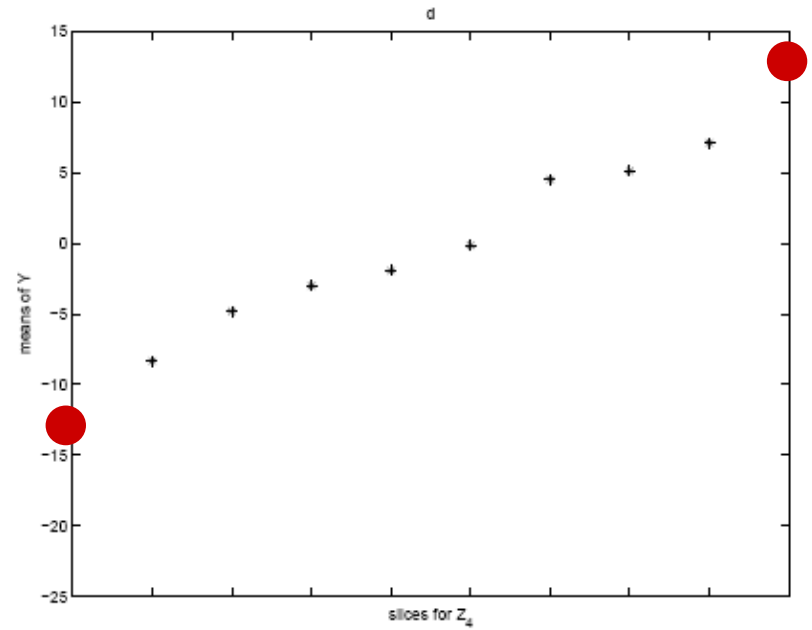
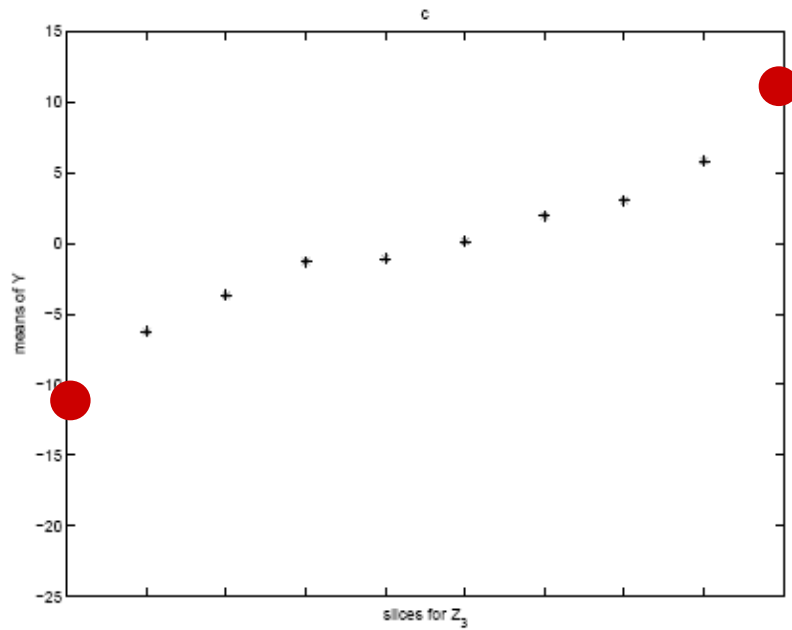
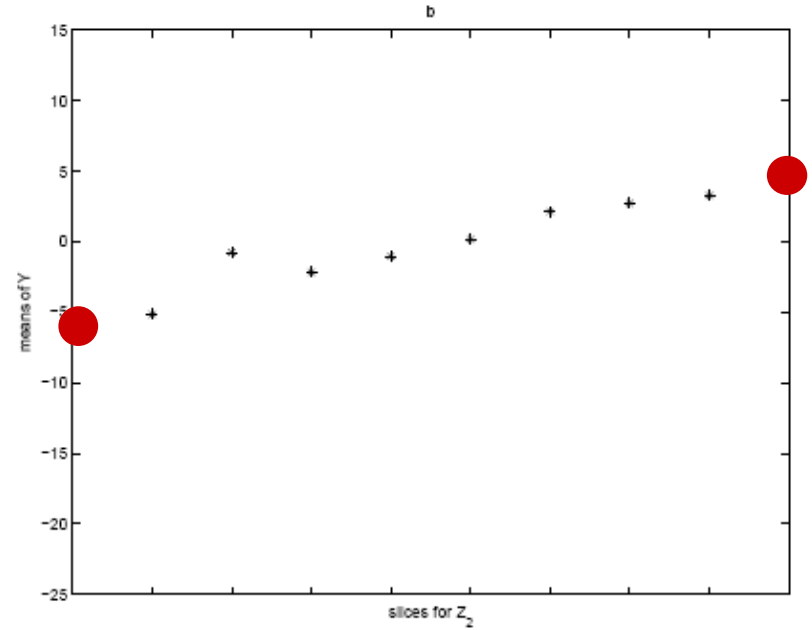
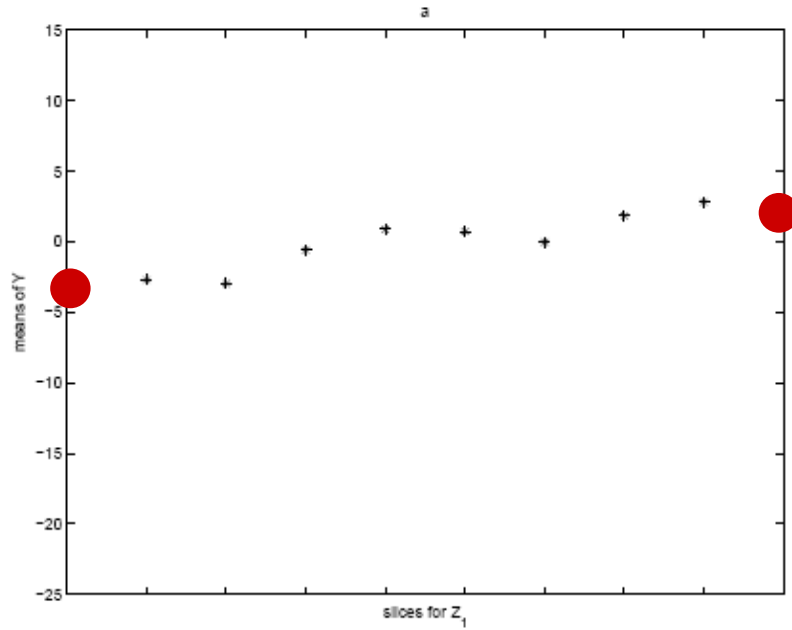
The points are always the same!



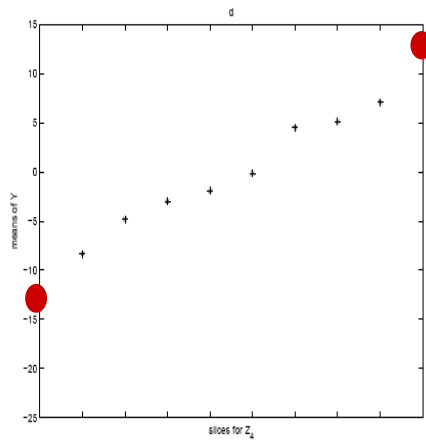
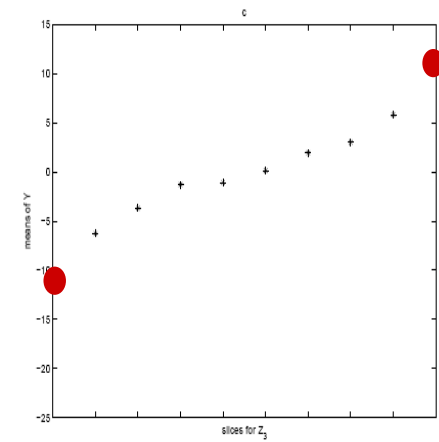
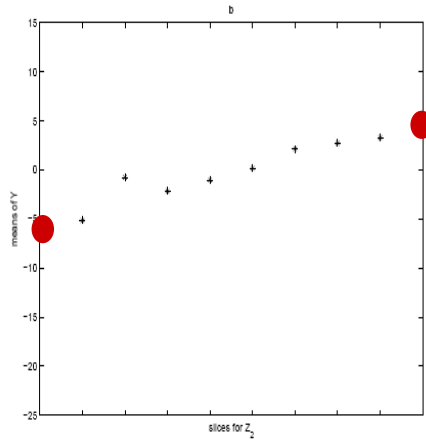
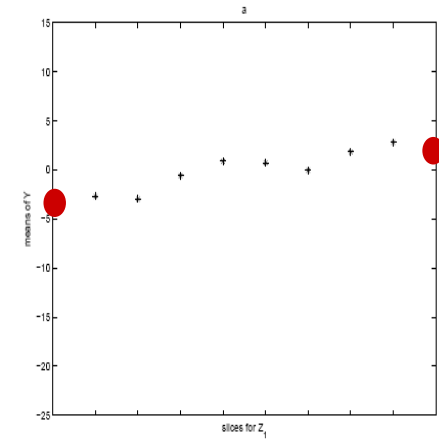
# Cutting into slices...



# Average of $Y$ versus $X_i$ – same scale for $Y$

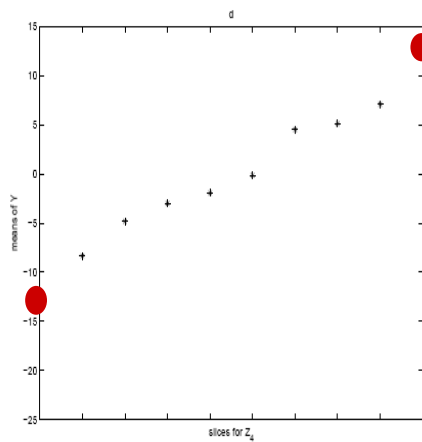
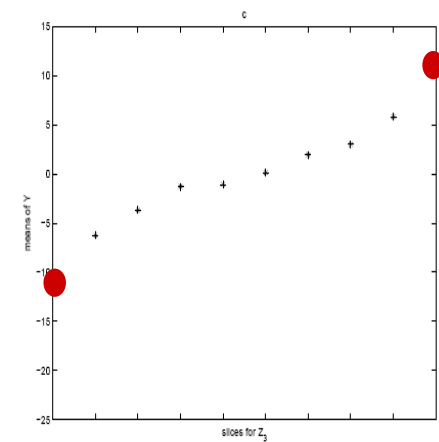
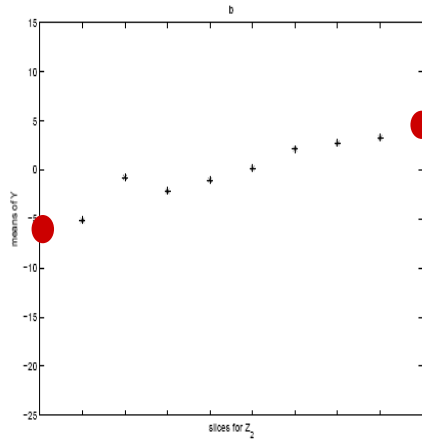
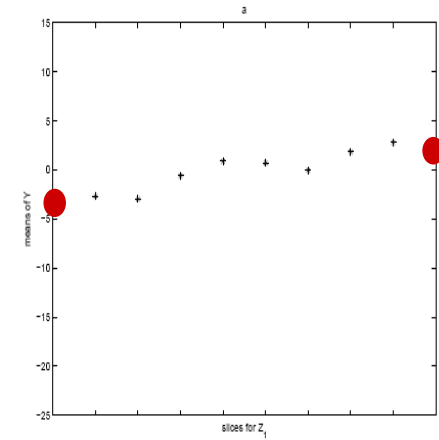






This shows the variance of  $Y$  across the slices: greater for  $X_4$  than for  $X_1$

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$



If the model is linear:

$$\frac{V_{X_i} \left( E_{\mathbf{X}_{\sim i}} \left( Y | X_i \right) \right)}{V} \approx \beta_{X_i}^2$$



$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

First order effect, or top marginal  
variance=

= the expected reduction in variance  
than would be achieved if factor  $X_i$   
could be fixed.

For additive systems one can decompose the total variance as a sum of first order effects

$$\sum_i V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) = V(Y)$$

... and a powerful variance based measure is also available for non-additive models ...

From this ...

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} \left( Y | X_i \right) \right)$$



This is a first order effect, or top marginal variance.

The expected reduction in variance than would be achieved if factor  $X_i$  could be fixed.

... to this

This is a total order effect, or bottom marginal variance.

The expected variance than would be left if all factors but  $X_i$  could be fixed.



$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} \left( Y | X_i \right) \right)$$

This has an  
intuitive  
interpretation (the  
scatterplots)

$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

How About this?

## Variance decomposition (ANOVA)

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) = V_i$$
$$V_{X_i X_j} \left( E_{\mathbf{X}_{\sim ij}} (Y | X_i X_j) \right) =$$
$$= V_i + V_j + V_{ij}$$

...

## Variance decomposition (ANOVA)

$$V(Y) =$$

$$\sum_i V_i + \sum_{i, j > i} V_{ij} + \dots + V_{123\dots k}$$

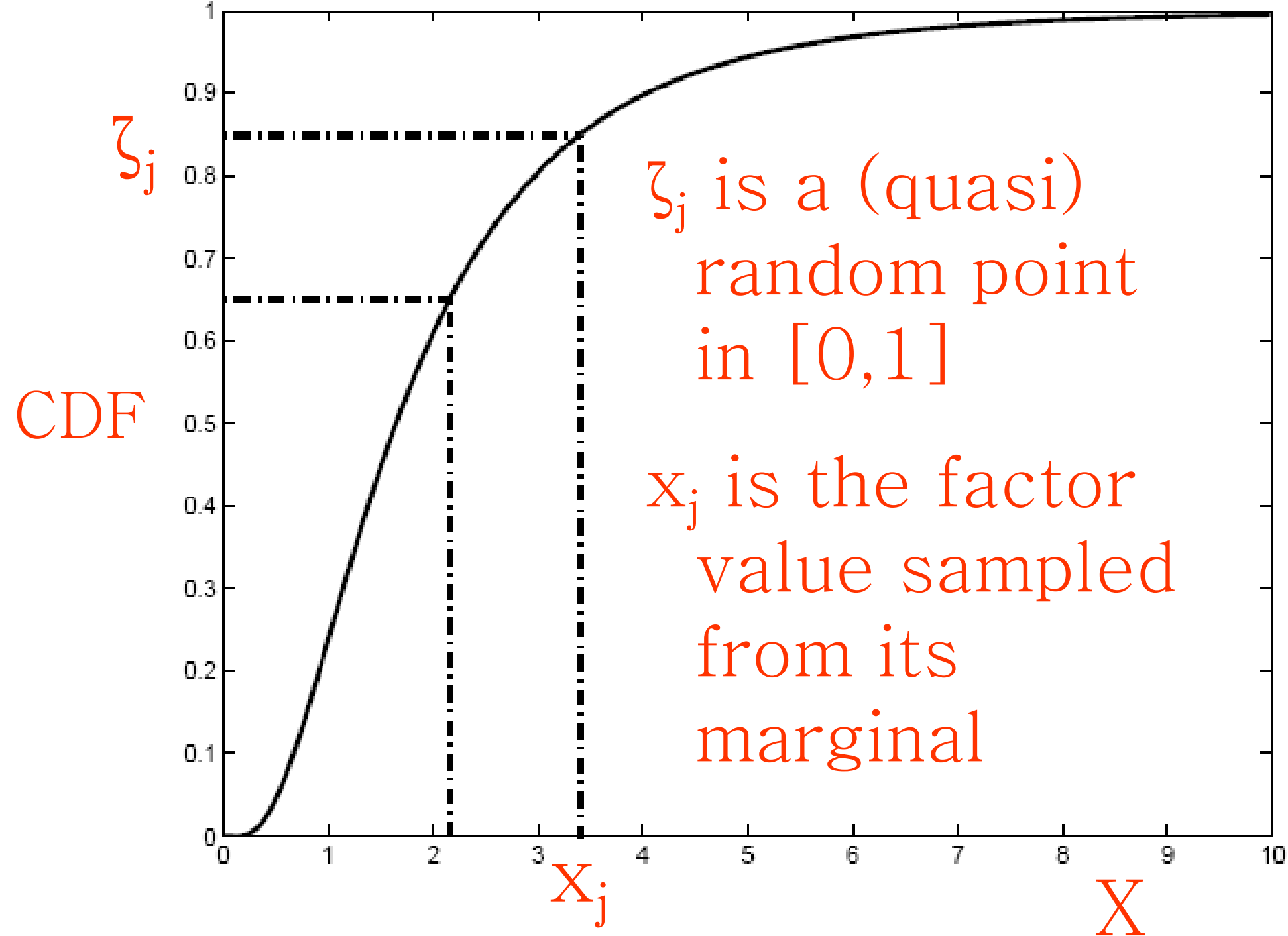
## Variance decomposition (ANOVA)

When the factors are independent the total variance can be decomposed into main effects and interaction effects up to the order  $k$ , the dimensionality of the problem.

When the factors are not independent the decomposition loses its unicity (and hence its appeal!)

# Sampling in the unit hypercube





## From main effect to total effect

From

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} \left( Y | X_i \right) \right)$$

Main effect of  
factor  $X_i$

replacing  $X_i$  with  $\mathbf{X}_{\sim i}$

To main effect of non-  $X_i$

$$V_{\mathbf{X}_{\sim i}} \left( E_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

BUT:

$$V_{\mathbf{X}_{\sim i}} \left( E_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right) +$$
$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right) = V(Y)$$

Easy to prove using  $V(\cdot) = E(\cdot)^2 - E^2(\cdot)$

$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

Main effect on non- $X_i$



$$V_{\mathbf{X}_{\sim i}} \left( E_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

... all remaining variance **must be**  
due to  $X_i$  and its interactions

Main effects

Residuals

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

$$E_{X_i} \left( V_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

$$V_{\mathbf{X}_{\sim i}} \left( E_{X_i} (Y | \mathbf{X}_{\sim i}) \right)$$

$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} (Y | \mathbf{X}_{\sim i}) \right)$$

Main (or first order) effect of

Main effects

$X_i$

Residuals

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)) + E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i)) = V(\mathbf{Y})$$

$$V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y|\mathbf{X}_{\sim i})) + E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i})) = V(\mathbf{Y})$$

Total (or total order) effect of  $X_i$

Rows add up to  $V(\mathbf{Y})$ ; diagonal terms equal for additive models.

$$\frac{V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)}{V(Y)} = S_i$$

$$\frac{E_{\mathbf{X}_{\sim i}} \left( V_{X_i} (Y | \mathbf{X}_{\sim i}) \right)}{V(Y)} = S_{Ti}$$

Rescaled to  $[0,1]$ , under the name of first order and total order sensitivity coefficient

This can be estimated without 'double loop'

$$\begin{aligned} V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) &= \\ &= E_{\mathbf{X}\mathbf{X}'_{\sim i}} (ff') - f_0^2 \end{aligned}$$

... simply as product of function values (single loop)



And this can be computed as follows – generate a (quasi) random numbers matrix of row dimension  $2k$  and column length  $N$

$$\begin{array}{cccc} x_{11} & x_{12} & \dots & x_{1(2k)} \\ x_{21} & x_{22} & \dots & x_{2(2k)} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{N(2k)} \end{array}$$

Split into two:

$$\mathbf{A} = \begin{matrix} x_{11} & x_{12} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2k} \\ \dots & \dots & \dots & \dots \\ x_{N1} & x_{N2} & \dots & x_{Nk} \end{matrix} \quad \mathbf{B} = \begin{matrix} x_{1(k+1)} & x_{1(k+2)} & \dots & x_{1(2k)} \\ x_{2(k+1)} & x_{2(k+2)} & \dots & x_{2(2k)} \\ \dots & \dots & \dots & \dots \\ x_{N(k+1)} & x_{N(k+2)} & \dots & x_{N(2k)} \end{matrix}$$

And generate a third matrix which is all-A but one column (column i) which is from B

$$\mathbf{A}_i^B = \begin{matrix} x_{11} & x_{12} & \dots & x_{1(k+i)} & \dots & x_{1k} \\ x_{21} & x_{22} & \dots & x_{2(k+i)} & \dots & x_{2k} \\ \dots & \dots & & \dots & & \dots \\ x_{N1} & x_{N2} & \dots & x_{N(k+i)} & \dots & x_{Nk} \end{matrix}$$

(call it a quasi-A matrix)

Finally we compute  $V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right) =$

$$= E_{\mathbf{X}_{\sim i}} (ff') - f_0^2 =$$

as:

$$= \frac{1}{N} \sum_{j=1}^N f_j^B \left( f_j^{A_i^B} - f_j^A \right)$$

Where:

$f_j^B$  is computed from row  $j$  of

$$\mathbf{B} = \begin{matrix} x_{1(k+1)} & x_{1(k+2)} & \cdots & x_{1(2k)} \\ x_{2(k+1)} & x_{2(k+2)} & \cdots & x_{2(2k)} \\ \cdots & \cdots & \cdots & \cdots \\ x_{N(k+1)} & x_{N(k+2)} & \cdots & x_{N(2k)} \end{matrix}$$

and  $f_j^{A_i^B}$  from the quasi-A matrix:

$$\mathbf{A}_i^B = \begin{matrix} x_{11} & x_{12} & \cdots & x_{1(k+i)} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2(k+i)} & \cdots & x_{2k} \\ \cdots & \cdots & & \cdots & & \cdots \\ x_{N1} & x_{N2} & \cdots & x_{N(k+i)} & \cdots & x_{Nk} \end{matrix}$$

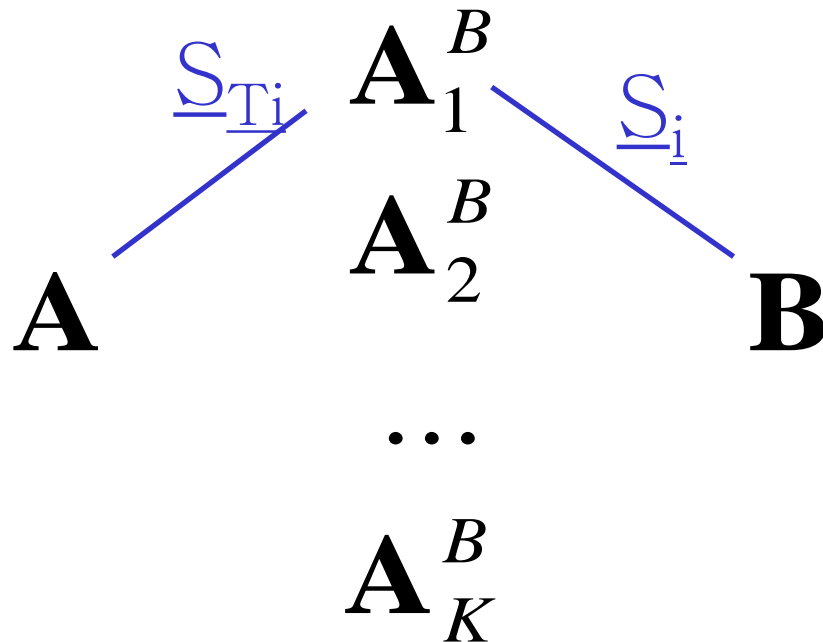
In summary one can compute the first order terms from one matrix A and B each and k matrices  $A_i^B$  i.e. using function values

$$f_j^A \quad f_j^B \quad f_j^{A_i^B}$$

The entire story can be repeated for the total effect index, which can be computed from

$$f_j^A \quad f_j^{A_i^B}$$

Thus with  $k$  quasi-A matrices and the two matrices  $A$  and  $B$  one can compute for a total of  $k+2$  matrices all total and first order effects



In three dimensions ( $k=3$ ), three points ( $N=3$ )

$$\mathbf{A} = \begin{matrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{matrix} \quad \mathbf{B} = \begin{matrix} x_{1(3+1)} & x_{1(3+2)} & x_{1(3+3)} \\ x_{2(3+1)} & x_{2(3+2)} & x_{2(3+3)} \\ x_{3(3+1)} & x_{3(3+2)} & x_{3(3+3)} \end{matrix}$$

Rewriting B:

$$\mathbf{B} = \begin{matrix} x_{14} & x_{15} & x_{16} \\ x_{24} & x_{25} & x_{26} \\ x_{34} & x_{35} & x_{36} \end{matrix}$$

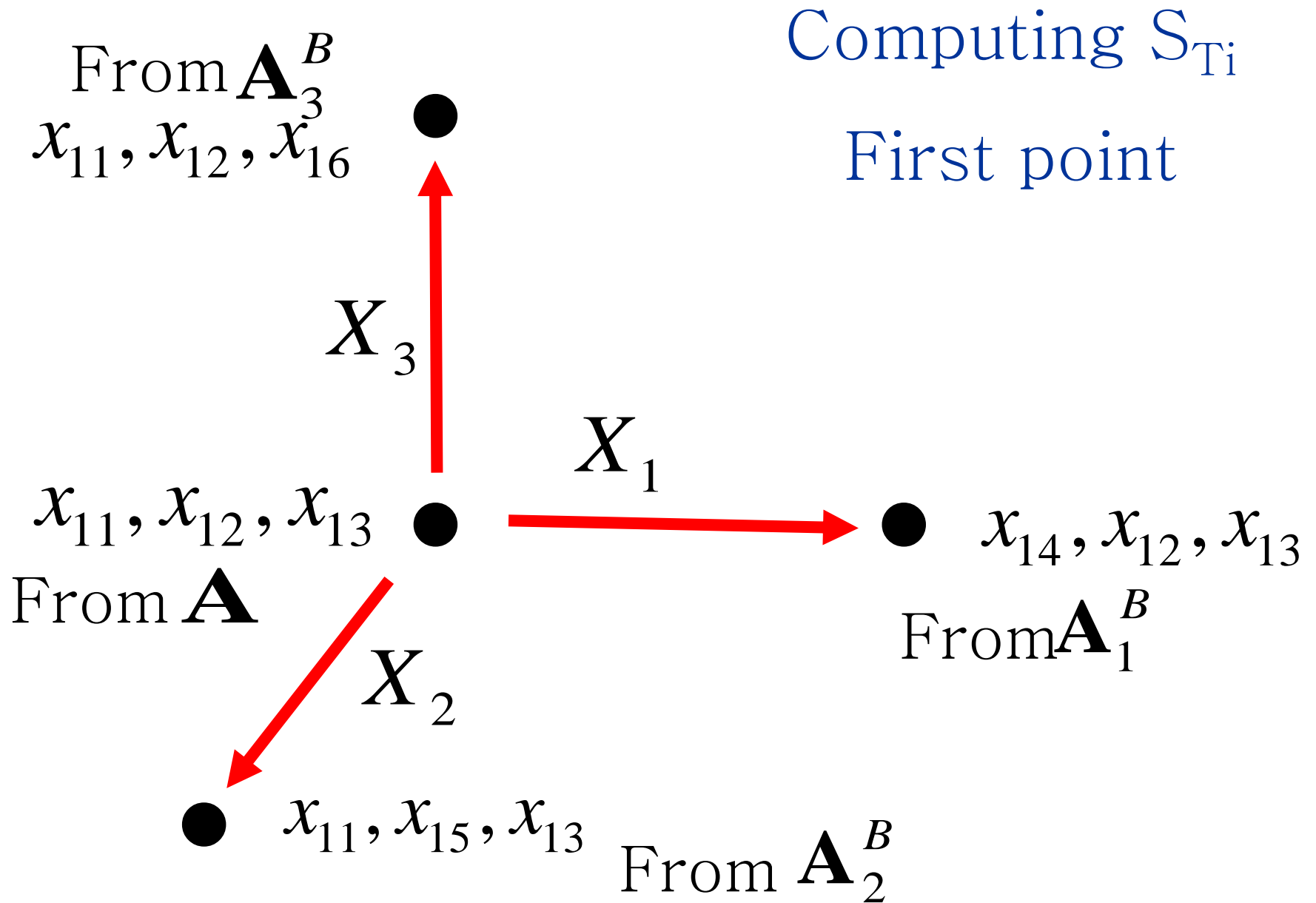


## Generate the 3 quasi-A matrices

$$\mathbf{A}_1^B = \begin{array}{ccc} x_{14} & x_{12} & x_{13} \\ x_{24} & x_{22} & x_{23} \\ x_{34} & x_{32} & x_{33} \end{array}$$

$$\mathbf{A}_2^B = \begin{array}{ccc} x_{11} & x_{15} & x_{13} \\ x_{21} & x_{25} & x_{23} \\ x_{31} & x_{35} & x_{33} \end{array}$$

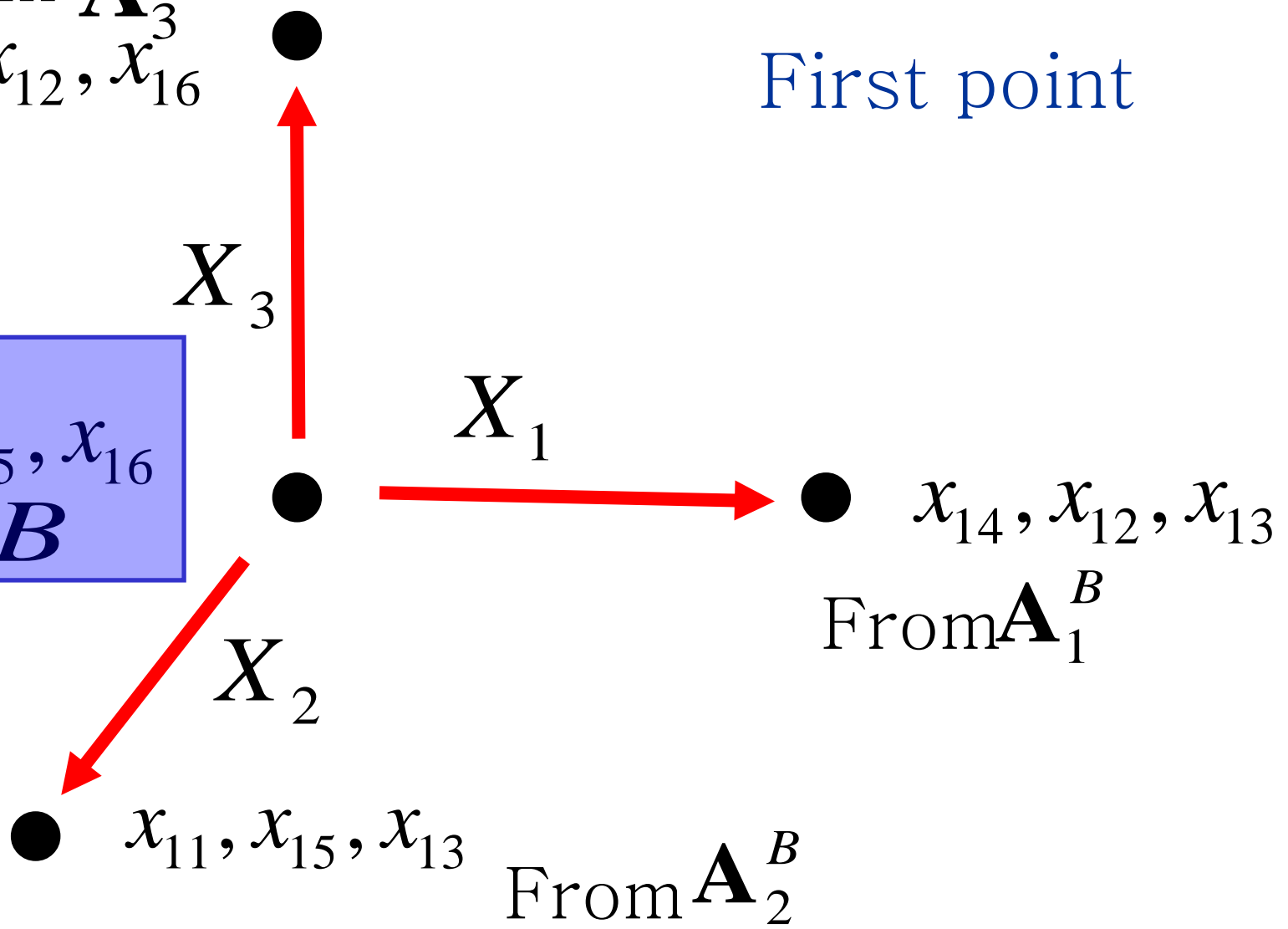
$$\mathbf{A}_3^B = \begin{array}{ccc} x_{11} & x_{12} & x_{16} \\ x_{21} & x_{22} & x_{26} \\ x_{31} & x_{32} & x_{36} \end{array}$$



From  $\mathbf{A}_3^B$   
 $x_{11}, x_{12}, x_{16}$

Computing  $S_i$   
First point

$x_{14}, x_{15}, x_{16}$   
From  $B$



# Reading about estimators

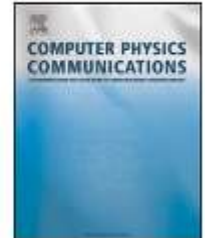
Computer Physics Communications 181 (2010) 259–270



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Computer Physics Communications

[www.elsevier.com/locate/cpc](http://www.elsevier.com/locate/cpc)



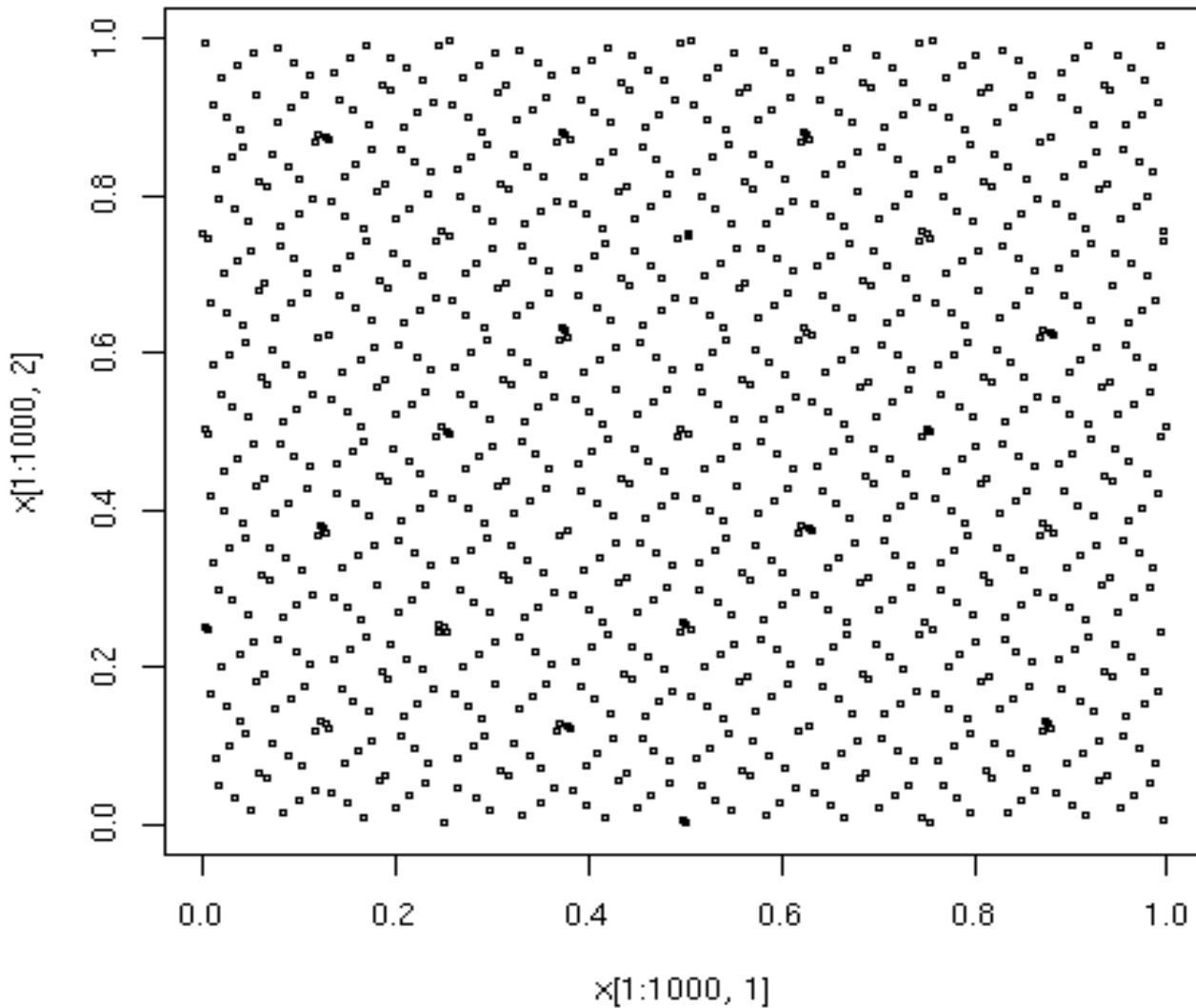
Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index

Andrea Saltelli, Paola Annoni\*, Ivano Azzini, Francesca Campolongo, Marco Ratto, Stefano Tarantola

*Joint Research Centre of the European Commission, Institute for the Protection and Security of the Citizen, Ispra, Italy*

What you have seen so far  
has been optimized as to have  
a maximum of coordinates  
from A and a minimum of  
coordinates from B.

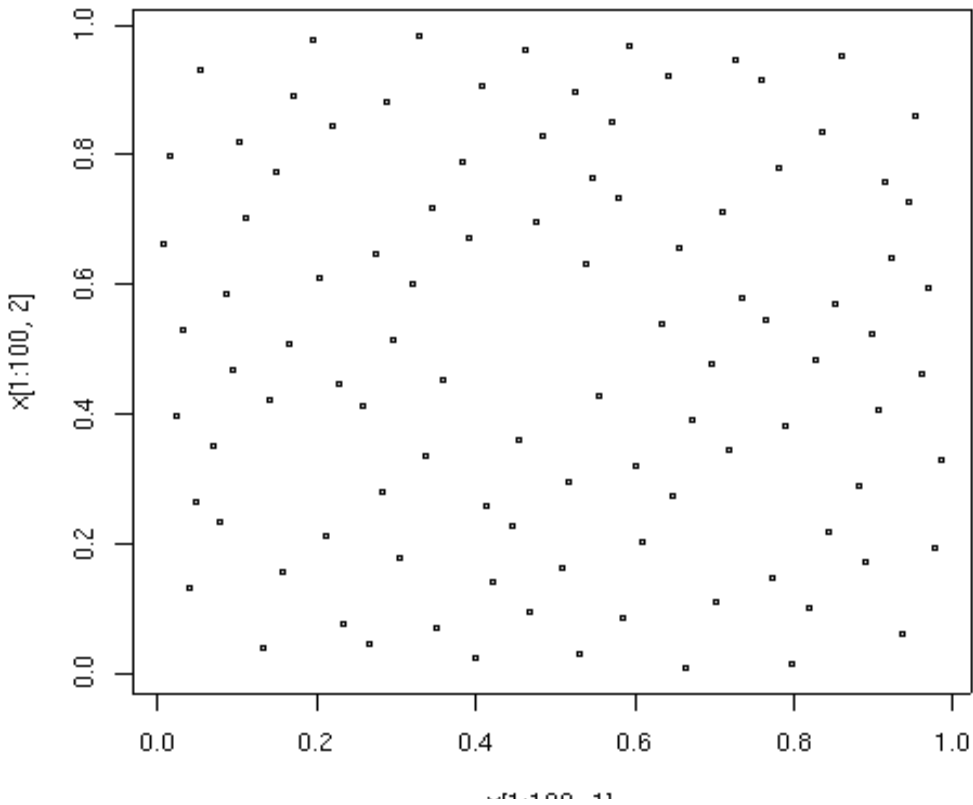
Why?



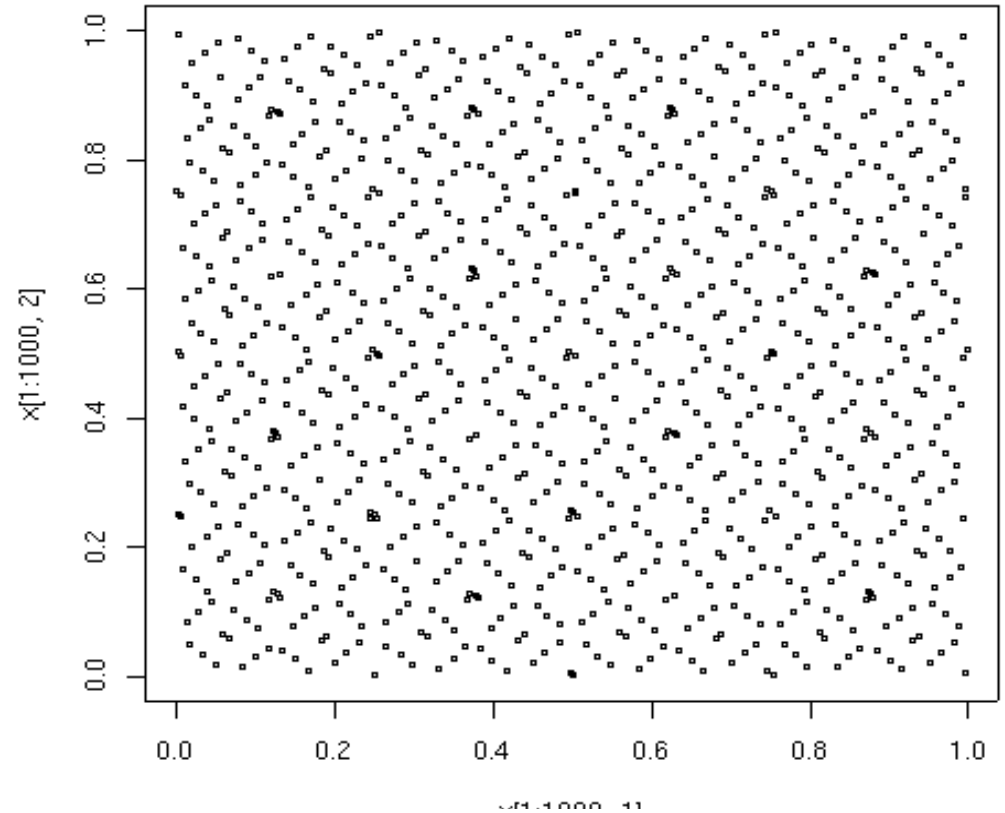
An  $LP_\tau$  sequence



We normally use low discrepancies sequences developed by I.M Sobol' – these are known as LP-TAU sequences

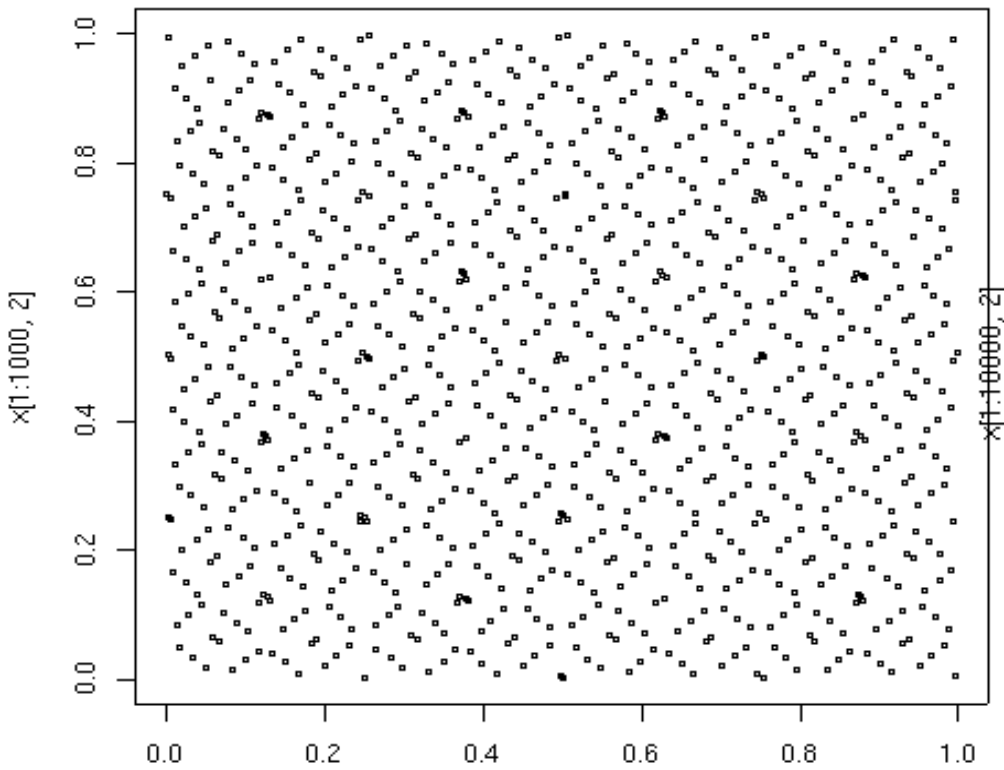


X1,X2 plane, 100 Sobol' points

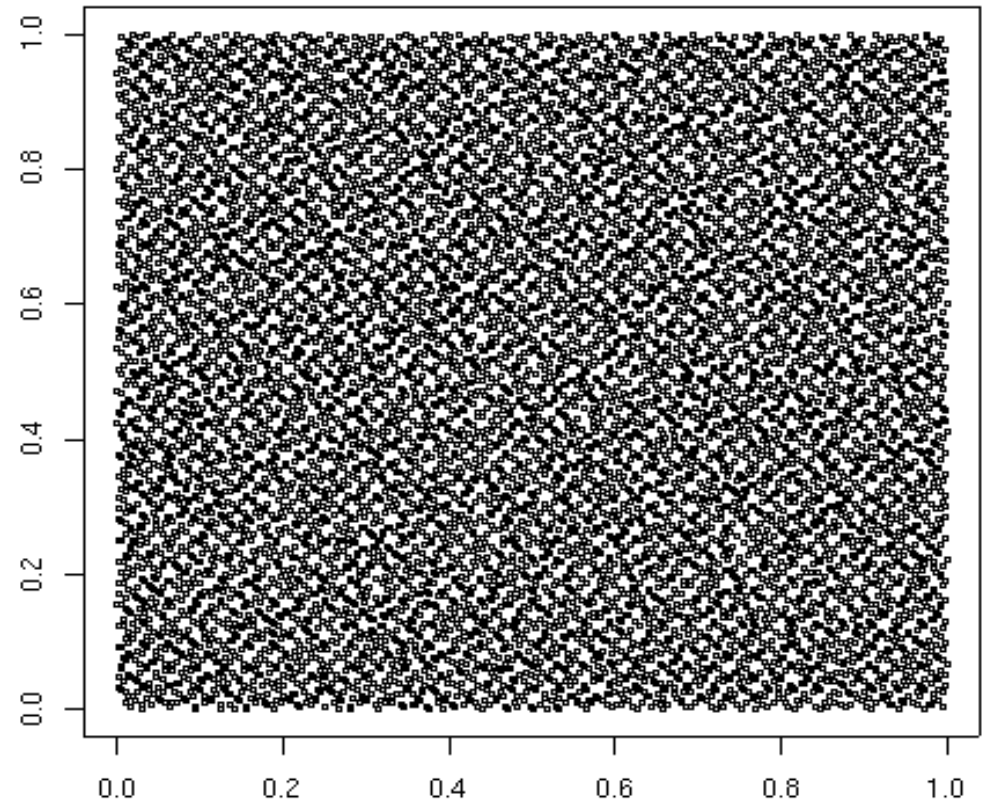


X1,X2 plane, 1000 Sobol' points

Sobol' sequences of quasi-random points



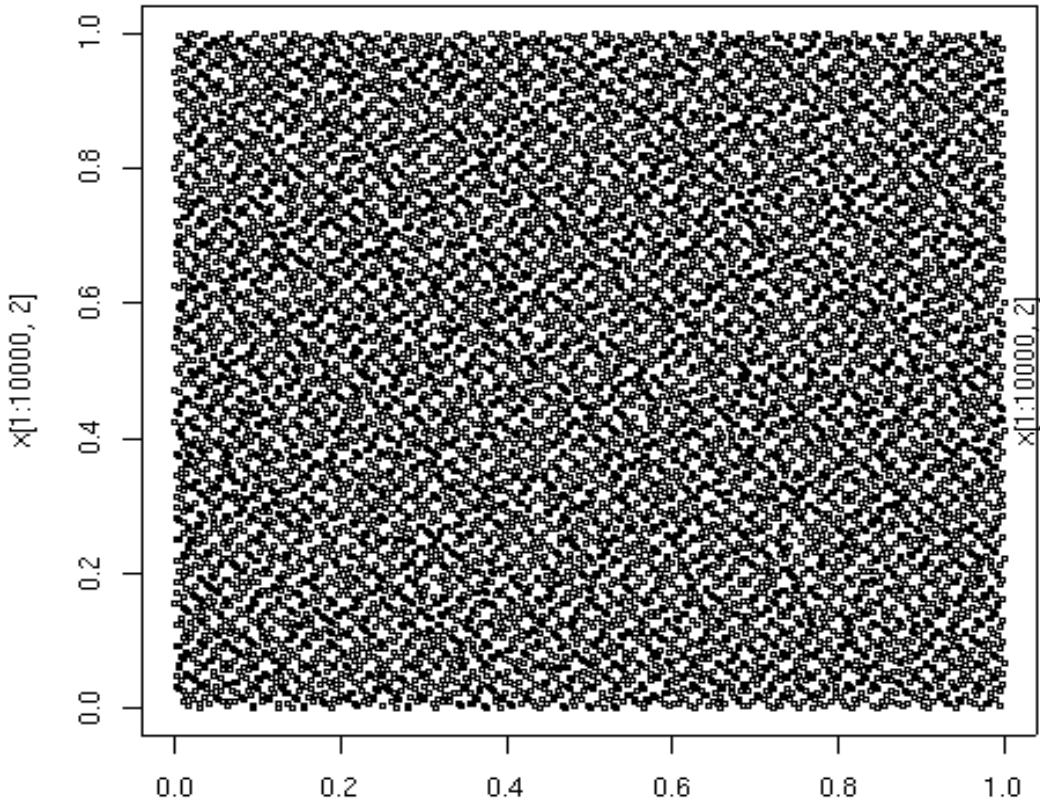
X1,X2 plane, 1000 Sobol' points



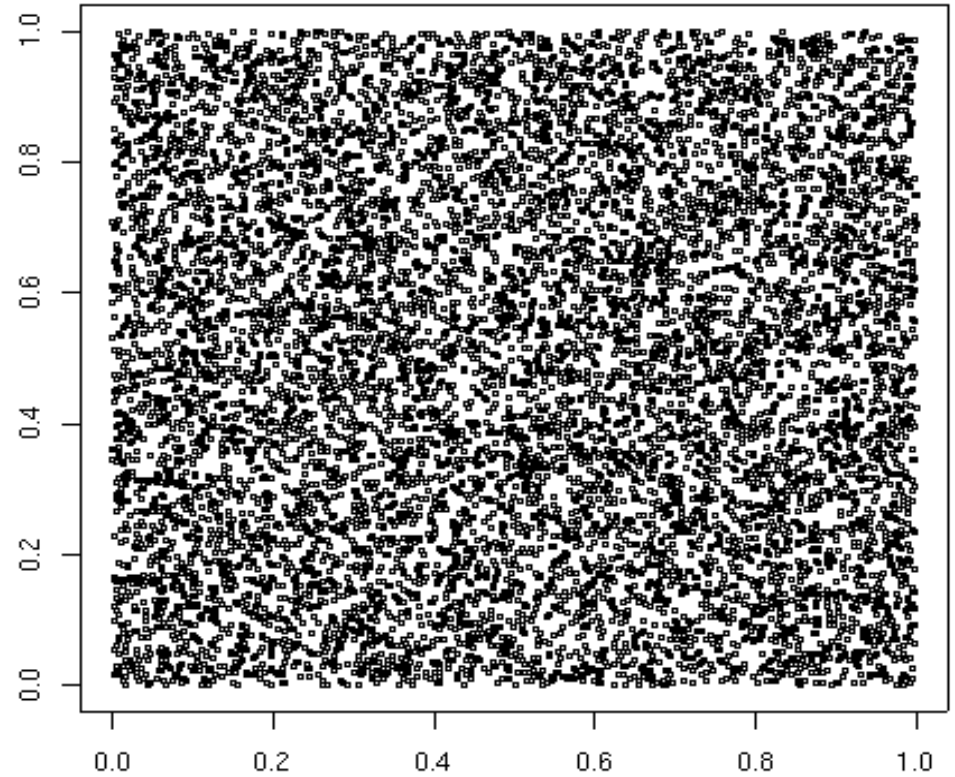
X1,X2 plane, 10000 Sobol' points

Sobol' sequences of quasi-random points





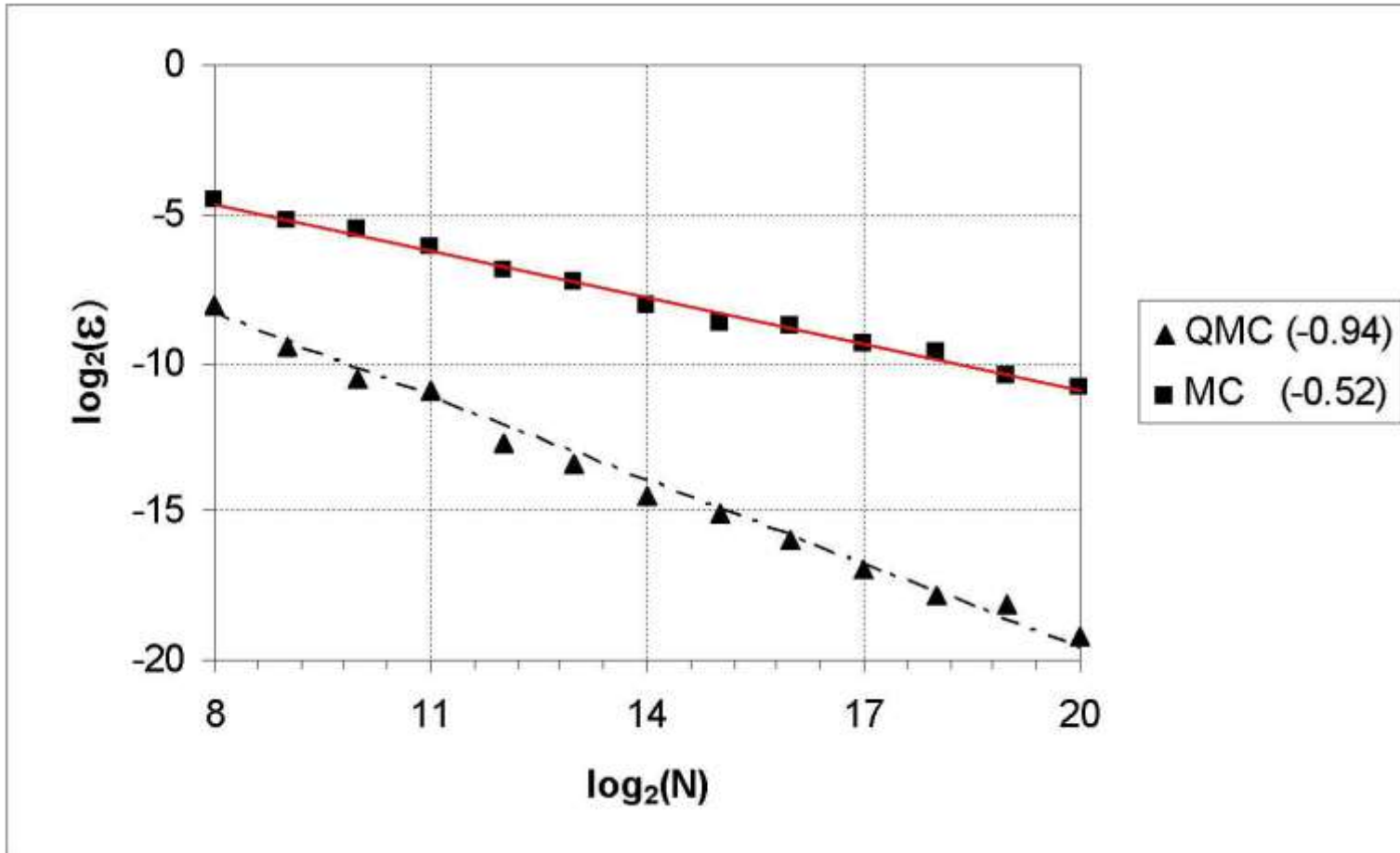
X1,X2 plane, 10000 Sobol' points



X1,X2 plane, 10000 random points

Sobol' sequences of quasi-random points  
against random points

# Why quasi-random



Source: Mauntz and Kucherenko, 2005

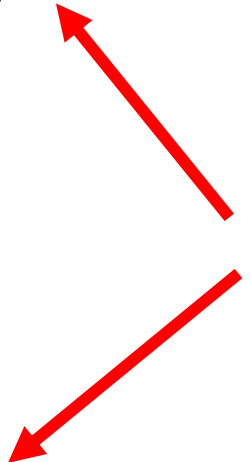
Why estimate using as much as possible from  $A$  and quasi- $A$  matrices?

The lower the column number the better its discrepancy property

→ quasi-MC trick: if possible put important variables on the left

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} (Y | X_i) \right)$$

$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} (Y | \mathbf{X}_{\sim i}) \right)$$



Equal to one  
another when the  
model is additive

Why these two measures?

$$V_{X_i} \left( E_{\mathbf{X}_{\sim i}} \left( Y | X_i \right) \right)$$

Factors prioritization

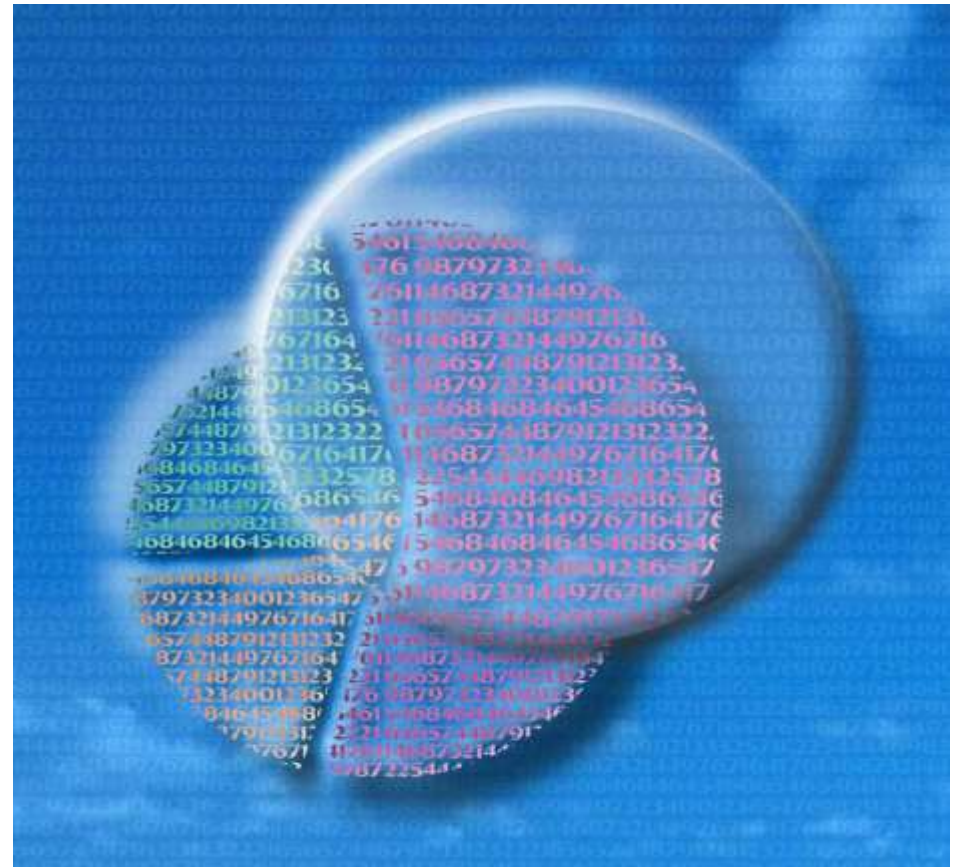
$$E_{\mathbf{X}_{\sim i}} \left( V_{X_i} \left( Y | \mathbf{X}_{\sim i} \right) \right)$$

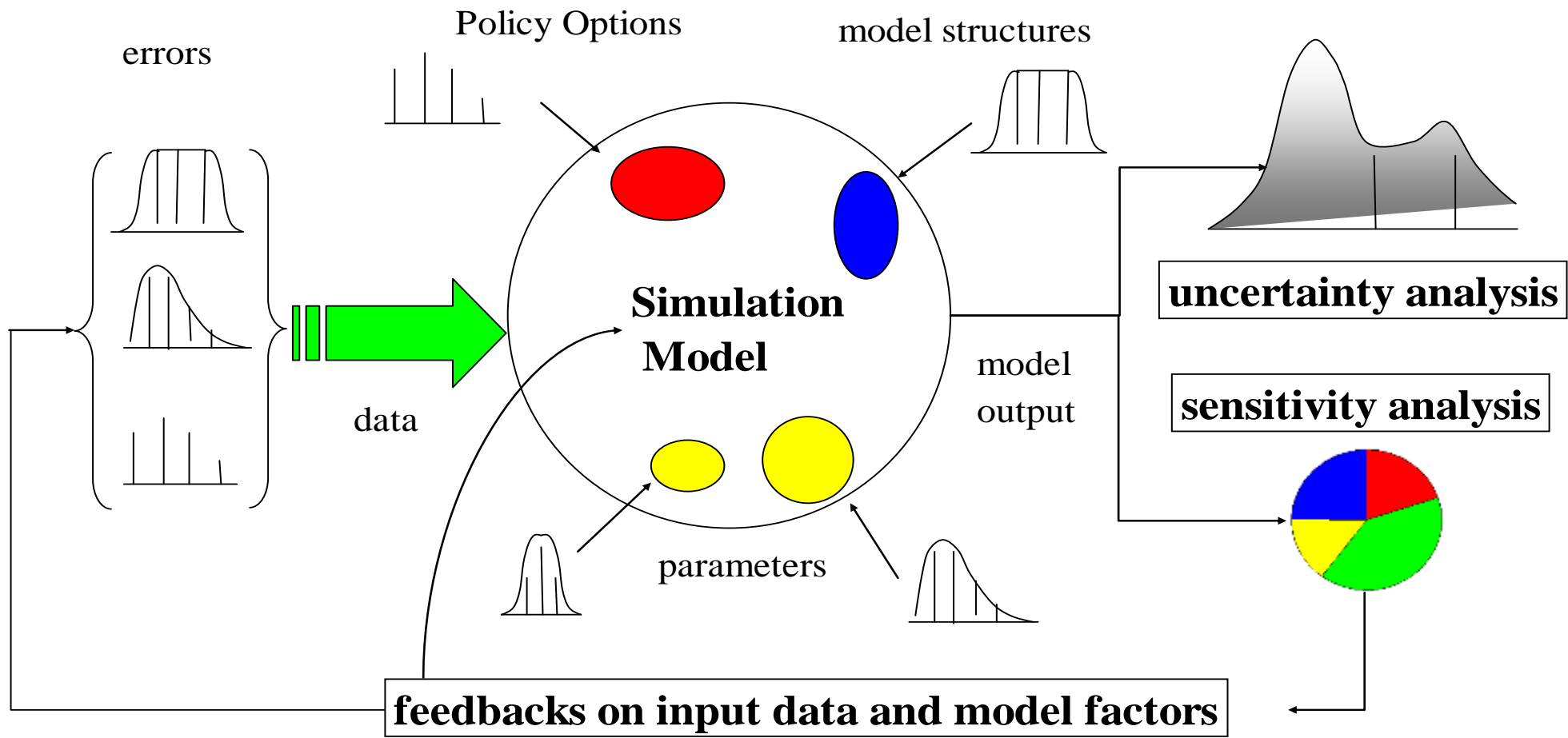
Fixing (dropping) non  
important factors

## Computational details:

1. Easy-to-code, Monte Carlo – better on quasi-random points. Estimate of the error available.
2. The main effect can be made cheap; its computational cost does not depend upon  $k$ .
3. The total effect is expensive; its computational cost is  $(k+1)N$  where  $N$  is one of the order of one thousand.

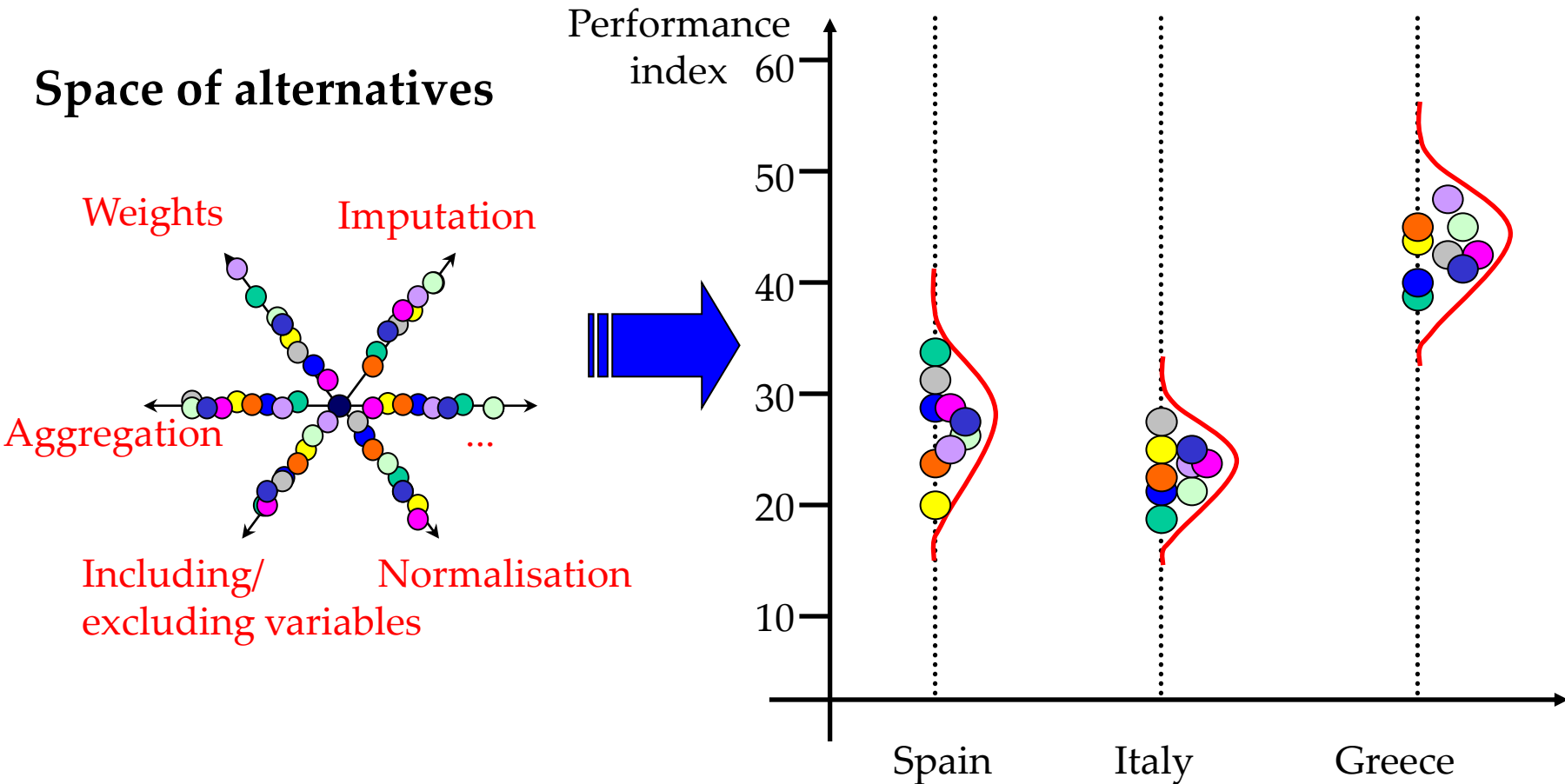
# Applications







*Uncertainty analysis can be used to assess the robustness of composite indicators ...*



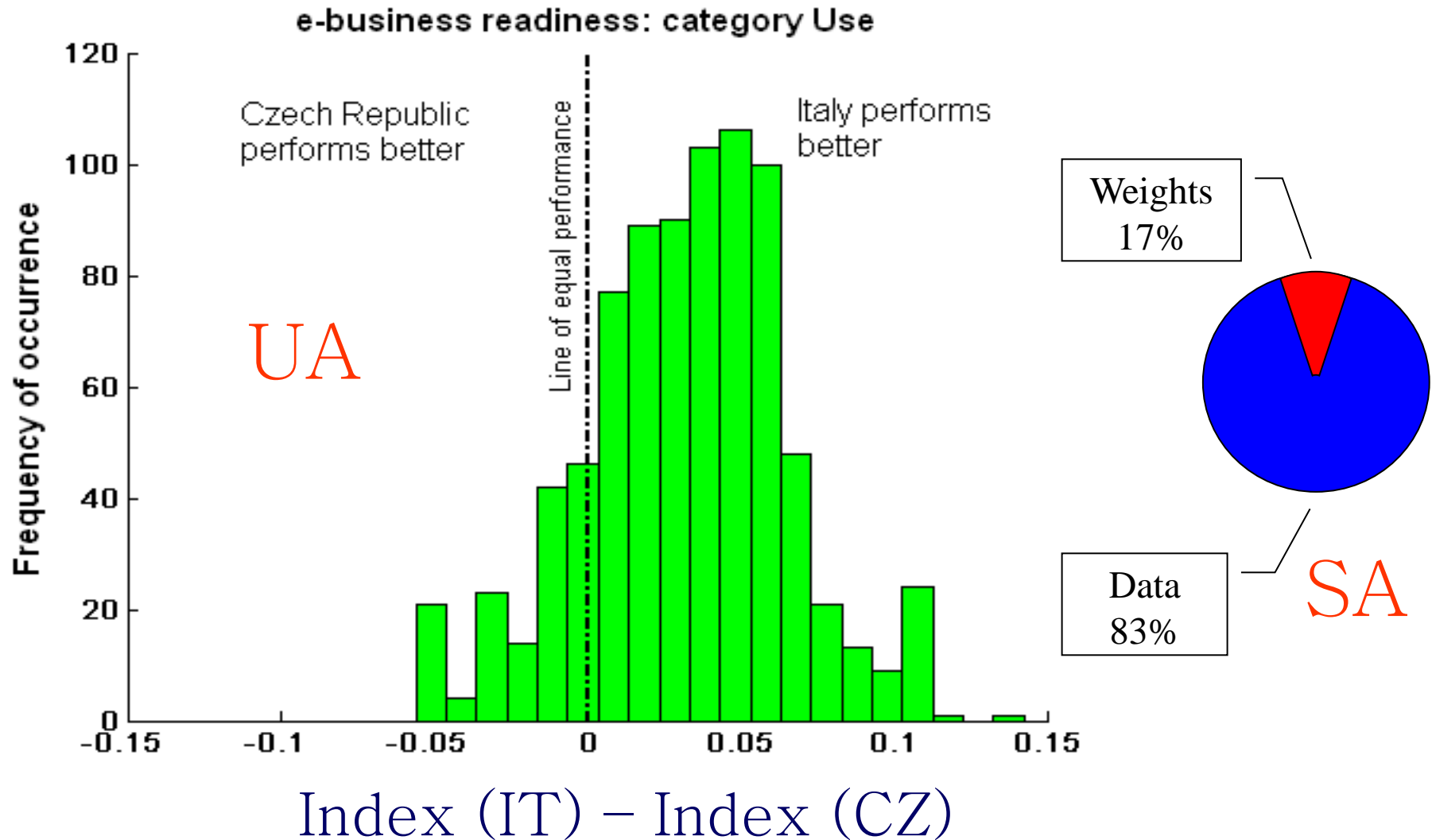
Methodology from:  
Joint OECD-JRC  
**handbook.**



Handbook on Constructing Composite Indicators. METHODOLOGY AND USER GUIDE



# Uncertainty and sensitivity (UA, SA)



# Reading about university ranking and sensitivity analysis

Research Policy 40 (2011) 165–177



Contents lists available at ScienceDirect

Research Policy

journal homepage: [www.elsevier.com/locate/respol](http://www.elsevier.com/locate/respol)



## Rickety numbers: Volatility of university rankings and policy implications

Michaela Saisana\*, Béatrice d'Hombres, Andrea Saltelli

*Econometrics and Applied Statistics, Joint Research Centre, European Commission, Enrico Fermi 2749, 21027 Ispra, Italy*

# SJTU rank

500

400

300

200

100

0

university name

(SJTU rank range, median rank [95% confidence interval for the median rank])

or

(SJTU rank, median rank [95% confidence interval for the median rank])

Univ St Andrews in UK  
(201-302, 171[154, 201])

Univ California - Davis  
(48, 98 [71, 116])

0

100

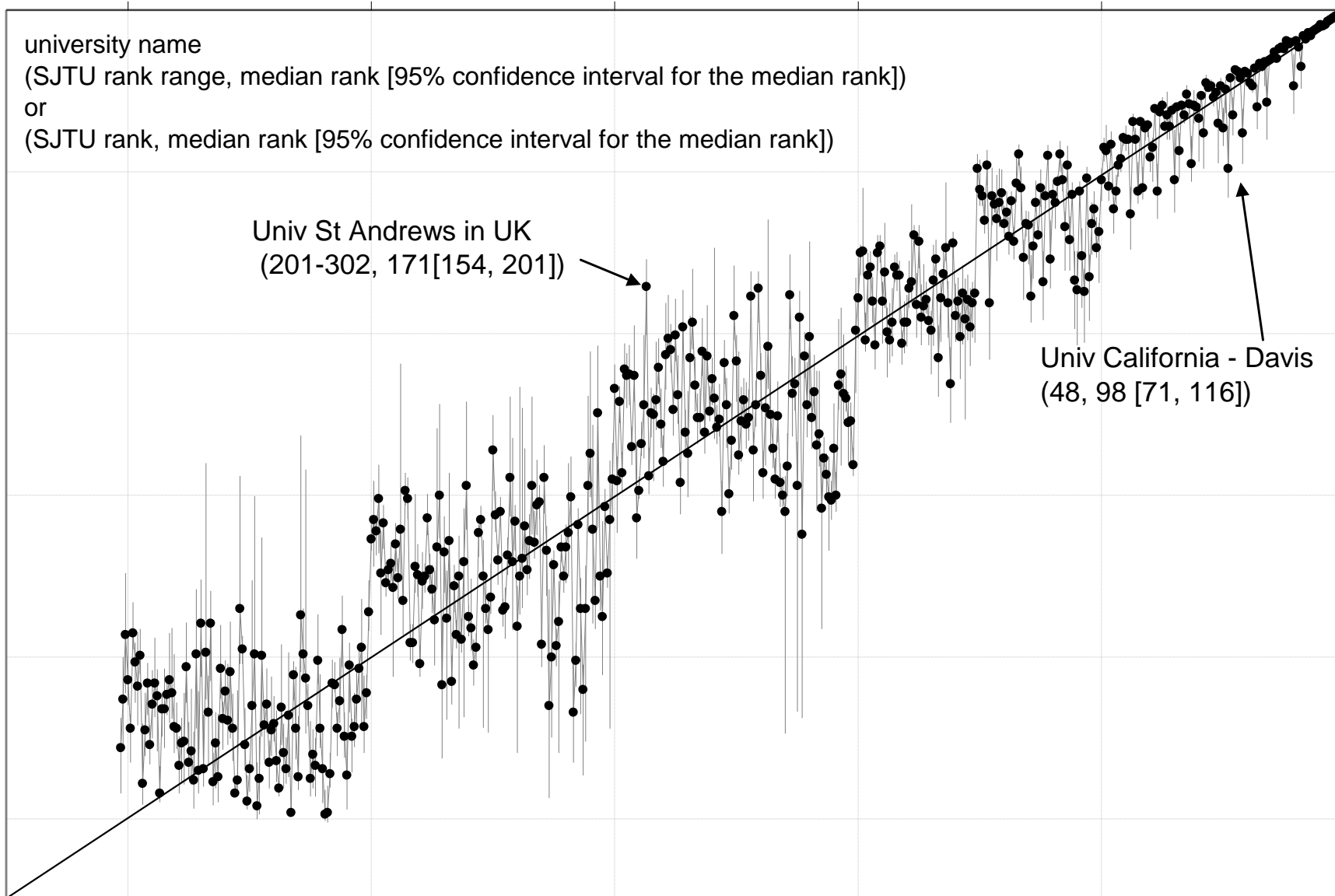
200

300

400

500

Median rank (and 95% confidence interval) accounting for methodological uncertainties



### Simulated rank range

	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70	71-75	76-80	81-85	86-90	91-95	96-100	101-105	106-110	111-115	116-120	121-125	126-130	SJTU rank
Harvard Univ	100																										1 USA
Stanford Univ	89	11																									2 USA
Univ California - Berkeley	97	3																									3 USA
Univ Cambridge	90	10																									4 UK
Massachusetts Inst Tech (MIT)	74	26																									5 USA
California Inst Tech	27	53	19	1																							6 USA
Columbia Univ	23	77																									7 USA
Princeton Univ		71	9	11	7	1																					8 USA
Univ Chicago		51	34	13	1																						9 USA
Univ Oxford		99	1																								10 UK
Yale Univ		47	53																								11 USA
Cornell Univ		27	73																								12 USA
...	...																										...
Univ California - San Francisco			14	9	14	3	11	3	7	10	4	3	3	3		6				1	6		1				18 USA
...	...																										...
Duke Univ				10	6	13	11		6	3	7	6	3	1	3	1	9	9	7	1	3				1		32 USA
Rockefeller Univ			4	10	23	26	1		3	3	3	3	3	3	4	4	6	3	1	1				1			32 USA
Univ Colorado - Boulder					19	39	30	11	1																		34 USA
Univ British Columbia					20	60	20																				35 Canada
Univ California - Santa Barbara					9	9	10	3	10	6	7	6		11	4	6	3	4	7			1	1				36 USA
Univ Maryland - Coll Park					6	37	44	9	4																		37 USA
...	...																										...
Ecole Normale Super Paris					7	9	4	6	7	6	4	9	6	7	4	3	3	4	3	3				1	6	4	73 France
Univ Melbourne												1	20	17	31	23	1	6									73 Australia
Univ Rochester							1	10	7	16	24	14	10	10	6	1											73 USA
Univ Leiden							3	6	9	23	24	13	14	9													76 Netherlands
...	...																										...
Univ Sheffield										1	21	26	21	9	13	7	1										77 UK
Tohoku Univ										4	1	7	1		4	17	19	3	3	3		19	7	3	4	4	79 Japan
Univ Utah											4	4	6	1	4	9	6	16	7	13	4	9	6	6	1		79 USA
King's Coll London											4	6	9	29	17	14	10	1	6	3	1						81 UK
Univ Nottingham											1	6	10	21	21	10	17	7	4	1							82 UK
Boston Univ													3	1	6	3	6	11	1	4	3	13	14	10	10	10	83 USA
...	...																										...

**Legend:**

Frequency lower 15%	
Frequency between 15 and 30%	
Frequency between 30 and 50%	
Frequency greater than 50%	

It is beyond doubt that Harvard, Stanford,  
Berkeley, Cambridge, and MIT are top 5

(both in the original SJTU and in more than 80%  
of our simulations) ...

... Still for 96% of the universities, the range of ranks is greater than 10 positions.

## Examples of rank variation

- 92 positions (Univ Autonoma Madrid) and 277 positions (Univ Zaragoza) in Spain,
- 71 positions (Univ Milan) and 321 positions (Polytechnic Inst Milan) in Italy,
- 22 positions (Univ Paris 06) and 386 positions (Univ Nancy 1) in France.



# Reading about evolution of SA (including software)



Environmental Modelling & Software

Volume 137, March 2021, 104954



Position Paper

## The Future of Sensitivity Analysis: An essential discipline for systems modeling and policy support

Saman Razavi<sup>a</sup>  , Anthony Jakeman<sup>b</sup>, Andrea Saltelli<sup>c</sup>, Clémentine Prieur<sup>d</sup>,  
Bertrand Iooss<sup>e</sup>, Emanuele Borgonovo<sup>f</sup>, Elmar Plischke<sup>g</sup>, Samuele Lo Piano<sup>h</sup>, Takuya Iwanaga<sup>b</sup>,  
William Becker<sup>i</sup>, Stefano Tarantola<sup>j</sup>, Joseph H.A. Guillaume<sup>b</sup>, John Jakeman<sup>k</sup>, Hoshin Gupta<sup>l</sup>,  
Nicola Melillo<sup>m</sup>, Giovanni Rabitti<sup>n</sup>, Vincent Chabridon<sup>e</sup>, Qingyun Duan<sup>o</sup>, Xifu Sun<sup>b</sup>,  
Stefán Smith<sup>h</sup>...Holger R. Maier<sup>u</sup>

# Ongoing work



Cornell University

We gratefully acknowledge s

arXiv > stat > arXiv:2206.13470

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**Statistics > Applications**

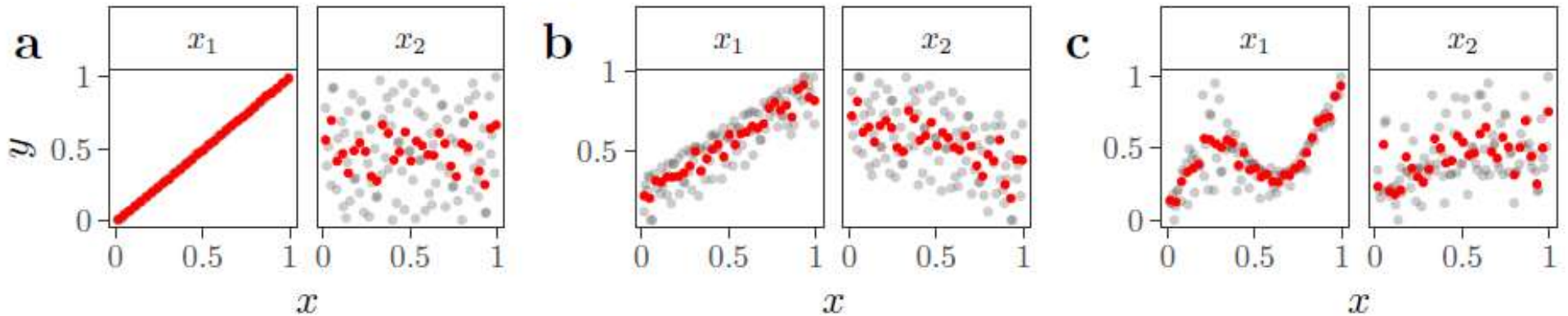
*[Submitted on 27 Jun 2022 (v1), last revised 17 Mar 2023 (this version, v2)]*

## **Discrepancy measures for sensitivity analysis**

[Arnald Puy](#), [Pamphile T. Roy](#), [Andrea Saltelli](#)

While sensitivity analysis improves the transparency and reliability of mathematical models, its uptake by modelers is still scarce. This is partially explained by its technical requirements, which may be hard to understand and implement by the non-specialist. Here we propose a sensitivity analysis approach based on the concept of discrepancy that is as easy to understand as the visual inspection of input-output scatterplots. Firstly, we show that some discrepancy measures are able to rank the most influential parameters of a model almost as accurately as the variance-based total sensitivity index. We then introduce an ersatz-discrepancy whose performance as a sensitivity measure matches that of the best-performing discrepancy algorithms, is simple to implement, easier to interpret and orders of magnitude faster.

# Input-output scatterplots



## Discrepancy measures for sensitivity analysis

Arnaud Puy, Pamphile T. Roy, Andrea Saltelli

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# Input-output scatterplots

Hand-waiving description of discrepancy: how many points are in a selected subspace versus how many should be there if the distribution were perfectly uniform

Investigation:  
compute  
“discrepancies”  
of these bi-  
dimensional plots  
and see if they  
are a good proxy  
of the total  
sensitivity index



The screenshot shows the top portion of an arXiv paper page. On the left, the Cornell University logo is visible. In the center, the arXiv logo is followed by the text 'stat > arXiv:2206.13470'. On the right, there is a search bar with the text 'Search...' and a 'Help | Ad' link below it. The background is a dark red color.

Statistics > Applications

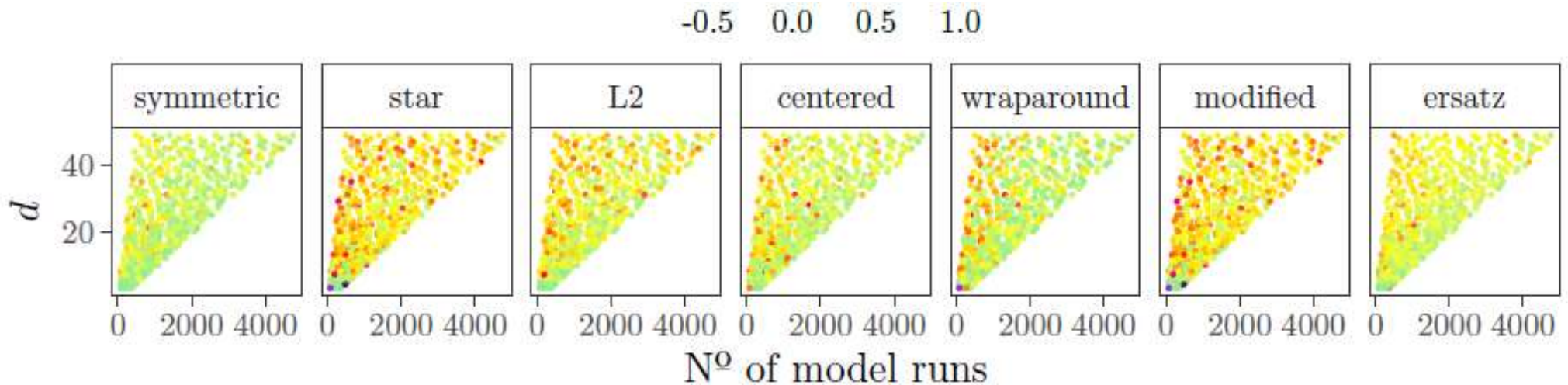
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# Input-output scatterplots



Distribution of the Pearson correlation  $r$  between the savage scores-transformed ranks yielded by each discrepancy measure and the savage scores-transformed ranks produced by the total sensitivity index

Investigation:  
compute  
“discrepancies”  
of these bi-  
dimensional plots  
and see if they  
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sensitivity index

The screenshot shows the top portion of an arXiv page. On the left is the Cornell University logo. In the center, the text reads 'arXiv > stat > arXiv:2206.13470'. On the right, there is a search bar and links for 'Help' and 'Ad'. Below the header, the page title 'Discrepancy measures for sensitivity analysis' is visible, along with the authors' names: 'Arnald Puy, Pamphile T. Roy, Andrea Saltelli'. A submission date is also present: '[Submitted on 27 Jun 2022 (v1), last revised 17 Mar 2023 (this version, v2)]'.

## Discrepancy measures for sensitivity analysis

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[Arnald Puy](#) , [Razi Sheikholeslami](#), [Hoshin V. Gupta](#), [Jim W. Hall](#), [Bruce Lankford](#), [Samuele Lo Piano](#), [Jonas Meier](#), [Florian Pappenberger](#), [Amilcare Porporato](#), [Giulia Vico](#) & [Andrea Saltelli](#)

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## Models with higher effective dimensions tend to produce more uncertain estimates

[ARNALD PUY](#) , [PIERFRANCESCO BENEVENTANO](#), [SIMON A. LEVIN](#) , [SAMUELE LO PIANO](#) , [TOMMASO PORTALURI](#), AND [ANDREA SALTELLI](#)  [Authors Info &](#)

[Affiliations](#)



Published August 25, 2023

The strong principle for the real world is: never use a model if you don't know its limitations and side effects. In fact, you must know what it can't do for you better than what it can do. I am glad this project is taking place: a long-awaited examination of the role—and obligation—of modeling.

**Nassim Nicholas Taleb, Distinguished Professor of Risk Engineering,  
NYU Tandon School of Engineering. Author of the  
five-volume Incerto series (*The Black Swan*)**

